Magnetic Oscillations in Magnetars

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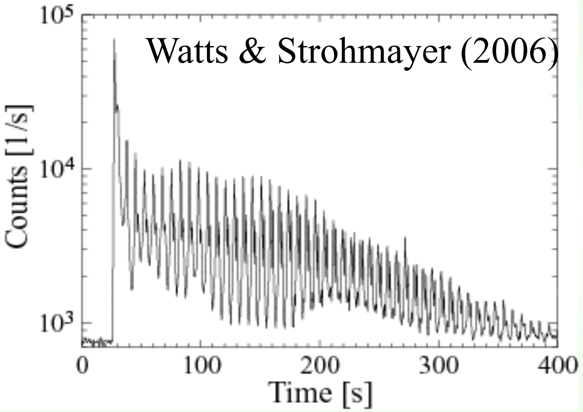
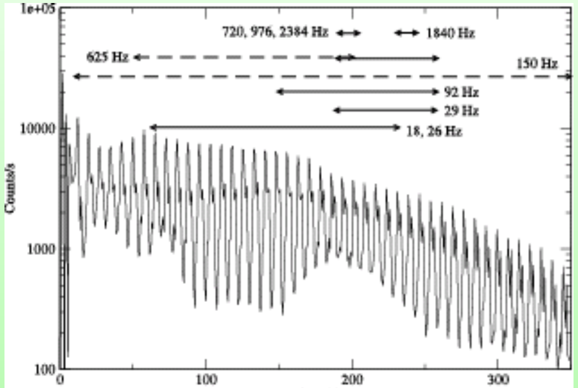
Observations

- Soft gamma Repeaters and Anomalous X-ray Pulsars are candidates of **Magnetars**, which are neutron stars with strong magnetic fields.
- **Soft gamma Repeaters** (SGRs)
  - radiating sporadic X- and gamma-ray bursts (~ 10^{41} erg/s)
- **Giant Flare from SGRs (10^{44}-10^{46} ergs/s)**
  - SGR 0526–66 in March.5.1979
  - SGR 1900+14 in August.27.1998
  - **SGR 1806–20** in December.27.2004
- In the decaying tail after the flare, QPOs are found !!

  → Barat et.al. (1983); Israel et.al. (2005);
  Watts & Strohmayer (2005, 2006)


Watts & Strohmayer (2006)

  - SGR 0526-66 : **23ms (43Hz)**, $B \sim 4 \times 10^{14}$G, $L \sim 10^{44}$ ergs/s
  - SGR 1900+14 : $B > 4 \times 10^{14}$G, **28, 54, 84, and 155 Hz**
  - SGR 1806–20 : $B \sim 8 \times 10^{14}$G, $L \sim 10^{46}$ ergs/s
    18, 26, 30, 92.5, 150, 626.5, and 1837 Hz
    (also 720Hz ?? and 2384 Hz ??)

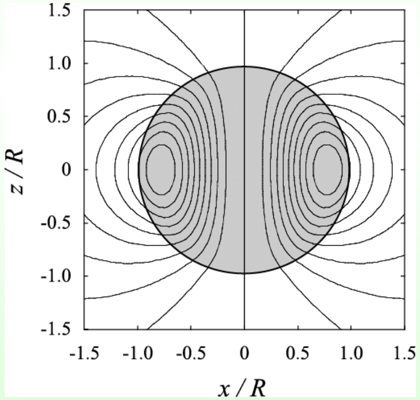


Strohmayer & Watts (2006)

Models of Magnetar

- Ideal MHD approximation

  → Electric fields are zero for comoving observer.
- The stellar deformation due to the magnetic fields are neglect.
  - Magnetic energy / gravitational energy ~ 10^{-4} $(B/10^{16}[G])^2$
  - Equilibrium configuration : static spherically symmetric
- Axisymmetric poloidal magnetic fields



Perturbations

- Linearizing the equation of motion and Maxwell equations
  - Axisymmetric perturbation ($m = 0$)

    → polar perturbation is independent from axial perturbation
  - Cowling approximation ($\delta g_{\mu\nu} = 0$)

How to explain QPOs

- **QPOs of SGRs are due to the crust torsional oscillations ??**
  - In Newtonian; Hansen & Cioffi (1980), McDermott et al. (1998), Carroll et al. (1986), Storhmayer (1991), …

→ the case without magnetic fields

$$\boxed{{}_\ell t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)}\ \mathrm{Hz} \qquad {}_\ell t_n \sim \frac{\sqrt{\mu/\rho}}{2\Delta r} \sim 500 \times n\ \mathrm{Hz}}$$

→ ${}_2t_0 = 39,\ {}_3t_0 = 55,\ {}_4t_0 = 72,\ {}_5t_0 = 88,\ {}_6t_0 = 104,\ \cdots,\ {}_\ell t_1 = 500,\ \cdots$

  - In GR; Schumaker & Thone (1983), Leins (1994), Samuelsson & Andersson (2006), Sotani, Kokkotas & Stergioulas(2007)

- This attempt was *partially* successful.
  - The stellar models with stiff EOS and massive star are favored.
  - However, it is found the difficulty to explain all observed frequencies of QPOs.
  - Explanation for lower frequencies could be impossible with only using the crust torsional oscillations.

