

# Relativistic magnetic turbulence and "magneto-luminescence"

Jonathan Zrake (Stanford / KIPAC)  
Krakow Jet Meeting  
22 April, 2015

# Who?

Roger Blandford

William East

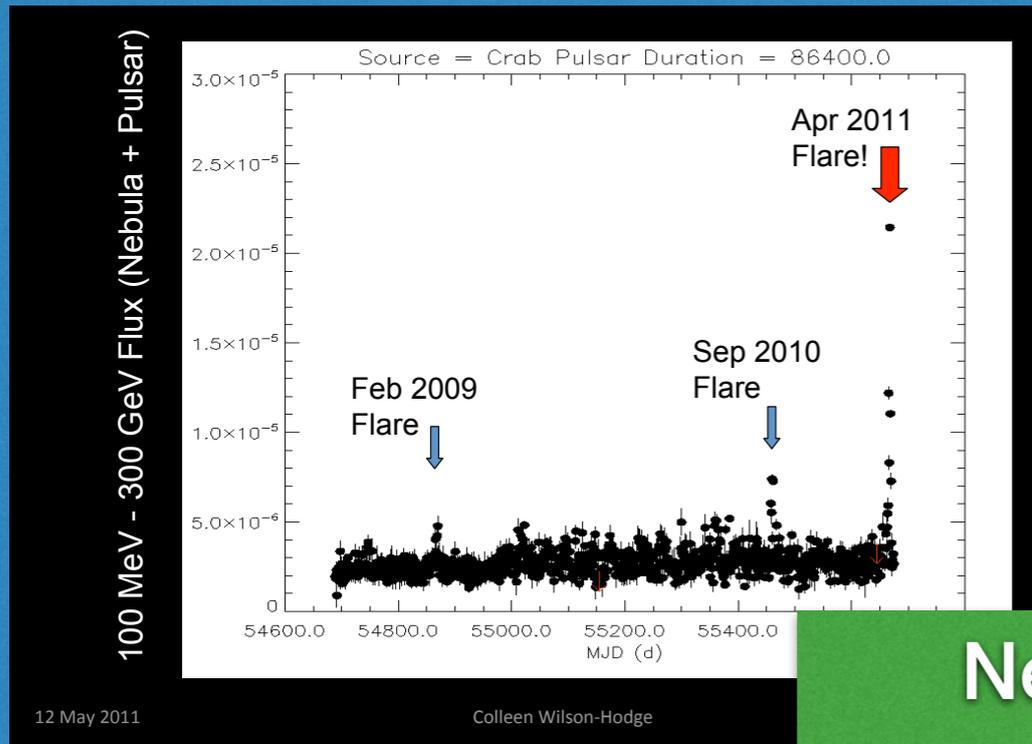
Yajie Yuan

Krzysztof Nalewajko

Jonathan Zrake

# Gamma-ray bursts

## Crab nebula



Need to convert **all magnetic energy** into high energy particles and radiation, impulsively!

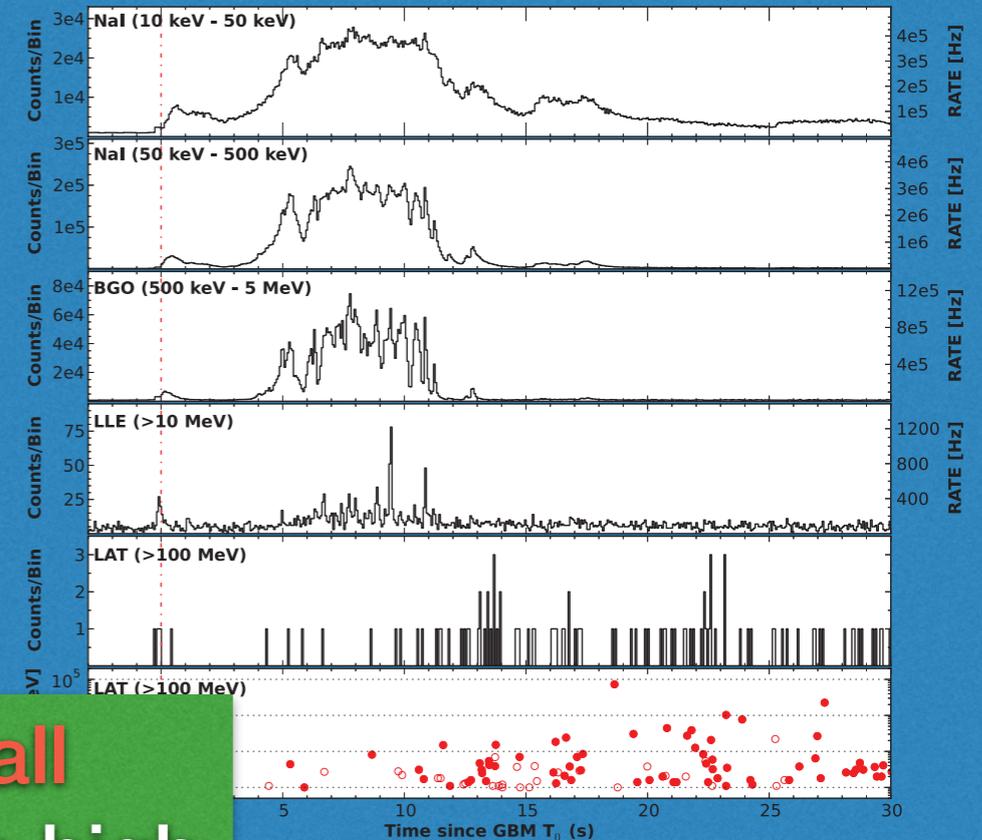


Figure 17: Light curves of GRB 17. The top panels show the light curves of the NaI and BGO detectors. The bottom panel shows the light curves of the LAT detectors. The open circles in the bottom panel represent the individual LAT "transient" class photons and their energies; the solid circles indicate photons with a >0.9 probability of being associated with this burst (17).

VOL 343 SCIENCE

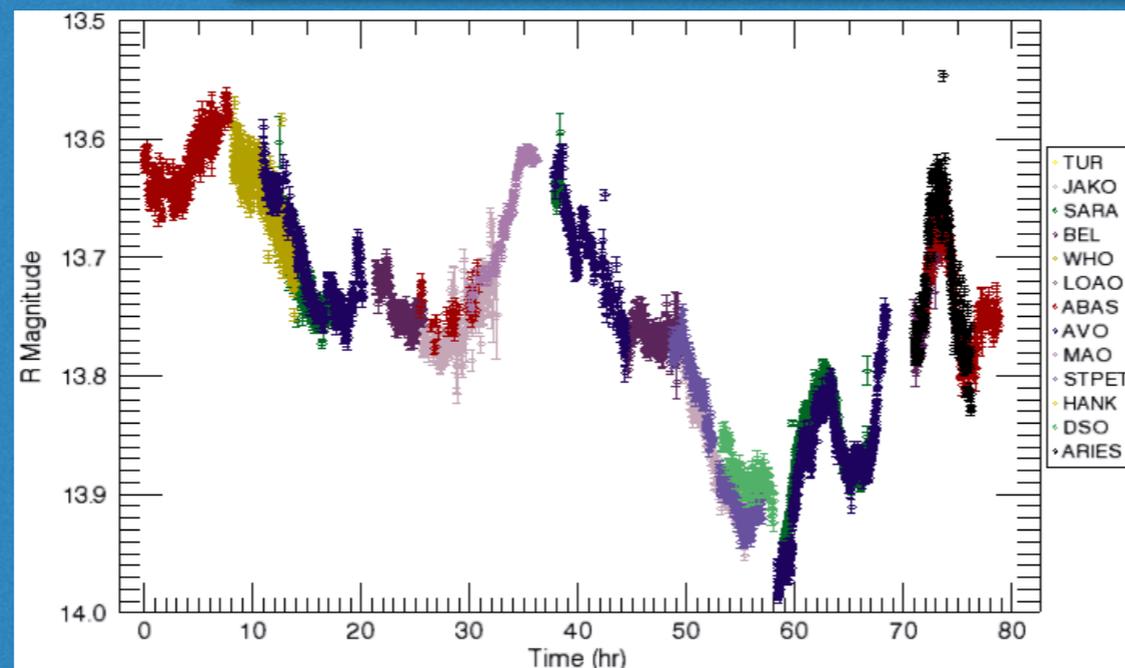


Figure 1: Raw light curve of S5 0716+714 obtained by the compilation of some of the high quality data by major contributors. The light curves contributed from each observatory is plotted together with different symbols indentifying the observatory according to the codes given in Table 1.

- Critical **magnetic reconfiguration**
- that triggers **volumetric** magnetic energy conversion
- and energizes particles **impulsively** ( $E \sim B$ )

# What?

**Magneto-luminescence:**

*Rapid conversion of magnetic energy into radiation*

In this talk, I'll focus on the hydro-magnetic (RMHD), and magneto-dynamic (FFE) aspects.

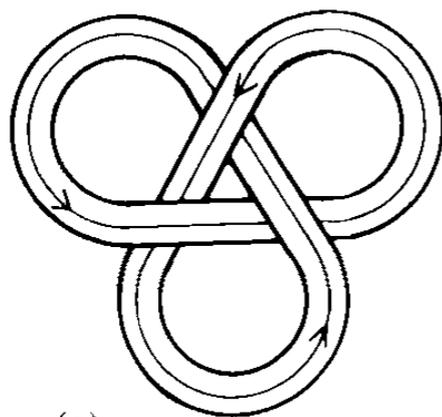
# Vocabulary

$$H = \int \mathbf{A} \cdot \mathbf{B} dV \quad \leftarrow \quad = 2U/k \quad \text{if equilibrium}$$

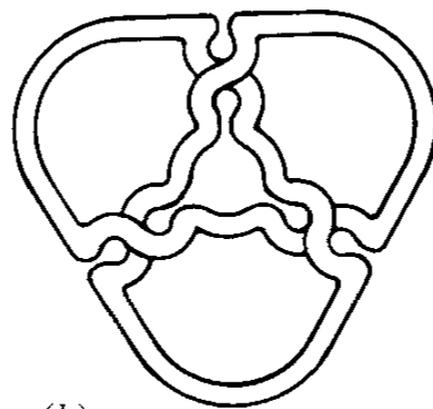
$$\dot{H} = -2 \int \mathbf{E} \cdot \mathbf{B} dV \quad \leftarrow \quad = 0 \quad \text{if ideal}$$

