Proceedings of The Magnetized Plasma in Galaxy Evolution
Sep. 27th – Oct. 1st 2004, Jagiellonian University, Kraków, Poland
Eds. K.T. Chyży, K. Otmianowska-Mazur, M. Soida and R.-J. Dettmar

Magnetised plasma turbulence in clusters of galaxies

Alexander A. Schekochihin,[1] S. C. Cowley,[2,3] R. M. Kulsrud,[4]
G. W. Hammett[4], and P. Sharma[4]

[1]DAMTP, University of Cambridge, Cambridge CB3 0WA, UK; as629@damtp.cam.ac.uk
[2]Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA
[3]Plasma Physics, Imperial College London, London SW7 2BW, UK
[4]Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543-0451, USA

Abstract

Cluster plasmas are magnetised already at very low magnetic field strength. Low collisionality implies that conservation of the first adiabatic invariant results in an anisotropic viscous stress (Braginskii viscosity) or, equivalently, anisotropic plasma pressure. This triggers firehose and mirror instabilities, which have growth rates proportional to the wavenumber down to scales of the order of ion Larmor radius. This means that MHD equations with Braginskii viscosity are not well posed and fully kinetic description is necessary. In this paper, we review the basic picture of small-scale dynamo in the cluster plasma and attempt to reconcile it with the existence of plasma instabilities at collisionless scales.

The intracluster medium (ICM) is a unique environment for studying astrophysical turbulence: it is made of amorphous diffuse fully ionised plasma with almost no net rotation or shear. The ICM is, therefore, in a certain sense, a case of "pure" turbulence. The presence of magnetic fields in the ICM with $B \sim 1...10\ \mu$G and field scale ~ 1 kpc is well documented (e.g., Carilli & Taylor 2002). Because of the availability of the rotation measure data from extended radio sources, it is possible to infer the magnetic-energy spectra for some of the clusters with spatial resolution of ~ 0.1 kpc (Vogt & Enßlin 2003, Vogt et al. 2004). This is due to improve dramatically with the arrival of LOFAR and, especially, SKA (Gaensler et al. 2004). At the same time, there is a rising interest among astronomers in measuring the velocity spectra of the cluster turbulence using instruments both future (ASTRO-E2, Sunyaev et al. 2003) and present (XMM-Newton, Schuecker et al. 2004). Once we have both kinetic and magnetic-energy spectra, a clear observational picture of the magnetised cluster turbulence will emerge. The purpose of this paper is to review our theoretical understanding of this turbulence.

There is no consensus on the exact mechanism that drives turbulence in clusters. The main candidates are galaxy wakes, jets, and cluster mergers. We will leave a discussion of these outside the scope of this paper. The important point for us is that all these mechanisms imply the outer scale of the cluster turbulence in the range $l_0 \sim 10^2...10^3$ kpc with outer scale velocities comparable to the thermal (sound) speed $v_{\rm th} = (T/m_i)^{1/2} \sim 10^3$ km/s, where m_i is ion mass (assuming hydrogen plasma) and $T \sim 10^8$ K the plasma temperature (assuming for simplicity $T_i = T_e$). This gives the outer eddy timescale of order $l_0/v_{\rm th} \sim 10^9$ yr.

The ion kinematic viscosity is $\nu \sim v_{\rm th}^2/\nu_{ii}$, where $\nu_{ii} = 4\pi n e^4 \ln\Lambda m_i^{-1/2} T^{-3/2}$ is the ion-ion collision frequency, $n \sim 10^{-2}...10^{-3}$ cm^{-3} is the ion number density (for cluster cores), e is the electron charge, and $\ln\Lambda \sim 20$ is the Coulomb logarithm (Spitzer 1962). This gives Reynolds numbers in the range $\mathrm{Re} \sim 10^2...10^3$. As the turbulent velocities at the outer scales are $\sim v_{\rm th}$, the turbulence in the inertial range will be subsonic. It is natural to assume that Kolmogorov's dimensional theory should apply at least approximately. Then the viscous scale of the turbulence is $l_\nu \sim l_0 \mathrm{Re}^{-3/4} \sim 10...30$ kpc. These numbers appear to agree quite well with observations of turbulence in the Coma cluster via pressure maps (Schuecker et al. 2004).

