Blandford-Znajek mechanism is a Penrose process

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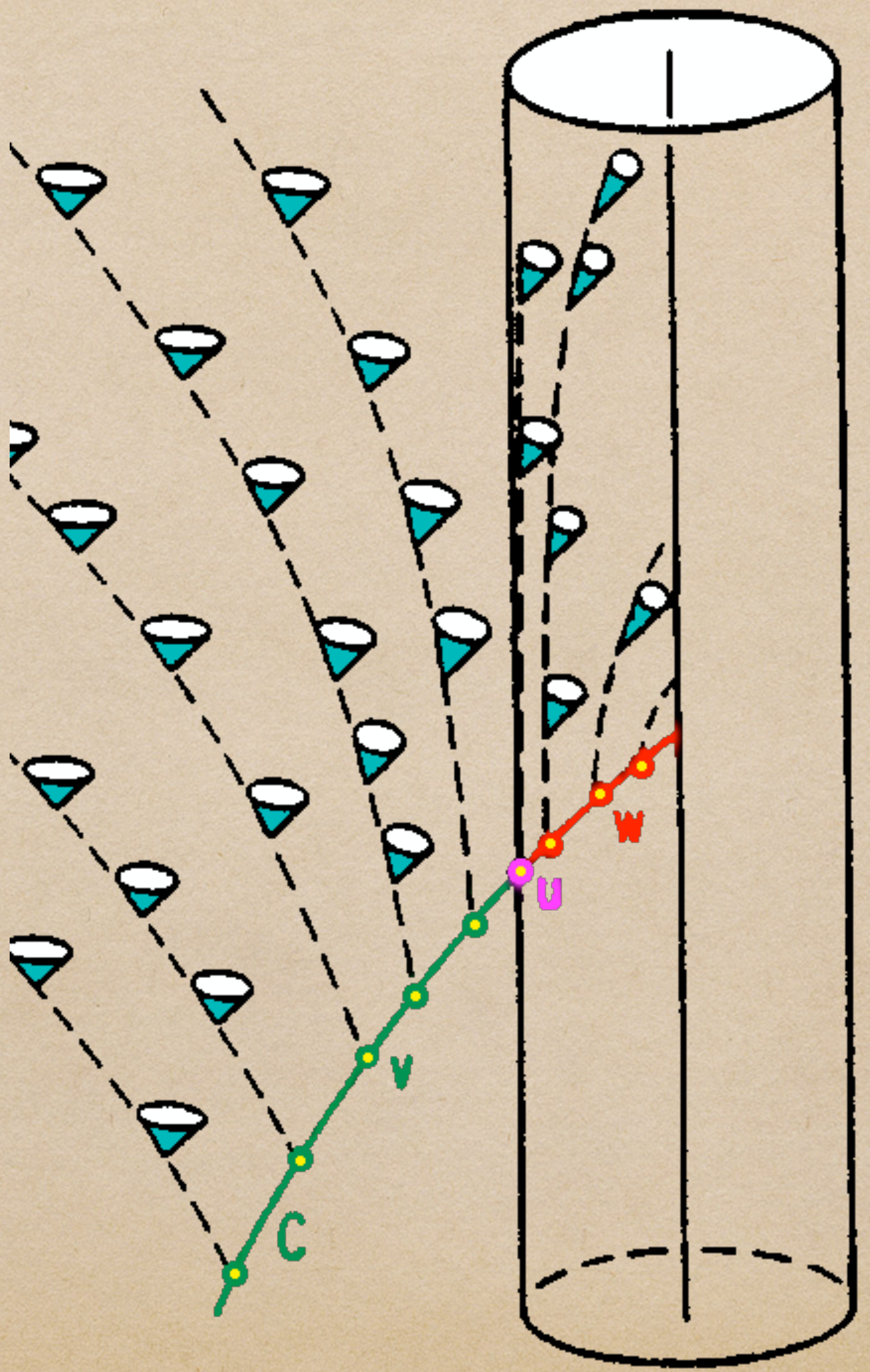
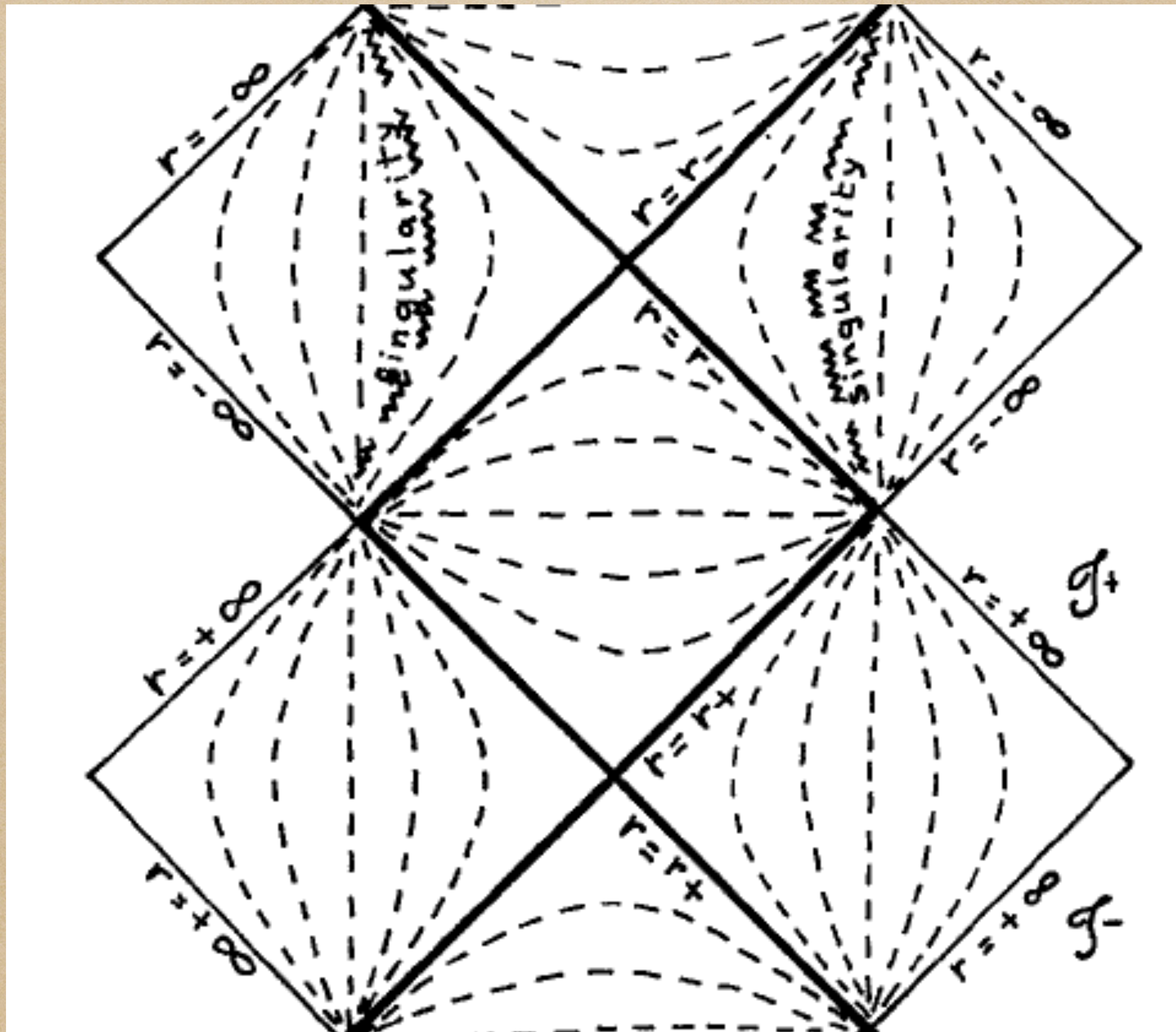
Based on
Lasota, Gourgoulhon, Abramowicz, Tchekhovskoy & Narayan ; Phys. Rev. D 89, 024041 (2014)

