

$$i\hbar \frac{\partial \Psi(x, t)}{\partial t} = H \Psi(x, t)$$

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V(r)$$

$$i \frac{\partial \Psi(x, t)}{\partial t} = (-\nabla^2 + V(x)) \Psi(x, t)$$

$$\Psi(x, t) = e^{-i(-\nabla^2 + v(x))t} \Psi(x, t) = e^{i\nabla^2 t} e^{-iVt} \Psi(x, 0)$$

$$e^{i\Delta t \nabla^2} \simeq \frac{1 + i\frac{\Delta t}{2} \nabla^2}{1 - i\frac{\Delta t}{2} \nabla^2}$$

$$\Psi(x, t + \Delta t) = \frac{1 + i\frac{\Delta t}{2} \nabla^2}{1 - i\frac{\Delta t}{2} \nabla^2} e^{-iV \Delta t} \Psi(x, t)$$

$$\Psi(x, t + \Delta t) = \left(\frac{2}{1 - i\frac{\Delta t}{2}\nabla^2} - 1 \right) e^{-iV\Delta t}\Psi(x, t)$$

$$\Psi(x, t + \Delta t) = \Psi^* - e^{-iV\Delta t}\Psi(x, t)$$

$$\Psi^* = \frac{2e^{-iV\Delta t}\Psi(x, t)}{1 - i\frac{\Delta t}{2}\nabla^2}$$

$$\left(1 - i\frac{\Delta t}{2}\nabla^2\right)\Psi^* = 2e^{-iV\Delta t}\Psi(x, t)$$

$$\Psi_i^{*n+1} = \frac{i + \Delta t/\Delta^2}{1 - \Delta t^2/\Delta^4} \left(\Psi_{i-1}^{*n} + \Psi_{i+1}^{*n} \right) \frac{\Delta t}{2\Delta^2} + 2e^{-iV_i\Delta t} \frac{1 - i\Delta t/\Delta^2}{1 - \Delta t^2/\Delta^4} \Psi_i$$

$$\Psi(x, 0) = Ae^{-\frac{(x-x_0)^2}{2\sigma^2} - ikx}$$