Because the ICM plasma is fully ionised, the viscous cutoff is, in fact, quite different from the usual Laplacian viscosity that normally appears in MHD equations. It can be shown that at frequencies below the ion cyclotron frequency $\Omega_i = eB/cm_i$ and scales above the ion Larmor radius $\rho_i = v_{\mathrm{th}}/\Omega_i$, fluid velocity and magnetic field satisfy

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}(p_\perp - p_\parallel)\right] + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi}, \quad (1)$$

$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\nabla\cdot\mathbf{u} + \eta\nabla^2\mathbf{B}, \quad (2)$$

where $d/dt = \partial/\partial t + \mathbf{u}\cdot\nabla$, $\rho = m_i n$ is mass density, $\hat{\mathbf{b}} = \mathbf{B}/B$, $p_\perp$ and $p_\parallel$ are the plasma pressures perpendicular and parallel to the magnetic field, $\eta \sim T^{-3/2} m_e^{1/2} e^2 c^2 \ln\Lambda/4\pi$ is the magnetic diffusivity (Spitzer 1962; we ignore the order-unity difference between $\eta_\parallel$ and $\eta_\perp$). Eq. (1) is the appropriate description for the fluid motions in the ICM provided the ions are magnetised, i.e., $\rho_i \ll \lambda_{\mathrm{mfp}}$, where $\lambda_{\mathrm{mfp}} \sim v_{\mathrm{th}}/\nu_{ii} \sim 1...10$ kpc is the ion mean free path. This requirement is satisfied if $B \gg 10^{-18}$ G, which is far below the observed field strengths of 1...10 μG. Note that 10 μG is roughly the field strength corresponding to the energy of the viscous-scale eddies ($\sim \rho v_{\mathrm{th}}^2 \mathrm{Re}^{-1/2}$), which is the lower bound for dynamically important fields. Thus, even for dynamically weak fields, the plasma is already very well magnetised.

The plasma pressures in Eq. (1) must be calculated kinetically. At low frequencies ($\omega \ll k v_{\mathrm{th}}$) and at scales above λ_{mfp}, the total isotropic pressure $p_\perp + B^2/8\pi$ can be determined from incompressibility, $\nabla\cdot\mathbf{u} = 0$, while the pressure anisotropy is calculated perturbatively from the kinetic theory (Braginskii 1965):

$$p_\perp - p_\parallel = 3\rho\nu\,\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u} \equiv \rho v_{\mathrm{th}}^2 \Delta, \quad (3)$$

where $\nu \sim v_{\mathrm{th}}\lambda_{\mathrm{mfp}} \sim v_{\mathrm{th}}^2/\nu_{ii}$ is the ion viscosity and Δ is the dimensionless measure of the pressure anisotropy. The pressure anisotropy is proportional to (the parallel component of) the rate of strain, so it is determined by the viscous-scale eddies' turnover rate u_ν/l_ν. For Kolmogorov turbulence with outer-scale velocity $\sim v_{\mathrm{th}}$, we have $u_\nu/v_{\mathrm{th}} \sim \mathrm{Re}^{-1/4}$ and $l_\nu/\lambda_{\mathrm{mfp}} \sim \mathrm{Re}^{-1/4}$, so $\Delta \sim u_\nu\lambda_{\mathrm{mfp}}/v_{\mathrm{th}} l_\nu \sim \mathrm{Re}^{-1/2}$.

Eqs. (1-3) plus the incompressibility condition $\nabla\cdot\mathbf{u} = 0$ are a closed system that we will call the Braginskii MHD. Two key properties of these equations should be noted. First, velocity gradients transverse to the magnetic field (e.g., shear-Alfvén-polarised fluctuations) are not dissipated. Velocity fluctuations can now penetrate below the viscous cutoff. Second, the velocities that are dissipated are those that change the strength of the magnetic field: indeed, from Eq. (2),

$$\frac{1}{B}\frac{dB}{dt} = \hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla\mathbf{u}. \quad (4)$$

Physically, the Braginskii viscosity encodes a fundamental property of charged particles moving in a magnetic field: the conservation of the first adiabatic invariant $\mu = mv_\perp^2/2B$, which is only weakly broken when $\lambda_{\mathrm{mfp}} \gg \rho_i$. This means that any change in B must be accompanied by a proportional change in $p_\perp$. Therefore, the emergence of the pressure anisotropy is a natural consequence of the changes in the magnetic-field strength and vice versa.

