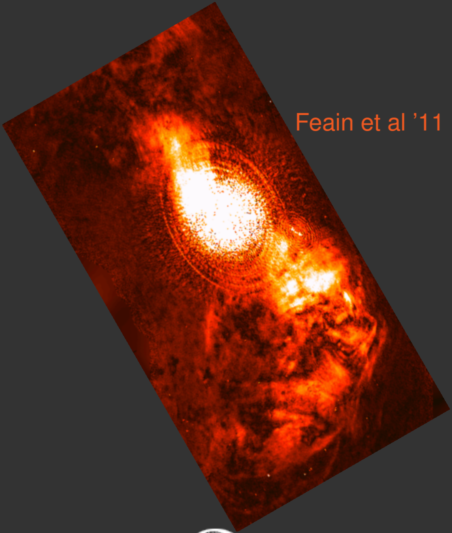
Numerical Approaches to Particle Acceleration in Astrophysical Plasmas



Feain et al ’11

Brian Reville

Queen’s University Belfast

Relativistic Jets:
Creation, Dynamics, and Internal Physics,
Krakow, 20-24 April 2015

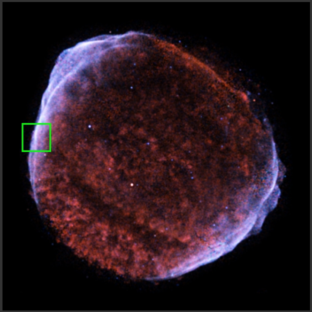
Queen's University Belfast

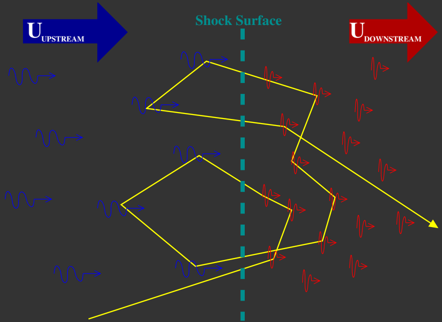
Diffusive shock acceleration

Let's consider SNR as a test laboratory

Krymskii 77, Axford et al. 77,
Bell 78, Blandford & Ostriker 78

$U_{\rm DS} < U_{\rm US} \ll v \sim c$





Scattering provided by MHD fluctuations
(Assuming particle can escape thermal pool, $v \gg u_{\rm sh}$)
Energy approx. conserved in scatterings provided $M_A \gg v/v_{\rm sh}$
Energy gain measured in local fluid frame at each crossing ($\Delta p/p \sim u/v$)

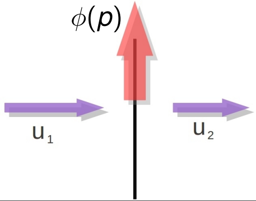
Steady-state particle spectrum

Consider the transport eqn at a velocity discontinuity (an M_1 closure for energetic particles c.f. Narayan's talk)

$$\frac{\partial f_0}{\partial t} + u\frac{\partial f_0}{\partial x} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial f_0}{\partial x}\right) + \frac{1}{3}\frac{du}{dx}p\frac{\partial f_0}{\partial p}$$

acceleration only occurs at the shock surface.

Integrating the transport eqn across the shock gives upward flux in mom. space



$$\phi(p) = \frac{4\pi}{3}p^3 f(p)(u_1 - u_2)$$

In steady-state, escape balances acceleration

$$\frac{\partial \phi}{\partial p} + 4\pi p^2 f_2 u_2 = 0$$

which can be rearranged as: $$\left.\frac{\partial \ln f}{\partial \ln p}\right|_{x_{\rm sh}} = -3\left[1 + \frac{u_2}{u_1 - u_2}\frac{f_2}{f(x_{\rm sh})}\right]$$

Particle spectrum – not always p^{-4}

$$\left.\frac{\partial \ln f}{\partial \ln p}\right|_{x_{\rm sh}} = -3\left[1 + \frac{u_2}{u_1 - u_2}\frac{f_2}{f(x_{\rm sh})}\right]$$

