Dynamical α quenching and helicity fluxes on spherical $\alpha\Omega$ dynamos

G. Guerrero
A. Brandenburg P. Chatterjee

NORDITA

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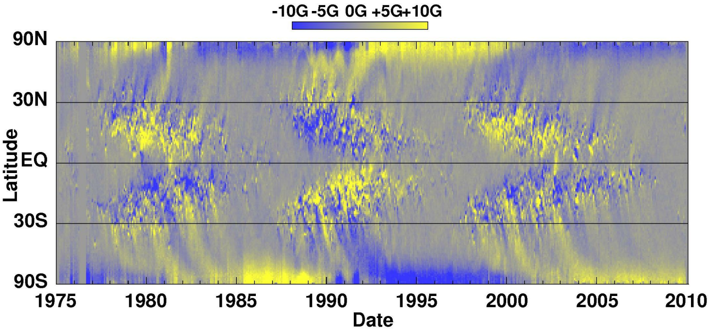
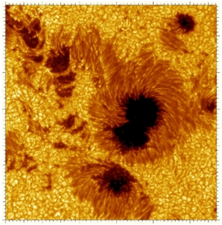
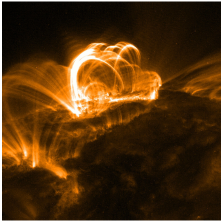
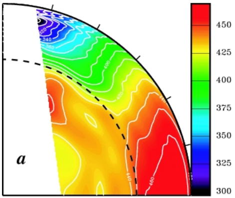
Outline

11-years solar cycle









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- Mean-field induction equation (*Steenbeck & Krause, 1969*):

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta_{\rm m} \boldsymbol{\nabla} \times \overline{\boldsymbol{B}} \right) , \tag{1}$$

with $\overline{\boldsymbol{\mathcal{E}}} = \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \mu_0 \overline{\boldsymbol{J}}$,

- Spherical coordinates and axisymmetry: $\overline{\boldsymbol{B}} = B\hat{\boldsymbol{e}}_\phi + \nabla \times (A\hat{\boldsymbol{e}}_\phi)$ and $\overline{\boldsymbol{U}} = r \sin\theta \Omega \hat{\boldsymbol{e}}_\phi + \boldsymbol{u}_{\rm p}$
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Dynamo saturation

- Heuristic (*Stix, 1976*), back-reaction of $\boldsymbol{B}$ on $\boldsymbol{U}$.

$$\alpha_{\rm K} \rightarrow \alpha_{\rm K} \left(1 + B^2/B_{\rm eq}^2\right)^{-1}, \quad B_{\rm eq} = (\mu_0 \rho \overline{\boldsymbol{u}^2}) \tag{4}$$

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  $R_{\rm m} \gg 1$ (10^8 sun, 10^{18} galaxy), $\rightarrow$, Catastrophic α quenching (*Vainshtein & Cattaneo, 1992, Cattaneo & Hughes, 1996*).

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  - Interface dynamo (*Parker 1993*),
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Consistent physical interpretation of dynamo saturation (*Pouquet et al. 1976*).

$$\alpha = \alpha_{\rm K} + \alpha_{\rm M} = \tfrac{1}{3}\tau(\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}} + \rho^{-1}\overline{\boldsymbol{j}\cdot\boldsymbol{b}}) \tag{6}$$

Magnetic helicity conservation (*e. g. Blackman & Brandenburg, 2002*):

$$\frac{d}{dt}\langle\boldsymbol{A}\cdot\boldsymbol{B}\rangle = -2\eta\mu_0\langle\boldsymbol{J}\cdot\boldsymbol{B}\rangle \rightarrow 0, \quad \mathrm{For} \quad R_{\rm m} \gg 1(\eta \rightarrow 0) \tag{7}$$