# Magnetic helicity

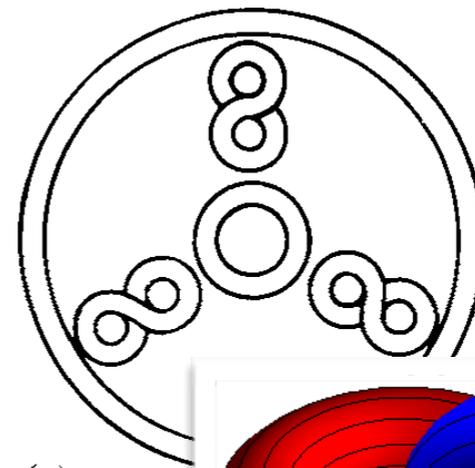
*M. A. Berger and G. B. Field*



(a)

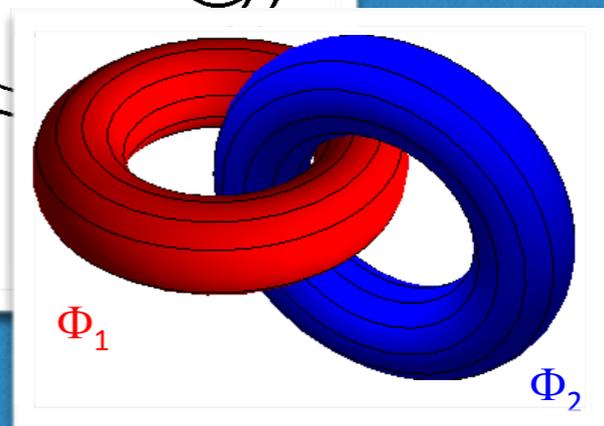


(b)



(c)

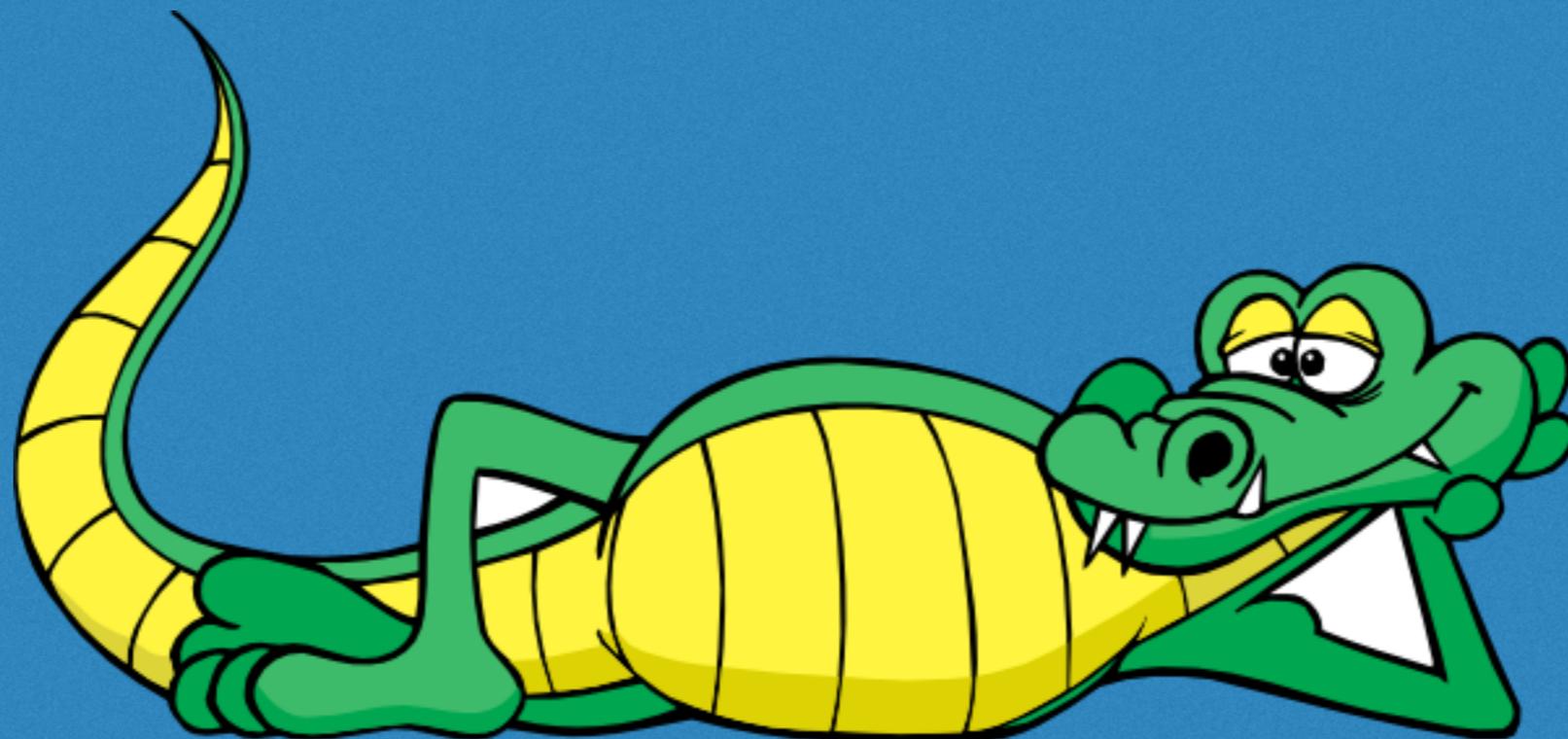
FIGURE 2. A trefoil knot with  $H_K = -3\Phi^2$ .



See poster by Yajie Yuan

- What characterizes the **ground state**?
- How long to attain it?
- Are magneto-static equilibria stable?

## Magnetic relaxation



**“Ground state”**: lowest energy allowed by topology

Electrical current is everywhere parallel to magnetic field.

**“Linear Beltrami fields”**

$$\nabla \times \mathbf{B} = \alpha \mathbf{B}$$

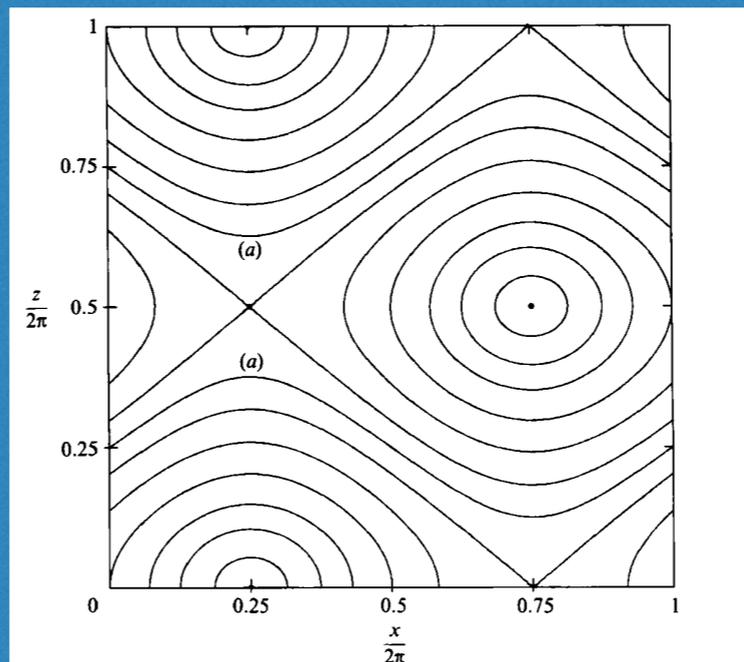
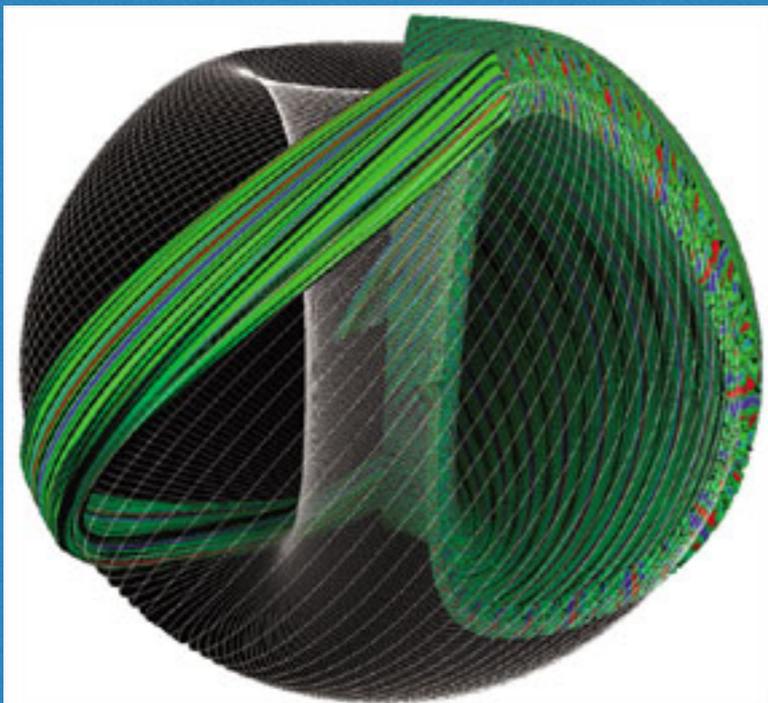


FIGURE 3. The integrable case: projection of streamlines on (x, z)-plane.

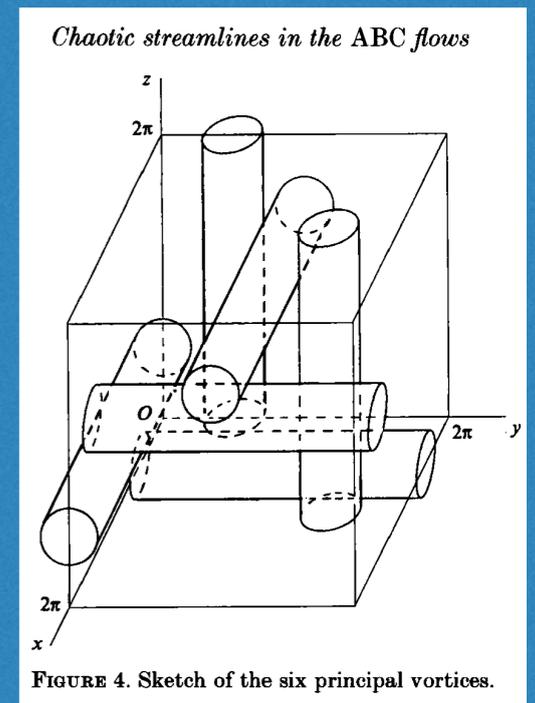


FIGURE 4. Sketch of the six principal vortices.

