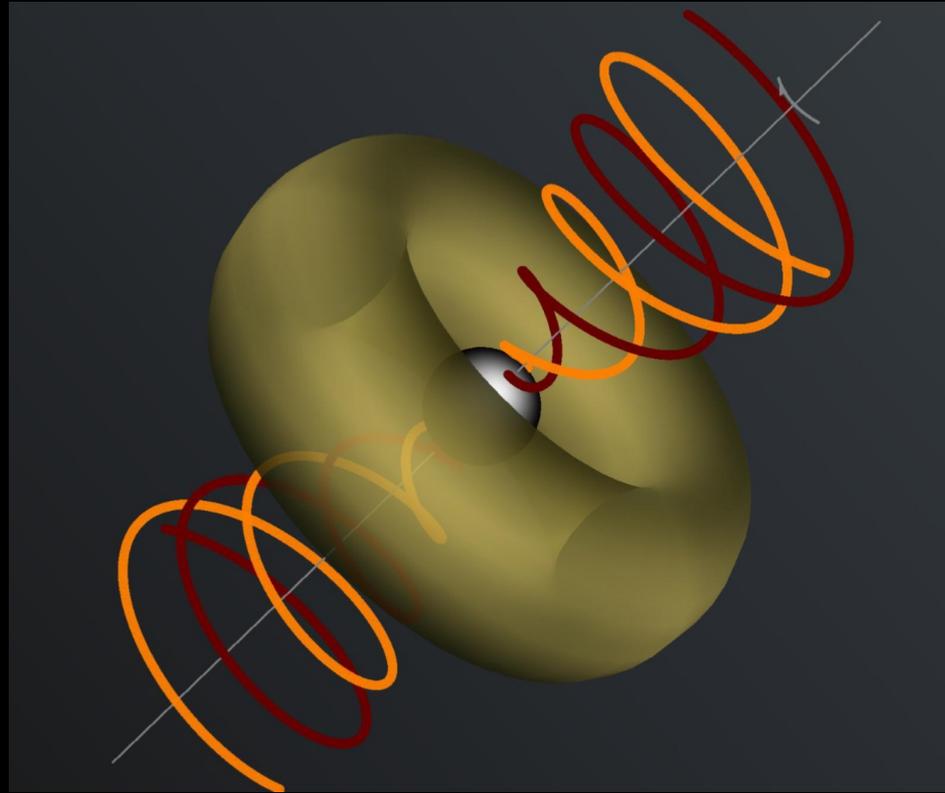


# Black hole magnetospheres: how they work and application to GRBs

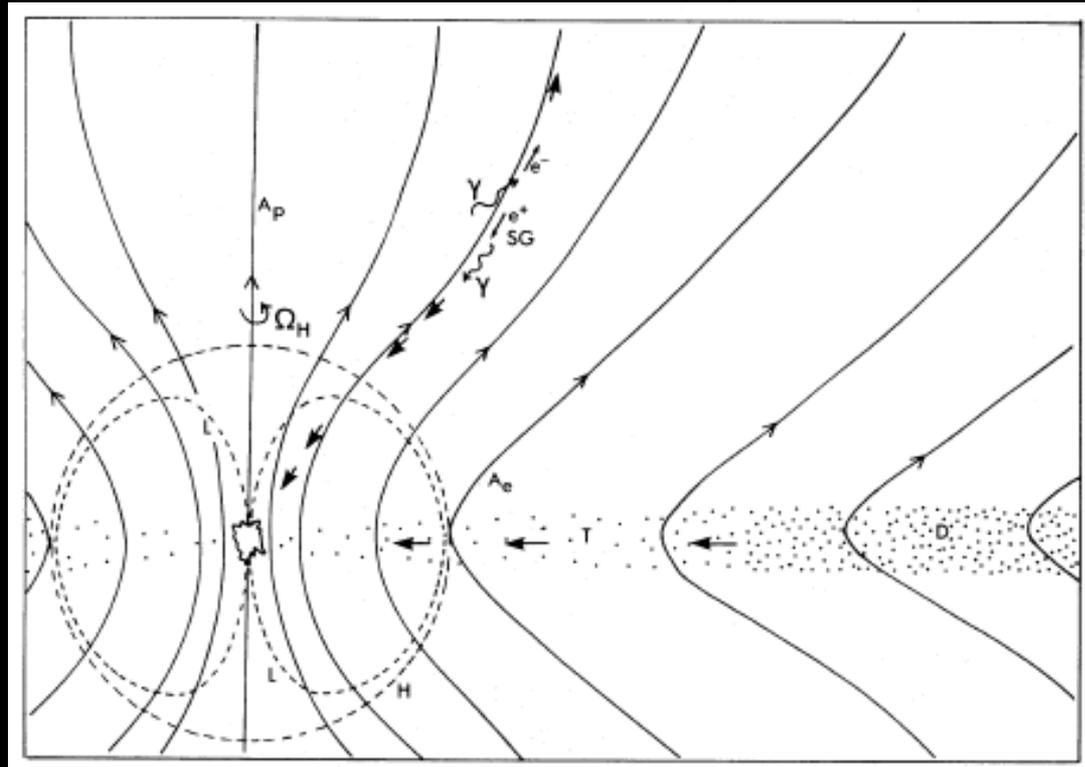


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Krakow April 2015

# Blandford & Znajek 1977

$$\mathcal{E}_{EM} \propto \omega(\Omega_{BH} - \omega)\Psi_m^2 \sim \Omega_{BH}^2 \Psi_m^2$$

Magnetic field is supported by external currents in an accretion disc



# Ingredients to describe the problem

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -\alpha^2 dt^2 + \frac{A \sin^2 \theta}{\Sigma} (d\phi - \Omega dt)^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 \end{aligned}$$

Kerr metric

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = 4\pi \rho_e$$

$$\nabla \times (\alpha \mathbf{B}) = 4\pi \alpha \mathbf{J}$$

$$\nabla \times (\alpha \mathbf{E}) = 0 ,$$

Macdonald & Thorne 1982

# Ingredients to describe the problem

$$\mathbf{E} \cdot \mathbf{B} = 0$$

Ideal

$$\rho_e \mathbf{E} + \mathbf{J} \times \mathbf{B} = 0$$

force-free

$$\mathbf{B}(r, \theta) = \frac{1}{\sqrt{A} \sin \theta} \left\{ \Psi_{,\theta} \mathbf{e}_{\hat{r}} - \sqrt{\Delta} \Psi_{,r} \mathbf{e}_{\hat{\theta}} + \frac{2I\sqrt{\Sigma}}{\alpha} \mathbf{e}_{\hat{\phi}} \right\}$$

$$\mathbf{E}(r, \theta) = \frac{\Omega - \omega}{\alpha\sqrt{\Sigma}} \left\{ \sqrt{\Delta} \Psi_{,r} \mathbf{e}_{\hat{r}} + \Psi_{,\theta} \mathbf{e}_{\hat{\theta}} + 0 \mathbf{e}_{\hat{\phi}} \right\}$$

# Blandford & Znajek equation

$$\begin{aligned}
 & \left\{ \Psi_{,rr} + \frac{1}{\Delta} \Psi_{,\theta\theta} + \Psi_{,r} \left( \frac{A_{,r}}{A} - \frac{\Sigma_{,r}}{\Sigma} \right) - \frac{\Psi_{,\theta} \cos \theta}{\Delta \sin \theta} \right\} \left[ 1 - \frac{2Mr}{\Sigma} + \frac{4Ma\omega r \sin^2 \theta}{\Sigma} - \frac{\omega^2 A \sin^2 \theta}{\Sigma} \right] \\
 & + \left( \frac{2Mr}{\Sigma} - \frac{4Ma\omega r \sin^2 \theta}{\Sigma} \right) \left( \frac{A_{,r}}{A} - \frac{1}{r} \right) \Psi_{,r} + \left( \frac{\Sigma_{,r}}{\Sigma} - \frac{A_{,r}}{A} \right) \Psi_{,r} \\
 & - \left( 2 \frac{\cos \theta}{\sin \theta} + \frac{A_{,\theta}}{A} - \frac{\Sigma_{,\theta}}{\Sigma} \right) \omega A (\omega - 2\Omega) \frac{\Psi_{,\theta} \sin^2 \theta}{\Delta \Sigma} \\
 & - 2\omega\Omega \frac{\Psi_{,\theta} A_{,\theta}}{\Delta A} - 2Mr \Sigma_{,\theta} \frac{\Psi_{,\theta}}{\Delta \Sigma^2} \\
 & - \frac{\omega' A \sin^2 \theta}{\Sigma} (\omega - \Omega) \left( \Psi_{,r}^2 + \frac{1}{\Delta} \Psi_{,\theta}^2 \right) \\
 & = -\frac{4\Sigma}{\Delta} II'
 \end{aligned}$$

