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The primordial magnetic field transfer at decoupling and its spatial energy spectrum

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Abstract

The primordial magnetic field transition between a radiative and a matter-dominated universe is analyzed. The gravitational structure formation affects its evolution and energy spectrum. The structure excitation can trigger magnetic field amplification and the steepening of its energy density spectrum.

1 Introduction

The magnetic fields of $\sim 10^{-6}$ G are observed at galactic and cluster scales and suspected in galaxy supercluster. The origin of these coherent fields at cosmological scales is not well understood. Either the initial (primordial) fields of strength $\sim 10^{-9}$ G substantially contribute to the observed values or alternatively, the weak seed fields of $\sim 10^{-20}$ G are amplified by a dynamo mechanism. It is however uncertain whether this mechanism can operate effectively at larger than galactic scale. Both the theoretical (i.e. the lack of universal mechanism able to simultaneously explain the magnetogenesis at the different spatial scales) difficulty and the observational evidence of intergalactic field correlation with large scale structure motivates the study of the primordial fields.

Two forthcoming experiments are expected to obtain some constraints on the primordial field nature. The presence of the magnetic field at the last scattering surface have an impact on the physics of CMB radiation. The PLANCK mission will be able to measure the CMB polarization with sufficient sensitivity to test the possible existence of the large-scale magnetic field prior to recombination. On the other hand the data provided by SKA are supposed to allow tracing the magnetic field associated with the cosmic structure.

In parallel, there are several theoretical questions which need to be explored. The principal axes are:

- what is the origin of the large-scale magnetic field; when it could occur and
- what is the relation between the structure formation and the evolution of the primordial magnetic field?

The latter one provides a great number of questions to be answered (see, e.g. Kim et al. 1996, Wasserman 1978). It is vital, for example, to understand whether the structure origin and its subsequent evolution may change the initial spectrum of the energy density of the primordial magnetic field. Does the structure appearance 'define' the ultimate shape of the magnetic energy spectrum?

In particular, the most important is the problem of primordial magnetic field transfer at decoupling and its later linear evolution. We attempt to clarify this issue here.

2 Basic equations and assumptions

The main purpose of this paper is the investigation of the primordial field development at decoupling and directly thereafter. Assuming that magnetic fields were created in the early universe by some particle physics processes and that the fields were not correlated with the density fluctuations, the following questions are raised.

1. Can the correlation between the magnetic field and matter density growth be established at recombination, i.e. if initially we have a random distribution of magnetic energy, then, does it become more organized later,

2. how the final energy distribution is determined by the initial magnetic spectrum.

The motivation for these questions is that the velocity which exists at recombination affects the magnetic field through the induction equation. Thus if structure formation develops hierarchically, does it mean that the cosmological magnetic field also amplifies as a bottom-up phenomenon resulting in an 'inverse cascade' type of the magnetic energy transfer?

To study this we need to consider the magnetic evolution described by the induction equation, which is a part of the MHD set of equations. Below, we apply the full set of the pressureless MHD equations (see, e.g. Siemieniec and Woszczyna 2004b MCW)

$$\dot{\rho} = -\rho\vartheta \tag{1}$$

$$\dot{\vartheta} = -\frac{1}{3}\vartheta^2 - 4\pi\kappa\rho - 2\sigma^2 + 2\omega^2 + \left(\frac{1}{\rho}(H^i H_j - \frac{1}{2}H^2\delta^i_j)^{;j}\right)_{;i} \tag{2}$$

$$\dot{H}^i = \sigma^i_j H^j + \omega^i_j H^j - \frac{2}{3}\vartheta H^i, \tag{3}$$

where ρ, ϑ, H are the density, expansion rate and the magnetic strength of the matter; the hydrodynamic scalars σ and ω measure the shear and the rotation. Semicolon means the covariant derivative with respect to the space coordinates, dot stands for the time convective derivative.

In order to study the cosmological development of the primordial magnetic field in the linear regime, we solve the above set of equations, assuming the following:

1. The evolution of the magnetic field is investigated in the post-recombination phase of a spatially flat FRW universe.

2. We analyze the magnetic field perturbations (small with respect to density perturbations) in the framework of the model (Siemieniec and Woszczyna 2004a AI) describing the density perturbation transition between the radiation to the matter-dominated era.

3. The transition is instantaneous. In this oversimplified approach the recombination coincides with the decoupling.

4. Density and velocity perturbations evolve in both epochs. The perturbations of the primordial magnetic field δH are induced by the density and velocity field perturbations $\delta\rho$, $\delta\vartheta$, and since it is attributed to the structure formation process, they have their origin at decoupling. The magnetogenesis mechanism producing the primordial seed field is expected to occur prior to recombination i.e. created before the 'initialization' of structure formation.