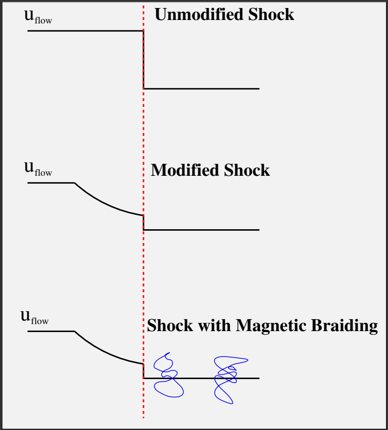
Consider some typical cases:

1. strong shock $u_1 = 4u_2$,
   diffusion approx. : $f_2 = f(x_{\rm sh})$

   $$f_\infty \propto p^{-4}$$

2. modified shock,
   higher mom. sample larger vel. jump
   particle distribution concave, $f \propto p^{-s(p)}$
   $s(p < mc) > 4$ and $s(p \gg mc) < 4$
   (Eichler, Malkov, Blasi, etc.)

3. magnetic bottles enhance particle transport downstream
   $f_2 > f(x_{\rm sh}) \Rightarrow$ steep spectra.
   (Duffy et al. 95, Kirk et al. 96)



Can we explore these effects numerically ?

The Vlasov-Fokker-Planck approach

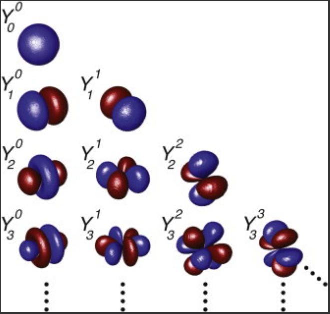
Solve VFP equation in the mixed coordinate frame

$$\frac{\partial f}{\partial t} + (\boldsymbol{u} + \boldsymbol{v}) \cdot \nabla f + \dot{\boldsymbol{u}} \cdot \frac{\partial f}{\partial \boldsymbol{p}} - [(\boldsymbol{p} \cdot \nabla)\boldsymbol{u}] \cdot \frac{\partial f}{\partial \boldsymbol{p}} + e\boldsymbol{v} \cdot \left(\boldsymbol{B} \times \frac{\partial f}{\partial \boldsymbol{p}} \right) = \mathcal{C}(f)$$

But this is a 6D problem!!
One option: use spherical harmonic expansion of the distribution

$$f(p, \theta, \varphi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} f_\ell^m(p) P_\ell^{|m|}(\cos\theta) e^{im\varphi}$$

In local frame, $\mathcal{C}(f_\ell) = \frac{\nu}{2}\nabla^2 f_\ell \sim \ell(\ell+1) f_\ell$.
We can typically truncate expansion after a relatively small number of terms.



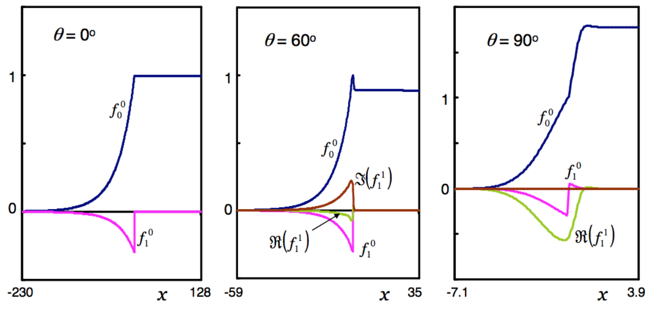
Y_ℓ^m iso-surfaces (Tzoufras et al. 2011)

Step 1:

Solve equations for 1D shock profile with uniform B-field
Examine $(u_{\rm sh}, \theta_B, \nu_{\rm coll})$ phase space of steady state solns.