# Spontaneous decay of periodic magnetostatic equilibria

William E. East, Jonathan Zrake, Yajie Yuan, and Roger D. Blandford

*Lawrence Berkeley National Laboratory, University of California, Berkeley, California 94720, USA*  
**Magneto-static equilibrium**

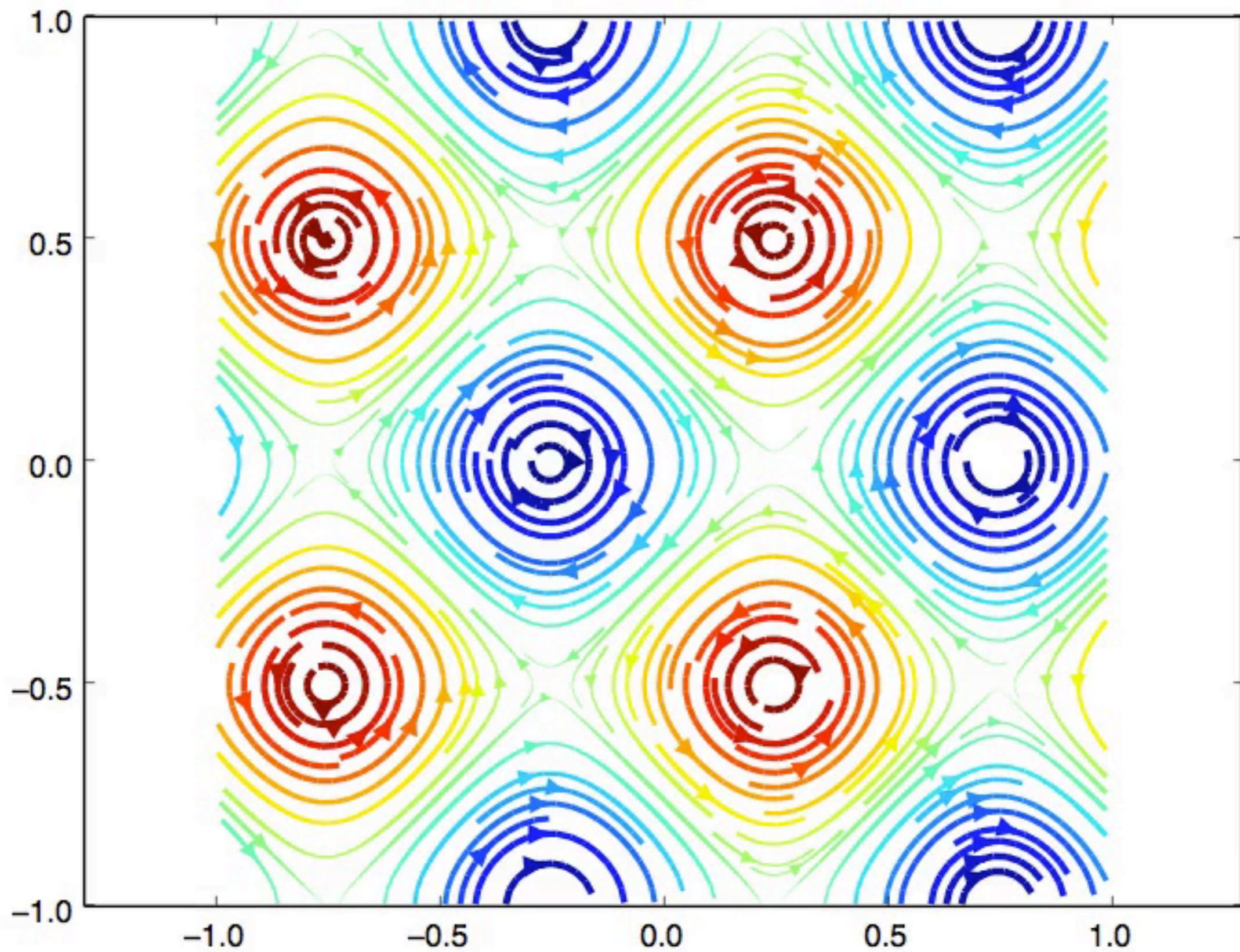
Submitted to PRL

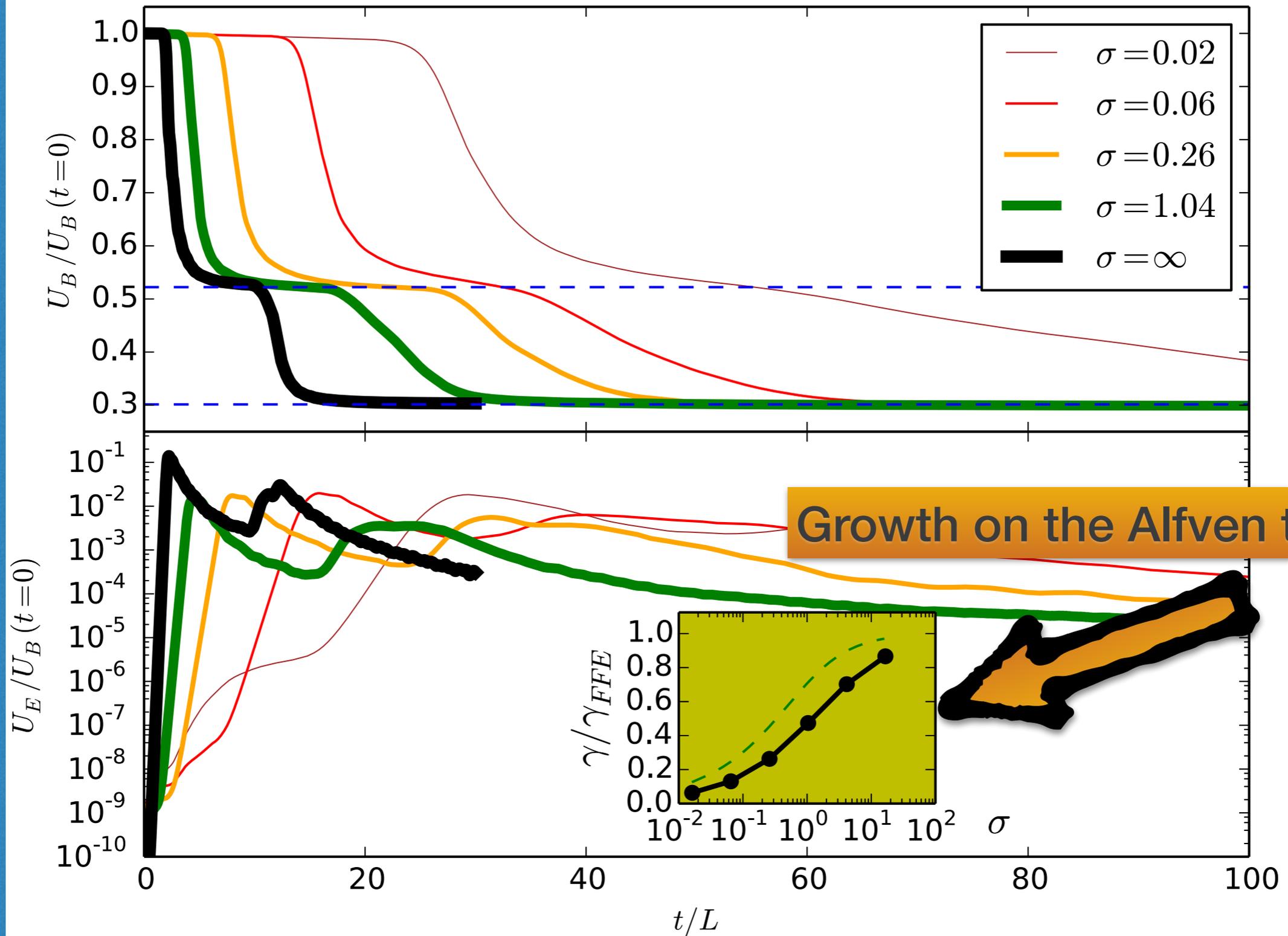
arXiv:1503.04793v2 [astro-ph.HE] 23 Mar 2015

**What is the mechanical stability of the following field?**

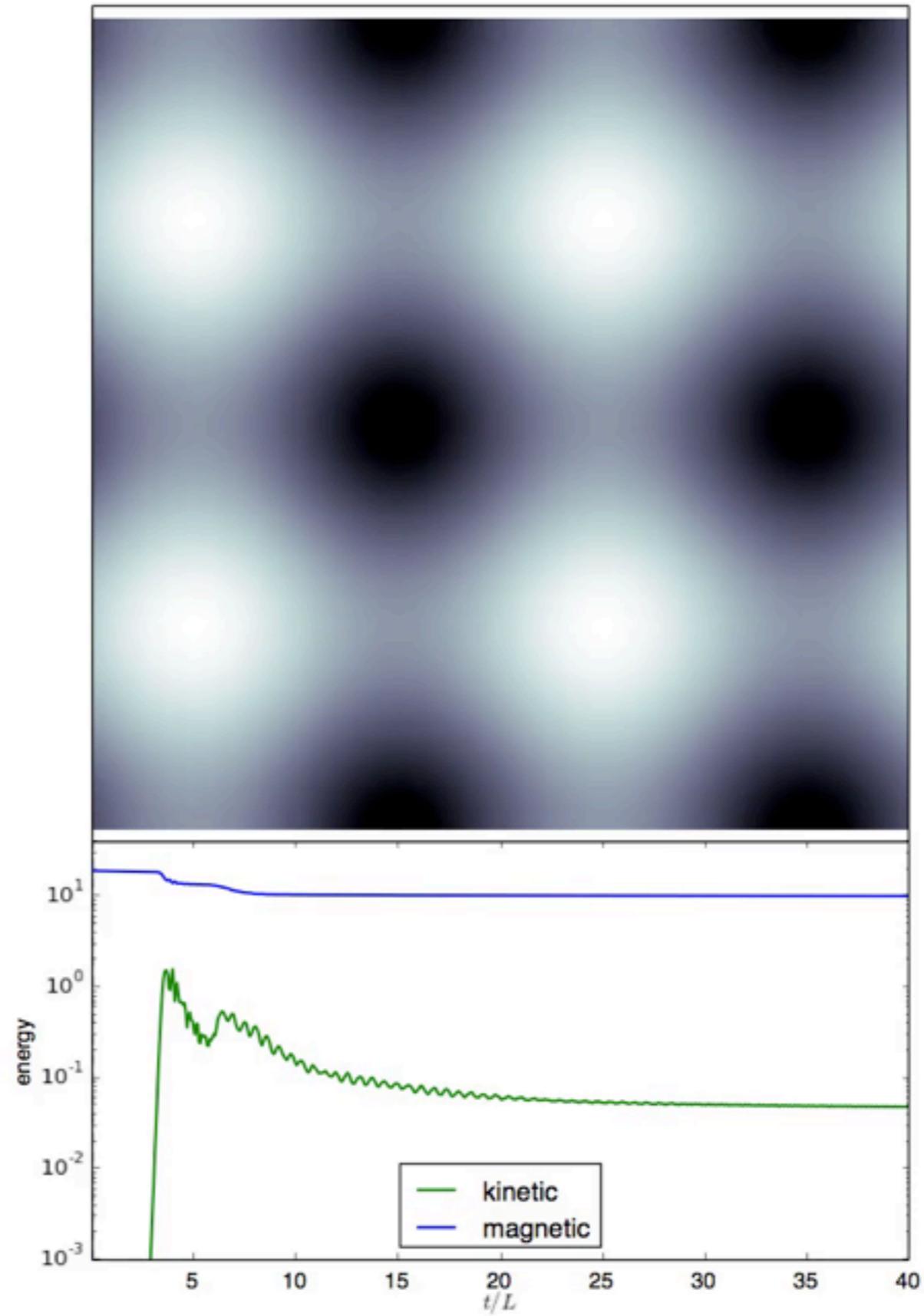
$$\mathbf{B}^E = \left( \begin{array}{l} B_3 \cos \alpha z - B_2 \sin \alpha y, \\ B_1 \cos \alpha x - B_3 \sin \alpha z, \\ B_2 \cos \alpha y - B_1 \sin \alpha x \end{array} \right)$$