Let us ignore for a moment the inability of the Braginskii viscosity to damp all velocity fluctuations and consider the turbulent motions between the outer scale and the viscous scale and the random stretching of the magnetic field by these motions, or the small-scale dynamo. Such a view might appear justified because the Braginskii viscosity cuts off any motions that change the field strength, so the small-scale dynamo action is associated only with velocities above the viscous scale. The turbulent small-scale dynamo is a problem that we have studied extensively (for the full account of our work and a list of relevant references, see Schekochihin et al. 2004). Here we reiterate briefly the main points.

In the weak-field regime (no back reaction on the flow), the turbulent stretching is done primarily by the viscous-scale eddies because they turn over the fastest. The magnetic energy grows exponentially roughly at their turnover rate $u_\nu/l_\nu \sim 10^{-8}$ yr^{-1}. The turbulence tends to arrange the magnetic fields in long thin flux sheets (or ribbons): the field reverses its direction at the smallest scale l_B available to it (if the magnetic cutoff is modelled by the Ohmic diffusivity η, it is the resistive scale: $l_B \sim l_\eta \sim l_\nu \mathrm{Pm}^{-1/2}$, where $\mathrm{Pm} = \nu/\eta$) but field lines curve at the scale of the flow ($l_\parallel \sim l_\nu$, the viscous scale) except in the sharp bends of the folds. The field is in the state of reduced tension everywhere: both in straight and bent parts of the folds, $BK^{1/2} \sim$ const, where $K = |\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}|$ is the field-line curvature. Thus, the field strength and the field-line curvature are anticorrelated.

The tendency to fold the field is a kinematic property of random stretching. A key numerical finding is that the folded structure is also a feature of the nonlinear saturated state of the dynamo. The folded structure allows the small-scale direction-reversing magnetic field to back react on the flow in a spatially coherent way. The back-reaction mechanism we have proposed consists in the velocity gradients becoming locally anisotropic with respect to the direction of the folds (with $\hat{\mathbf{b}}\hat{\mathbf{b}} : \nabla \mathbf{u}$ partially suppressed) so that the dynamo saturates at the marginally stable balance between reduced "parallel" stretching and "perpendicular" mixing.

In the kinematic regime, the characteristic fold length is the viscous scale, $l_\parallel \sim l_\nu$. In the saturated state, the folds tend to elongate to the outer scale, $l_\parallel \sim l_0$. We have proposed that the saturation is controlled by the anisotropisation of the outer-scale eddies by the folded fields, while the inertial range ($l_0 > l > l_\nu$) is populated by Alfvén waves that propagate along the folds with the dispersion relation $\omega^2 = \mathbf{kk} : \hat{\mathbf{b}}\hat{\mathbf{b}} \langle B^2 \rangle / 4\pi\rho$.

The physical considerations outlined above provide a qualitative framework for interpreting the saturated spectra of magnetic and kinetic energies that are seen in the numerical simulations of forced isotropic MHD turbulence: magnetic energy significantly exceeds kinetic energy at small scales and, in the case of large Pm, tends to concentrate below the viscous scale (see, e.g., Schekochihin et al. 2004, Haugen et al. 2004). Such a magnetic spectrum makes sense if it is thought of as a superposition of folds with field reversals at the resistive scale and and Alfvén waves propagating along the folds.

Recently published magnetic-energy spectra based on RM measurements from extended radio sources for Abell 400, 2255, 2634, and Hydra A clusters (see Vogt & Enßlin 2003, Vogt et al. 2004, and Enßlin's contribution in these Proceedings) show magnetic energy to be peaked at ~ 1 kpc with the overall shape of the spectra quite similar to those that emerge in numerical simulations cited above. Does this mean that these simulations correctly describe cluster turbulence? The answer to this question is probably in the negative. All existing simulations either use isotropic viscosity and Ohmic diffusivity to model kinetic and magnetic cutoffs or rely on numerical dissipation. Clearly, simulations with a magnetic cutoff at or not far below 1 kpc (i.e., about a decade below the viscous scale) are likely to produce magnetic spectra that will look roughly like the observed ones. However, if we estimate the resistive cutoff scale based on the formula for Ohmic diffusivity given above, we will get $l_\eta \sim 10^4$ km — many orders of magnitude below 1 kpc. Furthermore, no published numerical study has included the effect of anisotropic viscosity to address the undamped velocity fluctuations at subviscous scales. In what follows, we show that the Braginskii viscosity not only fails to efficiently damp kinetic energy at the viscous scale but triggers fast-growing instabilities at subviscous scales. This fundamentally alters the structure of the turbulence at these scales.