→ Observed frequencies in SGR 1806-20; 18, 26, and 30Hz

→ *The interval of observed frequencies is much smaller than that expected by the torsional oscillation with different values of l.*

How to explain QPOs

- **Alfven oscillations in the core region ??**
  - Levin(2006), Glampedakis et al. (2006)
  - Levin(2007) : the QPO frequencies are enhanced at their edges or turning point.
  - Sotani et al. (2008) : find the two families in Alfven QPOs
  - Colaiuda et al. (2009), Cerda-Duran et al. (2009) : more detailed studies
- Time evolution of two dimensional wave equations
  - Spectrum of Alfven oscillation becomes ***continuum***.
  - There exist two families; ***upper and lower QPOs***.
  - $f_{Ln} \approx 0.6 \times f_{Un}$, $f_{Un} \approx (n+1) \times f_{U0}$, $f_{Ln} \approx (n+1) \times f_{L0}$, $f \propto B$.
- Observed frequencies of QPOs in SGRs
  - SGR 1900+14 : 28, 54, 84, 155 Hz

    ×2 ×3 **crust torsional oscillation ? or polar ones ??**
  - SGR 1806-20 : 18, 26, 30, 92.5, 150 Hz

    0.6 **×3 ×5**

    crust torsional oscillation ?

→ ex) EOS L: ${}_2t_0$=25.8; f_{L0}=17.5, f_{U0}=30.0, f_{U3}=90.1, f_{U5}= 50.2 Hz; B=2.94×10^{15} G

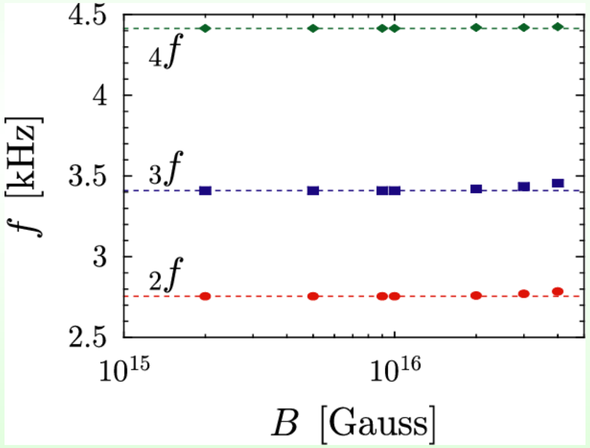
Observed QPOs ➡ restrict on the stellar model and/or magnetic field strength !

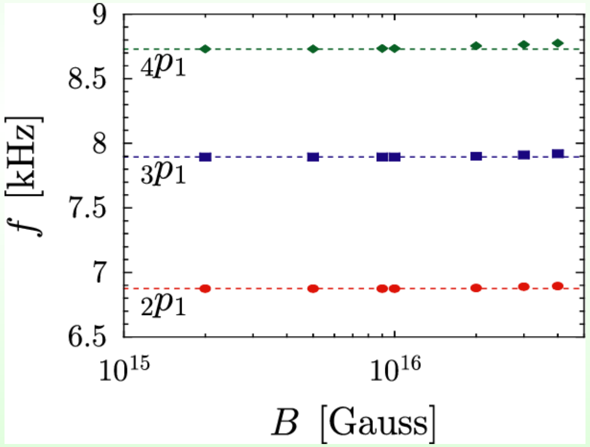
How about Polar Oscillations ?

- Previous works for polar oscillations in magnetized stars
  - Sibahashi & Takata (1993) : only fluid oscillations in Ap stars
  - Rincon & Rieutord (2003) : oscillations in spherical *incompressible* shell
  - Lee (2007) : oscillations in crust region of Newtonian models
- These previous discussions have not studied in detail of spectrum of *polar Alfven oscillations* in relativistic stellar models.
- In order to know the spectrum feature of the polar Alfven oscillations, we check two points;
  - *FFT* at the various points inside the star
  - *Phase* of the specific frequencies in FFT
- We can find that *the polar Alfven oscillations could be discrete ones*.
  - The frequencies of peak in FFT are independent of the observer positions.
  - Phase with the frequency of peak in FFT is constant.

Polar Oscillations in Magnetars

- Dependence of polar Alfven modes:
  - Typical frequencies are *around a few hundred Hz*.
  - These might be possible to explain the observed higher frequencies of QPOs.
- Dependence of fluid modes frequencies:

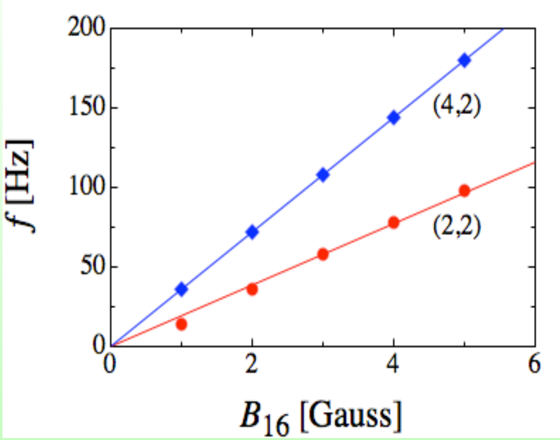




  - We can see the effect of the magnetic field lager than 10^{16} G.
  - That deviation is only less than a few %.

Non-axisymmetric Oscillations ?

- The axial oscillations are coupled with the polar ones even for non-rotating magnetars.
  - As a first step, we consider the only axial type oscillations.
- Non-axisymmetric axial Alfven oscillations are discrete oscillations.
- Those frequencies are smaller than that of axial Alfven oscillations.
  - Axisymmetric axial; minimum frequency is around 15 Hz for $B=4\times10^{15}$G
  - Non-axisymmetric axial; f_{22}=7.7 and f_{42}=14.4Hz for $B=4\times10^{15}$G
- This type of oscillations could be important to explain theoretically the observed evidence of QPOs in the SGR
  - To fit the possible stellar model with the observations in SGRs, it is necessary to produce more oscillation frequencies with different value of (l,m) for the stellar models constructed with different EOSs.



Conclusion

- QPOs are found in the SGRs.
- Crust torsional oscillations is partially successful to explain the observed frequencies, still those are impossible to explain the all.
- Taking into account the axial Alfven oscillations, it could be possible to explain the all, whose spectrum could be continuum.
- Polar Alfven oscillations are not continuum but discrete modes.
- Fluid modes in magnetar are almost independent from the magnetic field strength.
- Frequencies of non-axisymmetric axial Alfven oscillations become smaller than those of axisymmetric axial ones.