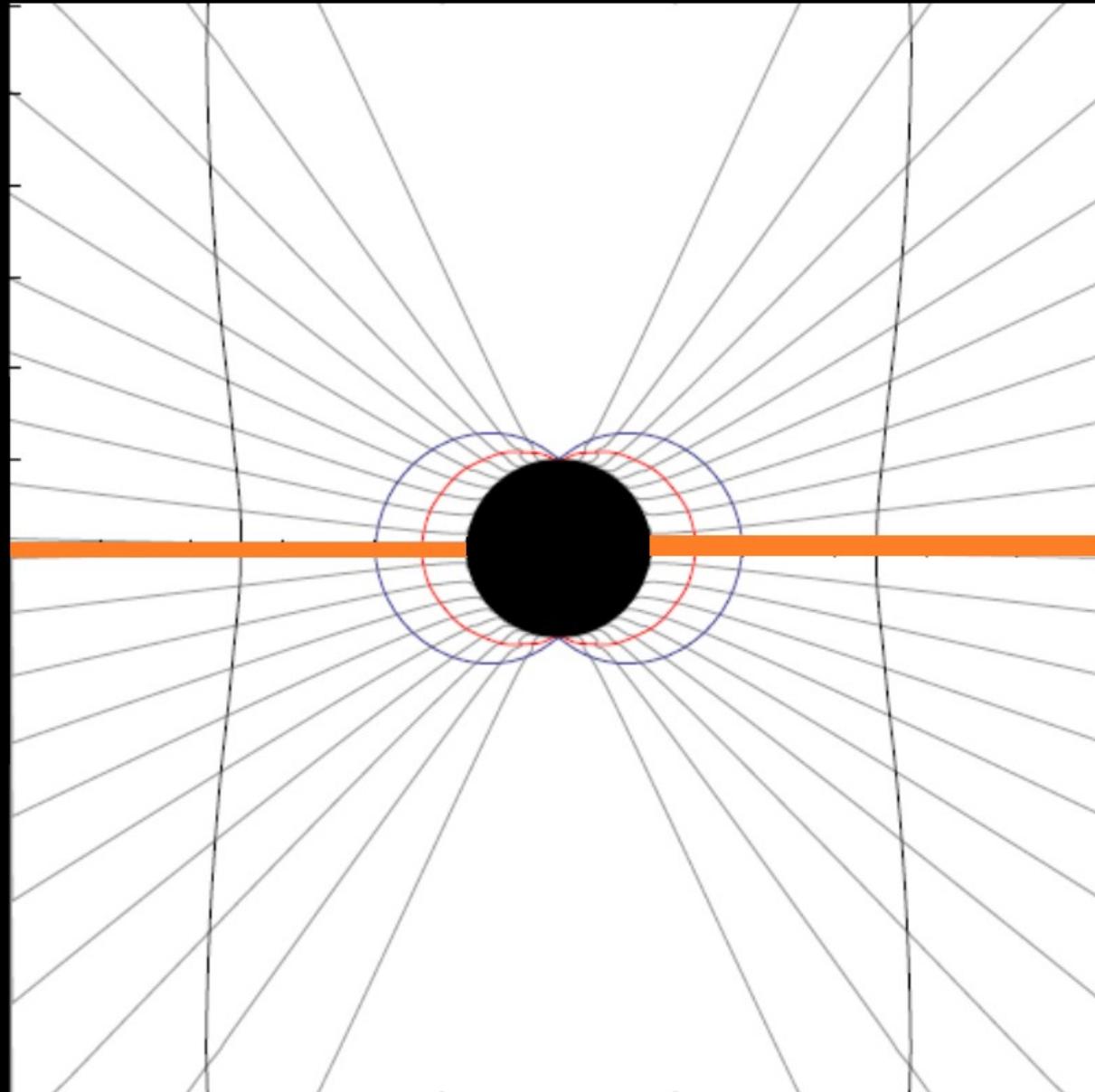
# Blandford & Znajek equation

$$\begin{aligned}
 & \left\{ \Psi_{,rr} + \frac{1}{\Delta} \Psi_{,\theta\theta} + \Psi_{,r} \left( \frac{A_{,r}}{A} - \frac{\Sigma_{,r}}{\Sigma} \right) - \frac{\Psi_{,\theta} \cos \theta}{\Delta \sin \theta} \right\} \left[ 1 - \frac{2Mr}{\Sigma} + \frac{4M\omega r \sin^2 \theta}{\Sigma} - \frac{\omega^2 A \sin^2 \theta}{\Sigma} \right] \\
 & + \left( \frac{2Mr}{\Sigma} - \frac{4M\omega r \sin^2 \theta}{\Sigma} \right) \left( \frac{A_{,r}}{A} - \frac{1}{r} \right) \Psi_{,r} + \left( \frac{\Sigma_{,r}}{\Sigma} - \frac{A_{,r}}{A} \right) \Psi_{,r} \\
 & - \left( 2 \frac{\cos \theta}{\sin \theta} + \frac{A_{,\theta}}{A} - \frac{\Sigma_{,\theta}}{\Sigma} \right) \omega A (\omega - 2\Omega) \frac{\Psi_{,\theta} \sin^2 \theta}{\Delta \Sigma} \\
 & - 2\omega\Omega \frac{\Psi_{,\theta} A_{,\theta}}{\Delta A} - 2Mr \Sigma_{,\theta} \frac{\Psi_{,\theta}}{\Delta \Sigma^2} \\
 & - \frac{\omega' A \sin^2 \theta}{\Sigma} (\omega - \Omega) \left( \Psi_{,r}^2 + \frac{1}{\Delta} \Psi_{,\theta}^2 \right) \\
 & = -\frac{4\Sigma}{\Delta} II'
 \end{aligned}$$

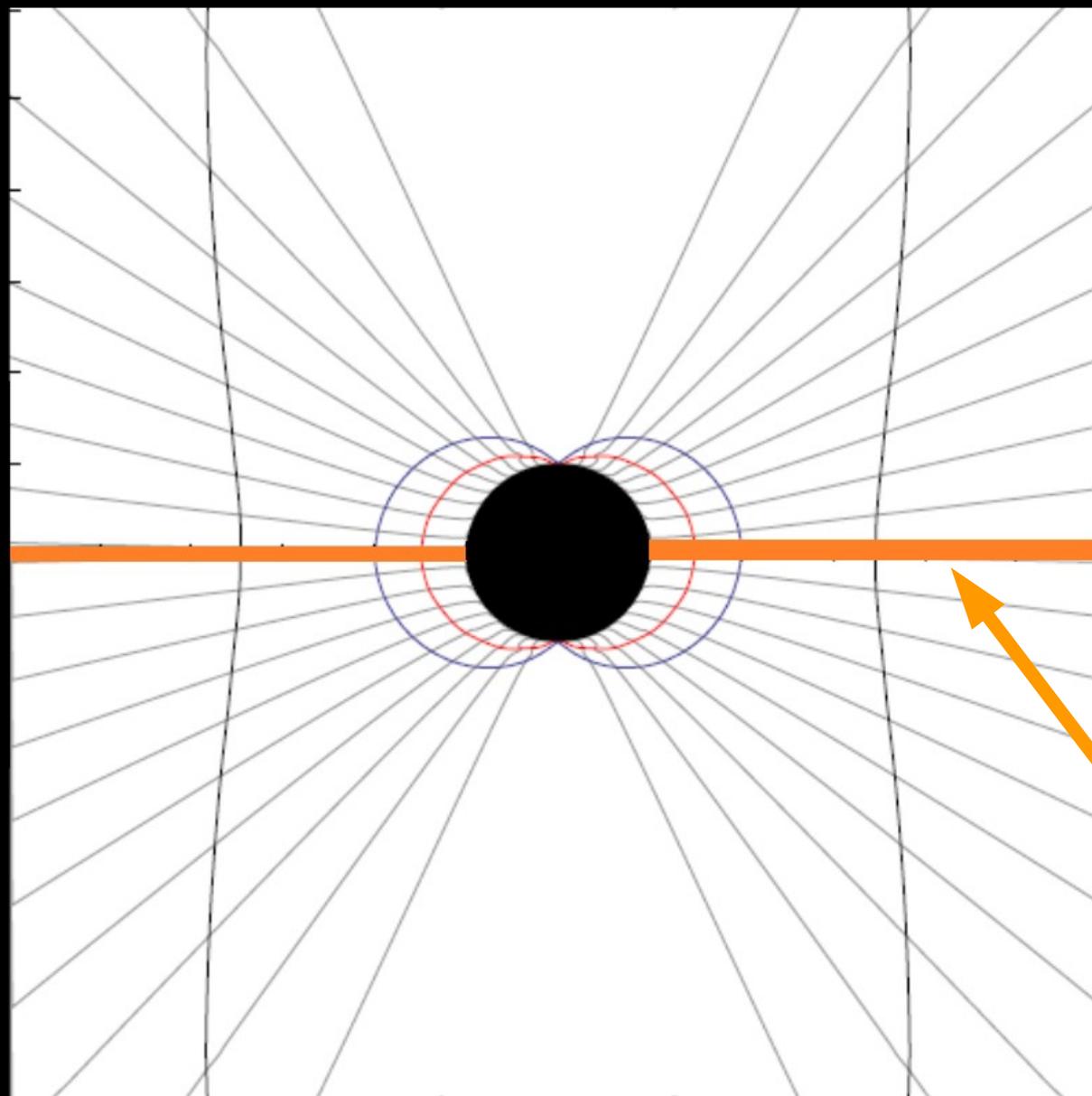
# The Black Hole Problem has two light surfaces

