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Eds. K.T. Chyży, K. Otmianowska-Mazur, M. Soida and R.-J. Dettmar

Magnetized Turbulence

Mordecai-Mark Mac Low

Dept. of Astrophysics, American Museum of Natural History, 79th Street at Central Park W., New York, NY 10024-5192, USA

Abstract

Several topics in the theory of magnetized turbulence are reviewed with application to star formation and the interstellar medium. The density, pressure, and temperature distribution in a turbulent interstellar medium is described in comparison to a medium dominated by thermal instability. The derivation of the empirical theory for the energy spectrum of hydrodynamic and MHD turbulence is outlined, and comparisons are given to numerical models for the magnetized case. Next discussed is how density fluctuations modify observations of velocity fluctuations in line centroids, coincidentally cancelling the effects of projection smoothing. The analytic description of velocity structure functions as a function of the dimensionality of the dissipative structures is then covered. Finally the question of whether turbulence can support against gravitational collapse is addressed. Turbulence both prevents and promotes collapse, but its net effect is to inhibit collapse, so it does not trigger star formation in any significant sense.

1 Observations

For more than half a century, evidence for the turbulent nature of the interstellar medium (ISM) has been known (von Weiszäcker 1943, 1951; Münch 1958). Armstrong, Rickett, & Spangler (1995) compiled observations of the ionized ISM that appear to show that the magnetized gas has a fluctuation power spectrum consistent with Kolmogorov (1941) turbulence over scales ranging from 100 pc to small fractions of an AU.

Other observations pointing to the presence of supersonic, magnetized turbulence in the ISM include the presence of superthermal linewidths in the diffuse ionized medium, the warm and cold neutral gas, and molecular clouds. Line centroid variations also show correlations with separation suggestive of turbulence. Magnetic field strengths are measured using the Zeeman effect or, more recently, the Chandrasekhar-Fermi (1953) method (Heitsch et al. 2001; Ostriker, Stone, & Gammie 2001; Matthews, Fiege, & Moriarty-Schieven 2002; Crutcher et al. 2004). Small scale fluctuations are measured using refractive and diffractive pulsar scintillation, and fluctuations in rotation measure and dispersion measure.

This review briefly discusses the density and temperature structure of turbulent gas, as a counterpoint to multi-phase models of the ISM. Analytic descriptions of magnetized turbulence are then treated, as well as numerical tests of those descriptions. The observational consequences of density fluctuations on the line centroid velocity fluctuation spectrum are briefly noted. Finally the question of whether magnetized turbulence can support against gravitational collapse is reviewed. Topics that are not covered that fall under the general topic include magnetized turbulence generated by the magnetorotational instability, dynamos and field structure in turbulent flow, and diffusion of cosmic rays or tracers such as heavy elements.

2 Density and Temperature Structure

Classical theories of the interstellar medium have emphasized the thermal phases produced by the structure of the cooling curve as a function of temperature (Field, Goldsmith, & Habing 1969; for a modern example see Wolfire et al. 1995). At constant pressure, isolated points of stable equilibrium can be found on the pressure-density plane, giving phase structure to the ISM. However, turbulent flows generate continuous distributions of pressure (Mac Low et al. 2004; Avillez & Breitschwerdt 2004), a point also emphasized by Breitschwerdt and Avillez in these proceedings. Although thermal instability still acts in such flows, broad regions on the pressure-density plane end up occupied, rather than isolated phases.

3 Scaling Relations in Magnetized Turbulence

Our understanding of magnetized turbulence rests on the foundations of the Kolmogorov (1941) theory for the energy spectrum of incompressible hydrodynamical turbulence. This theory requires two assumptions: that energy input at large scale and energy dissipation at small scale balance, and that energy transfer between spatial scales occurs only between neighboring scales in the intervening inertial range. Or, to quote Richardson,

> Big whirls have little whirls, which feed on their velocity,
> and little whirls have lesser whirls, and so on to viscosity.

The energy of incompressible turbulence $E \propto v^2/2$, and the crossing time of a perturbation over any scale ℓ is $\Delta t = \ell / v_\ell$, where v_ℓ is the average velocity difference across structures at that scale. The assumptions of constant flow between neighboring scales requires that the energy flow through any scale

$$\dot{E}_\ell \propto \frac{E_\ell}{\Delta t} \propto \frac{v_\ell^3}{\ell} \tag{1}$$

must be constant. Thus, the velocity must scale as

$$v_\ell \propto \ell^{1/3}. \tag{2}$$

In wavenumber space, $v_k \propto k^{-1/3}$. We can derive the energy spectrum in wavenumber space $E(k)$ in either 1D or 3D. In 1D, the energy in some portion of the inertial range

$$E = \int E(k) dk \propto v_k^2. \tag{3}$$

Substituting the scaling relation derived from equation 1, $kE(k) \propto k^{-2/3}$, so $E(k) \propto k^{-5/3}$. In 3D, $E = \int E(k) d^3k$, so $k^3 E(k) \propto k^{-2/3}$, and $E(k) \propto k^{-11/3}$.