20th of April 2015

Black holes are (pure) general-relativistic objects



Kerr black-hole solution: the Carter-Penrose diagram



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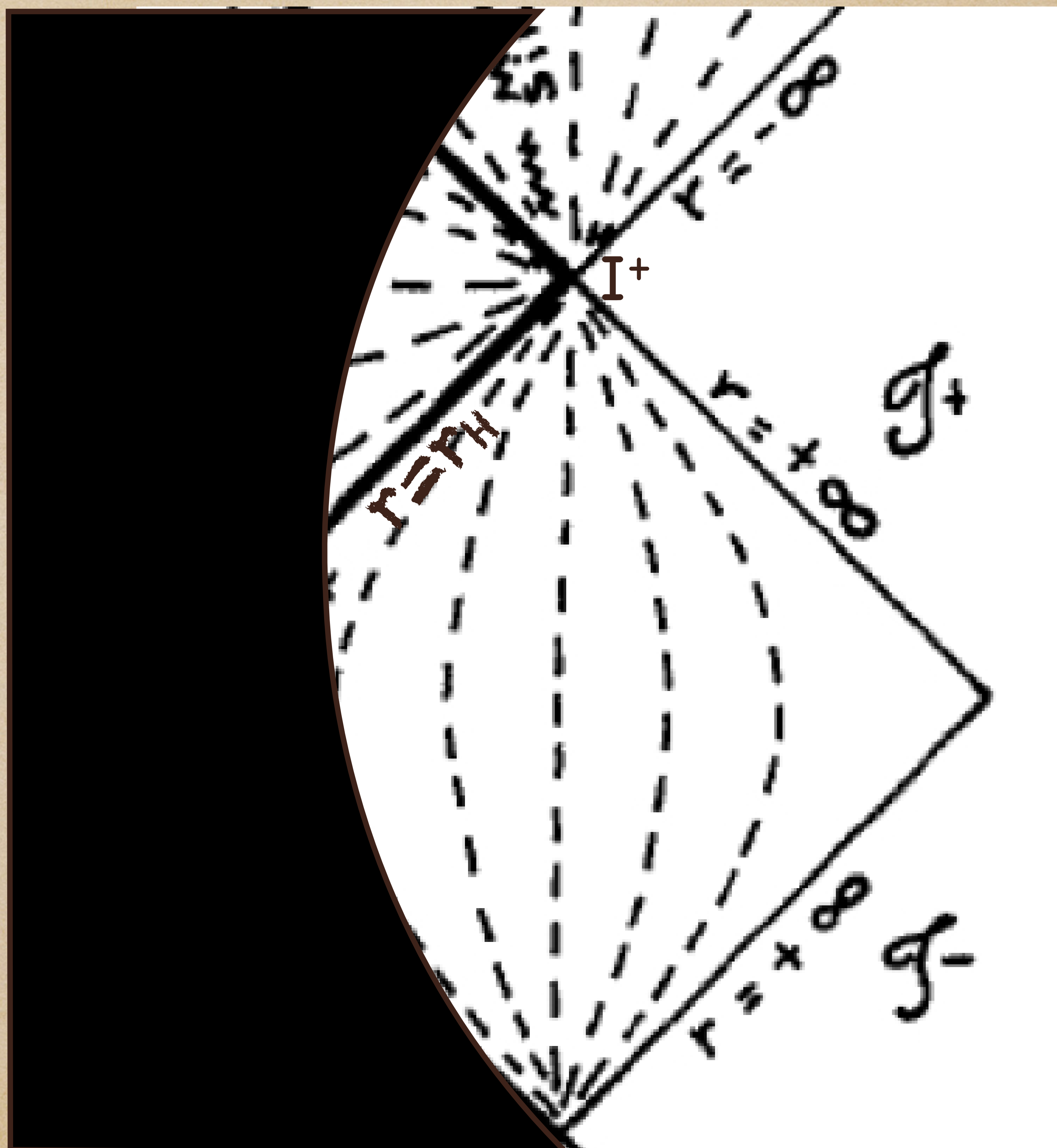