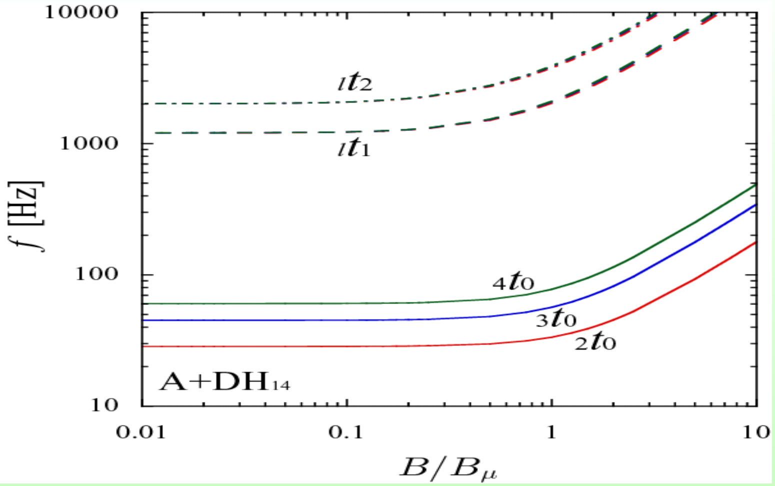
- Dependence of the toroidal magnetic field on the oscillation frequency ??
- Still, there is open problem for the oscillation spectra in magnetars when the **crust effect** would be taken into account.
- Coupling between the axial and polar oscillations.

Thanks for your attentions

Crust torsional oscillations

H.S., K.D.Kokkotas, & N.Stergioulas (2007)

- We examine the torsional oscillations *restricted in the crust region.*
  → we omit the coupling with the core oscillations.
- We can see the magnetic effect for $B > B_\mu$, where $B_\mu = 4 \times 10^{15}$ G.
- Frequencies for fundamental mode **depend strongly** on the stellar properties, where they change in 30 ~ 50 %
- Frequencies for overtones are **independent** from the harmonic index ℓ.
  - Frequencies depend strongly on the both EOS of crust and core.
    → ex) first overtones are in the range of 500 – 1200 Hz.



We find the empirical formula such as

$$\frac{{}_\ell t_n}{{}_\ell t_n^{(0)}} \sim \left[1 + {}_\ell\alpha_n \left(\frac{B}{B_\mu}\right)^2\right]^{1/2}$$

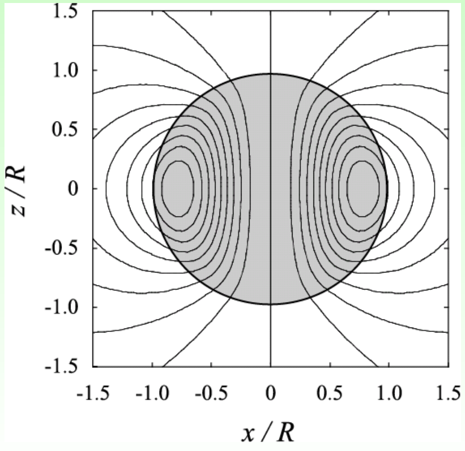
depend on the stellar models

The attempt to explain the observational data is partially successful, but it may be difficult to explain all data.

especially 18, 26, 30 Hz for SGR 1806-20

Calculations for Alfven Oscillations

H.S., K.D.Kokkotas, & N.Stergioulas (2008)



- We calculate the Alfven oscillations
  - On the whole star **without the crust regions**
  - Axisymmetric poloidal magnetic fields
  - 2D evolutions for axisymmetric torsional oscillations with numerical viscosity (Kreiss-Oliger dissipation)
- Perturbed variable : $\delta u^{\phi} = e^{-\Phi}\partial_t \mathcal{Y}(t,r,\theta)$
- Two dimensional evolutionary equations

$$\mathcal{A}_{tt}\frac{\partial^2\mathcal{Y}}{\partial t^2} = \mathcal{A}_{20}\frac{\partial^2\mathcal{Y}}{\partial r^2} + \mathcal{A}_{11}\frac{\partial^2\mathcal{Y}}{\partial r\partial\theta} + \mathcal{A}_{02}\frac{\partial^2\mathcal{Y}}{\partial\theta^2} + \mathcal{A}_{10}\frac{\partial\mathcal{Y}}{\partial r} + \mathcal{A}_{01}\frac{\partial\mathcal{Y}}{\partial\theta} + \varepsilon_D\mathcal{D}_4\mathcal{Y}$$