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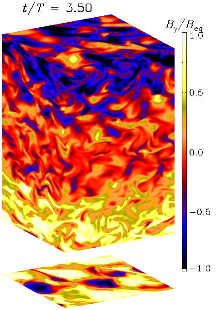
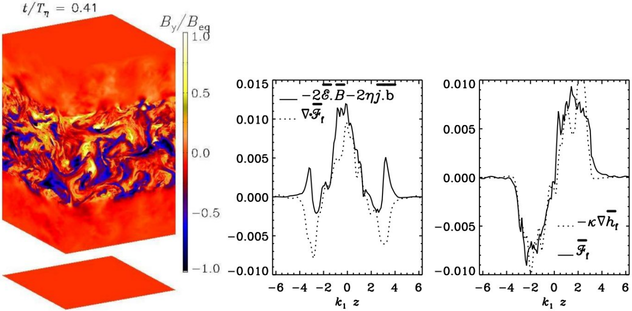
Magnetic helicity fluxes, diffusive flux

Mitra et al. 2010



Hubbard & Brandenburg, 2010

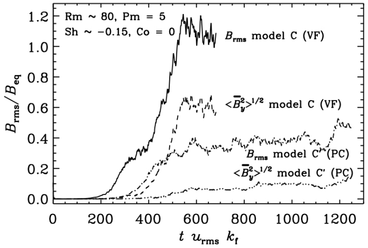


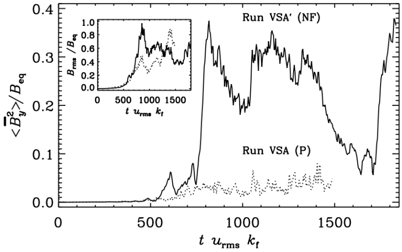
- h_f (or α_M) diffuses following a Fickian diffusion law: $\mathcal{F}_\mathrm{D} = -\kappa_\alpha \boldsymbol{\nabla} \alpha_\mathrm{M}$
- $\kappa_\alpha \sim (0.1 - 0.3)\eta_\mathrm{t}$
- Diffusion of h_f is gauge independent

(see Simon Candelaresi poster)

Magnetic helicity fluxes, Vishniac-Cho flux

DNS: convection + shear $= \alpha\Omega$ dynamo.





- Magnetic field grows faster in simulations with open boundary conditions allowing magnetic helicity flux (e.g. *Käpylä* et al. 2008).
- *Vishniac & Cho, 2001, Brandenburg & Subramanian, 2005*

$$\overline{\mathcal{F}}_i^{\mathrm{VC}} = C_{\mathrm{VC}}\epsilon_{ijl}\overline{\mathsf{S}}_{lk}\overline{B}_j\overline{B}_k \quad , \qquad (10)$$

where $\overline{\mathsf{S}}_{lk} = \frac{1}{2}(\overline{U}_{l,k} + \overline{U}_{k,l})$, C_{VC} is a scaling factor.

Model (*Guerrero et al. 2010*)

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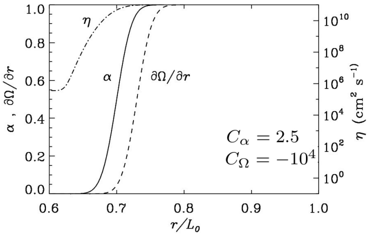
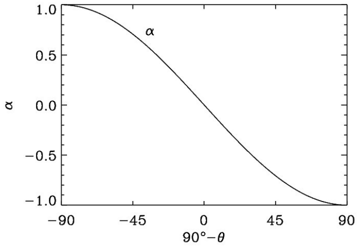
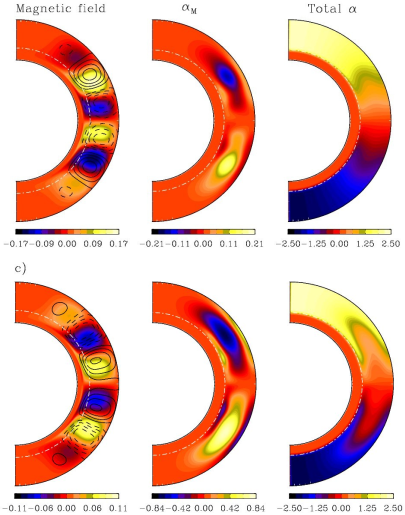
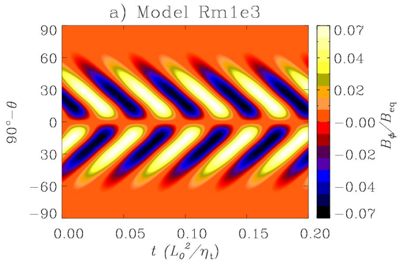
BC's: $A = B = 0$ (poles), $A = \partial(rB)/\partial r = 0$ (bottom), $(\nabla^2 - s^{-2})A = 0$ (top).