$A/B=1, \alpha=2$

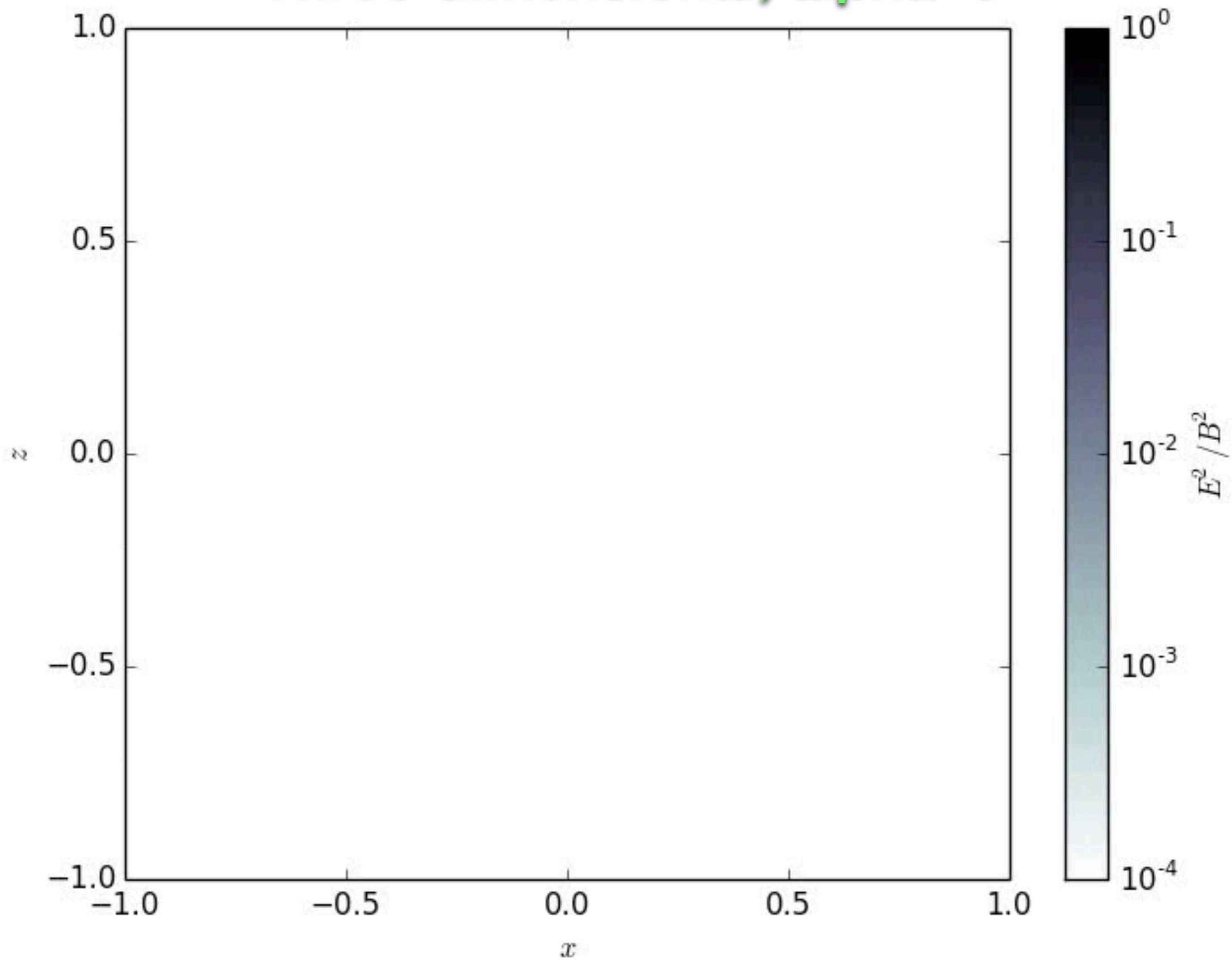




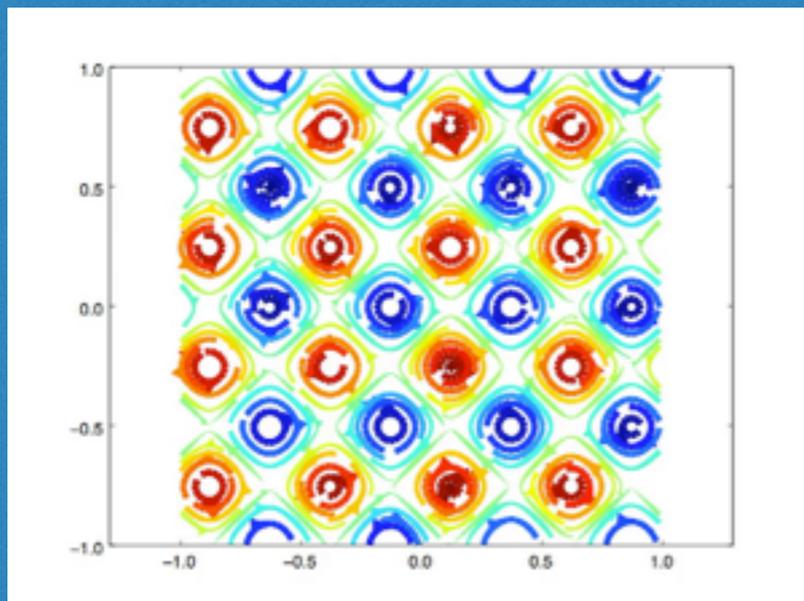
$j_z$  (current out-of-page)



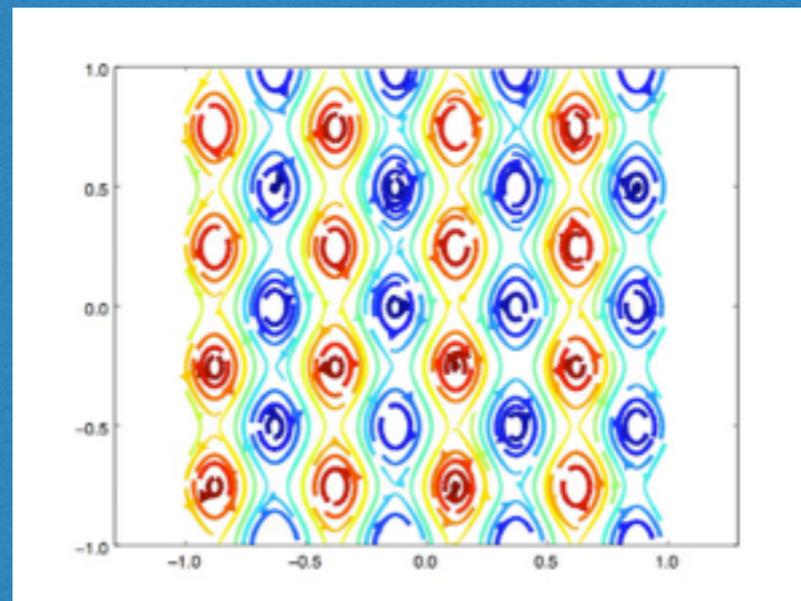
$t=0.00$  Three-dimensional,  $\alpha=3^{1/2}$