Test-particle simulations of oblique shocks

$$\theta = \cos^{-1}(B_x/B), \quad u_{\rm sh} = c/10, \quad \nu = 0.1\omega_{\rm g}$$

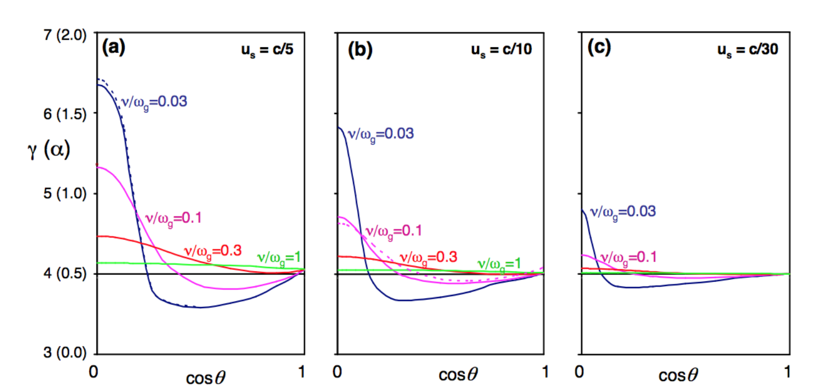


At oblique shocks, matching conditions can not be met in diffusion approximation

Recall: $$\gamma \equiv -\frac{\partial \ln f(0)}{\partial \ln p} = 3\left[1 + \frac{u_2}{u_1 - u_2}\frac{f(\infty)}{f(0)}\right]$$

Bell, Schure & Reville '11

Resulting spectra – $f \propto p^{-\gamma}, S_\nu \propto \nu^{-\alpha}$



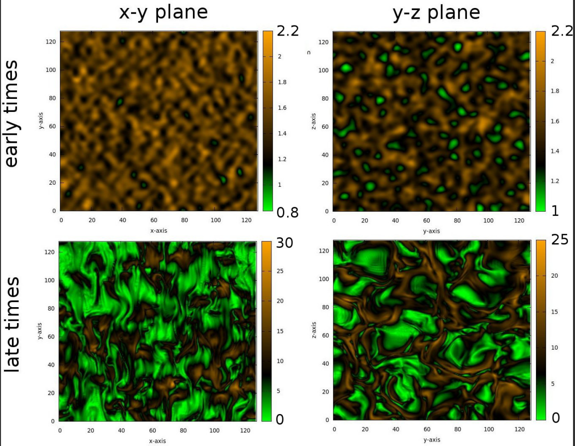
Note for highly oblique shocks, faster means steeper spectra, unless $\omega_g \tau \sim 1$ (Bohm), or of course $\omega\tau \ll 1$ (e.g. Weibel mediated shocks)

Scattering was used as free parameter for these simulations .
Can we investigate self-generated scattering field?

Step 2:
Extend VFP technique to 3D and couple to MHD code (Reville & Bell 13)

Magnetic field growth driven by cosmic-ray flux

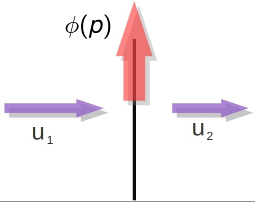
$B_{\rm rms}$ in fluid element far upstream of a **parallel** shock (BR& Bell 2013)



- $r_g = 256/B_0$, so twice box size at t=0, but magnetised at late times $r_g/\lambda \sim 1$ (in localised incoherent structures – NOT Alfvén waves)
- if enough time available ($\sim 5\gamma_{\rm NR}^{-1}$), cosmic-rays can self confine, otherwise they escape to infinity. This gives the maximum energy.

Maximum energy from escape point of view

Recall, upward flux in momentum space:



$$\phi(p) = \frac{4\pi}{3} p^3 f(p)(u_1 - u_2)\delta(x)$$

Assume highest energy particles escape **upstream** unless self-confining fields have been generated

Equate accelerating flux with upstream escape flux:

$$j_{\rm cr} = e\phi(p_{\rm max}) = e\pi p_{\rm max}^3 f_0(p_{\rm max}) u_{\rm sh} \approx \frac{3}{4} \frac{e}{p_{\rm max} c} \frac{P_{\rm cr}}{\rho u_{\rm sh}^2} \frac{\rho u_{\rm sh}^3}{\ln(p_{\rm max}/mc)} \quad ,$$