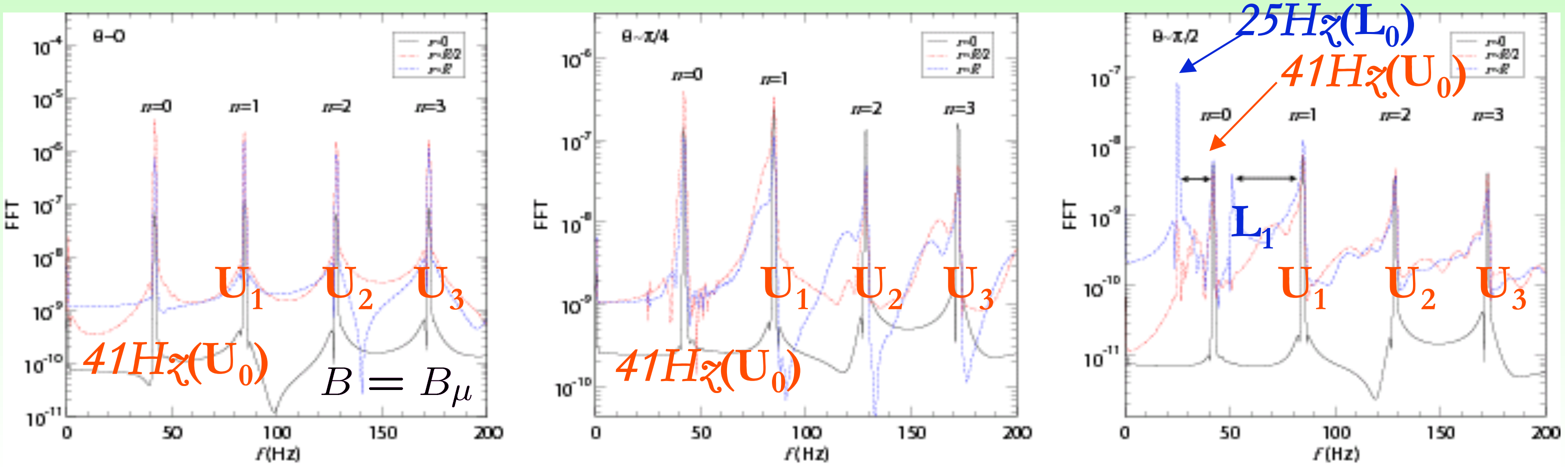
4th-order Kreiss-Oliger dissipation

- Boundary conditions
  - $\mathcal{Y} = 0$ at $r = 0$ (regularity condition)
  - $\mathcal{Y}_{,r} = 0$ at $r = R$ (vanishing traction)
  - $\mathcal{Y}_{,\theta} = 0$ at $\theta = 0$ (axisymmetry)
  - $\mathcal{Y} = 0$ at $\theta = \pi/2$ (equatorial symmetry of even ℓ)

* more detail studies :
Colaiuda, et al. (arXiv:0902.1401)
Cerda-Duran, et al. (arXiv:0902.1472)

Feature of Oscillations

- FFT on the different position in the magnetar



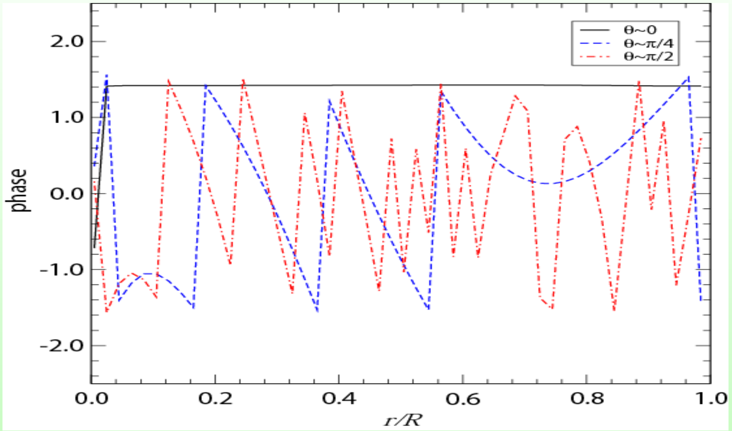
  - The both edges of continuum are enhanced, which are corresponding to 25 and 41 Hz for the fundamental QPOs.

→ two families; **upper** and **lower QPOs**

  - The oscillations on the axis are same phase, but those on the other stellar location have different phase

    → **not normal-modes but *continuum***

  - we can find that the feature of continuum arise near the equatorial plane.



Empirical Formula

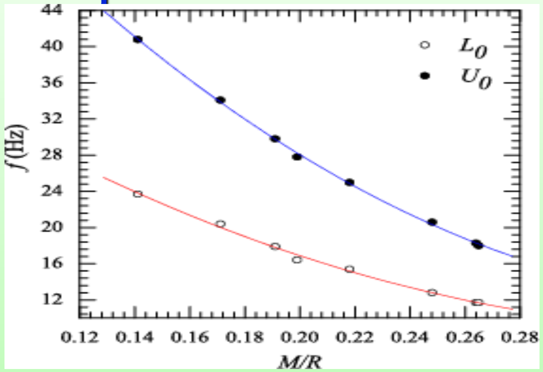
- The lower and upper Alfven QPO frequencies with realistic stellar models

Model	M/R	f_{L_0} (Hz)	f_{U_0} (Hz)	ratio	f_{L_1} (Hz)	f_{U_1} (Hz)	ratio	f_{U_2} (Hz)
A+DH$_{14}$	0.218	15.4	25.0	0.616	30.7	49.4	0.621	74.4
A+DH$_{16}$	0.264	11.7	18.3	0.639	23.5	35.7	0.658	54.0
WFF3+DH$_{14}$	0.191	17.9	29.8	0.601	36.2	59.2	0.611	89.8
WFF3+DH$_{18}$	0.265	11.7	18.0	0.650	23.5	35.5	0.662	53.3
APR+DH$_{14}$	0.171	20.4	34.1	0.598	41.3	68.6	0.602	104.6
APR+DH$_{20}$	0.248	12.8	20.6	0.621	26.0	40.3	0.645	61.0
L+DH$_{14}$	0.141	23.7	40.8	0.581	47.5	81.6	0.582	123.8
L+DH$_{20}$	0.199	16.4	27.8	0.590	33.1	54.7	0.605	82.6

- the ratio of f_{L_n}/f_{U_n} are almost **0.6** independently of the stellar models.
- the frequencies of overtones are nearly integer multiples of the fundamental one;

$$\boxed{f_{L_n} \simeq (n+1) f_{L_0}, \;\; f_{U_n} \simeq (n+1) f_{U_0}}$$

- the frequencies are proportional to the magnetic field strength.
- **Empirical formula** for the frequencies of lower and upper Alfven QPOs



$$f_{L_n}(\text{Hz}) \simeq 48.9(n+1)\left[1-4.51\left(\frac{M}{R}\right)+6.18\left(\frac{M}{R}\right)^2\right]\left(\frac{B}{4\times10^{15}\text{G}}\right)$$

$$f_{U_n}(\text{Hz}) \simeq 86.1(n+1)\left[1-4.58\left(\frac{M}{R}\right)+6.06\left(\frac{M}{R}\right)^2\right]\left(\frac{B}{4\times10^{15}\text{G}}\right)$$

Attempt to fit to Observational data

- Now we have empirical formula for the frequencies of Alfven QPOs and of crust torsional modes.