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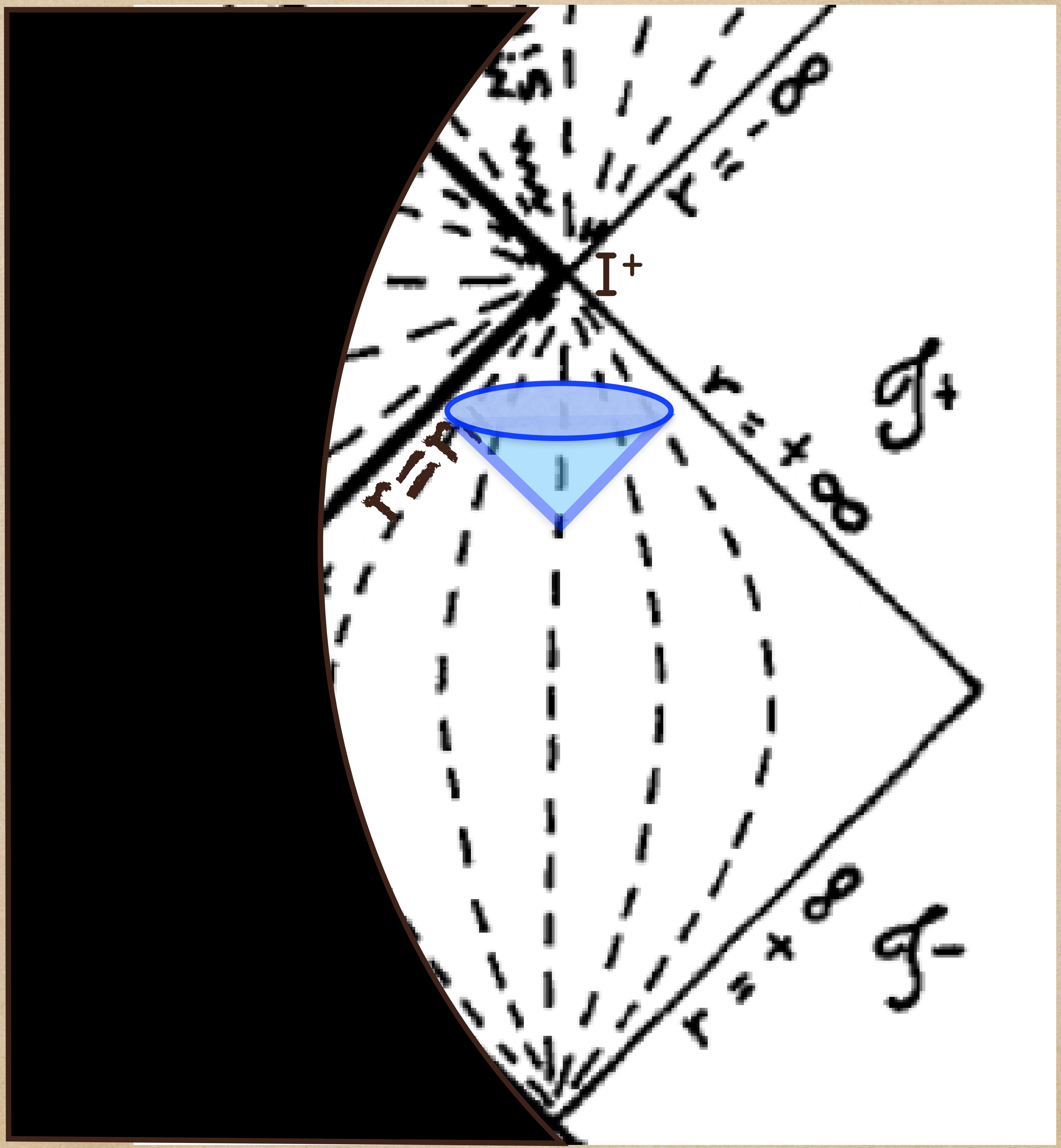