5. The perturbations are rotationless but the shear introduced by collapse is important. Referring to MCW, we postulate the symmetry in shear tensor of the form $\sigma^i_j H^j = \alpha\delta\vartheta\delta^i_j H^j$. It means that if the fluid contraction is one-dimensional i.e. $\alpha = \frac{2}{3}$ for the field component which is not contracted (parallel to the velocity) and $\alpha = -\frac{1}{3}$ for both components lying in the pancake plane. Thus if the perturbation $\delta\vartheta$ is known, then the induction equation can be separated from the whole MHD system.

6. The spectral distributions of the hydrodynamic quantities (velocity and density fields) are determined by the unknown spectrum of the primordial density perturbations $P(k)$. The primordial magnetic field spectrum is also unknown. Both come from the early universe processes and are supposed to be generated independently.

3 Solution of the MHD equations

The density and velocity contrasts $\delta = \frac{\delta\rho}{\rho_{(0)}}$, $\Theta = \frac{\delta\vartheta}{\vartheta_{(0)}}$ calculated in AI are 'functionals' of the primordial density spectrum $P(k)$. The linearized hydrodynamic evolution, eqs. (1) and (2), after introducing the perturbations,

$$\rho = \rho_{(0)} + \delta\rho, \quad \vartheta = \vartheta_{(0)} + \delta\vartheta, \quad H^i = H^i_{(0)} + \delta H^i$$

is governed by

$$\dot{\delta\rho} = -\vartheta_{(0)}\delta\rho - \rho_{(0)}\delta\vartheta, \quad \dot{\delta\vartheta} = -4\pi\kappa\delta\rho - \frac{2}{3}\vartheta_{(0)}\delta\vartheta, \tag{4}$$

while the linearized induction equation (3) for δH^i, according to assumption 5, becomes

$$(\delta\dot{H}^i) = \left(\alpha - \frac{2}{3}\right)\delta\vartheta H^i_{(0)} - \frac{2}{3}\vartheta_{(0)}\delta H^i. \tag{5}$$

The primary result of the transition model (AI) applied here is structure excitation with the features of acoustic shock formation at decoupling. The changes in density and velocity field (due to this transition) are felt by the magnetic field. Thus one can expect that the induced magnetic field δH should be substantially correlated with the structure.

3.1 Induction equation

Let us consider the propagation of the magnetic perturbations (5). For convenience we use here the conformal time $\eta = \int a(t)^{-1}dt$, where the scale factor $a(\eta)$ has the form (eq.(21) in AI) $a(\eta) = \frac{\sqrt{\mathcal{M}}(\eta+\eta_\Sigma)^2}{4\sqrt{3}\eta_\Sigma}$, $\mathcal{M}$ is the constant of motion in the Friedmann equation and η_Σ represents the transition time from a radiation to a matter-dominated era, $\vartheta_0 = 3\frac{da/dt}{a}$. Equation (5) rewritten in the variable η can be replaced by

$$\frac{(\eta+\eta_\Sigma)}{6}(\delta H^i)' = \beta\frac{\delta\vartheta}{\vartheta_0}H^i_{(0)} - \frac{2}{3}\delta H^i \tag{6}$$

To obtain its solution in the linear regime we express the primordial and perturbed magnetic field as well as the velocity perturbation $\delta\vartheta$ as Fourier series:

$$H_0 = \eta_\Sigma^{-2}(1+\tau)^{-4}\int d\mathbf{k}e^{i\mathbf{k}\mathbf{x}}f(k) + \text{c.c.}, \qquad \delta H = \eta_\Sigma^{-2}\int d\mathbf{k}e^{i\mathbf{k}\mathbf{x}}G(k,\tau) + \text{c.c.} \tag{7}$$

and according to eq. (45) in AI

$$\Theta \equiv \frac{\delta\vartheta}{\vartheta_0} = \int d\mathbf{k}e^{ik\mathbf{x}}\tilde{\Theta}(\tau,k) + \text{c.c.} \tag{8}$$

This form of $a(t)$ is a result of the joining conditions at the time of transition with the radiation scale factor $a(\eta) = \sqrt{\frac{\mathcal{M}}{3}}\,\eta$.