- crust torsional modes : $\frac{{}_\ell t_n}{{}_\ell t_n^{(0)}} \sim \left[1 + {}_\ell\alpha_n \left(\frac{B}{B_\mu}\right)^2\right]^{1/2}$
- lower Alfven QPOs : $f_{L_n}(\text{Hz}) \simeq 48.9(n+1)\left[1 - 4.51\left(\frac{M}{R}\right) + 6.18\left(\frac{M}{R}\right)^2\right]\left(\frac{B}{B_\mu}\right)$
- upper Alfven QPOs : $f_{U_n}(\text{Hz}) \simeq 86.1(n+1)\left[1 - 4.58\left(\frac{M}{R}\right) + 6.06\left(\frac{M}{R}\right)^2\right]\left(\frac{B}{B_\mu}\right)$

- On the other hand the observed frequencies of QPOs in SGRs
  - SGR 1900+14 : 28, 54, 84, 155 Hz

    x2 x3

    crust torsional oscillation ? or polar oscillation ?
  - SGR 1806-20 : 18, 26, 30, 92.5, 150 Hz

    x3 x5

    0.6

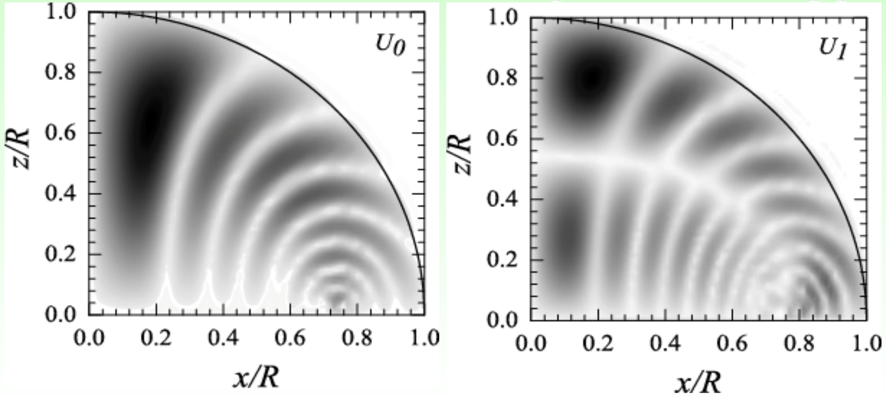
    crust torsional oscillation ?

EOS L : ${}_2t_0 = 25.8;\ f_{L_0} = 17.5,\ f_{U_0} = 30.0,\ f_{U_3} = 90.1,\ f_{U_5} = 150.2\text{Hz}$ for $B = 2.94\times10^{15}\text{G}$
EOS APR : ${}_2t_0 = 25.9;\ f_{L_1} = 17.7,\ f_{U_1} = 30.0,\ f_{U_5} = 90.1,\ f_{U_9} = 150.1\text{Hz}$ for $B = 1.77\times10^{15}\text{G}$

Observed QPOs ➡ restrict on the stellar model and/or magnetic field strength !

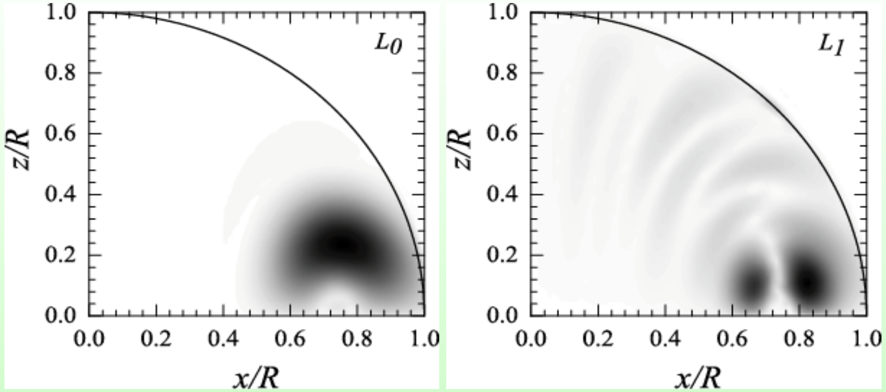
Numerical Results 2

- Distribution of effective amplitude for upper QPOs



- The maximum exists near the axis.
- There are nodal lines along certain magnetic field lines.
- For the overtone, the additional “horizontal” nodal line exits.

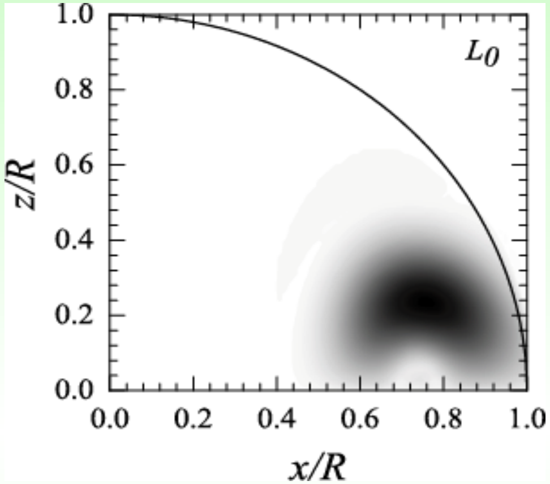
- Distribution of effective amplitude for lower QPOs