$$\left[ 1 - \frac{2Mr}{\Sigma} + \frac{4Ma\omega r \sin^2 \theta}{\Sigma} - \frac{\omega^2 A \sin^2 \theta}{\Sigma} \right]$$

The electric current  $I(\Psi)$  and angular velocity of the field lines  $\omega(\Psi)$  must be determined self-consistently

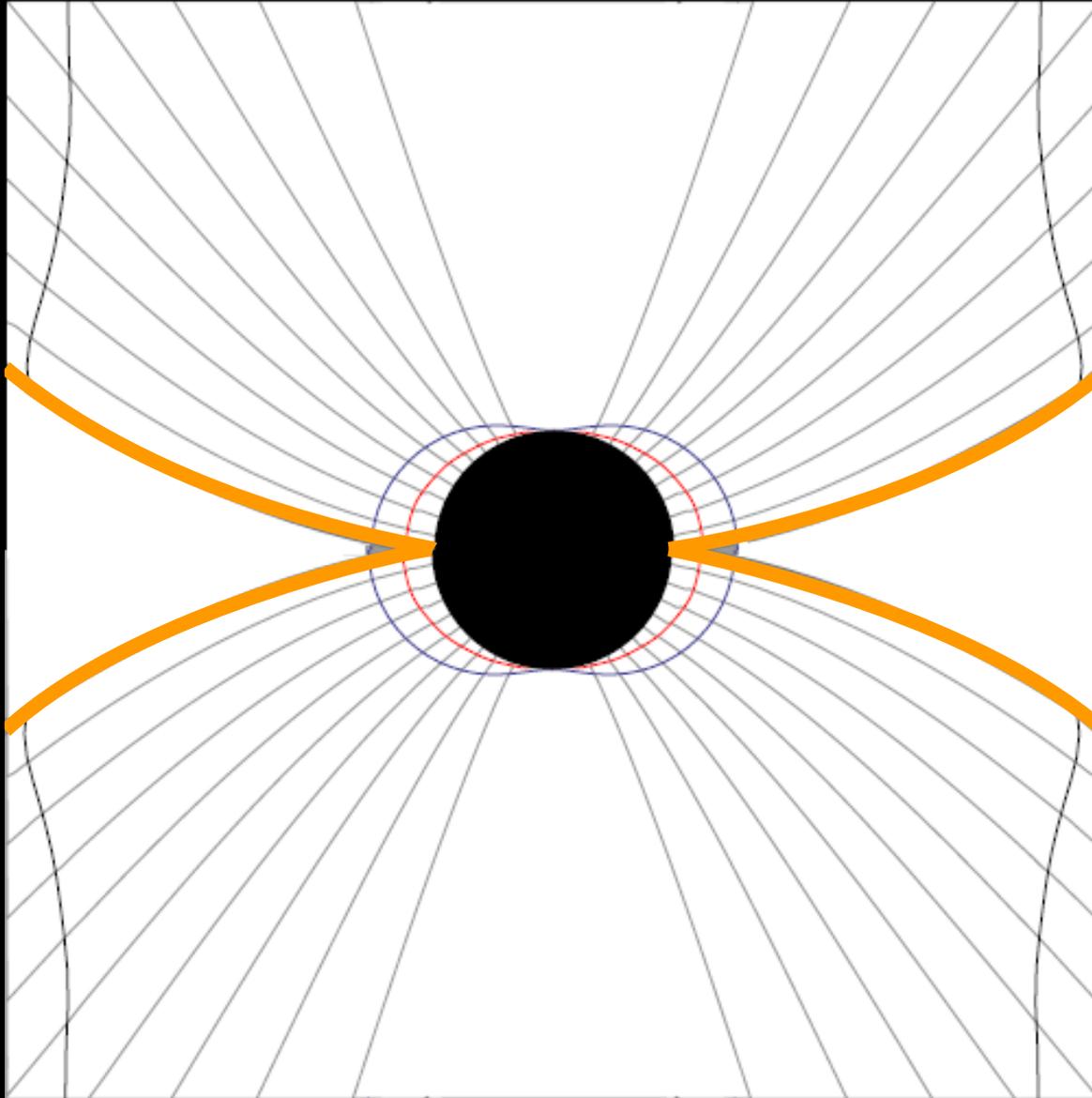


$\alpha = 0.999$

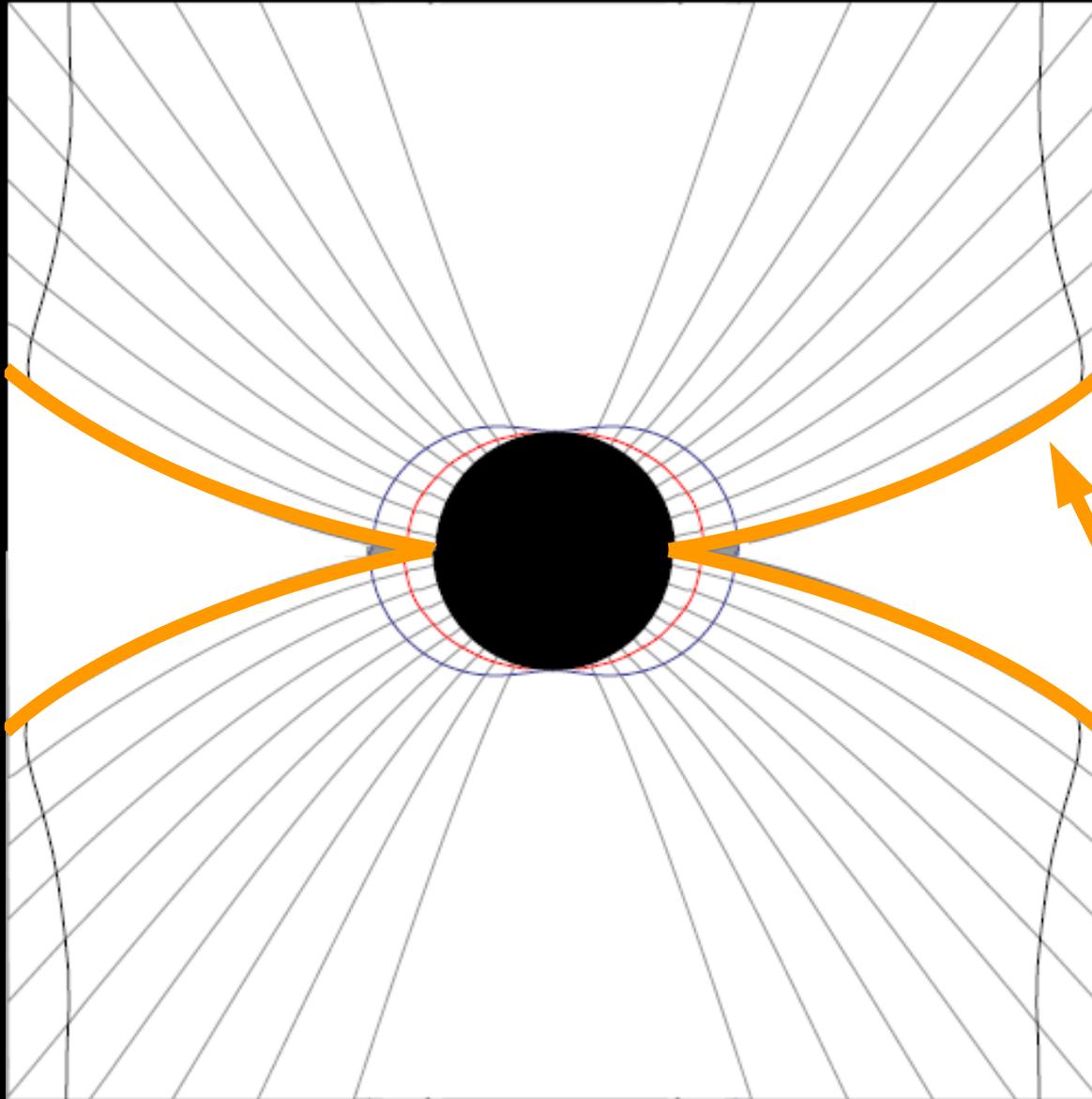


$\alpha = 0.999$

current  
sheet (CS)



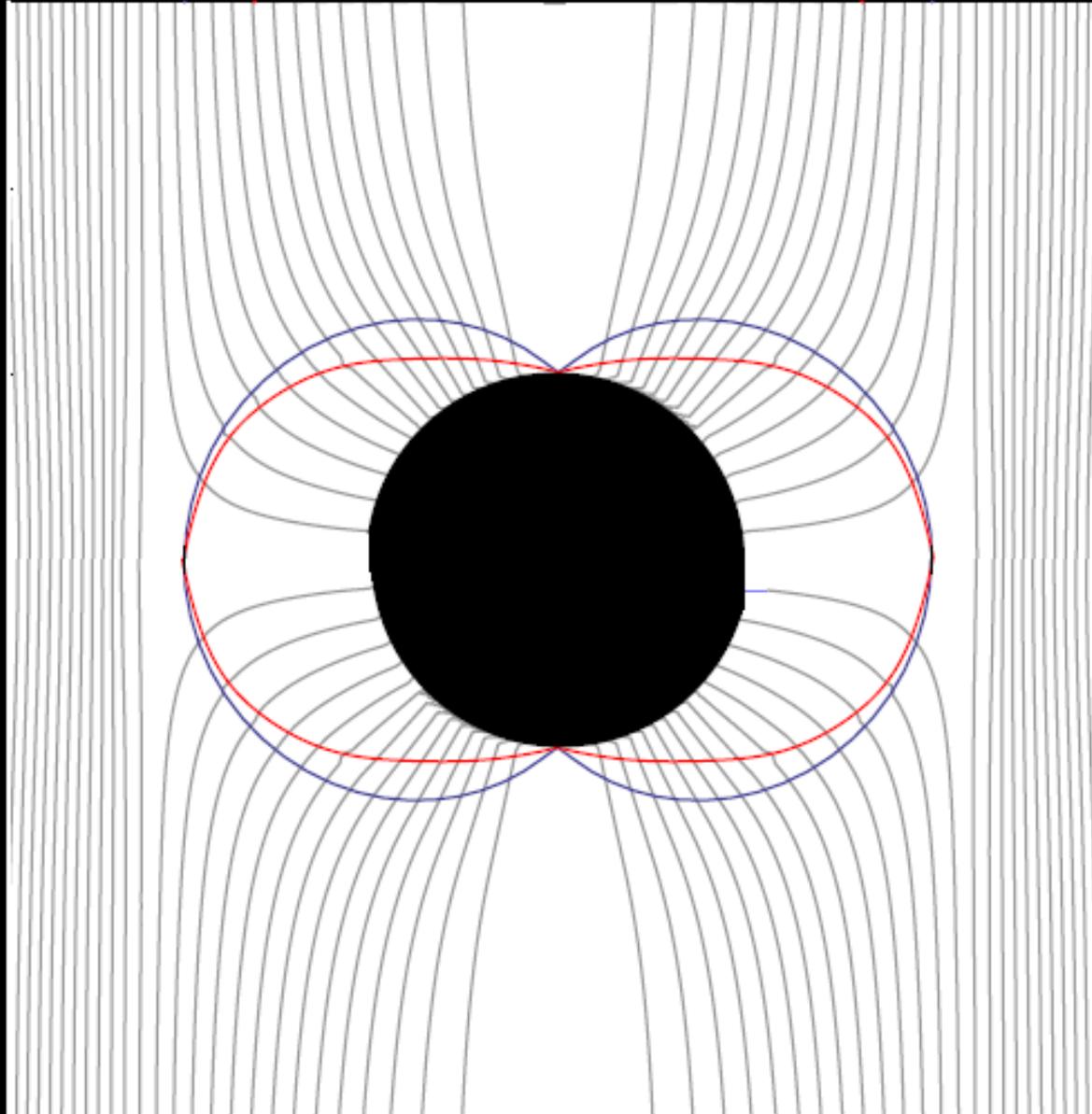
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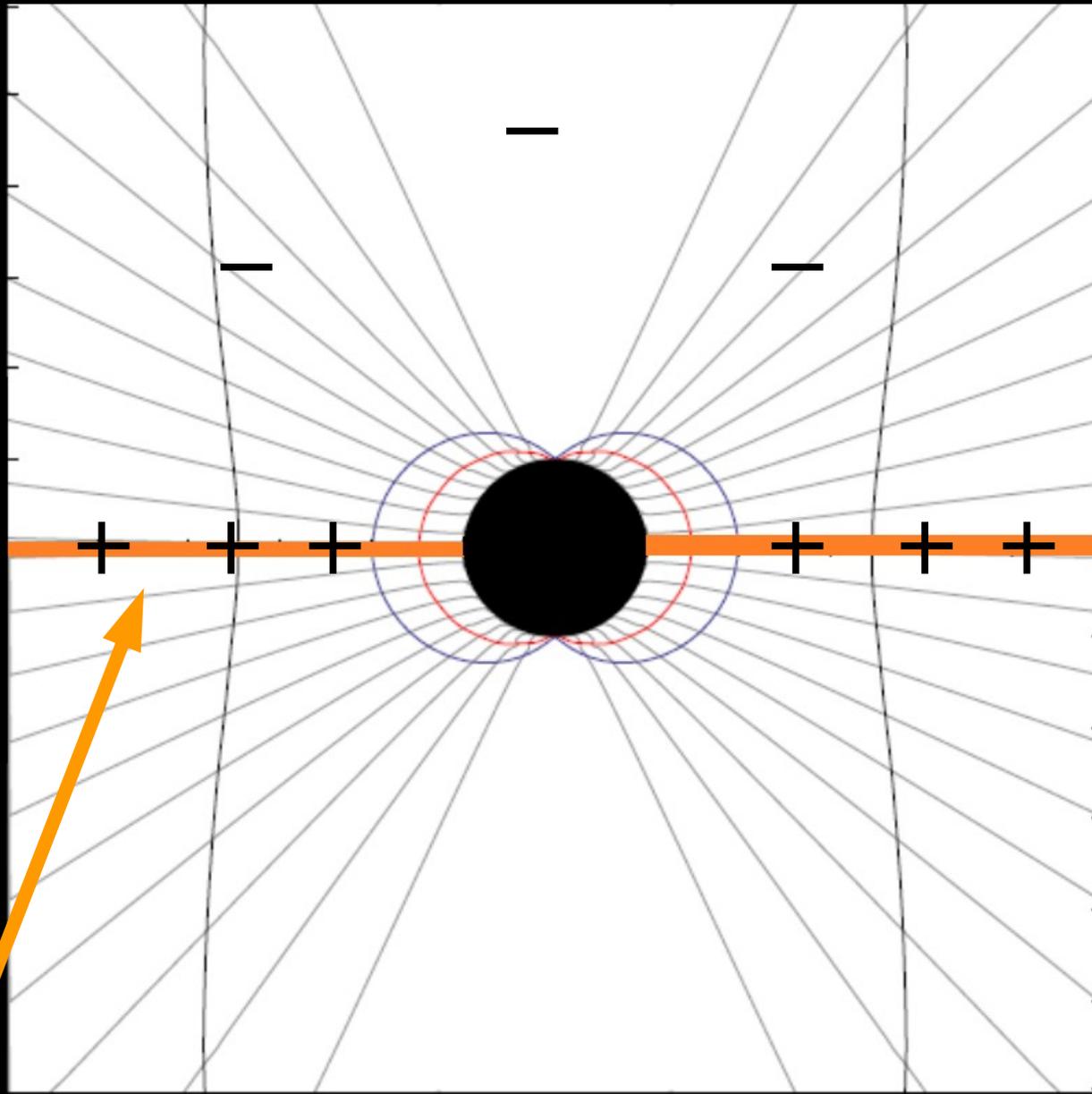
$\alpha = 0.999$