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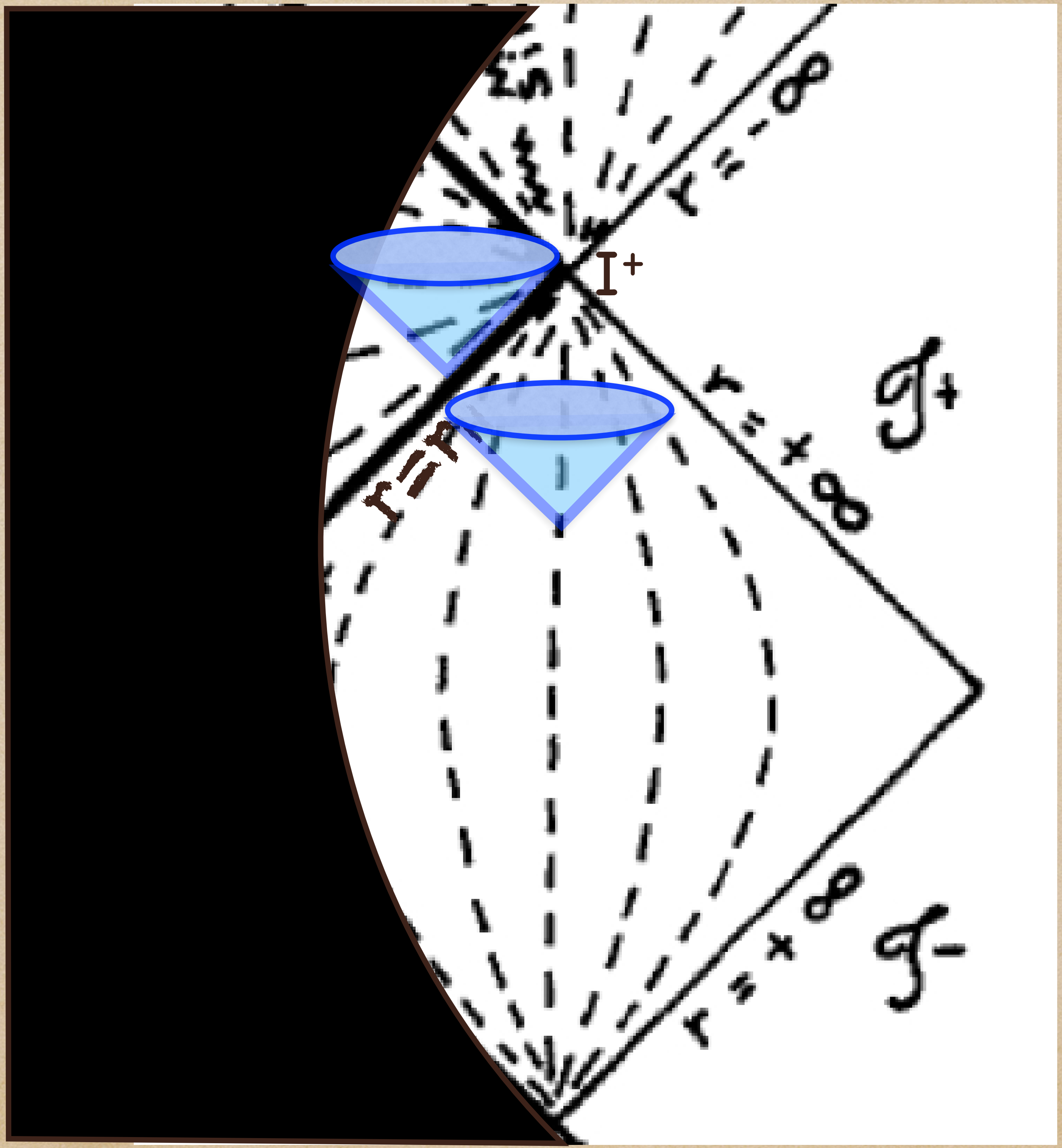