It has, in fact, been known for a long time that pressure anisotropy leads to plasma instabilities (Rosenbluth 1956, Chandrasekhar et al. 1958, Parker 1958, Vedenov & Sagdeev 1958). A vast literature exists on these instabilities, which we do not attempt to review. The standard approach is to postulate a bi-Maxwellian equilibrium distribution with $T_\perp \neq T_\parallel$ (see, e.g., Ferrière & André 2002 and references therein). We do not need to adopt such a description because Eqs. (1-3) incorporate the pressure anisotropy in a self-consistent way.

Consider a turbulent cascade that originates from the large-scale driving, extends down to the viscous scale, and gives rise to velocity and magnetic fields $\mathbf{u}$ and $\mathbf{B}$ that vary on time scales $\sim (\nabla u)^{-1}$ and on spatial scales $\sim l_\nu$. We study the stability of such fields. The presence of turbulent shear (velocity gradients) gives rise to the pressure anisotropy given by Eq. (3). We now look for linear perturbations $\delta\mathbf{u}$, $\delta\mathbf{B}$, $\delta p_\perp$, $\delta p_\parallel$ that have frequencies $\omega \gg \nabla u$, and wavenumbers $k \gg l_\nu^{-1}$. With respect to these perturbations, the unperturbed rate-of-strain tensor $\nabla\mathbf{u}$ can be viewed as constant in space and time. Linearising Eq. (1) and neglecting temporal and spatial derivatives of the unperturbed quantities, we get

$$\begin{aligned} -i\omega\rho\delta\mathbf{u} \quad &= \quad -i\mathbf{k}\,\delta\left(p_\perp + \frac{B^2}{8\pi}\right) + \left(p_\perp - p_\parallel + \frac{B^2}{4\pi}\right)\delta\big(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}\big) \\ &\quad + \hat{\mathbf{b}}\,\delta\left[\nabla_\parallel\left(p_\perp - p_\parallel\right) - \left(p_\perp - p_\parallel - \frac{B^2}{4\pi}\right)\frac{\nabla_\parallel B}{B}\right] \end{aligned} \tag{5}$$

and, from Eq. (2), $\delta\hat{\mathbf{b}} = -(k_\parallel/\omega)\delta\mathbf{u}_\perp$ and $\delta\big(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}\big) = ik_\parallel\delta\hat{\mathbf{b}}$. We have denoted $\nabla_\parallel = \hat{\mathbf{b}}\cdot\nabla$ and $k_\parallel = \hat{\mathbf{b}}\cdot\mathbf{k}$. In the resulting dispersion relation, it is always possible to split off the part that corresponds to the modes that have shear-Alfvén-wave polarisation, $\delta u_\parallel = 0$, $\mathbf{k}_\perp\cdot\delta\mathbf{u}_\perp = 0$:

$$\omega^2 = k_\parallel^2 v_{\rm th}^2\left(\Delta + 2\beta^{-1}\right), \tag{6}$$

where $\beta = 2v_{\rm th}^2/v_A^2 = 8\pi\rho v_{\rm th}^2/B^2$. When the magnetic energy is larger than the energy of the viscous-scale eddies, $\beta \ll |\Delta|^{-1} \sim \mathrm{Re}^{1/2}$, Eq. (6) describes shear Alfvén waves. In the opposite limit, $\beta \gg |\Delta|^{-1}$, an instability appears: when $\Delta < 0$, the growth rate is $\gamma = k_\parallel v_{\rm th}|\Delta|^{1/2} \sim k_\parallel v_{\rm th}\mathrm{Re}^{-1/4}$. Since $k_\parallel \gg l_\nu^{-1}$, the instability is faster than the rate of strain, $\gamma \gg \nabla u \sim v_{\rm th}\mathrm{Re}^{-1/4}/l_\nu$, in accordance with the assumption made in the derivation of Eq. (6). This instability, triggered by $p_\parallel > p_\perp$, is called the firehose instability. It does not depend on the way the pressure perturbations are calculated because it arises from the perturbation of the field-line curvature $\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}$ in Eq. (5): to linear order, it entails no perturbation of the field strength [from Eq. (2), $\delta B/B = -(k_\parallel/\omega)\delta u_\parallel$] and, therefore, does not alter the pressure.