current  
sheet (CS)

Only one Light Surface  $\omega(\Psi)$  is given

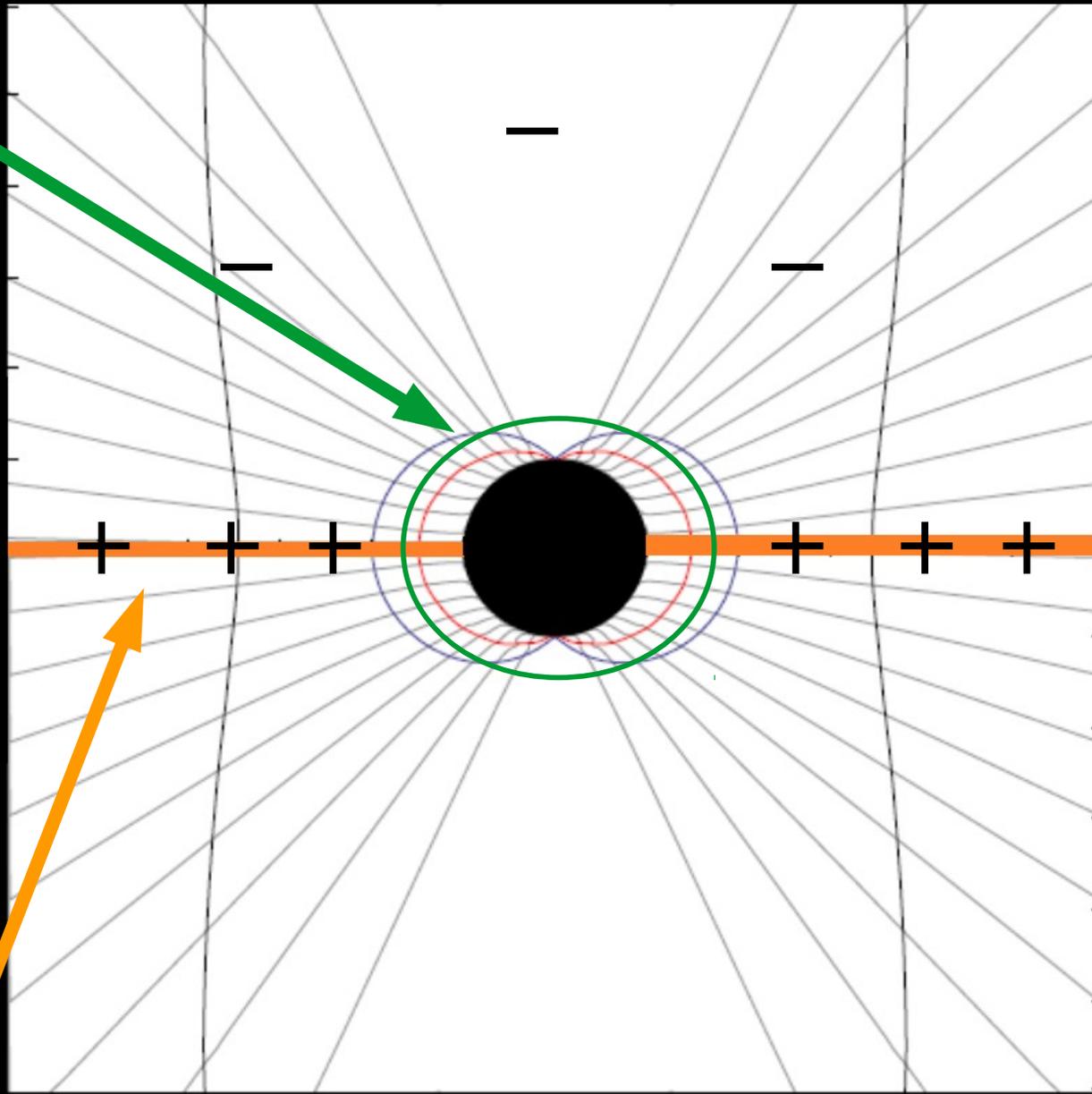


$\alpha = 0.999$



The CS parallel electric field will accelerate particles

zero charge surface



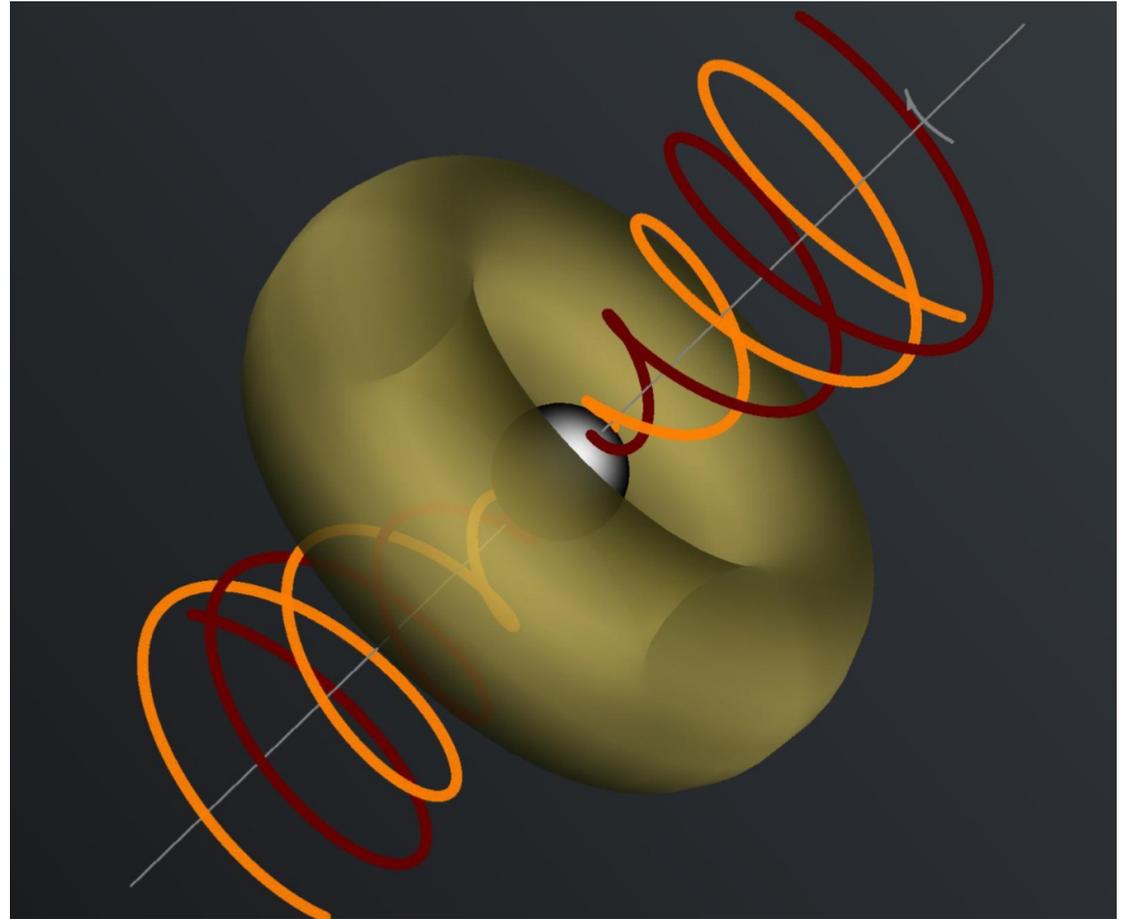
$E_{\parallel}$

The CS parallel electric field will accelerate particles

# Prime movers in GRBs?

$$\dot{E} \approx -\frac{1}{6\pi^2 c} \Psi_m^2 \Omega^2$$

BZ mechanism

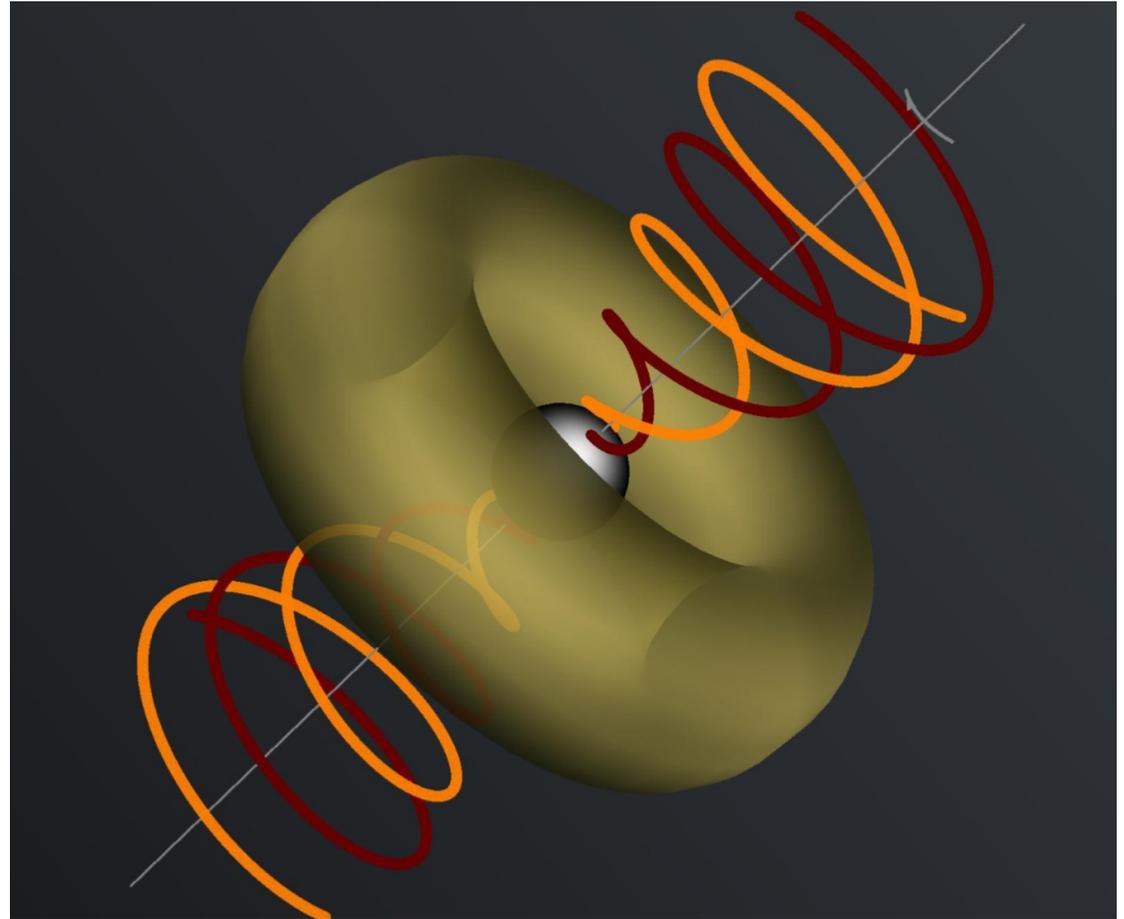


# Rotational Energy of the Black Hole

$$\dot{E} \approx -\frac{1}{6\pi^2 c} \Psi_m^2 \Omega^2$$

Equate with

$$\dot{E} = \frac{\mathcal{G} M^2}{c} \frac{d(a\Omega/M)}{dt}$$