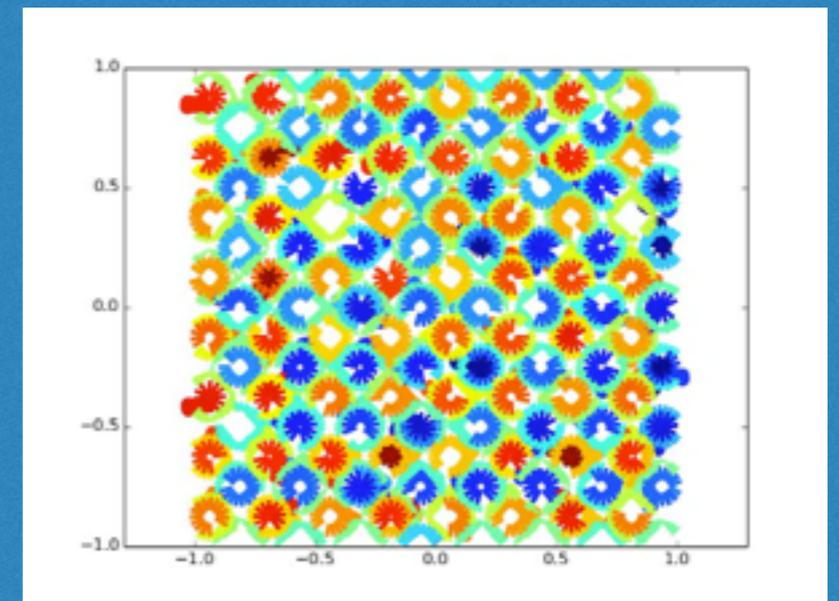
$A/B=1, \alpha=4$

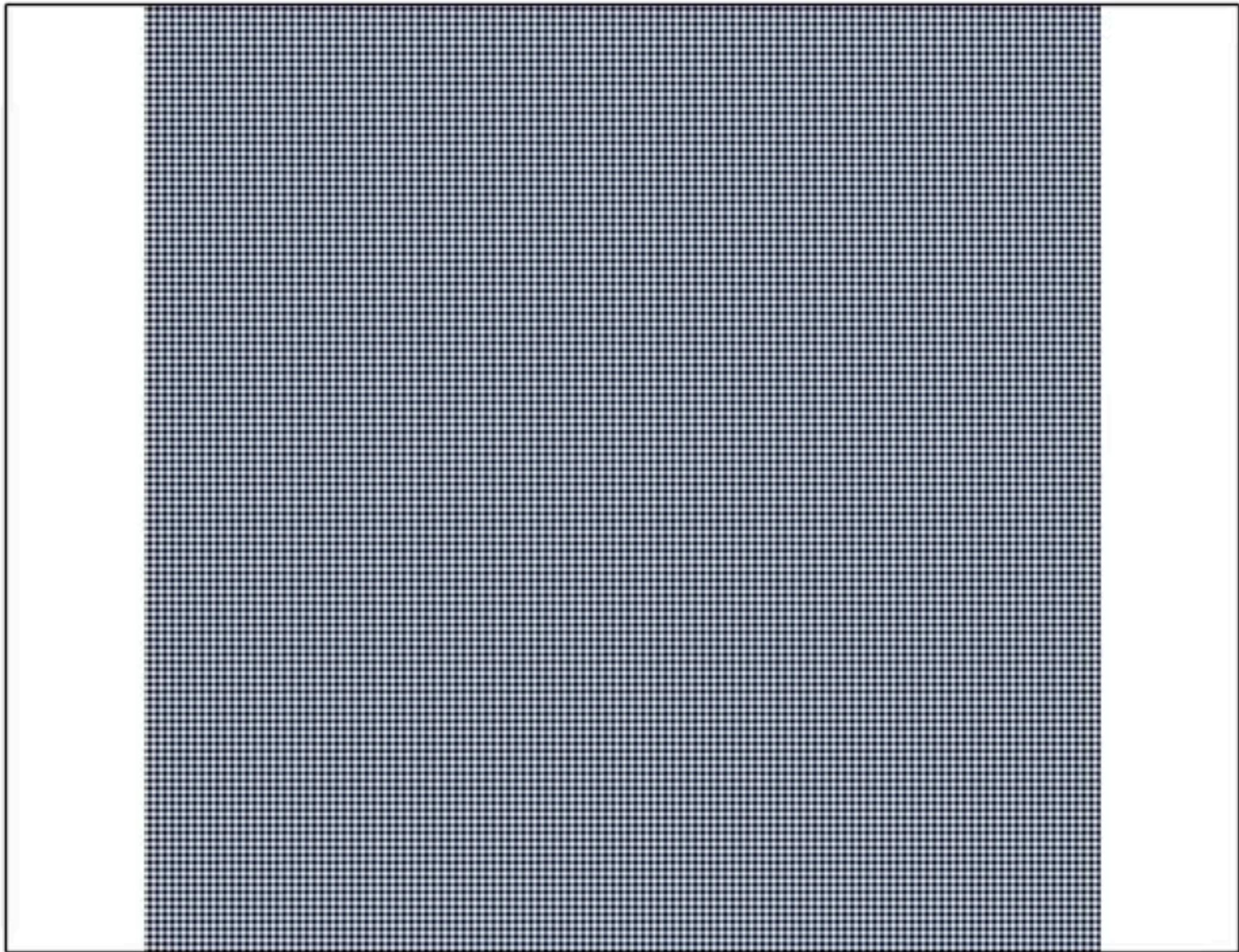


$A/B=2, \alpha=4$



$A/B=1, \alpha=8$





**Zrake, East, Yuan, Blandford (in prep)**

Equilibrium spectrum of magnetic energy strongly peaked at largest accessible scale.  
(Frisch et. al. 1975)

# Inverse cascade

Growing magnetic field at large wavelengths.

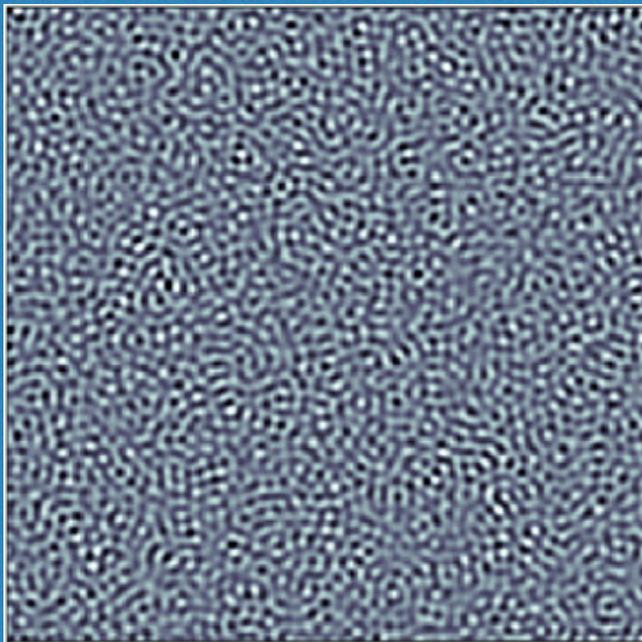
Typically believed to occur only when the magnetic helicity is non-zero.

## INVERSE CASCADE OF NONHELICAL MAGNETIC TURBULENCE IN A RELATIVISTIC FLUID

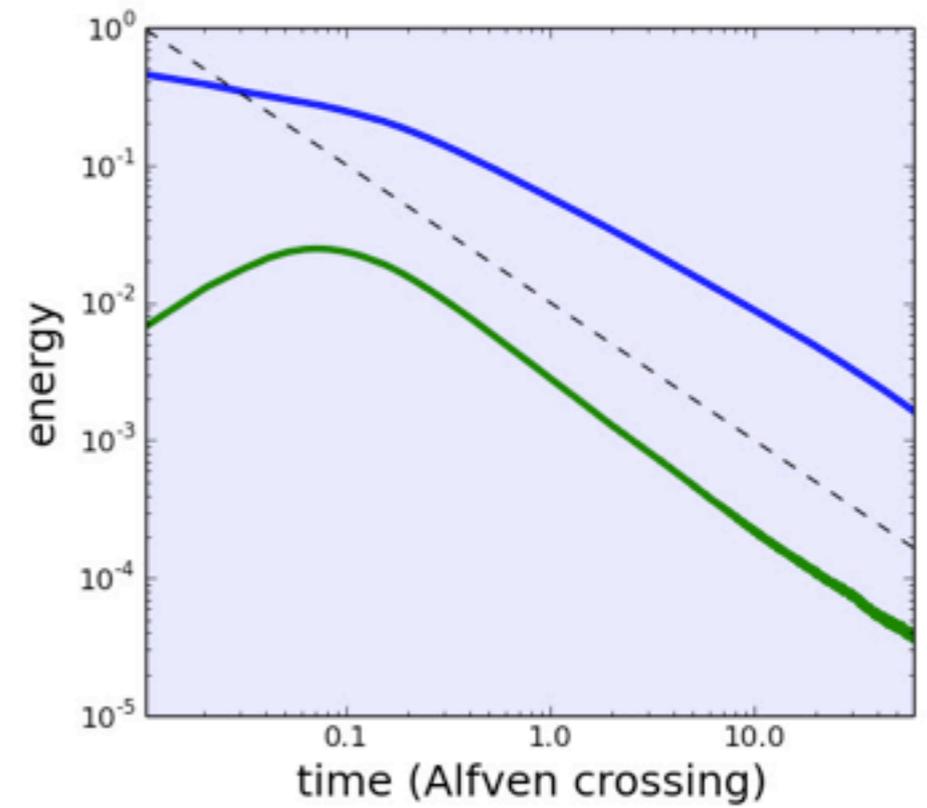
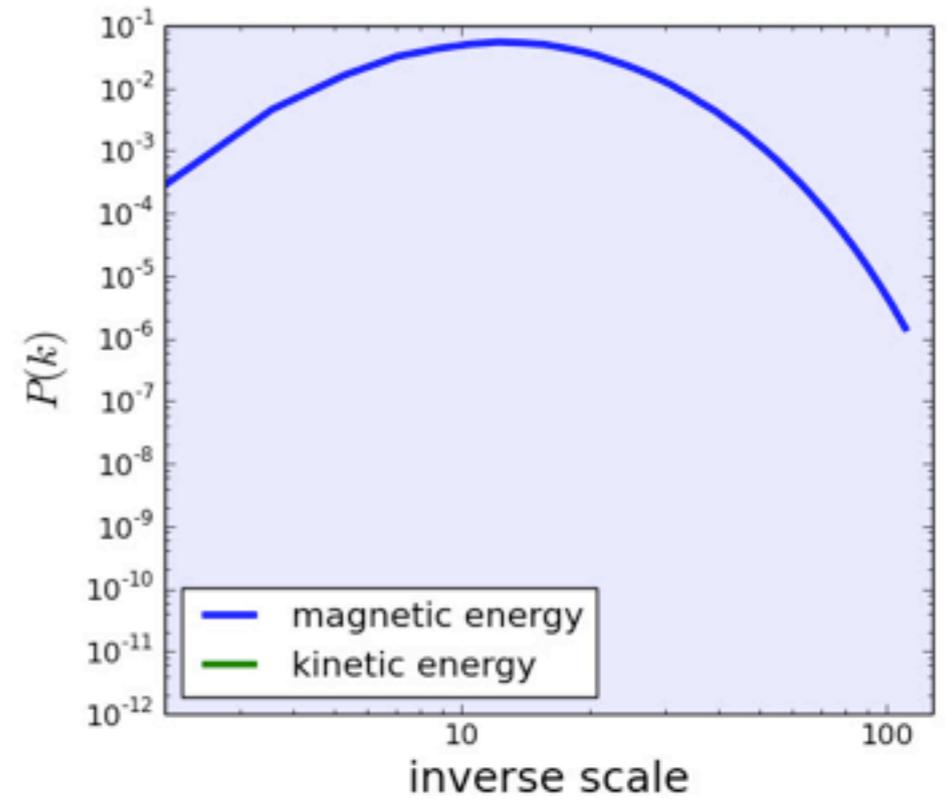
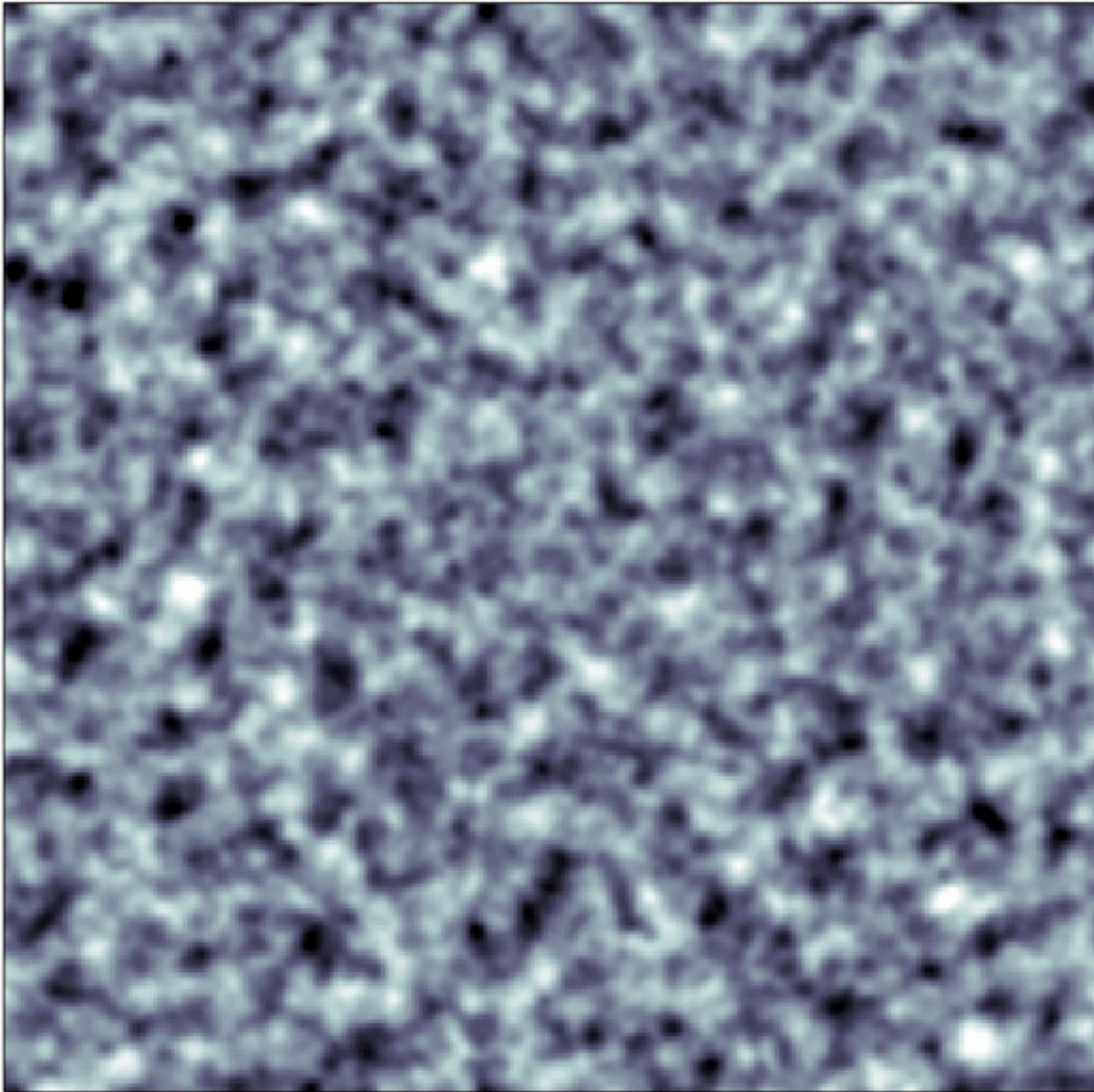
JONATHAN ZRAKE