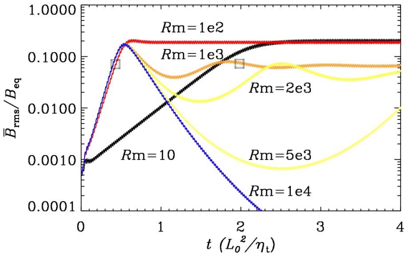




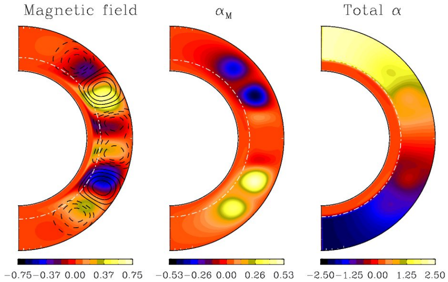
Results: *Dynamical* α *quenching*, $\overline{\mathcal{F}}_\alpha = 0$

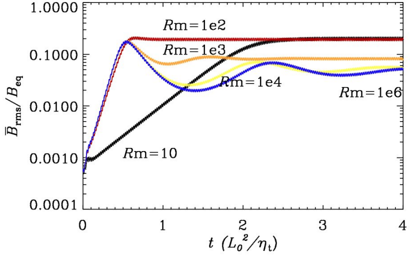


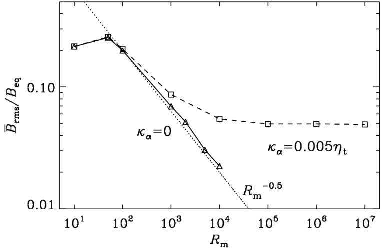


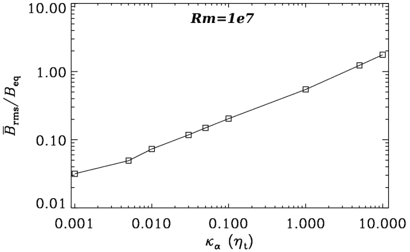


Results: *diffusive flux*, $\overline{\mathcal{F}}_\alpha = -\kappa_\alpha \nabla \alpha_{\rm M}$