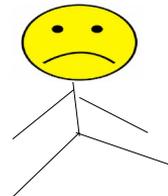


# Black Hole Spin Down

$$\dot{E} = \dot{E}_o \frac{W \left( -\frac{1}{2} e^{-\frac{1}{2} - \frac{t}{t_{BZ}}} \right)}{1 + W \left( -\frac{1}{2} e^{-\frac{1}{2} - \frac{t}{t_{BZ}}} \right)}$$

$W(x)$   
Lambert  $W$  function

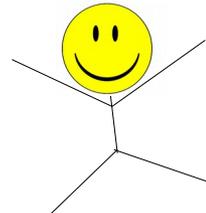
Approximate this please!



# Black Hole Spin Down

$$\dot{E} \approx \dot{E}_0 e^{-t/t_{BZ}}$$

Easy, That's an exponential!

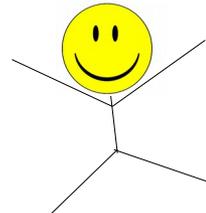


# Can we observe this in GRBs?

See poster

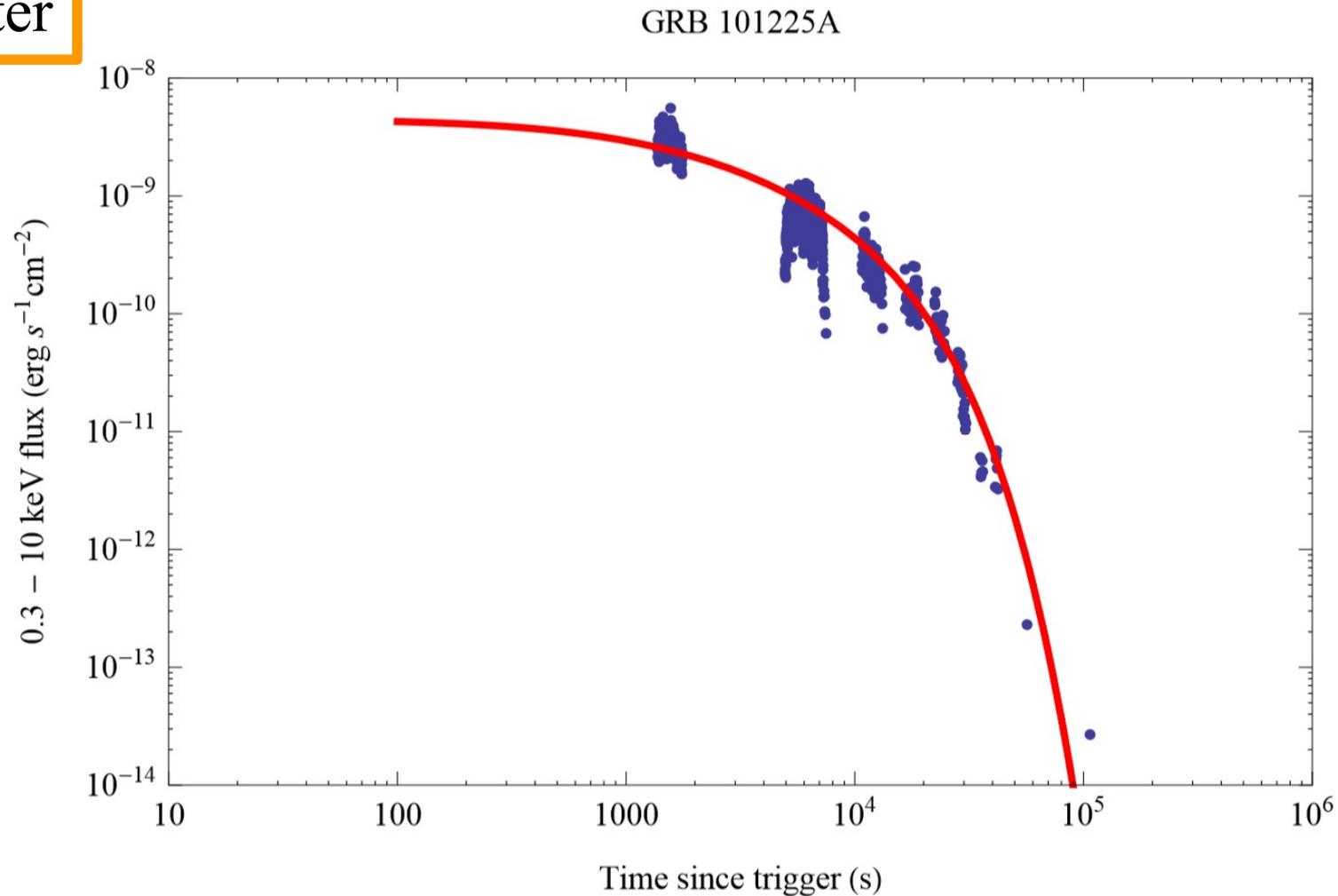
$$\dot{E} \approx \dot{E}_0 e^{-t/t_{BZ}}$$

Easy, That's an exponential!