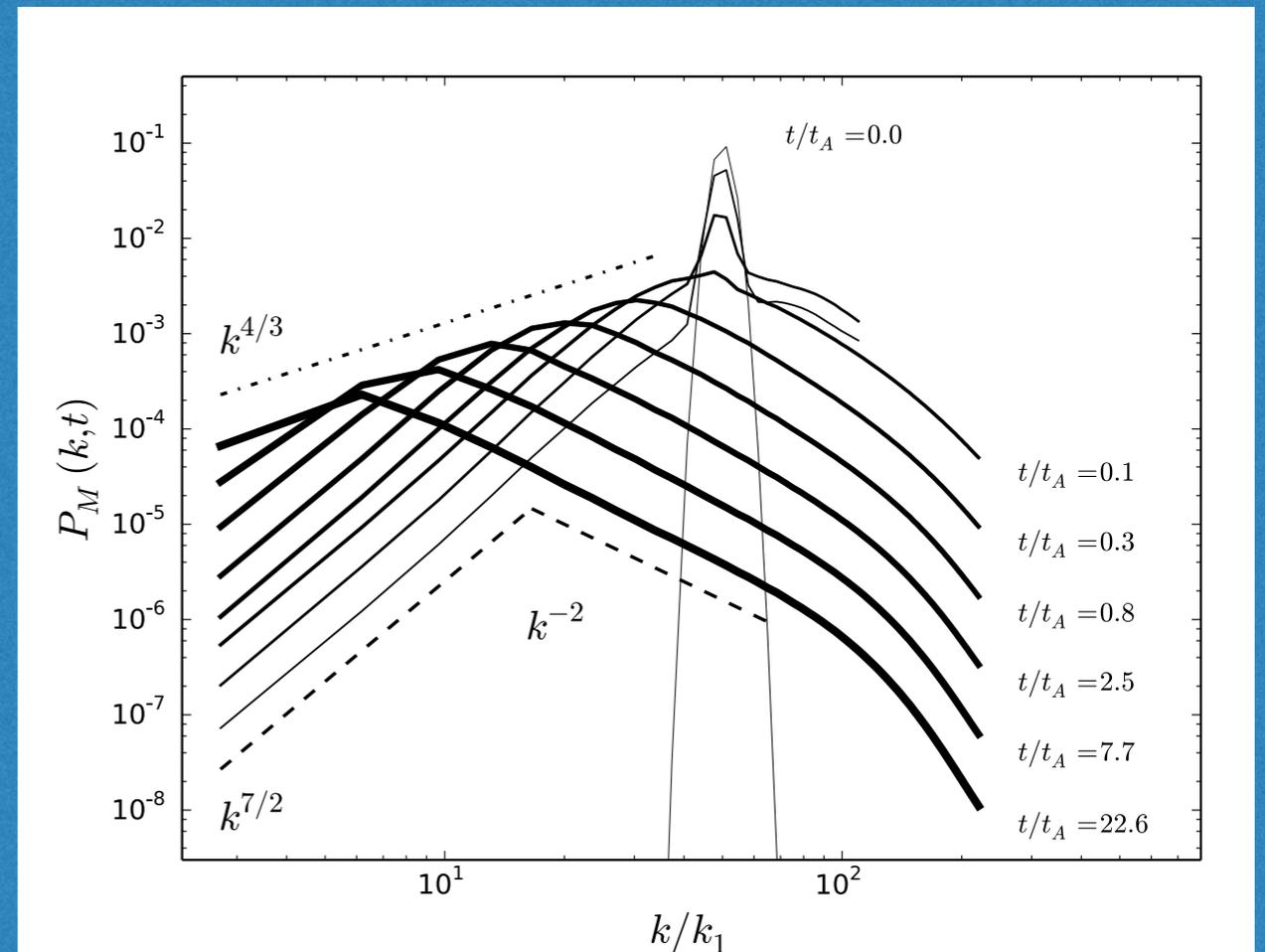
Kavli Institute for Particle Astrophysics and Cosmology, Stanford University, Menlo Park, CA 94025, USA

*Received 2014 July 21; accepted 2014 September 25; published 2014 October 8*



**Figure 1.** Two-dimensional slices of transverse magnetic field component showing the progression of magnetic field decay in a three-dimensional relativistic MHD turbulence. The leftmost panel shows the initial condition, and then from left to right the solution is shown at 4, 32, and 128 initial Alfvén crossing times of the simulation domain.





**Spectral evolution is self-similar:**

$$P_M(k, t) = s^{\gamma\beta + \delta} P_M(ks^{-\gamma}, t_A)$$

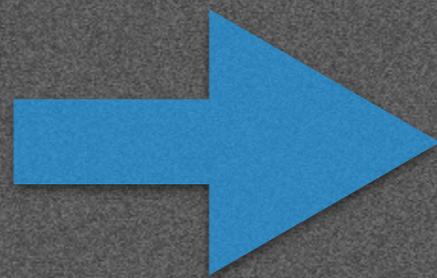
$$\lambda_t \propto t^{2/5}$$

Zrake (2014)

# Forward Alfven cascade

$$\dot{E} = -E/t_A$$

$$t_A = \lambda/v_A$$



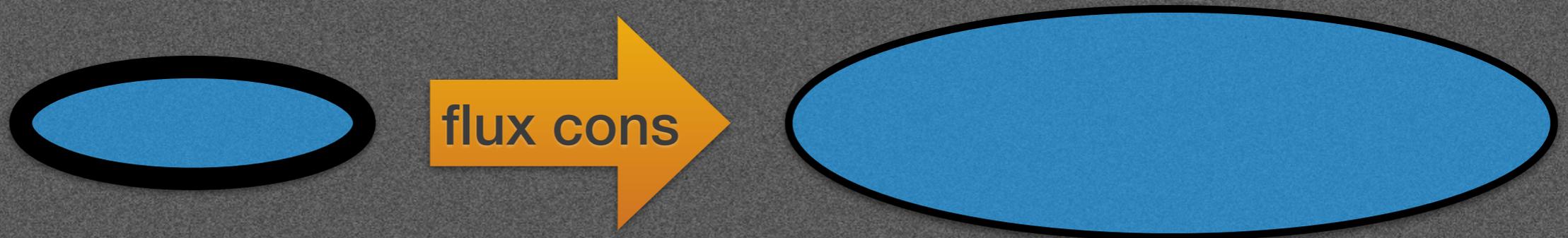
$$E \propto t^{-6/5}$$

$$v_A \propto E^{1/2}$$

$$\lambda \propto t^{2/5}$$

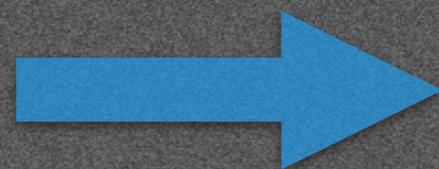
Too fast!

# “Untangling” alone



$$B \propto \lambda^{-1}$$

$$\lambda \propto t^{2/5}$$



$$E \propto t^{-4/5}$$

Too slow!