A theory of incompressible MHD turbulence was proposed by Iroshnikov (1963) and Kraichnan (1965) based on the idea that energy transfer between scales would still be local, but transferred by interactions between oppositely directed Alfvén waves rather than vortices. Isotropy was still assumed in their derivation, however, as was a small ratio of fluctuating to mean magnetic field strength. The interaction time between Alfvén waves at any scale $\delta t = \ell / v_A$ is determined by their propagation speed $v_A^2 = B^2/(4\pi\rho)$. The change in any energy from each interaction

$$\Delta E \propto \frac{dv^2}{dt} \delta t \propto v_\ell \dot{v}_\ell \delta t. \tag{4}$$

The perturbation crossing time Δt determines the change in velocity $\dot{v}_\ell = v_\ell/\Delta t \propto v_\ell^2/\ell$, so

$$\Delta E \propto \frac{v_\ell^3}{\ell}\frac{\ell}{v_A} \propto \frac{v_\ell^3}{v_A}. \tag{5}$$

The time required for energy to pass through an incoherent cascade can be shown to be

$$t_c \propto \left(\frac{v_\ell^2}{\Delta E}\right)^2 \delta t \propto \frac{\ell v_A}{v_\ell^2}, \tag{6}$$

so the energy flow $\dot{E} \propto v_\ell^2/t_c \propto v_\ell^4/(\ell v_A)$. If the energy flow through each scale is constant, then $v_\ell \propto \ell^{1/4}$ in this case. The energy spectrum in 1D can then be derived analogously to the hydrodynamic spectrum, yielding $E(k) \propto k^{-3/2}$.

However, MHD turbulence in the presence of a mean field with energy of the same order as the flow is not isotropic. Goldreich & Sridhar (1995, 1997) argued that the anisotropy can be taken into account by treating the energy cascade along parallel lines as Alfvén wave interactions with time scale $t_\parallel = \ell/v_A$, following Iroshnikov and Kraichnan, but treating the perpendicular cascade with hydrodynamic interactions that merely stir field lines with time scale $t_\perp = \ell/v_\ell$. If the cascades in the two directions maintain what is referred to as critical balance and proceed at equal rates, so that $t_\parallel = t_\perp$, then $k_\parallel v_A = k_\perp v_k$. From the hydrodynamic cascade, we know that $v_k \propto k_\perp^{-1/3}$, so the anisotropy of the turbulence can be derived to be scale dependent: $k_\parallel \propto k_\perp^{2/3}$. The parallel cascade is predicted to still be an Alfvén cascade with $E_\parallel(k) \propto k_\parallel^{3/2}$, while the perpendicular cascade looks more like a Kolmogorov cascade with $E_\perp(k) \propto k_\perp^{5/3}$. The 1D spectrum is predicted to be $E(k) \propto k^{5/3}$.

Maron & Goldreich (2001) confirmed the anisotropy predicted, but found a 1D spectral index of 3/2. Cho & Vishniac (2000), and Maron, Cowley, & McWilliams (2004) on the other hand, do find the predicted 5/3 spectral index. The difference appears to be the strength of the initial field. If the fluctuations in the magnetic field are small compared to the mean field, the local Alfvén cascade dominates, as found by Maron & Goldreich (2001). If the fluctuations are larger, though, the perpendicular hydrodynamic cascade takes effect, as seen by Cho & Vishniac (2000) and Maron et al. (2004). Finally, if the fields are weak compared to the flow, as in the case of the turbulent dynamo, the magnetic cascade is no longer local, and the power spectrum of the field is dominated by high-k modes, while the velocity power spectrum remains dominated by the driving scale (e.g. Maron et al. 2004).

4 Observations of Velocity Spectra

One diagnostic of turbulence is the fluctuation in velocity centroid from point to point across a map. This is a projection of the 3D velocity fluctuation spectrum into 2D. The 3D spectral exponent is $\Theta = 1/3$ in the incompressible hydrodynamical case (eq. 2), while for shock-dominated Burgers turbulence $\Theta = 1/2$ (Burgers 1974). The projection to 2D should reduce the value of Θ by 0.5 in the incompressible case (von Hoerner 1951, Brunt et al. 2003). However, the observed value of the 2D velocity centroid fluctuation spectrum is close to 0.5 (e.g. Brunt & Mac Low 2004), as if no projection smoothing had occurred.