Assuming $f_0 \propto p^{-4}$

We can combine this with our requirement for ~ 5 growth times ie. $\int \gamma_{\rm NR} dt \sim 5$, where from Bell 04:

$$\gamma_{\rm NR} = \sqrt{\frac{\pi}{\rho c^2}} j_{\rm cr} \quad \Rightarrow \quad Q_{\rm cr} = \int j_{\rm cr} dt \sim 5\sqrt{\frac{\rho c^2}{\pi}}$$

to find the maximum energy

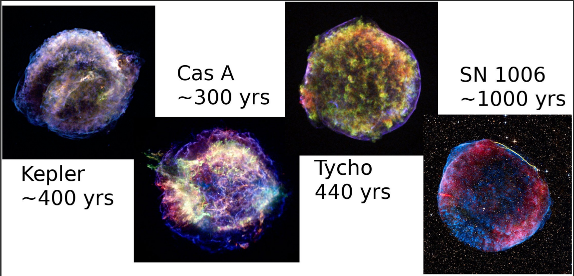
Maximum energy from escape point of view

Bell et al. 13, see also Zirakashvili & Ptuskin 08

$$E_{\rm max} \quad \sim \quad 10^{13} \frac{P_{\rm cr}}{\rho u_{\rm sh}^2} \frac{\sqrt{n}\, u_8^3\, t_{100}}{\ln(p_{\rm max}/mc)} \ {\rm eV}$$

Note: Unlike Hillas/Lagage Cesarsky, indep. of B field!!

- Since all historical SNR have $u_8 \sim 5$, $n \sim 1$

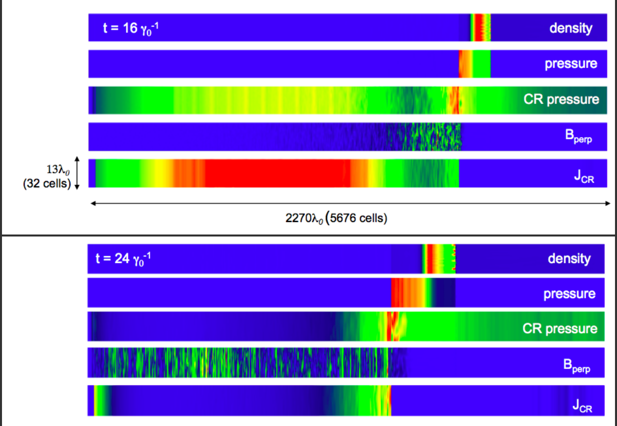


- Even with accel. efficiency $P_{\rm cr}/\rho u_{\rm sh}^2 \sim 0.3$ it would appear none are (currently) accelerating cosmic rays to the knee (few 10^{15} eV)

Full shock MHD-VFP simulations

Shock launched from RHS boundary using dense piston Bell et al. '13

$M_A = 200, n = 0.1 cm^{-3}, u_{\rm sh} = 6 \times 10^9 \text{ cm s}^{-1}, T_{\rm inj} = 100 \text{ TeV}, L \approx 0.25 \text{ pc}$



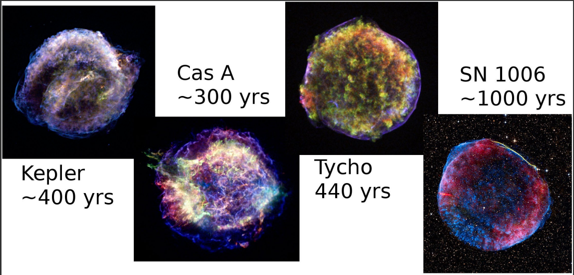
Confinement condition : $Q_{\rm cr} = \int j_{\rm cr} dt \sim 5\sqrt{\frac{\rho c^2}{\pi}}$

Theory : $Q = 3.46 \times 10^{-2}$, Simulation: $Q = 2.16 \times 10^{-2} \text{statcoulomb cm}^{-2}$

How to get beyond 10^{15} eV??

$$E_{\rm max} = 10^{13} \frac{P_{\rm cr}}{\rho u_{\rm sh}^2} \frac{\sqrt{n}\, u_8^3\, t_{100}}{\ln(p_{\rm max}/mc)}\ {\rm eV}$$

- Look much earlier in time? (Bell et al. 13, Schure & Bell 13)
- Can oblique magnetic fields help? In principle faster accelerators, but....