For short wavelength configurations (large  $\alpha$ ),

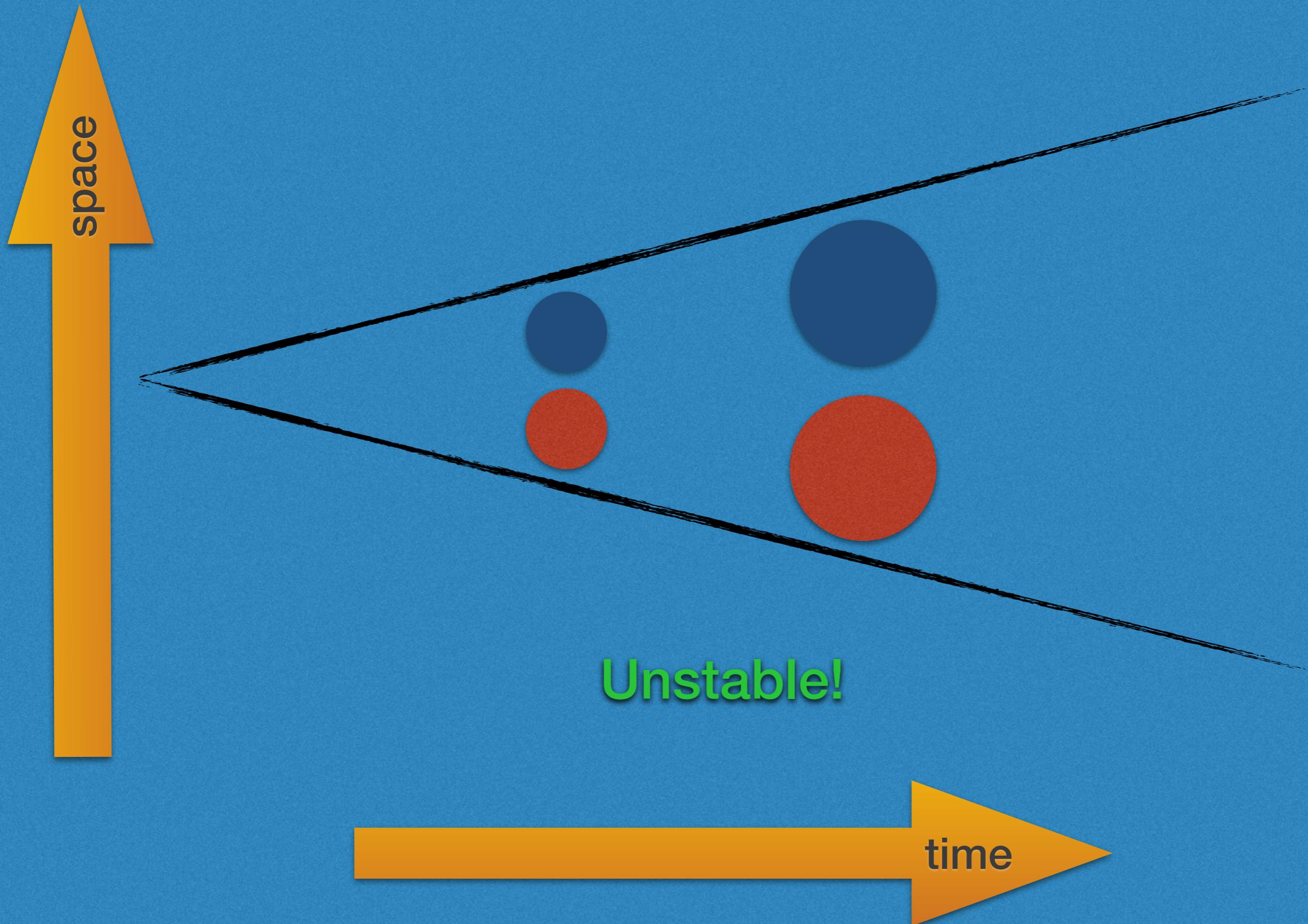
**2D is very different from 3D!**

In 3D, ground state is attained on a **dynamical** time-  
whereas in 2D, it takes **resistive** time

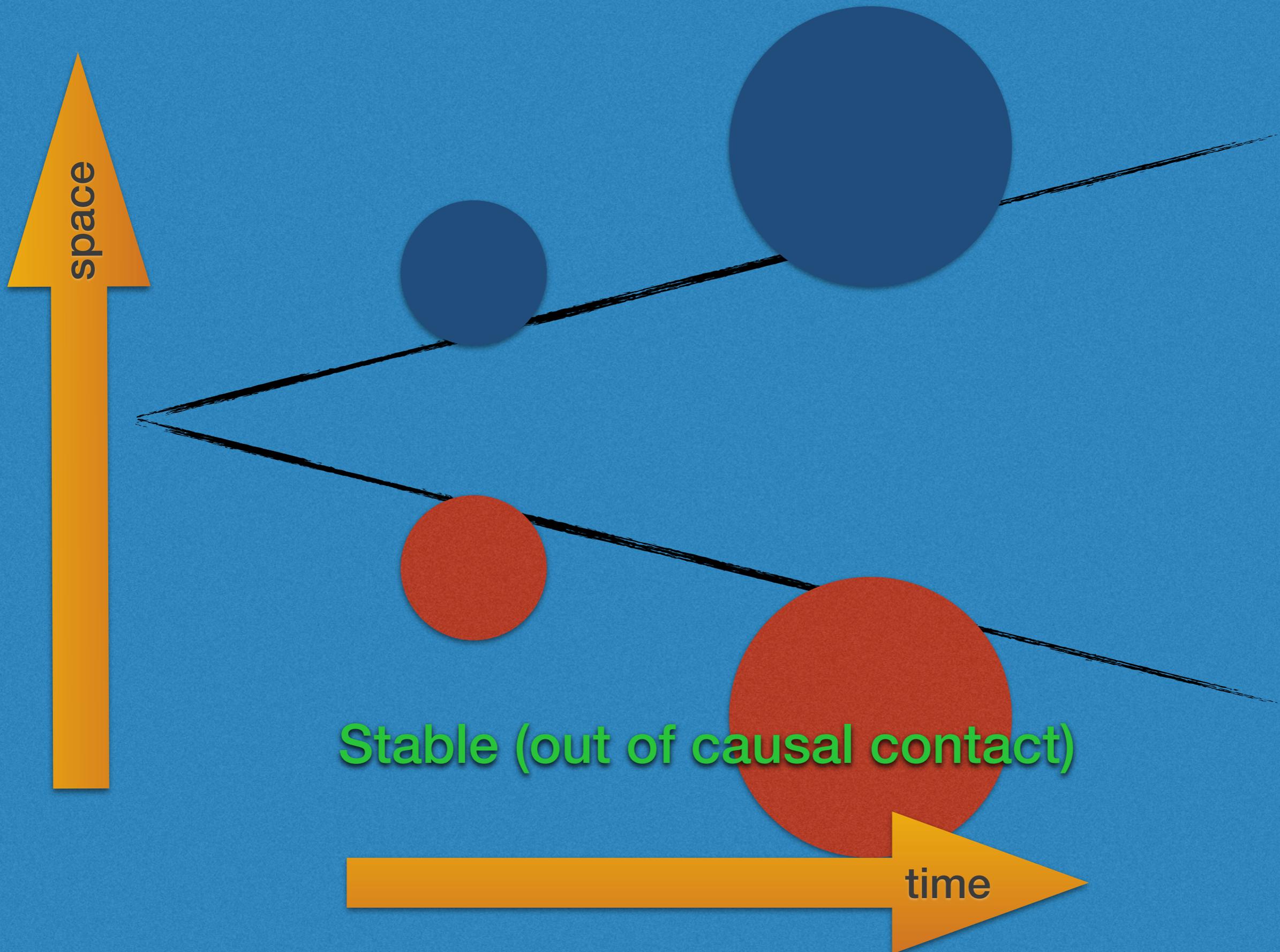
Zrake, East, Yuan, Blandford (in prep)