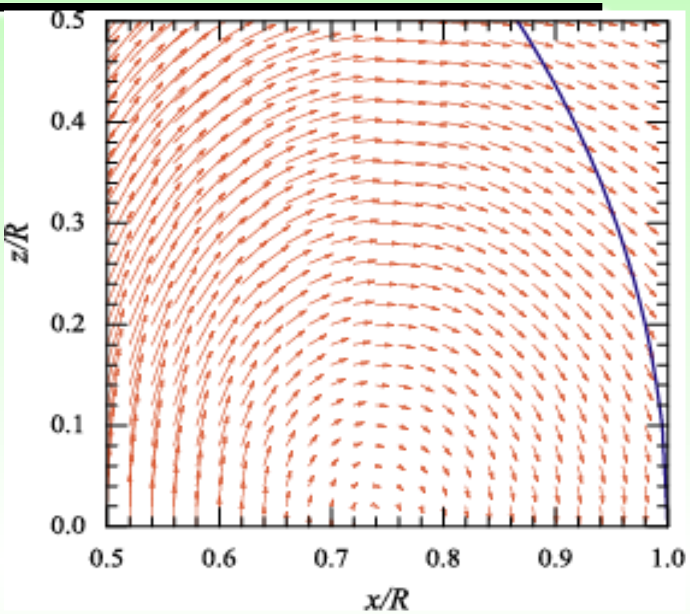


- The effective amplitude for lower QPOs is partially limited to a region around the closed magnetic field lines.
- For the overtone, the nodal line divides the region of maximum amplitude.

Numerical Results 3

- Magnetic lines of interior region





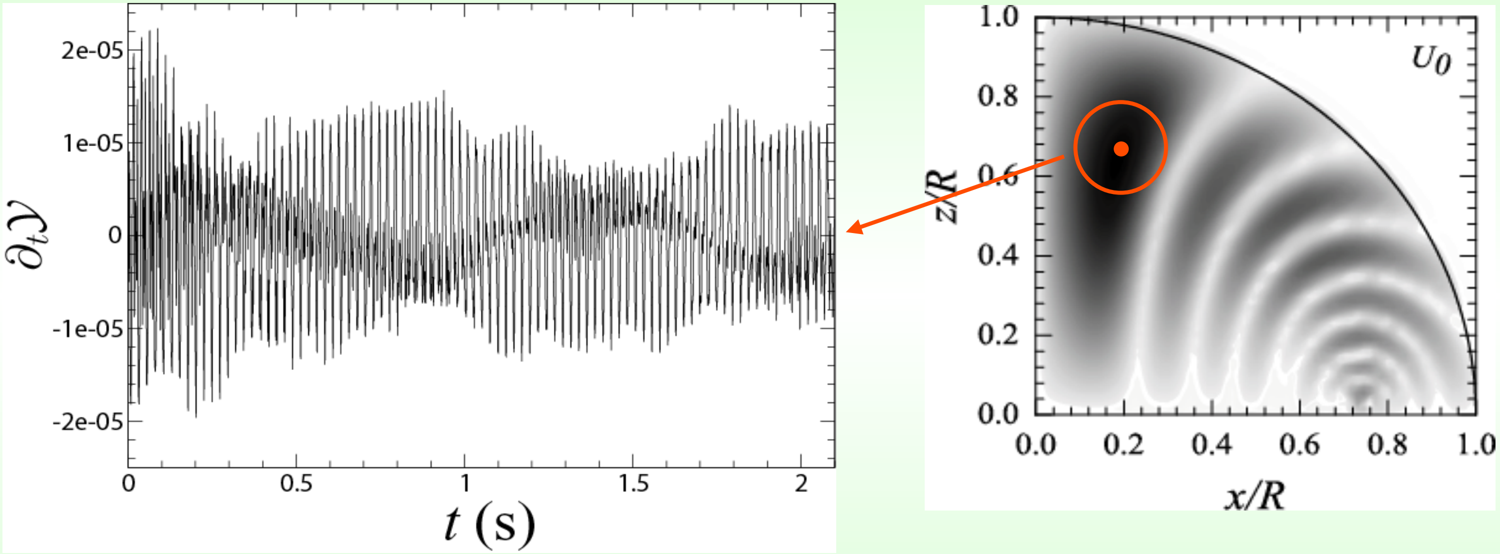
  - For the L0 QPOs, an oscillating region is restricted to roughly $0.5<x<1.0$, while it appears to be "touching" the surface for $0<z<0.3$.
  - We can see that the last closed field line inside stellar surface originates at about $x=0.58$.
  - The field lines that originate from the region $0.5<x<0.58$ are open and cross the surface at $0<z<0.3$

➔ **While the maximum oscillational amplitude is inside the region of closed lines, there is still a broader region, including open field lines.**

➔ **The information of oscillations with lower QPOs could leak out of the star.**