- Hillas limit : $E_{\rm max} < eZB\beta R$

$$E_{\rm max} < 10^{13} Z \left(\frac{u_{\rm shock}}{3000{\rm km/s}}\right) B_{\mu {\rm G}} R_{\rm pc}\ {\rm eV}$$

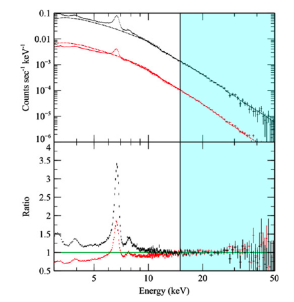
Field amplification still required.

NuSTAR observations of Cas A

Oblique shocks a necessity?

$$\frac{h\nu}{mc^2} = \frac{1}{2}\gamma^2\frac{B}{B_c} \longrightarrow \gamma = \sqrt{\frac{B_c}{5B}}\xi^{1/2}$$

$B_c = m^2c^3/e\hbar$ and $\xi = h\nu/50$ keV
Compare acceleration and cooling times for 50 keV synchrotron photon emitting electrons



Grefenstette et al. ’15

$$t_{\rm cool} = E/\frac{4}{3}c\sigma_T\beta^2\gamma^2U_B \propto \frac{1}{\gamma B^2} \propto B^{-3/2}$$

and

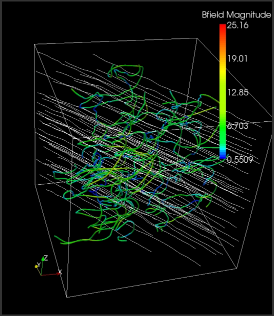
$$t_{\rm acc} = \eta\frac{r_g c}{u_{\rm sh}^2} \propto \frac{\gamma}{B} \propto B^{-3/2}$$

So $t_{\rm acc} < t_{\rm cool}$ only if

$$\eta < \frac{45}{4}\alpha_f^{-1}\left(\frac{u_{\rm sh}}{c}\right)^2\xi^{-1} \sim 0.4$$

Faster than Bohm acceleration required – oblique shocks

MHD-VFP Sims of precursor with oblique field

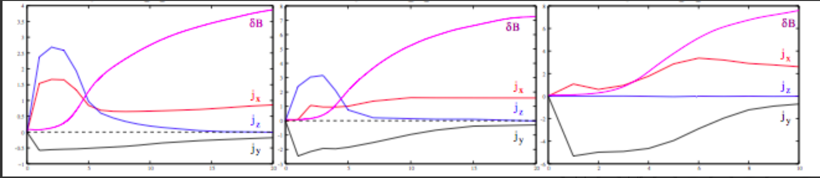


Magnetic field lines stretched by CR current (white lines) at perpendicular shock.

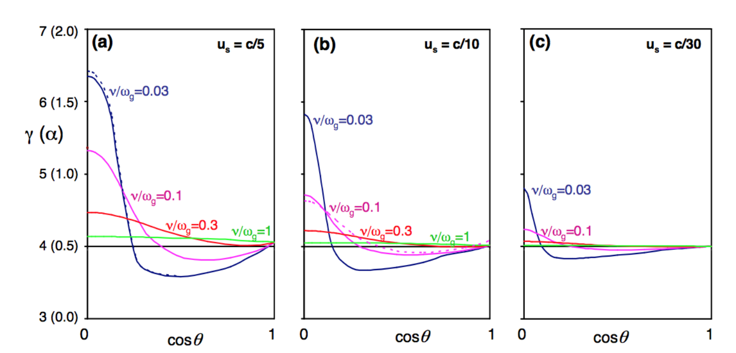
- situation is more complicated, as precursor scale is reduced, strong collision induced drifts
- **If** acceleration is efficient, the mean field can be disordered sufficiently to behave like a parallel shock (time-dependent)
- fields do appear to be amplified, by roughly 1 order of magnitude
- full shock simulations (similar to Bell et al 13) required. FAR more demanding.