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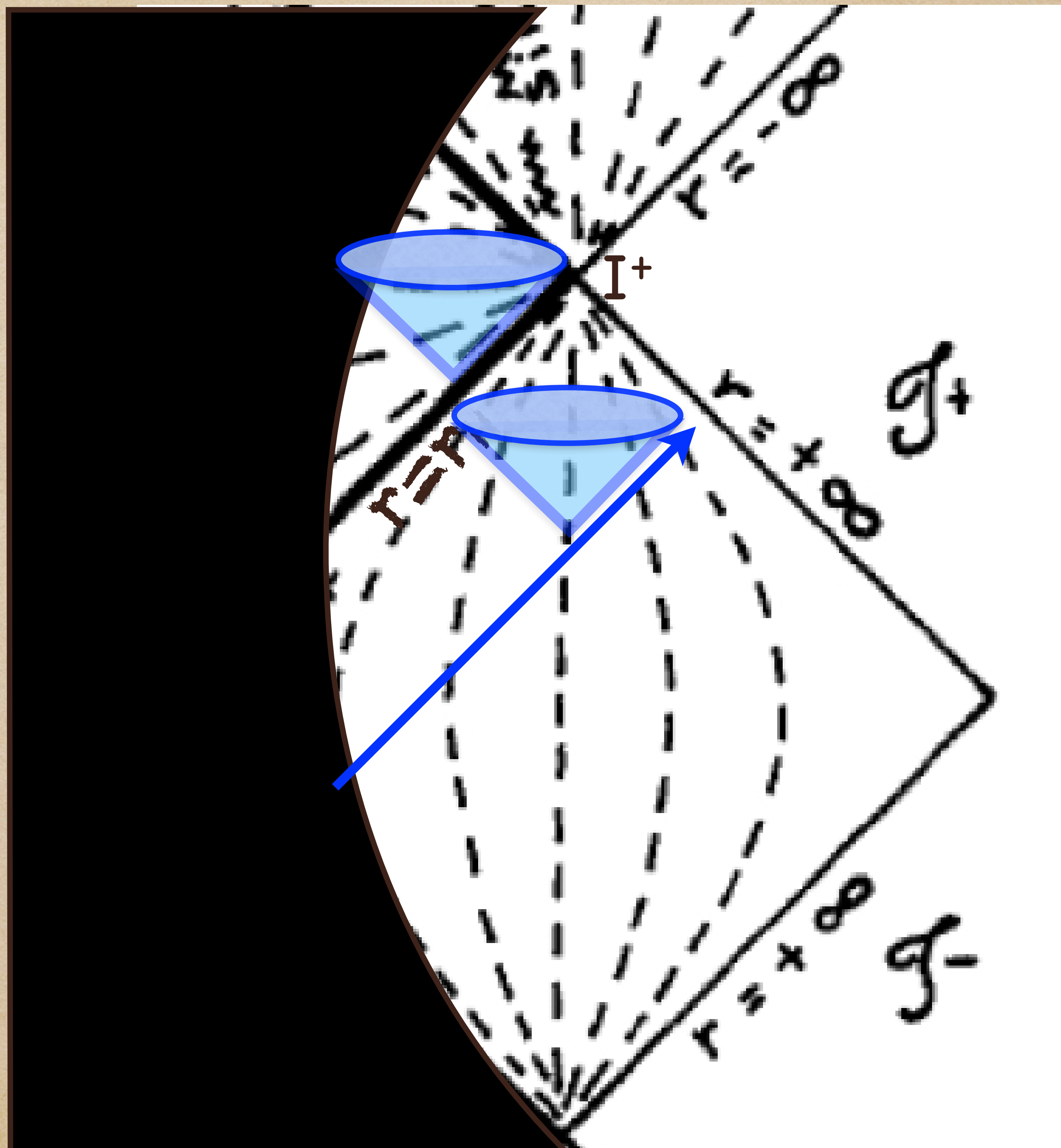

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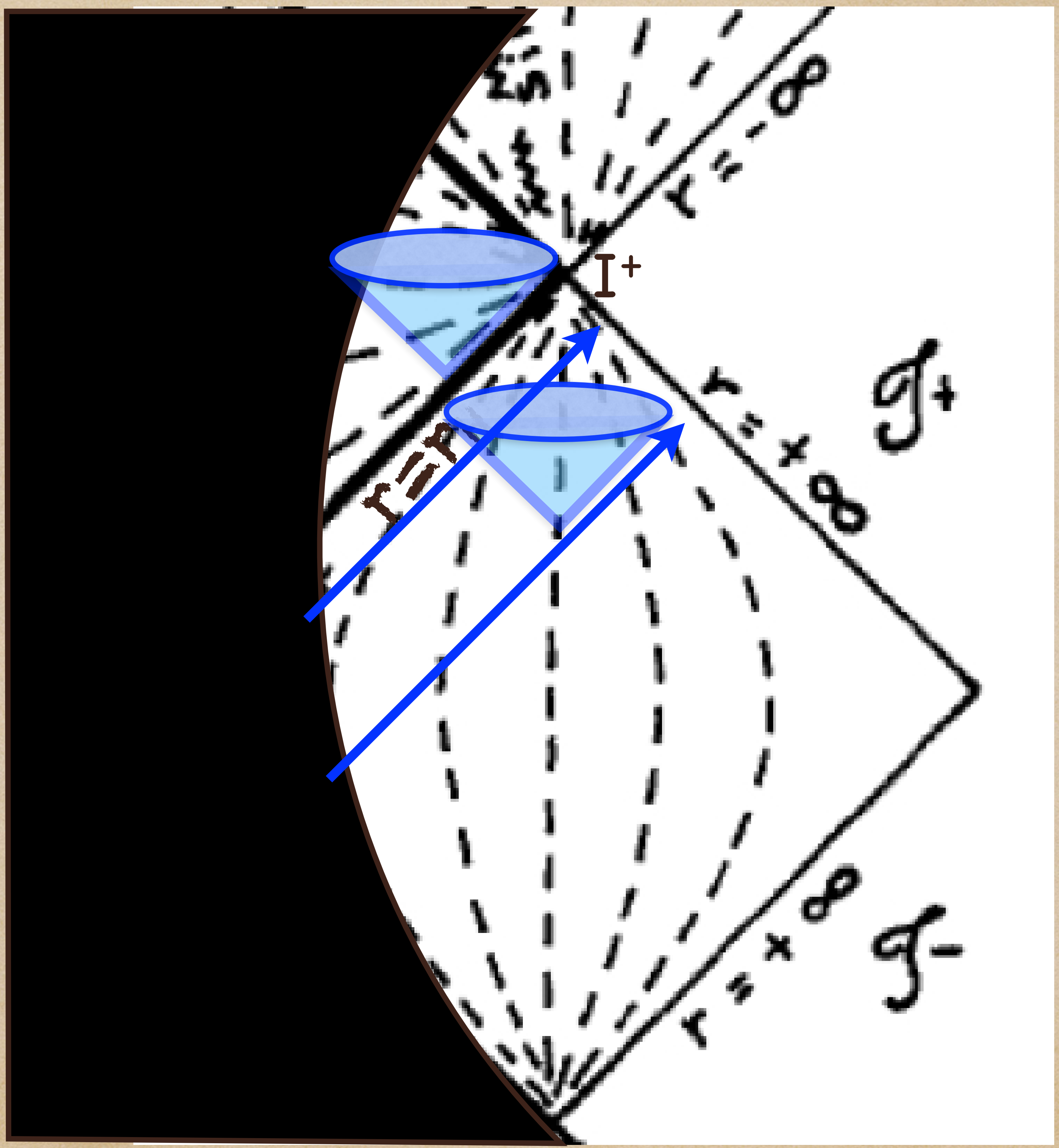