In order to determine the stability of perturbations with other polarisations, $\delta p_\perp$ and $\delta p_\parallel$ have to be computed. We consider scales smaller than the mean free path, $k\lambda_{\rm mfp} \gg 1$, where the plasma is collisionless. It is sufficient to look for "subsonic" perturbations such that $\omega \ll kv_{\rm th}$ because the high-frequency perturbations are subject to strong collisionless damping and cannot be rendered unstable by a small anisotropy. A straightforward linear kinetic calculation (Schekochihin et al. 2005) shows that the slow-wave-polarised perturbations are unstable with the growth rate

$$\gamma = \left(\frac{2}{\pi}\right)^{1/2} |k_\parallel| v_{\rm th} \left[\Delta\left(1 - \frac{k_\parallel^2}{2k_\perp^2}\right) - \beta^{-1}\left(1 + \frac{k_\parallel^2}{k_\perp^2}\right)\right]. \tag{7}$$

Modes with $k_\parallel > \sqrt{2}k_\perp$ are unstable if $\Delta < 0$ (slow-wave-polarised firehose instability); modes with $k_\perp > k_\parallel/\sqrt{2}$ are unstable if $\Delta > 0$ (mirror instability).

The instabilities we have described have growth rates $\propto k_\parallel$. Thus, *MHD equations with Braginskii viscosity do not constitute a well-posed problem.* This means, for example, that any numerical simulation of these equations will blow up at the grid scale unless additional isotropic viscosity is introduced (this has been confirmed by J. L. Maron, unpublished).

Eqs. (6) and (7) are valid in the limit $k\rho_i \to 0$. If finite ρ_i is taken into account, the growth rates peak at $k \sim \rho_i^{-1}$ with $\gamma_{\max} \sim |\Delta|\Omega_i$ for the mirror and $\gamma_{\max} \sim |\Delta|^{1/2}\Omega_i$ for the shear-Alfvén-polarised firehose. For clusters, this gives values of $\gamma_{\max}$ in the range $10^{-8}...10^{-7}(B/10^{-18}\ \mathrm{G})\ \mathrm{yr}^{-1}$ — much faster than the viscous-eddy turnover rate.

We have shown that, given sufficiently high β, the firehose and mirror instabilities occur in the regions of decreasing ($\Delta < 0$) and increasing ($\Delta > 0$) magnetic-field strength, respectively. The small-scale dynamo action by the viscous-scale motions associated with the turbulent cascade from large scales will produce regions of both types: the folded structure explained above contains growing straight direction-reversing fields and weakening curved fields in the corners of the folds. This structure is intrinsically unstable: straight growing fields to the mirror, curved weakening fields to the firehose instability. It seems plausible, however, that the magnetic field does grow until $\beta \sim |\Delta|^{-1} \sim \mathrm{Re}^{-1/2}$, so the instabilities are stabilised. This corresponds to $B \sim 10\ \mu\mathrm{G}$, which is comparable to the observed field strength in clusters.

After the instabilities are quenched, the fluid motions can be separated into two classes: above the viscous scale, there is the usual turbulent cascade, to which the picture of small-scale dynamo given in Schekochihin et al. 2004 and outlined above applies; below the viscous scale, the slow-wave-polarised motions are damped, while the shear Alfvén waves (the stable counterpart of the firehose instability) can exist. If we conjecture the survival of the folded structure, which is a generic outcome of random stretching, then the Alfvén waves will propagate along the folds. Note that the linear stability analysis outlined above can be recast in terms of the perturbations of striped direction-reversing fields represented by the tensor $\mathbf{BB}$. What we still lack is an estimate of the reversal scale.