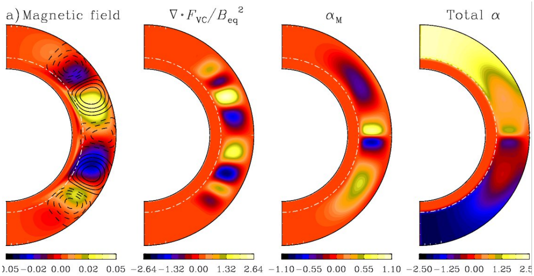


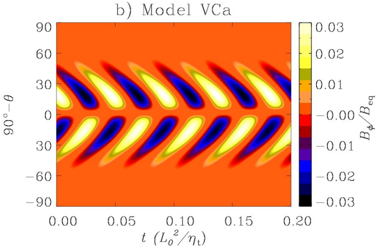


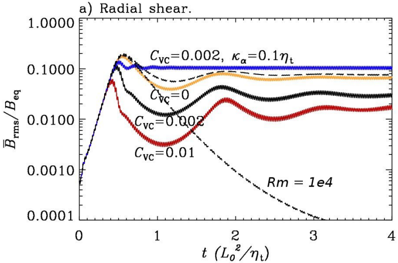




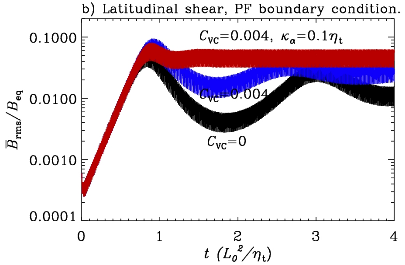
Results: *VC flux*: $\overline{\mathcal{F}}_\alpha = C_{\rm VC}\epsilon_{ijl}\overline{S}_{lk}\overline{B}_j\overline{B}_k$, radial shear

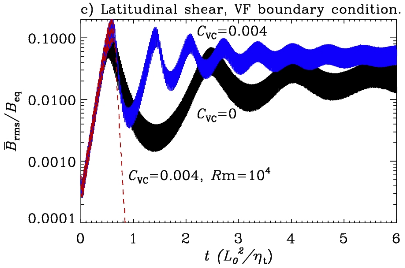


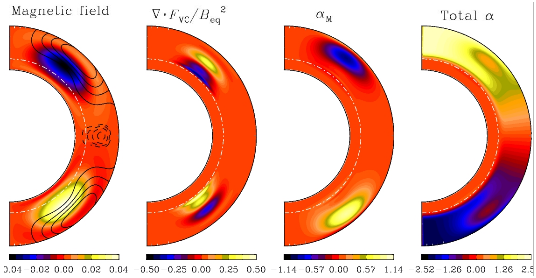




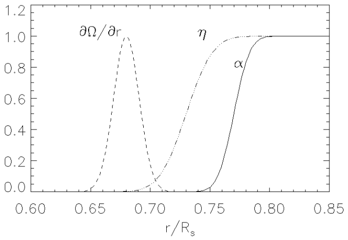
Results: *VC flux*: $\overline{\mathcal{F}}_\alpha = C_{\rm VC}\epsilon_{ijl}\overline{S}_{lk}\overline{B}_j\overline{B}_k$, latitudinal shear

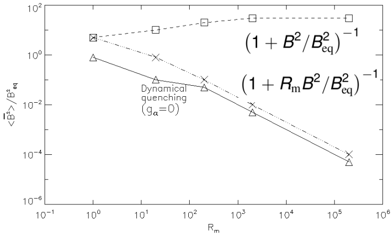


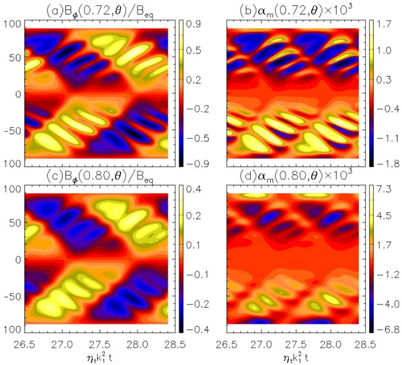




Results: *Parker's interface dynamo model*







$C_\alpha = 4C_\alpha^c$, $R_\mathrm{m} = 20$.
Secondary dynamo wave driven by the magnetic α effect.

Conclusions

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- Not the same for the Vishniac-Cho flux.
- VC-flux modifies the distribution of the magnetic field.
- For higher scaling factor, the VC-flux may develop local dynamo action.
- Catastrophic quenching is not alleviated by separating the dynamo layers. It implies that it is necessary to take into account a proper description of the quenching mechanism.

Ongoing and future work:

- Realistic solar dynamo models with differential rotation and meridional circulation profiles.
- Explore the effects of different helicity fluxes.
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