Equation (6) becomes now

$$\frac{\mathrm{d}G(k,\tau)}{\mathrm{d}\tau}+\frac{4}{\tau}G(k,\tau)=g(k,\tau),\quad \text{where}\quad \tau=(1+\frac{\eta}{\eta_\Sigma})\ ,\ \text{and} \tag{9}$$

$$g(k,\tau)=\frac{24\pi\beta}{\tau^5}\int_{-\infty}^{+\infty}\mathrm{d}k'k'^2(f(k-k')\tilde{\Theta}(k',\tau)+f(k+k')\tilde{\Theta}^*(k',\tau)) \tag{10}$$

The nonhomogeneous term $g(k,\tau)$ in the induction equations (9) shows the interaction of the mode $H_{(0)k}$ of the primordial field with the velocity modes Θ_k. In general, the nonvanishing source of eq. (9) gives the solution in the form

$$G(k,\tau)=\left(\frac{\tau}{2}\right)^{-4}C(k)+\left(\frac{\tau}{2}\right)^{-4}\int_2^\tau g(k,\tau)\left(\frac{\tau}{2}\right)^4\mathrm{d}\tau \tag{11}$$

where $C(k)$ is to be found from the boundary condition at decoupling i.e. $\delta H(\tau=2)=0$. It is seen from the above that the source modifies both the temporal and spatial (spectral) evolution of the primordial field.

3.2 The solution

The velocity field represented in this calculation by Θ is present at the decoupling time $\eta=\eta_\Sigma$. Together with the density contrast it propagates from the radiation era. Its evolution is determined by the primordial density fluctuation spectrum. The expansion contrast Θ is defined in the dust era by the joining conditions at η_Σ and by the input parameter $P(k)$, i.e. the energy spectrum of the acoustic field. According to AI, the general solution for Θ is expressed by eq. (45)

$$\Theta(\eta,\mathbf{x})=\int(\mathcal{A}_\mathbf{k}u_{\mathbf{k}(\vartheta)}(\eta,\mathbf{x})+\mathcal{A}^*_\mathbf{k}u^*_{\mathbf{k}(\vartheta)}(\eta,\mathbf{x}))\mathrm{d}\mathbf{k} \tag{12}$$

where the Fourier coefficients $\mathcal{A}_\mathbf{k}\mathcal{A}^*_\mathbf{k}\propto P(k)$, and the modes $u_{\mathbf{k}(\vartheta)}$ after transition are given by (eq. 47 in AI)

$$u_{\mathbf{k}(\vartheta)}\ =\ \frac{3^{1/4}}{\sqrt{2k}}e^{i\mathbf{kx}-ik\Sigma}\left(\frac{ik\Sigma}{40}\tau^2+\left(4+\frac{4}{ik\Sigma}+\frac{6ik\Sigma}{5}\right)\tau^{-3}\right), \tag{13}$$

where $\Sigma=\eta_\Sigma/\sqrt{3}$. The comparison of Fourier expansion (8) with the above allows one to obtain the transform $\tilde{\Theta}(\tau,k)$. For technical reasons we take in $\tilde{\Theta}(\tau,k)$ only the leading term $\propto\tau^2$ (we drop τ^{-3}), which is equivalent to the asymptotic form of the solution i.e. to a late time after recombination. For $\tilde{\Theta}(\tau,k)$ one finally gets

$$\tilde{\Theta}(k,\tau)=i\,C_1k^{\frac{1}{2}}\tau^2e^{-ik\Sigma},\ \text{where}\ C_1\equiv C\frac{3^{\frac{1}{2}}\Sigma}{2^{\frac{3}{2}}40} \tag{14}$$

The constant C above comes from the primordial fluctuations. For the sake of mathematical simplicity we adopt the primordial density spectrum of the white-noise type i.e. $P(k)=const$ which means $\mathcal{A}_\mathbf{k}=C$. For the initial primordial field, we do not indicate any particular magnetogenesis process with its specific initial spectrum . Below we assume that the spectrum generated by potentially different models of the early magnetogenesis can be described by the power law $f(k) = Ak^n$, $k_{\rm min} < k < k_{\rm max}$ and $f(k) = 0$ elsewhere. In the following calculations we restrict ourselves to the analysis of two cases i.e. similarly to density fluctuations, to the white-noise magnetic spectrum (n = 0) and to the case $f(k) \propto k^1$ (n = 1). For the postulated here both categories of the spectrum, the expression (10) becomes

$$g(k,\tau) = \Gamma\,\bar{g}(k)\,\tau^{-3} \tag{15}$$

where

$$\Gamma = 24\pi A\,C_1\beta,\;\; \bar{g}(k) = i\,I_n(k),\;\; I_n(k) = \int_{k_{\rm min}}^{k_{\rm max}} {\rm d}k'((k-k')^{\frac{5}{2}} - (k'-k)^{\frac{5}{2}})k'^n \exp(i(k'-k)\Sigma).$$

Finally, taking into consideration the initial condition $\delta H(\tau = 2)$ one obtains the solution for $G(k,\tau)$

$$G(k,\tau) = \frac{1}{2}\Gamma\,\bar{g}(k)(\tau^{-2} - 4\tau^{-4}) \tag{16}$$

It is important to note the structure of this solution. The temporal dependence of magnetic perturbations includes the typical solution of the homogeneous equation ($G \propto \tau^{-4} \propto a^{-2}$) and the additional component $\propto \tau^{-2} \propto a^{-1}$. Thus the sheared components, $\beta \neq 0$, of the induced magnetic fields are amplified in the linear regime much faster than those which are not contracted. The spatial dependence is determined by $I_n(k)$ i.e. by the convolution of a primordial magnetic spectrum with the kinematic quantity $\tilde{\Theta}(\tau,k)$.