Evolution of $\boldsymbol{j}_{\rm cr}$, from BR & Bell 13

$\theta = 60°$ $\theta = 80°$ $\theta = 90°$



So maybe oblique fields help, what else?



The above were all performed in the shock frame. As shock velocity approaches c, we inevitably move to $\cos\theta \to 0$

Spectra become steeper unless $\omega_g \tau \sim 1$ (Bohm), or of course $\omega_g \tau \ll 1$ (e.g. Weibel mediated shocks)

In fact, if $\Gamma \gg 1$, accel. switches off unless $\omega_g \tau < 1$ (Achterberg et al '01)

Scattering at relativistic shocks?

- Can we extend MHD-VFP to ultra-relativistic speeds? NO!
- CR density as measured in upstream ion frame $n_{cr} \sim \eta \Gamma_{\rm sh}^2 n_0$ can exceed background density.
- MHD not a good description of plasma immediately upstream of the shock.
- Three fluid (CR + e^-+p) analysis shows precursor to be Rayleigh-Taylor unstable (Reville & Bell 14) but can not achieve $\omega_g \tau < 1$ (for $\gamma \gtrsim \bar{\gamma}^2$).
- so Weibel instability must do all the work. Bad news for UHECRs

$$\omega_{\rm pi} \tau \sim \left(\frac{\gamma}{\bar{\gamma}}\right)^2 \left(\frac{\lambda}{c/\omega_{\rm pi}}\right)^{-1} \frac{4\pi\bar{\gamma} nmc^2}{B^2}$$

eg. Kirk & Reville '10

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eg. Kirk & Reville '10

- Clearly above some critical energy $\omega_g\tau$ will exceed unity

$$\gamma_{\rm d,max} < \bar{\gamma}\frac{\lambda_{\rm d}}{c/\omega_{\rm pp}}\sigma_{\rm d}\sigma_{\rm u}^{-1/2} = 10^5\left(\frac{\bar{\gamma}}{100}\right)\left(\frac{\lambda_{\rm d}}{10c/\omega_{\rm pp}}\right)\left(\frac{\sigma_{\rm d}}{10^{-2}}\right)\left(\frac{\sigma_{\rm u}}{10^{-8}}\right)^{-1/2}$$

using parameters from Sironi et al '13

BR & Bell 14, see also Lemoine's talk

Beyond shocks.....

X-ray (SUZAKU) hotspots in Cen A southern lobe (Stawarz et al. ’13)

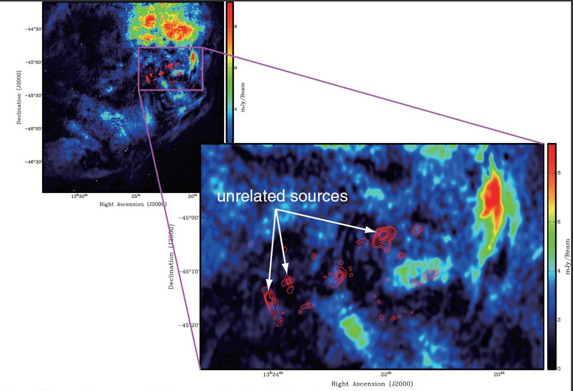
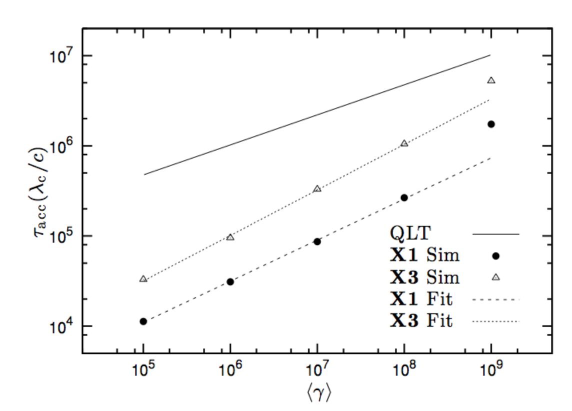


Figure 6. Polarized radio intensity map of the Centaurus A southern lobe with the X-ray *Suzaku* contours superimposed.