Numerical Results 4

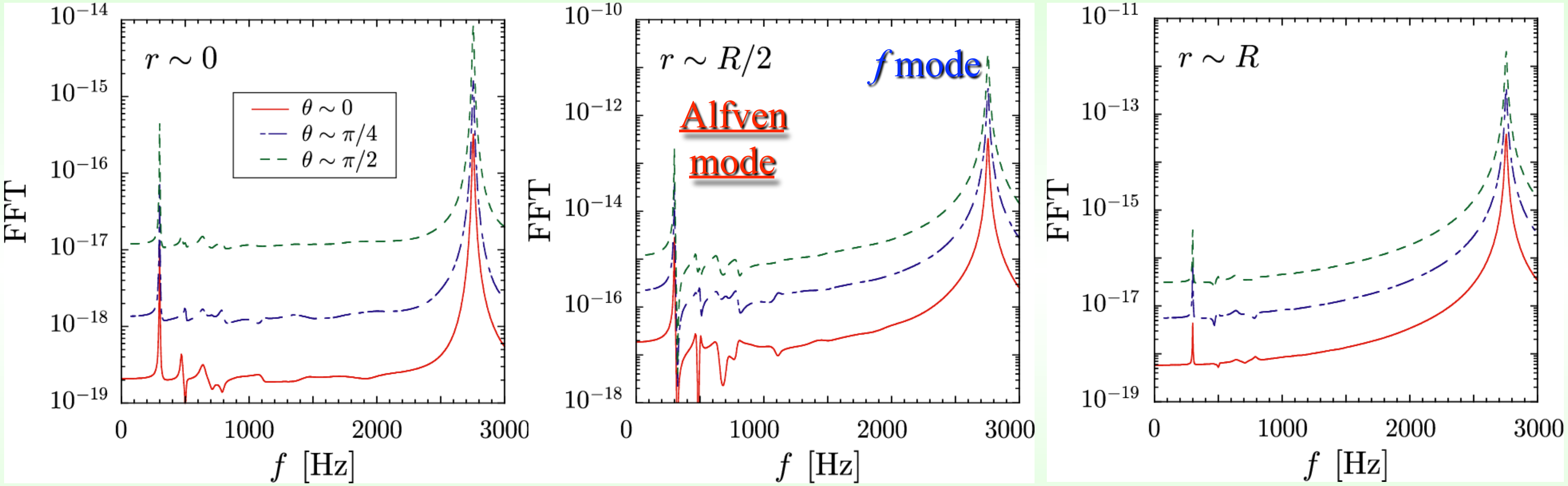
- At the location inside the star where the effective amplitude becomes maximum, time evolution of $\partial_t \mathcal{Y} \propto \delta u^\phi$ are …



  - We can see the **long-lived QPOs**

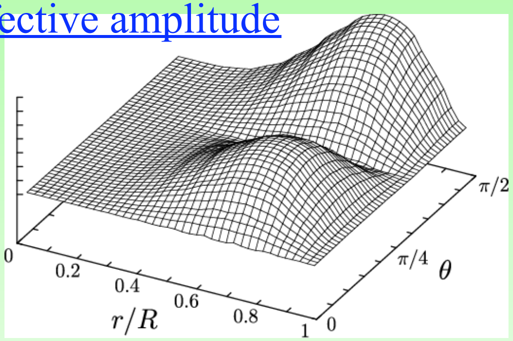
FFT for Polar Alfven Oscillations

- FFT at various points inside the star
  - B = 1 x 10^{16} Gauss

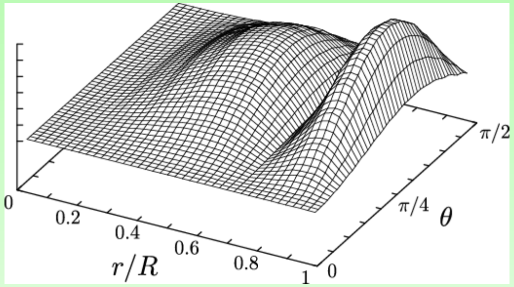


  - The frequencies of peak in FFT are independent from the observer positions.