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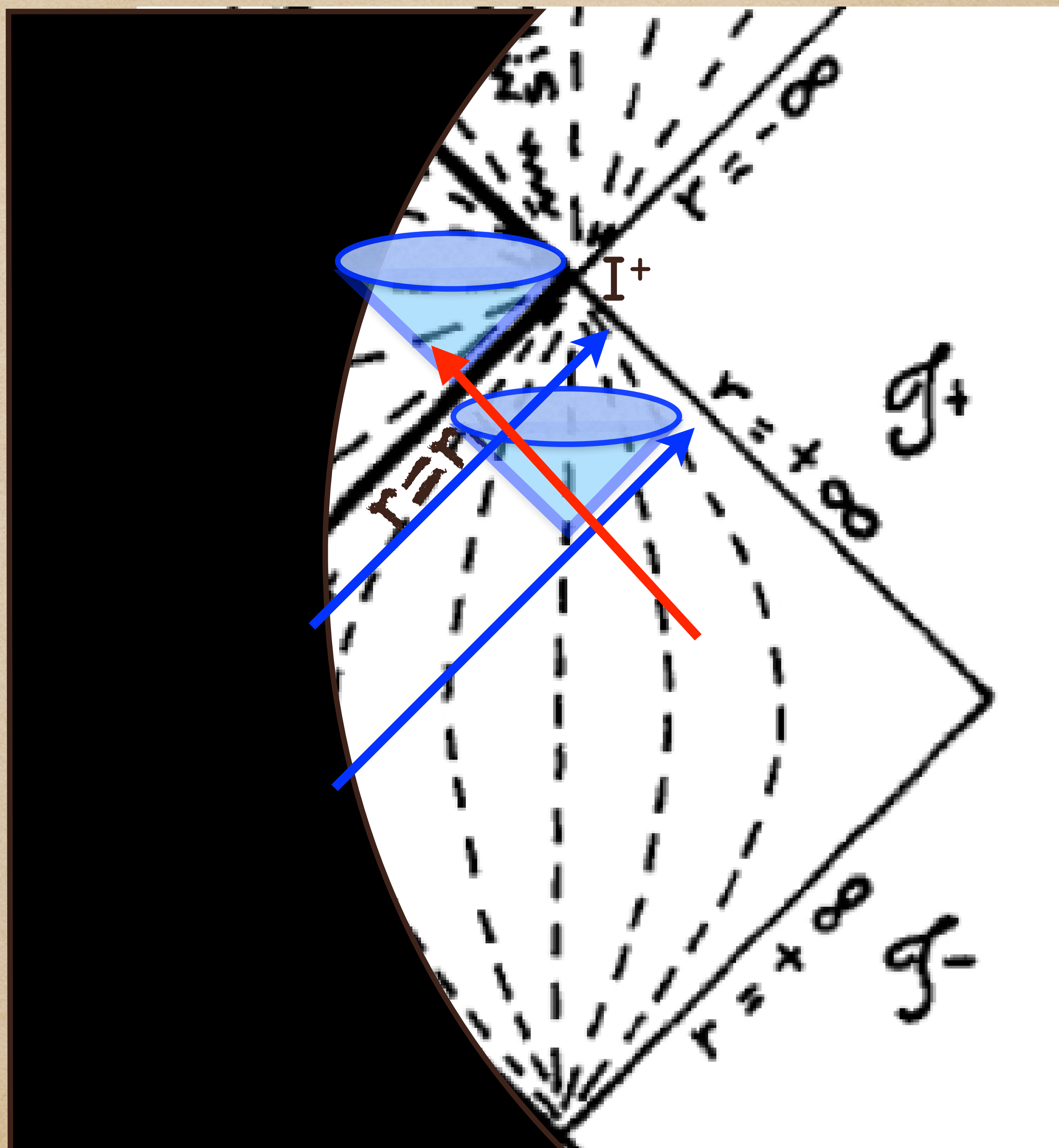
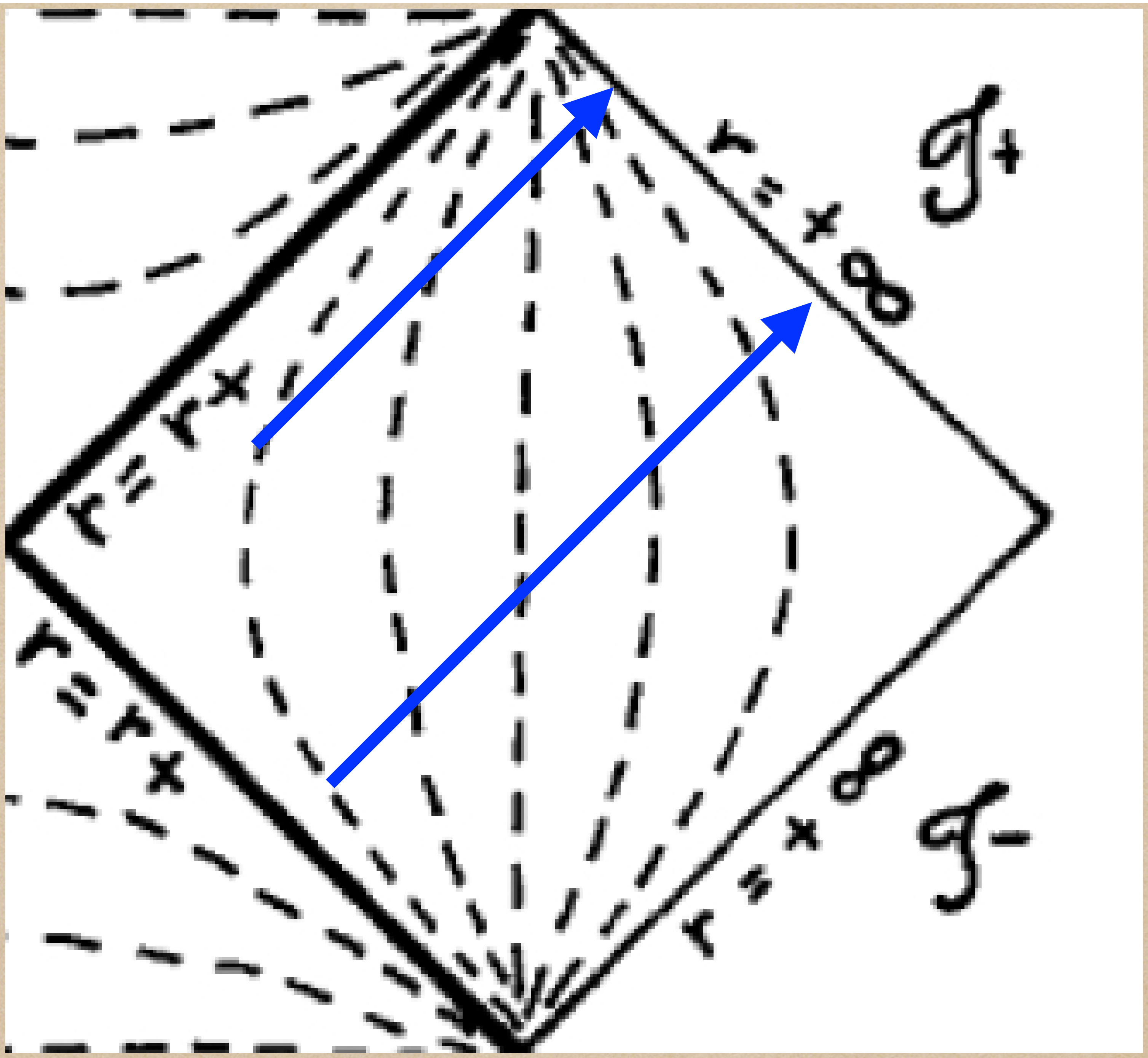