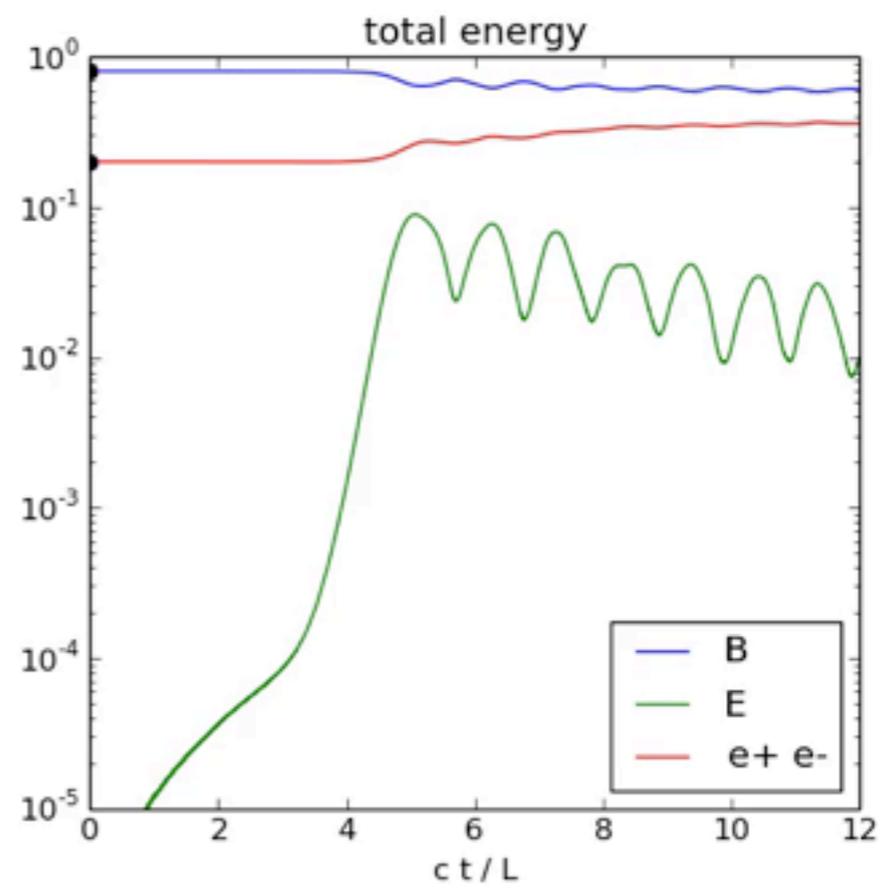
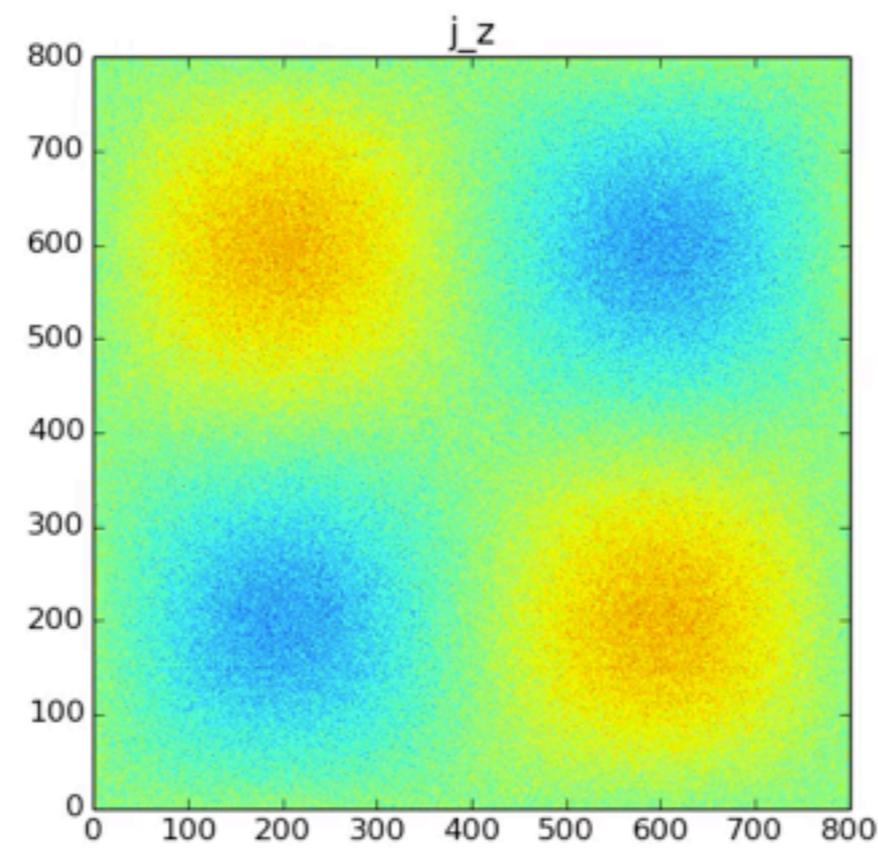
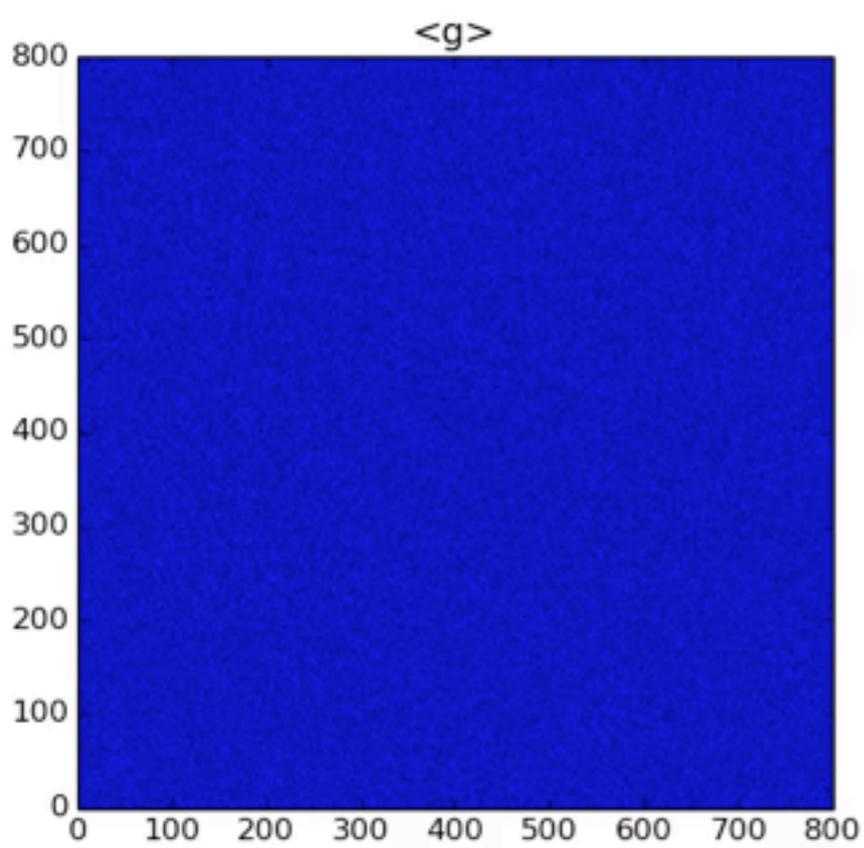
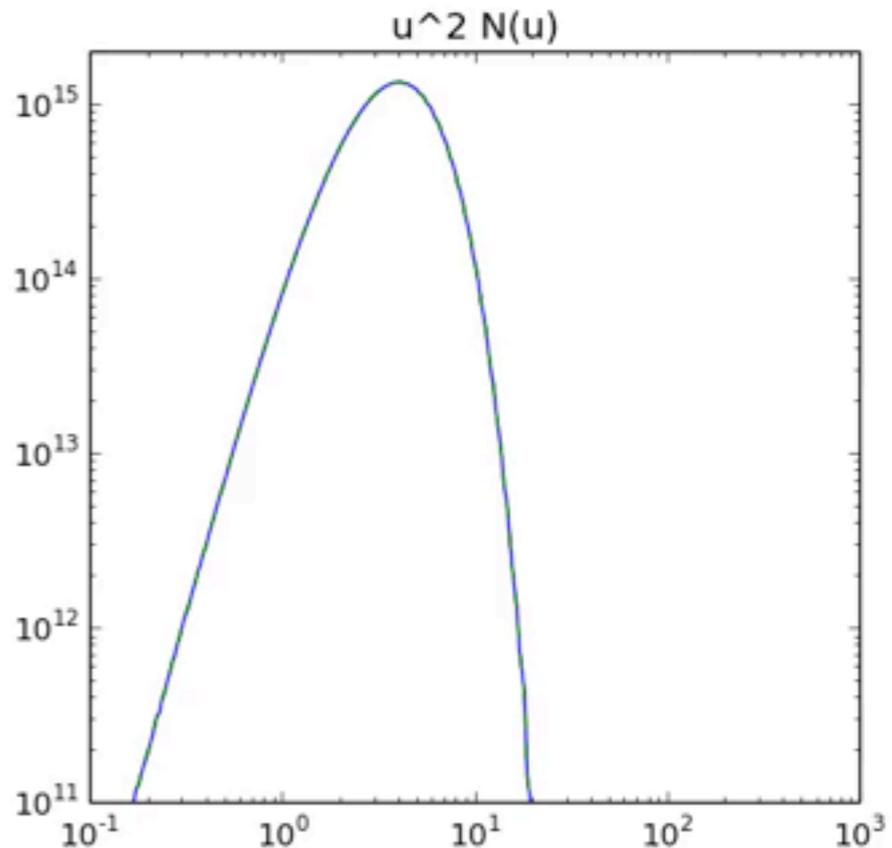
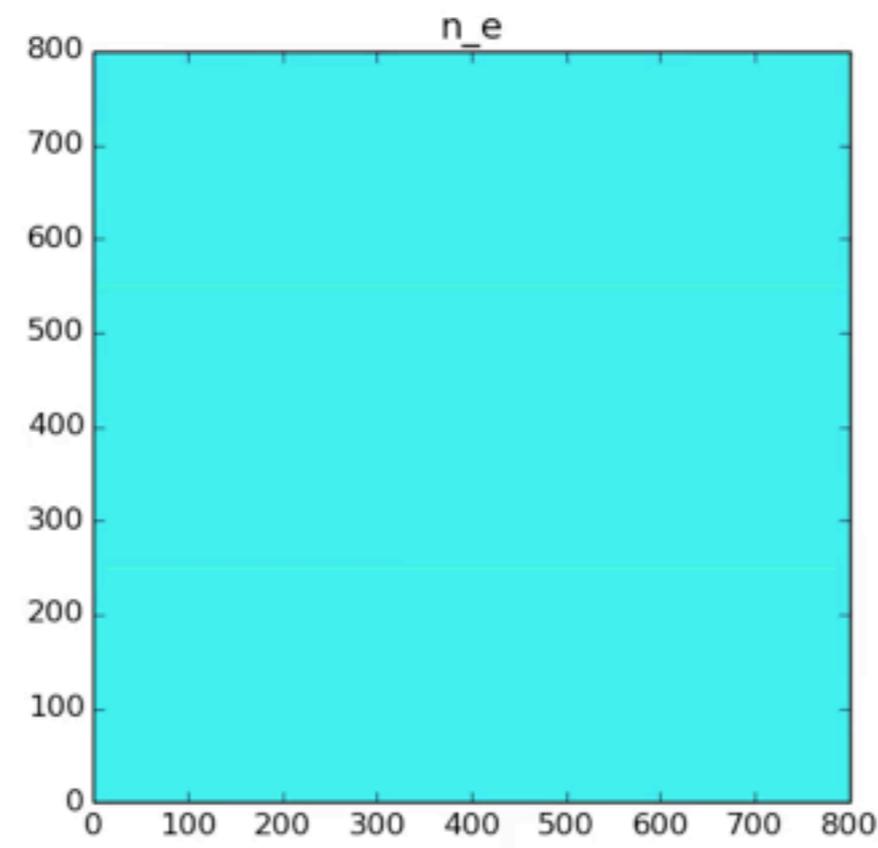
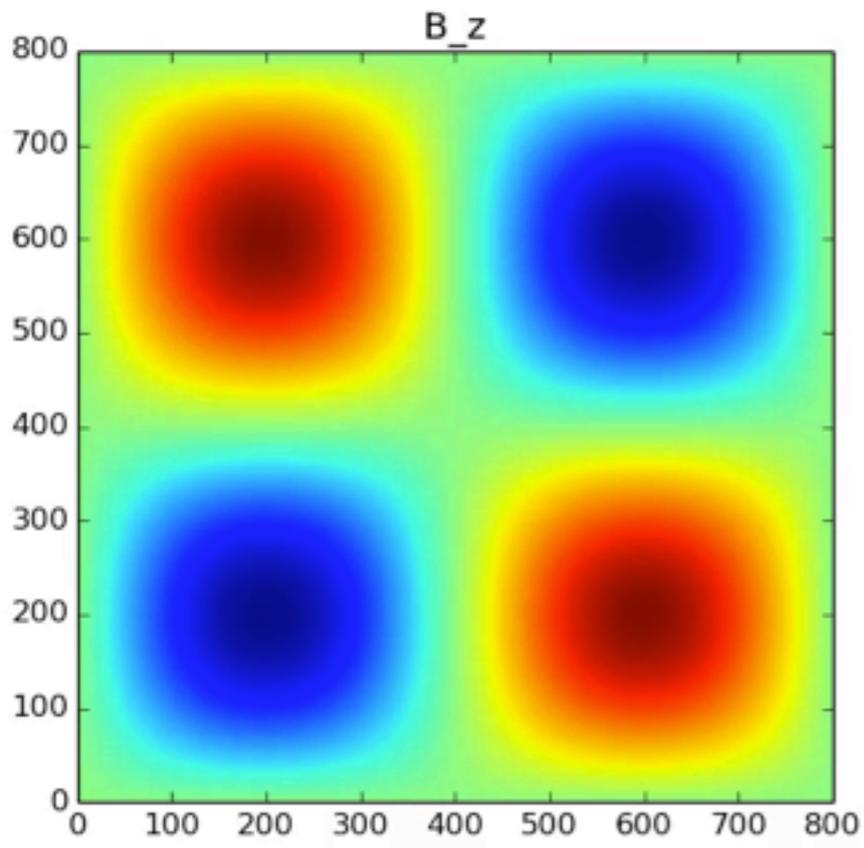
This process is **implosive**, *not* explosive!

Instability can be triggered by deceleration of the flow.



Instability can be triggered by deceleration of the flow.





Nalewajko, Zrake, East, Yuan, Blandford (in prep)

- Only ground-state equilibria are stable
- Transition to ground state is **bursty** when helical
- Smooth when non-helical
- **Inverse cascading** both with and without helicity
- Mutual **agreement** among [**analytic**, FFE, RMHD, PIC]
- $e^+/e^-$  form soft power-law tail in non-linear regime