effective amplitude

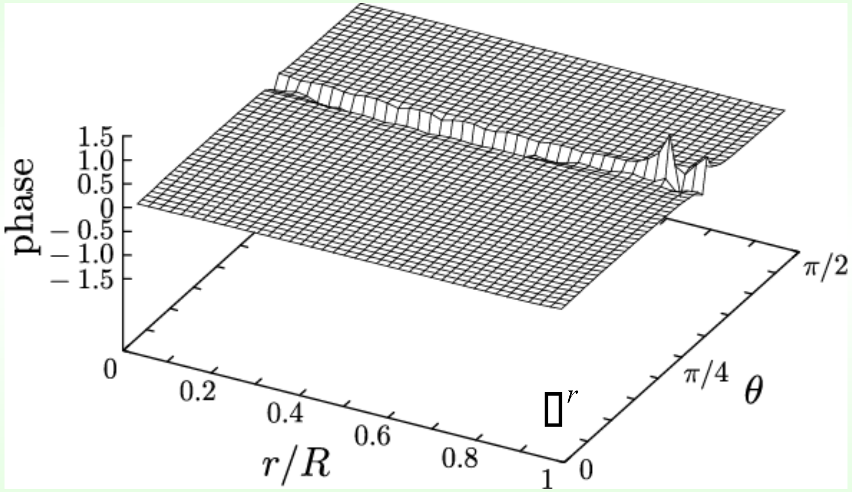


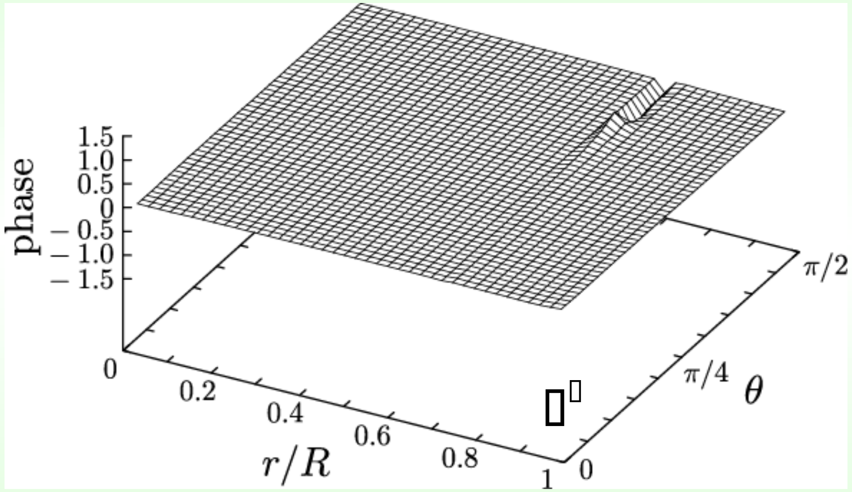
phase



effective amplitude

- The phase of each peak



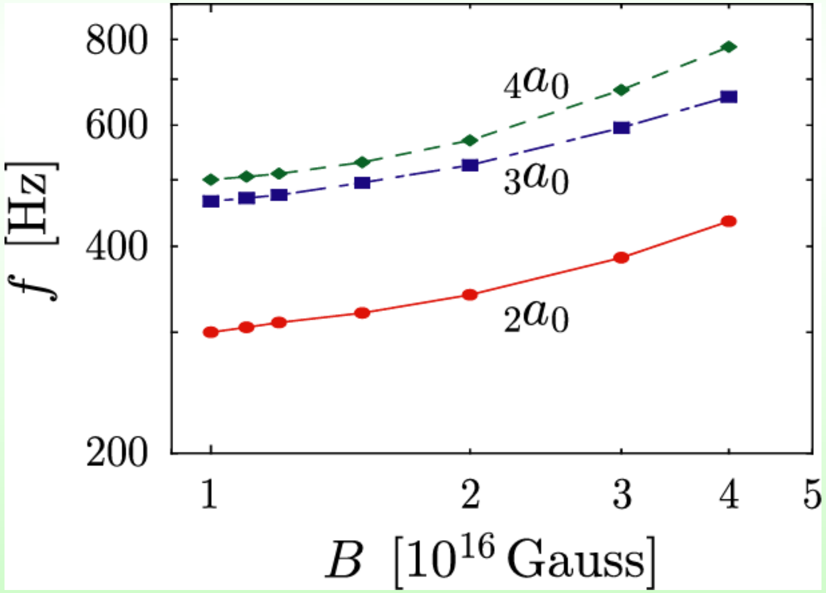


  - Phase with the frequency of peak in FFT is constant.

- With two results such as FFT and phase, the **polar Alfven oscillations** are not continuum but **discrete modes**.

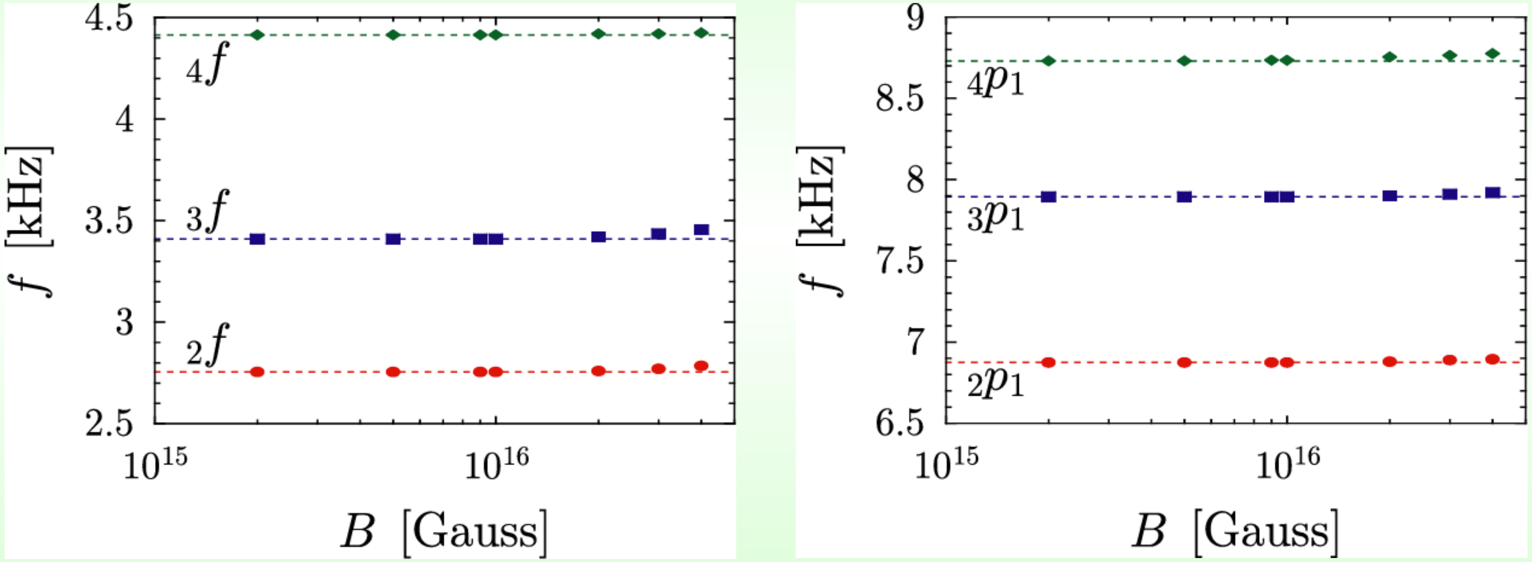
polar Alfven oscillations

- Dependence of polar Alfven modes on magnetic strength
  - Frequency ~ a few hundred Hz
  - These might be used to explain the observed higher QPOs frequencies.



fluid modes

- The dependence of fluid modes frequencies on the magnetic field strength
  - We can expect that these frequencies would be almost independent from the magnetic field strength unless the magnetic field are quite strong.



- We can see the effect of the magnetic field lager than 1x10^{16}Gauss.
  - That deviation from the frequencies without magnetic field are only less than a few %.

Open problems

- Axisymmetric Alfven oscillations in magnetars
  - Axial parity : continuum
  - Polar parity : discrete
- Crust oscillations : discrete (at least for non-magnetized star)

↓

- *How do the axial oscillations become in the case of existence of crust ??*
- *How about non-axisymmetric oscillation ??*