Supersonic turbulence drives density fluctuations. Velocity centroids depend on both velocity fluctuations and density fluctuations, as the emissivity at any particular velocity along a line of sight depends on the density at that point. These density fluctuations add extra power at small scales, increasing the slope of the velocity centroid fluctuation spectrum (Lazarian & Esquivel 2003; Brunt & Mac Low 2004). By a lucky coincidence, the density fluctuations in strongly supersonic turbulence cancel the effects of projection smoothing rather accurately. This does not hold for transsonic or subsonic turbulence however, and there is a transition region in the mildly supersonic regime where the cancellation is incomplete (Brunt & Mac Low 2004).

5 Velocity Structure Functions

Velocity structure functions offer another way of statistically characterizing turbulence that has proved tractable to analytic description. The structure function of order p at scale L is

$$S_p(L) = \langle |v(x+L) - v(x)|^p \rangle. \tag{7}$$

The structure function of order two can be directly related to the energy spectrum $E(k)$. In the inertial range of isotropic turbulence, structure functions scale as a power of the length scale $S_p(L) \propto L^{\zeta(p)}$. The behavior of the power-law exponents $\zeta(p)$ as a function of the order p is argued to depend on the dimension D of dissipative structures in the turbulence, along with the scaling exponents for $v(L) \propto L^{\Theta}$ and $t(L) \propto L/v \propto L^{\Delta}$ (She & Lévêque 1994; Dubrulle 1994). The form of the scaling is

$$\zeta(p) = \Theta(1-\Delta)p + C\left(1 - \Sigma^{p\Theta}\right), \tag{8}$$

where the codimension $C = 3 - D$, and $\Sigma = 1 - \Delta/C$. In Kolmogorov turbulence, equation (2) shows that $\Theta = 1/3$, so $\Delta = 2/3$.

In incompressible hydrodynamical turbulence, the dissipative structures are linear vortex tubes with dimension $D = 1$ (She & Lévêque 1994). Müller & Biskamp (2000) showed that incompressible MHD behaves as if its dissipative structures have dimension $D = 2$; they suggested that these structures are current sheets. Compressible MHD has also been examined. Boldyrev (2002) and Boldyrev, Nordlund, & Padoan (2002) suggest that the dissipative structures have $D = 2$ and are primarily shocks. Padoan et al. (2004) show that in super-Alfvénic but subsonic to transsonic turbulence $1 < D < 2$ depending on the sonic Mach number $\mathcal{M}_{\mathrm{rms}}$. However, Vestuto et al. (2001) raise a note of caution, as they find steeper power laws for energy spectra than would be predicted by the velocity structure function of order 2 under this formalism.

6 Gravitational Support

An important question for understanding star formation is whether magnetized turbulence can provide support against gravitational collapse. In order to provide significant support, it must act like a pressure in all directions, and it must last longer than a free-fall time. Neither of these requirements is clearly met by supersonic turbulence in molecular clouds.

Furthermore, hydrostatically supported objects do not form easily in supersonic turbulent flows. This is because an isothermal object in hydrostatic equilibrium requires a balancing external pressure to confine it. Too high a pressure sends it into collapse, while too low a pressure allows it to again expand. Vázquez-Semadeni, et al. (2004) have demonstrated numerically that these two paths are taken by virtually all density fluctuations in a magnetized turbulent flow, leaving very few objects that might be subject to ambipolar diffusion and loss of magnetic support over long time periods. Rather, the star formation rate appears to be limited by the necessity of assembling supercritical regions by moving gas along field lines in order to increase the mass-to-flux ratio, and by the global limitation of collapse by the turbulence, when that is effective.

If turbulence decays in less than a free-fall time, it will not provide significant support against gravitational collapse. For several decades, Arons & Max (1975) was interpreted to mean that magnetic fields could substantially reduce the decay rate of turbulence. In the late 1990's, three groups demonstrated numerically that even magnetized turbulence decays quickly (Mac Low et al. 1998, Stone, Ostriker, & Gammie 1998, Padoan & Nordlund 1999). Mac Low et al. (1998) showed that supersonic, trans-Alfvénic or super-Alfvénic turbulence decays as $E(t) \propto t^{-1}$, with a resolution study ranging from 32^3 to 256^3 zones.