Even for rms field strengths sufficient to quench the instabilities, there will always be regions where the field is weak. These regions can become unstable unless they are smaller than the local ion Larmor radius. We conjecture that the overall structure of the magnetic field is determined by the requirement that the field scale in the weak-field regions should be smaller than local value of ρ_i. The following rather tentative argument is an attempt to estimate the characteristic field scale based on this idea. As we mentioned above, the folded structure is characterised by the anticorrelation between the field strength and field-line curvature: $BK^{1/2} \sim \mathrm{const}$. The weak-field regions are the bending regions where the field is curved (see Fig. 1). A simple flux-conservation argument gives $B_{\rm bend}/B_{\rm rms} \sim l_B/l_\parallel$, where $B_{\rm rms}$ is the field in the strong-field regions, l_B is the reversal scale, and $l_\parallel$ is the fold length, which, in the nonlinear regime, can be as long as the outer scale l_0 of the turbulence. We estimate the curvature in the bending regions by $K_{\rm bend} \sim 1/\rho_{i,\rm bend}$, where $\rho_{i,\rm bend} = \rho_i B_{\rm rms}/B_{\rm bend}$ is the ion Larmor radius in the bending regions and $\rho_i = v_{\rm th}/\Omega_i = cm_i^{1/2}T^{1/2}/eB_{\rm rms}$ is the ion Larmor radius in the strong-field regions. The curvature in the strong-field regions is $\sim 1/l_\parallel$.

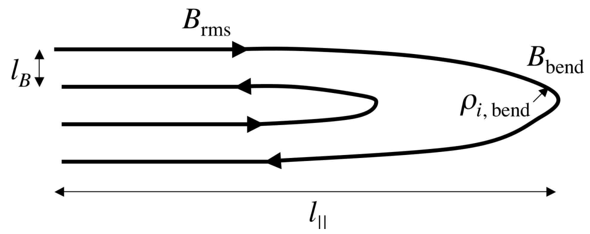

Figure 1: A sketch of the folded structure and scales referred to in the derivation of Eq. (8).

Then $B_{\rm bend}/B_{\rm rms} \sim (\rho_{i,\rm bend}/l_{\|})^{1/2}$. Assembling these relations and substituting numbers, we obtain the following estimate

$$l_B \sim \rho_i^{1/3} l_{\|}^{2/3} \sim 10^{-2} \left(\frac{T}{10^8\ \rm K}\right)^{1/6} \left(\frac{B_{\rm rms}}{1\ \mu\rm G}\right)^{-1/3} \left(\frac{l_{\|}}{1\ \rm Mpc}\right)^{2/3} \rm kpc. \tag{8}$$

This is still substantially smaller than the scale at which the observed spectrum peaks, but not entirely unreasonable, considering the simplistic nature of our argument. Note that the shear Alfvén waves can exist at scales below l_B, with a cutoff at $k_\perp \rho_i \sim 1$, so the turbulence spectra are likely to have extended power tails.

A somewhat more detailed exposition can be found in Schekochihin et al. 2005.

Acknowledgements

A.A.S. was supported by the UKAFF Fellowship. This work was supported in part by the NSF Grant No. AST 00-98670 and by the US DOE Center for Mutiscale Plasma Dynamics. G.W.H. and P.S. were supported by the US DOE Contract No. DE-AC02-76CH03073.

References

Braginskii S. I. 1965, Rev. Plasma Phys., 1, 205

Carilli C. L., Taylor, G. B. 2002, ARA&A, 40, 319

Chandrasekhar S., Kaufman A. N., Watson K. M. 1958, Proc. R. Soc. London A, 245, 435

Ferrière K. M., André N. 2002, J. Geophys. Res., 107, 1349

Gaensler B. M., Beck R., Feretti L. 2004, New Astron. Rev., in press (astro-ph/0409100)

Haugen N. E. L., Brandenburg A., Dobler W. 2004, Phys. Rev. E, 70, 016308

Parker E. N. 1958, Phys. Rev., 109, 1874

Rosenbluth M. N. 1956, Los Alamos Laboratory Report LA-2030

Schekochihin A. A., Cowley S. C., Taylor S. F., Maron J. L., McWilliams J. C. 2004, ApJ, 612, 276

Schekochihin A. A., Cowley S. C., Kulsrud R. M., Hammett G. W., Sharma P. 2005, ApJ, submitted (preprint on http://www.damtp.cam.ac.uk/user/as629)

Schuecker P., Finoguenov A., Miniati F., Böhringer H., Briel U. G. 2004, A&A, in press (astro-ph/0404132)

Spitzer L. 1962, Physics of Fully Ionized Gases (New York: Wiley)

Sunyaev R. A., Norman M. L., Bryan G. L. 2003, Astron. Lett., 29, 783

Vedenov A. A., Sagdeev R. Z. 1958, Soviet Phys.—Doklady, 3, 278

Vogt C., Enßlin T. A. 2003, A&A, 412, 373

Vogt C., Dolag K., Enßlin T. A. 2004, MNRAS, submitted (astro-ph/0401216)