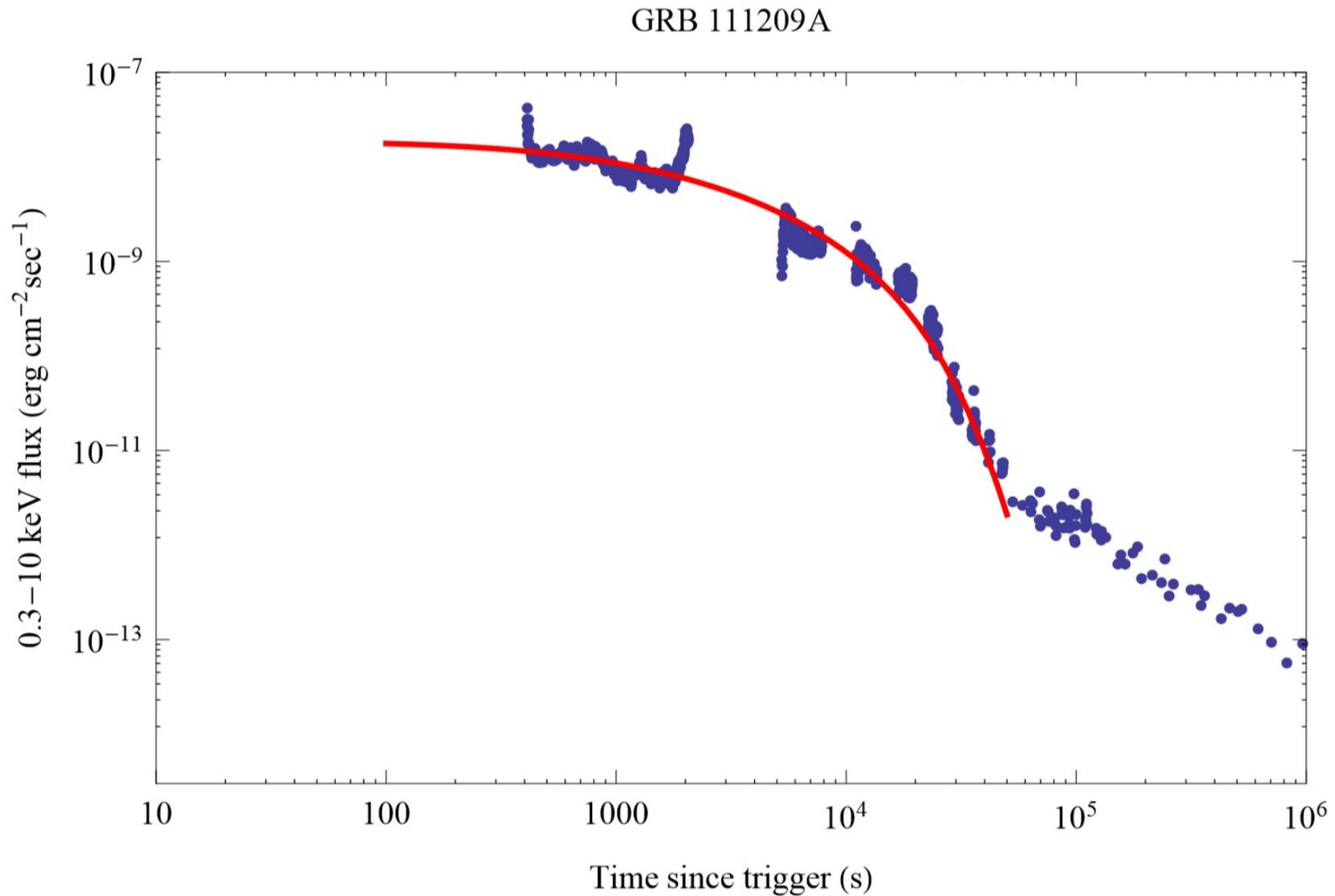
# Following the central engine activity

See poster

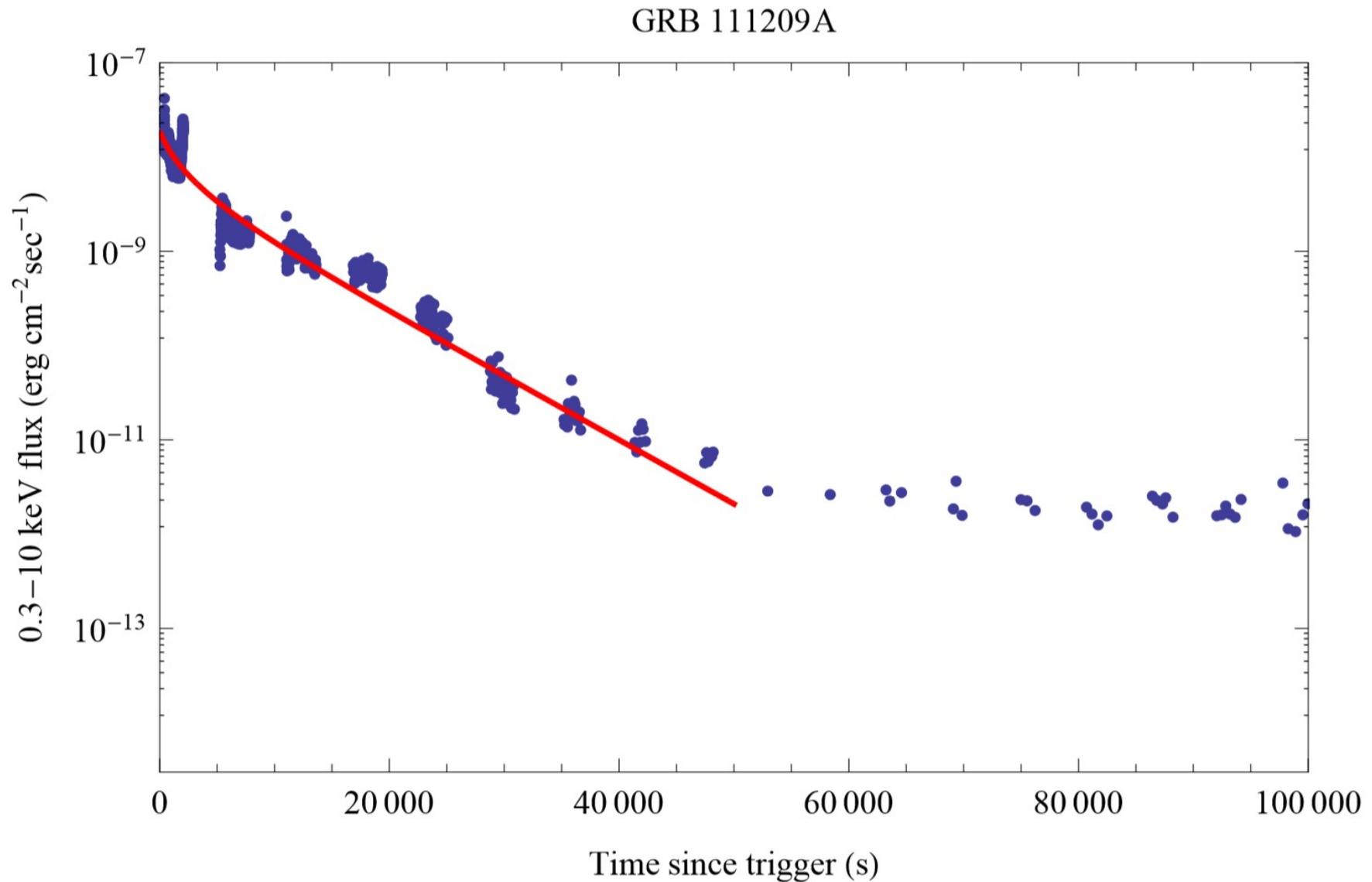


The end

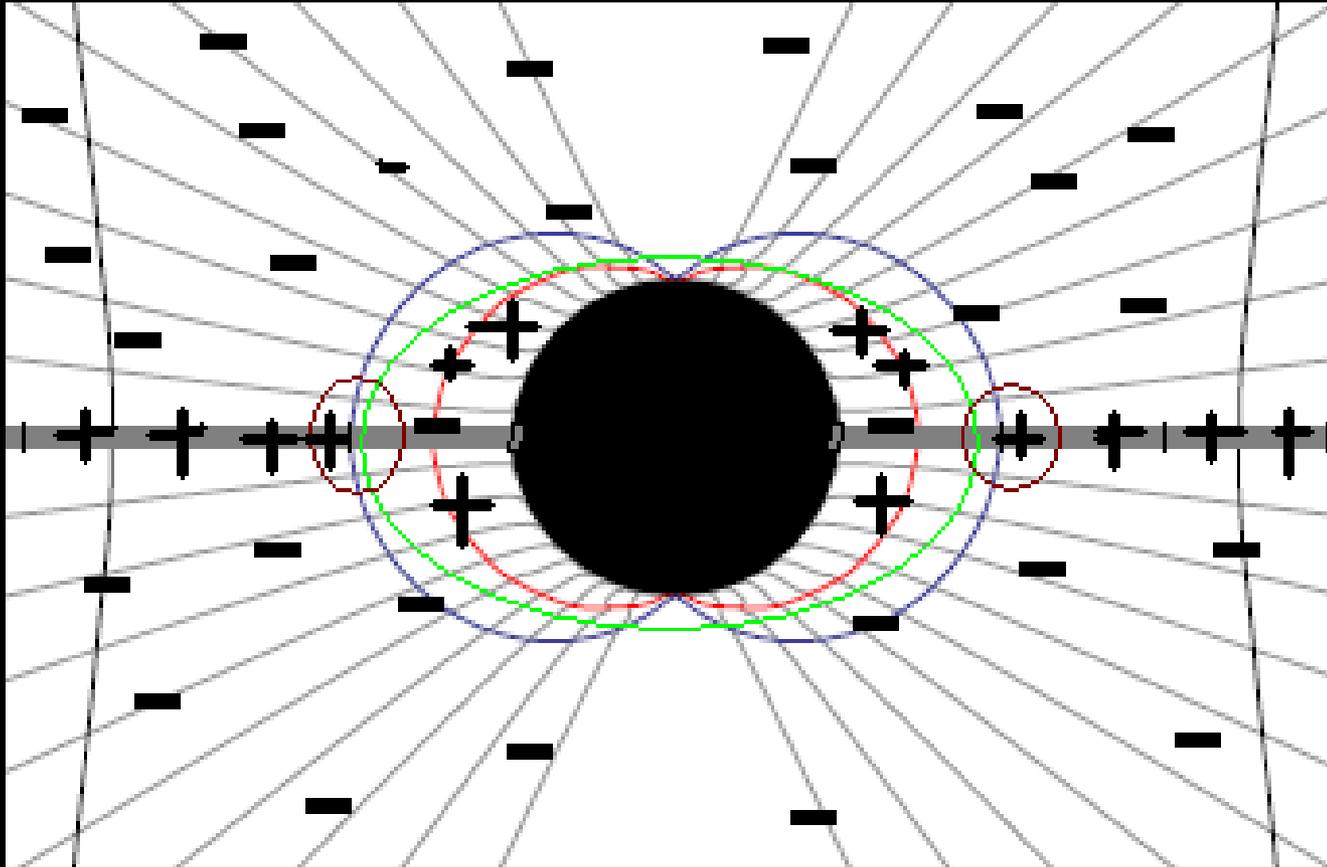
# Following the central engine activity