There are a couple of exceptions to this quick decay known, although their astrophysical relevance remains to be demonstrated. Biskamp & Müller (1999) used 512^3 simulations of incompressible MHD to show that $E \propto t^{-1}$ for flows with magnetic helicity $H = \vec{A} \cdot \vec{B} = 0$, but that $E \propto t^{-1/2}$ for flows with significant helicity. Unbalanced cascades of Alfvén waves have been shown to decay more slowly by Maron & Goldreich (2001) and Cho, Lazarian, & Vishniac (2002). However, to reduce decay significantly, Alfvén wave fluxes must be as much as an order of magnitude higher in one direction than the opposite.

Mac Low (1999, 2003) estimates that the dissipation rate for isothermal, supersonic turbulence is

$$\dot{e} \simeq -(1/2)\rho v_{\rm rms}^3/L_d, \tag{9}$$

where L_d is the driving scale. The dissipation time for turbulent kinetic energy

$$\tau_d = e/\dot{e} \simeq L/v_{\rm rms}, \tag{10}$$

which is just the crossing time for the turbulent flow across the driving scale (Elmegreen 2000).

Stone et al. (1998) and Mac Low (1999) showed that supersonic turbulence decays in less than a free-fall time under molecular cloud conditions, regardless of whether it is magnetized or unmagnetized. The hydrodynamical result agrees with the high-resolution, transsonic, decaying models of Porter & Woodward (1992) and Porter, Pouquet, & Woodward (1994). Mac Low (1999) showed that the formal dissipation time $\tau_{\rm d} = e/\dot{e}$ scaled in units of the free fall time $t_{\rm ff}$ is

$$\tau_{\rm d}/\tau_{\rm ff} = \frac{1}{4\pi\xi}\left(\frac{32}{3}\right)^{1/2}\frac{\kappa}{\mathcal{M}_{\rm rms}} \simeq 3.9\,\frac{\kappa}{\mathcal{M}_{\rm rms}}, \tag{11}$$

where $\xi = 0.21/\pi$ is the Kolmogorov energy-dissipation coefficient derived by Mac Low (1999), $\mathcal{M}_{\rm rms} = v_{\rm rms}/c_{\rm s}$ is the rms Mach number of the turbulence, and κ is the ratio of the driving wavelength to the Jeans wavelength $\lambda_{\rm J}$. In molecular clouds, $\mathcal{M}_{\rm rms}$ is typically observed to be of order 10 or higher. If the ratio $\kappa < 1$, as is probably required to maintain gravitational support (Léorat, Passot, & Pouquet 1990), then even strongly magnetized turbulence will decay long before the cloud collapses, and not markedly retard the collapse.

Supersonic turbulence both prevents and promotes collapse. It can provide support against collapse on scales larger than the driving scale. Classical theories treat small-scale turbulence as an isotropic pressure with effective sound speed $c_{s,\rm eff} = (c_s + \langle v^2\rangle/3)^{1/2}$ (Chandrasekhar 1951; von Weiszäcker 1951). By this argument, supersonic turbulence *increases* the Jeans mass to $M_{J,\rm eff} = (\pi/G)^{3/2}\rho^{-1/2}c_{s,\rm eff}^3$. However, supersonic flows drive strong density fluctuations on scales below the driving scale. Positive density fluctuations in isothermal turbulence reach densities $\rho' = \rho\mathcal{M}_{\rm rms}^2$. In these density enhancements, the Jeans mass *decreases* to $M_{J,{\rm eff}} = (\pi/G)^{3/2}\rho'^{-1/2}c_{s,{\rm eff}}^3$. If we express everything in terms of velocity, however,

$$M_{J,{\rm eff}} = (\pi/G)^{3/2}\rho'^{-1/2}c_{s,{\rm eff}}^3 \propto \frac{c_s}{v}\left(c_s + \langle v^2\rangle/3\right)^{3/2} \propto v^2. \tag{12}$$

Even though turbulence can locally promote collapse, on average it *inhibits* collapse when compared to the same conditions absent turbulence. Thus, although turbulence may give the appearance of locally triggering collapse and star formation, it is globally preventing star formation, acting as a counterbalance to gravitational instability. Turbulence controls star formation in opposition to gravity, not in alliance with gravity.