Stretched horizon or membrane



$$r_m = r_H + \delta r$$

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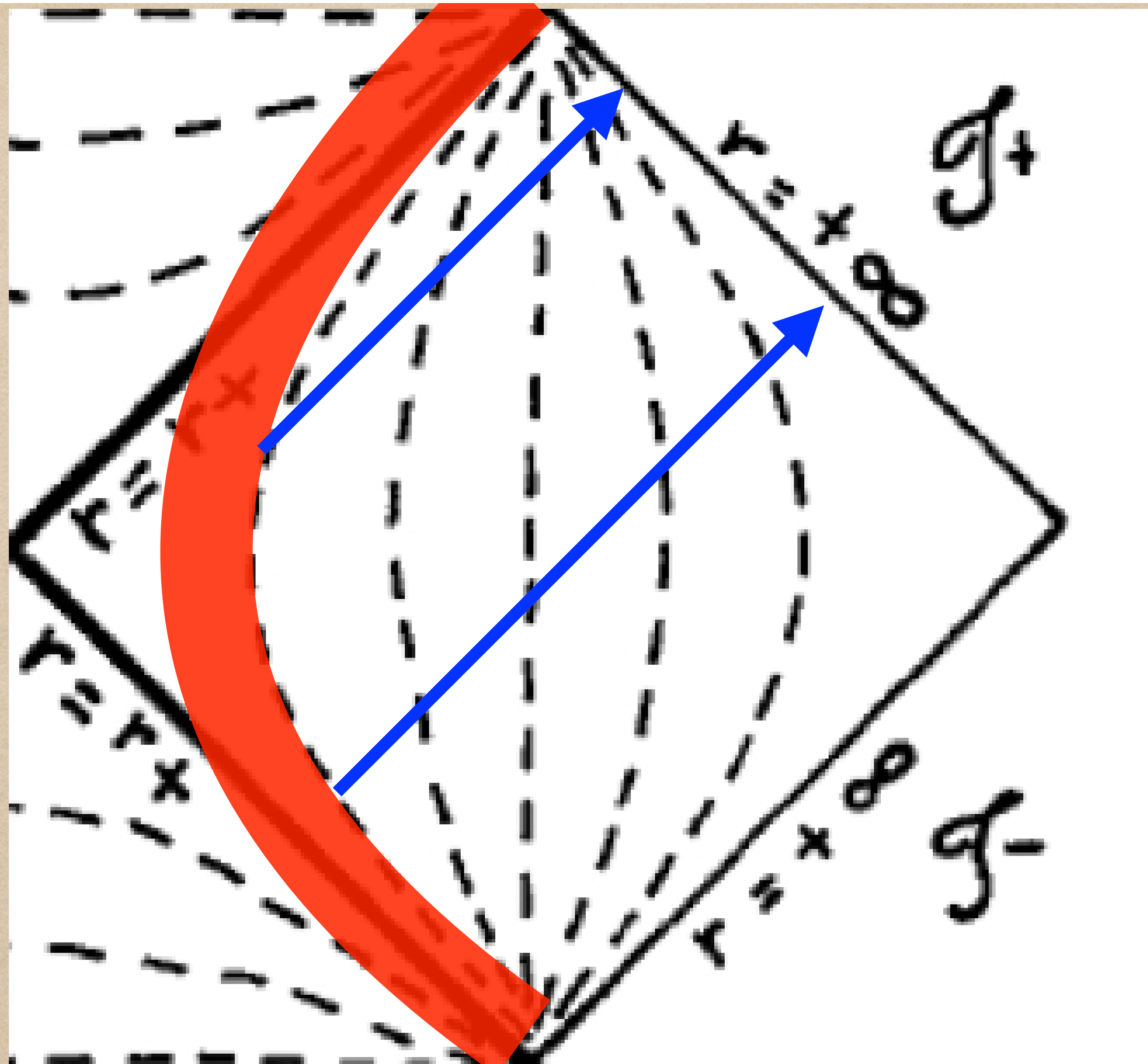