# Following the central engine activity

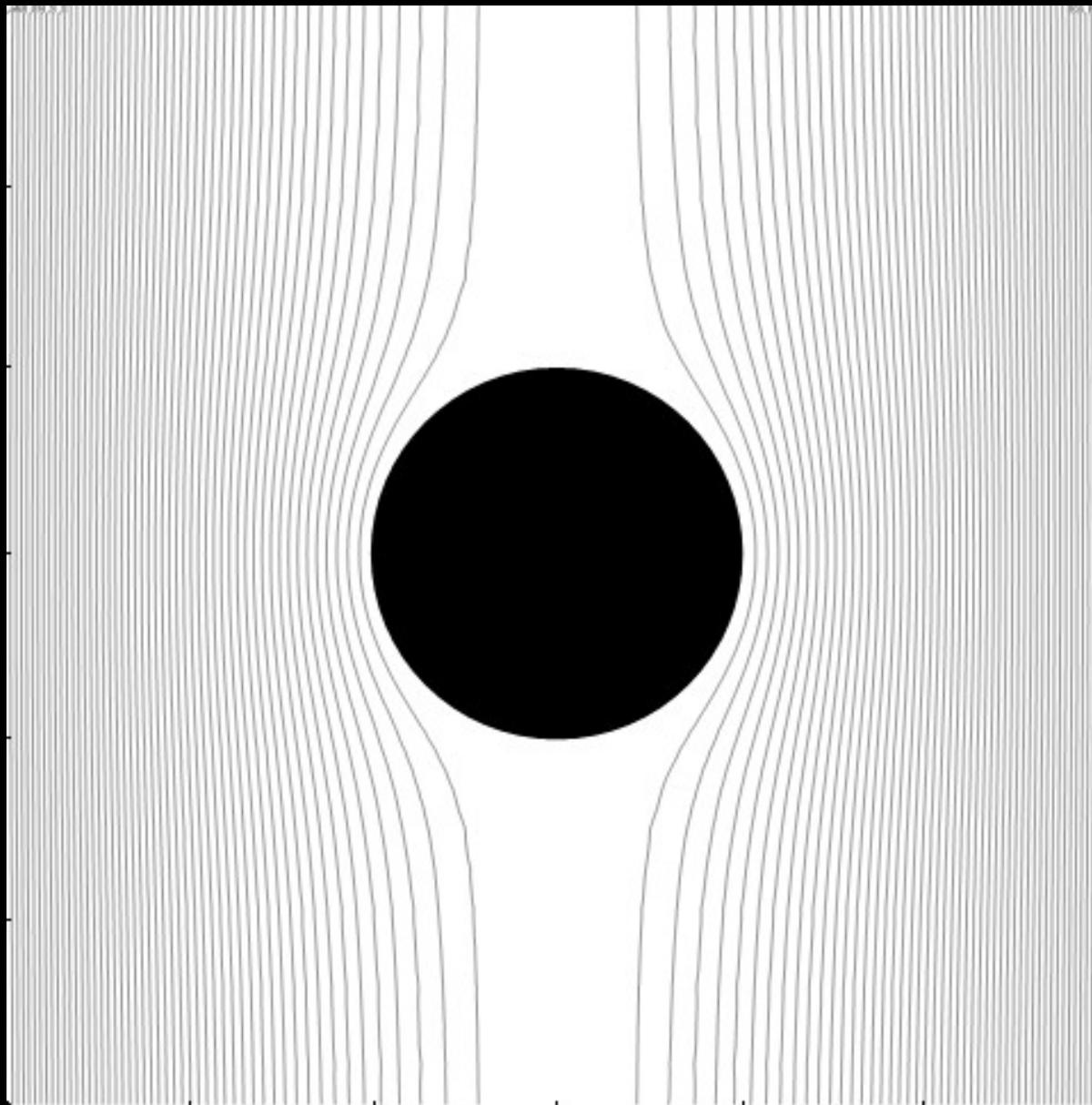


Green line: zero charge surface  
+ - Charges



a parallel electric field will accelerate particles

# Electrovacuum



$\alpha = 0.999$