4 Magnetic energy density spectrum

The evolutionary development of the magnetic field after recombination with no source term is well known. In the absence of velocity fields its adiabatic decrease occurs during the expansion of the universe (e.g. Rees Reinhardt 1972). The situation looks different if there is shear motion. In the considered model the gravitational structure begins to form at the recombination time. According to the solution (eq.16) the magnitude of the magnetic strength may substantially increase. The question arises whether generation of the large scale field is possible and what is the asymptotic form of its spectrum.

The calculation of the magnetic energy associated with the mode H_k, we restrict to the leading term in τ, obtaining $|H_k|^2 = \frac{1}{4}\eta_\Sigma^{-4}\,\bar{\Gamma}^2\,\tau^{-4}|I_n(k)|^2$. The resulting spectrum evolves through interaction between the velocity and magnetic modes which changes its initial form. The tendency of these changes is depicted by verification of two particular magnetic spectral forms. The analytical calculation has been performed for $|I_n(k)|^2$, (n = 0, 1). The resultant energy density spectra are presented below. For a given value of parameters k_{max}, Σ and for $k_{min} = 0$ we show in Fig. 1 the k-dependence of $|I_n(k)|^2$ - the quantity proportional to the magnetic energy density of the mode H_k. Two distinct features emerge: an energetic discrimination of $|I_n(k)|^2$ as a function of k and the large amplitudes of $|H(k)|^2 \propto \beta^2\,|I_n(k)|^2 \sim \beta^2\,10^{10}$. Depending on the initial magnetic spectrum, the magnetic energy is moved either to smaller and/or larger k. In the first case, there is an amplification of the 'magnetic noise' by shearing and the energy is carried to small and large scales. For the initial power spectrum $\propto f^2 \propto k^2$ the energy transfer goes backward i.e. from larger wave numbers to smaller wave numbers forming a sort of inverse cascade; which qualitatively shows the results of stretching. This in turn implies a correlation between the magnetic field and the density/velocity fields in structures.

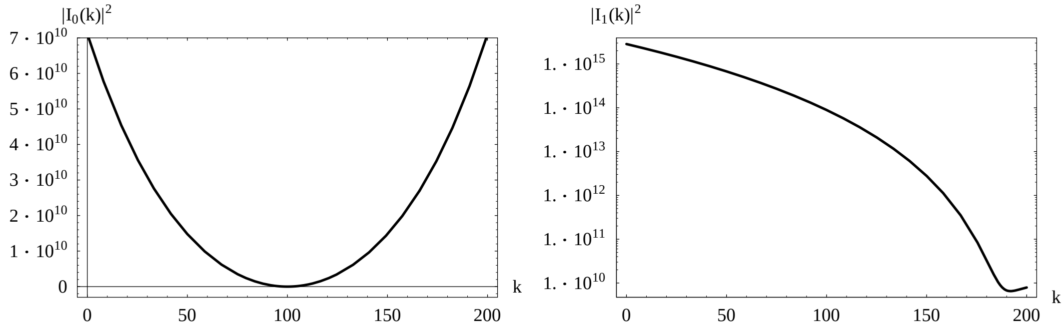

Figure 1: The quantities $|I_0(k)|^2$ (left panel) and $|I_1(k)|^2$ (right panel), proportional to the magnetic energy density spectrum as a function of the wave number. Axis of ordinates is in log scale.

4.1 Conclusions

In the presented model magnetic changes result from the coupling of velocity shear with the primordial magnetic field. The former appears due to the rapid initialization of structure collapse. The direct implication are

- field amplification and magnetic energy shift towards the larger scales (inverse cascading) and lead to much steeper magnetic spectra than the initial injection spectra.
- The shape of the final magnetic spectrum is a function of the initial magnetic spectrum (and implicitly depends on the primordial density fluctuations).
- The primordial field represents here the indispensable seed field for the future amplification (dynamo). Its strength at different length scales may differ by several orders of magnitude.
- Also a debate about the top-down versus bottom-up scenario for the magnetic field origin in structures may be rather unfounded. The seed field may appear simultaneously at all structure scales with strongly diverse strength.

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