Potentially of synchrotron origin from non-thermal electrons.
Requires localised field $B \sim 10\mu$G, electron energies of tens of TeV.
Needs a local rapid acceleration mechanism

Fermi II

Comparison of **proton** acceleration times, QLT (Schlickeiser '89) vs numerical particle tracing, in Alfvenic 'turbulence', in $\delta B = B_0$ limit (O'Sullivan, BR, Taylor 09)

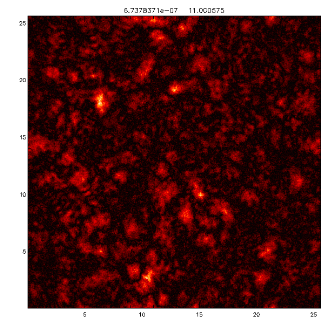


Acceleration of 10 TeV electrons $\sim$ 10Myr, cooling time $\sim$ 0.1 Myr.

How well do we understand plasma conditions?

Lobes are Hot, Tenuous, and Turbulent.
Dissipation is collisionless.

Plot of $|\boldsymbol{E} \cdot \boldsymbol{B}|$

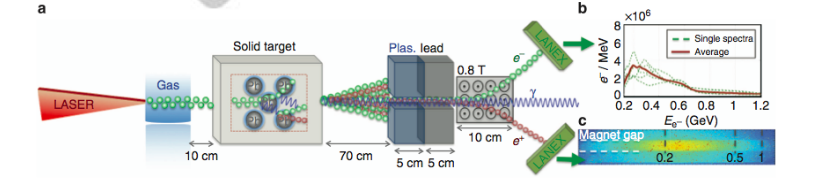


Perhaps worthwhile revisiting Fermi II in presence of reconnection mediated dissipation.

j_z from PIC simulation of 2D MHD cascade.
from P. Wu et al, 2013, PRL

Laboratory Simulations

Sarri et al, 2015 Nature Comm.
First ever generation of an $e^{\pm}$ plasma in the laboratory



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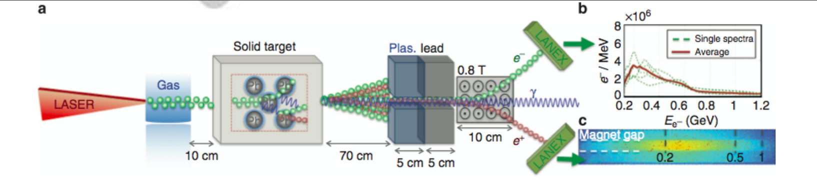
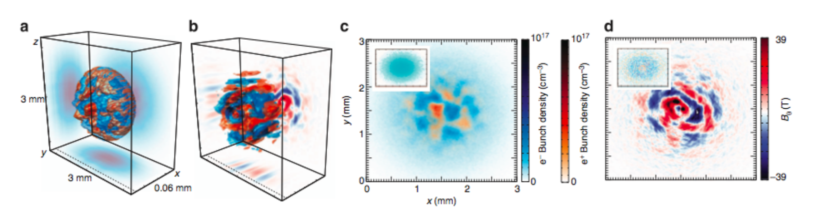


Supported by numerical experiments....



possibility of studying kinetic instabilities in a real pair plasma

Summary/Conclusions

- Vlasov-Fokker-Planck approach appealing for shock acceleration studies at non-relativistic shocks.
- the origin of CR to the knee still an unanswered question, but parameter space has been considerably reduced
- relativistic shocks unlikely to help much in this regard
- Or Fermi II
- But we can probably start pinning down parameters, try to match to observations
- When all else fails, we can always go back to the lab, and blow stuff up

Thank you.


Big Scary Laser
Do not look Into beam
with remaining eye
BD LSRFortessa
Special Order Research Product