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References

Armstrong, J. W., Rickett B. J., & Spangler, S. R., 1995, ApJ, 443, 209

Arons, J., & Max, C. E., 1975, ApJ (Letters), 196, L77

Avillez, M. A., & Breitschwerdt, D., 2004, A&A, 425, 899

Biskamp, D., & Müller, W.-C., 1999, Phys. Rev. Lett. 83, 2195

Boldyrev, S., 2002, ApJ, 569, 841

Boldyrev, S., Nordlund, Å., & Padoan, P., 2002, Phys. Rev. Lett., 89, 1102

Brunt, C. M., Heyer, M. H., Vázquez-Semadeni, E., & Pichardo, B., 2003, ApJ, 595, 824

Brunt, C. M., & Mac Low, M.-M., 2004, ApJ, 604, 196

Burgers, J. M., 1974, The Nonlinear Diffusion Equation (Dordrecht: Reidel)

Chandrasekhar, S., 1951, Proc. R. Soc. London A, 210, 26

Chandrasekhar, S., & Fermi, E., 1953, ApJ, 118, 113

Cho, J., Lazarian, A., & Vishniac, E. T., 2002, ApJ, 564, 291

Cho, J., & Vishniac, E. T., 2000, ApJ, 539, 273

Crutcher, R. M., Nutter, D. J., Ward-Thompson, D., & Kirk, J. M., 2004, ApJ, 600, 279

Dubrulle, B., 1994, Phys. Rev. Lett., 73, 959

Elmegreen, B. G., 2000, ApJ, 530, 277

Field, G. B., Goldsmith, D. W., & Habing, H. J., 1969, ApJ (Letters), 155, L149

Goldreich, P. & Sridhar, S., 1995, ApJ, 438, 763

Goldreich, P. & Sridhar, S., 1997, ApJ, 485, 680

Heitsch, F., Zweibel, E. G., Mac Low, M.-M., Li, P., & Norman, M. L., 2001, ApJ, 561, 800

Iroshnikov, P. S., 1963, Astron. Zh., 40, 742 (Engl. transl.: 1964, Sov. Astron., 7, 566)

Kolmogorov, A. N., 1941, Dokl. Akad. Nauk SSSR, 32, 141 (Engl. transl.: 1958, Amer. Math. Soc. Transl., Ser. 2, 8, 87.)

Kraichnan, R., 1965, Phys. Fluids, 8, 1385

Lazarian, A., & Esquivel, A., 2003, ApJ, 592, L37

Léorat, J., Passot, T. & Pouquet, A., 1990, MNRAS, 243, 293.

Mac Low, M.-M., 1999, ApJ, 524, 169

Mac Low, M.-M., 2003, in Simulations of magnetohydrodynamic turbulence in astrophysics, eds. T. Passot & E. Falgarone, Lecture Notes in Physics, 614, 182

Mac Low, M.-M., Balsara, D. S., Kim, J., & Avillez, M. A., 2004, ApJ, submitted (astro-ph/0410734)

Mac Low, M.-M., Klessen, R. S., Burkert, A., & Smith, M. D., 1998, Phys. Rev. Lett., 80, 2754

Maron, J., Cowley, S., & McWilliams, J., 2004, ApJ, 603, 569

Maron, J., & Goldreich, P., 2001, ApJ, 554, 1175

Matthews, B. C., Fiege, J. D., & Moriarty-Schieven, G., 2002, ApJ, 569, 304

Müller, W.-C., & Biskamp, D., 2000, Phys. Rev. Lett., 84, 475

Münch, G., 1958, Rev. Mod. Phys., 30, 1035

Ostriker, E. C., Stone, J. M., & Gammie, C. F., 2001, ApJ, 546, 980

Padoan, P., Jiminez, R., Nordlund, Å., & Boldyrev, S., 2004, Phys. Rev. Lett., 92, 191102

Padoan, P., & Nordlund, Å. 1999, ApJ, 526, 279

Porter, D. H., & Woodward, P. R., 1992, Phys. Rev. Lett., 68, 3156

Porter, D. H., Pouquet, A. & Woodward, P. R., 1994, Phys. Fl., 6, 2133

She, Z., & Lévêque, E., 1994, Phys. Rev. Lett., 72, 336

Stone, J. M., Ostriker, E. C., & Gammie, C. F., 1998, ApJ, 508, L99

Vázquez-Semadeni, E., Kim, J., Shadmehri, M., & Ballesteros-Paredes, J., 2004, ApJ, in press (astro-ph/0403134)

Vestuto, J. G., Ostriker, E. C., & Stone, J. M., 2003, ApJ, 590, 858

von Hoerner, S., 1951, Z. Astrophys., 30, 17

von Weizsäcker, C. F., 1943, Z. Astrophys., 22, 319

von Weizsäcker, C. F., 1951, ApJ, 114, 165

Wolfire, M., Hollenbach, D., McKee, C. F., Tielens, A. G. G. M., & Bakes, E. L. O., 1995, ApJ, 443, 152