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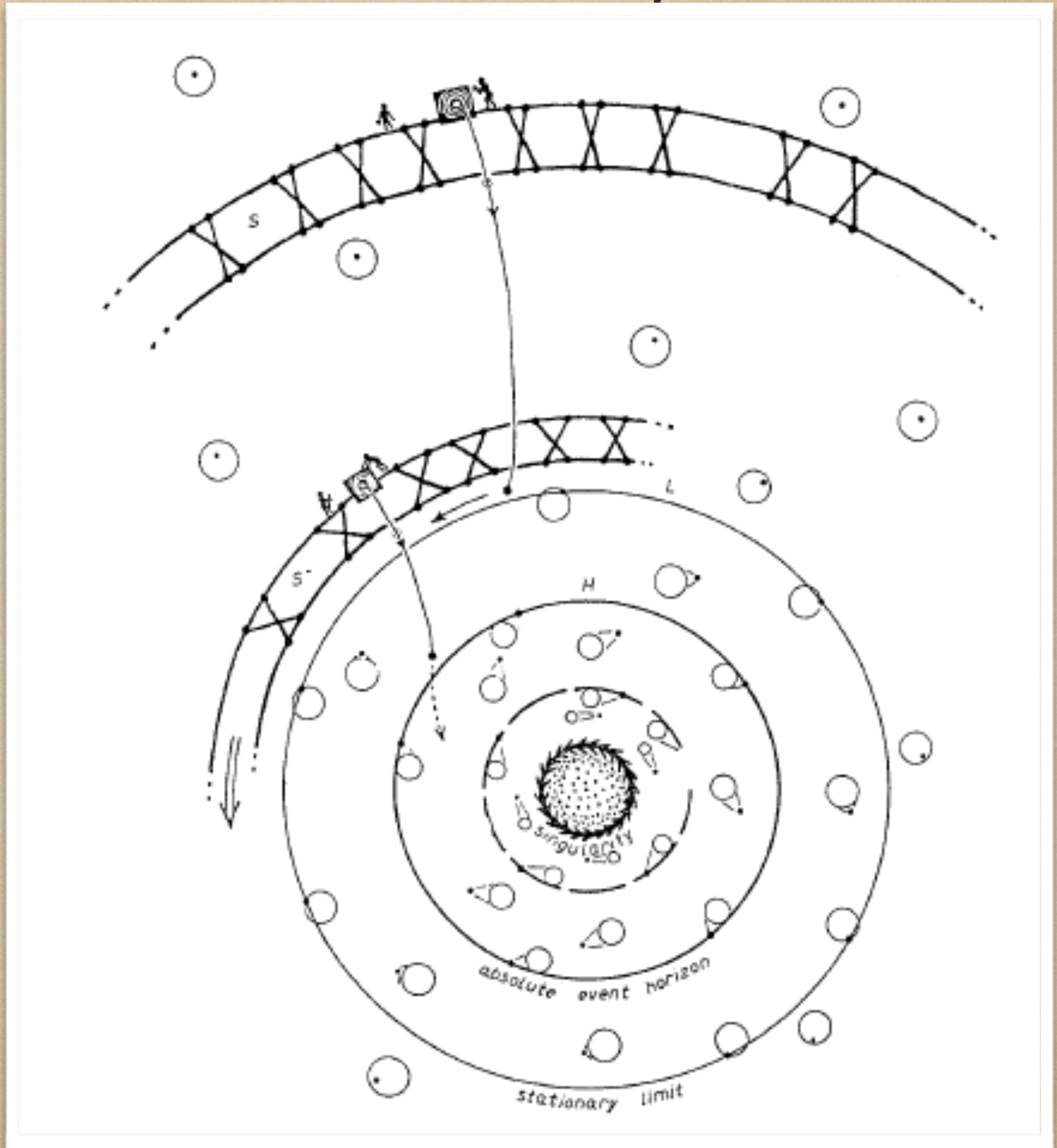
Rotation of Kerr space-time



Penrose '69

The whole of space-time rotates, including the horizon. It is impossible to change the rotation of external space-time without changing the black-hole rotation.

Horizon

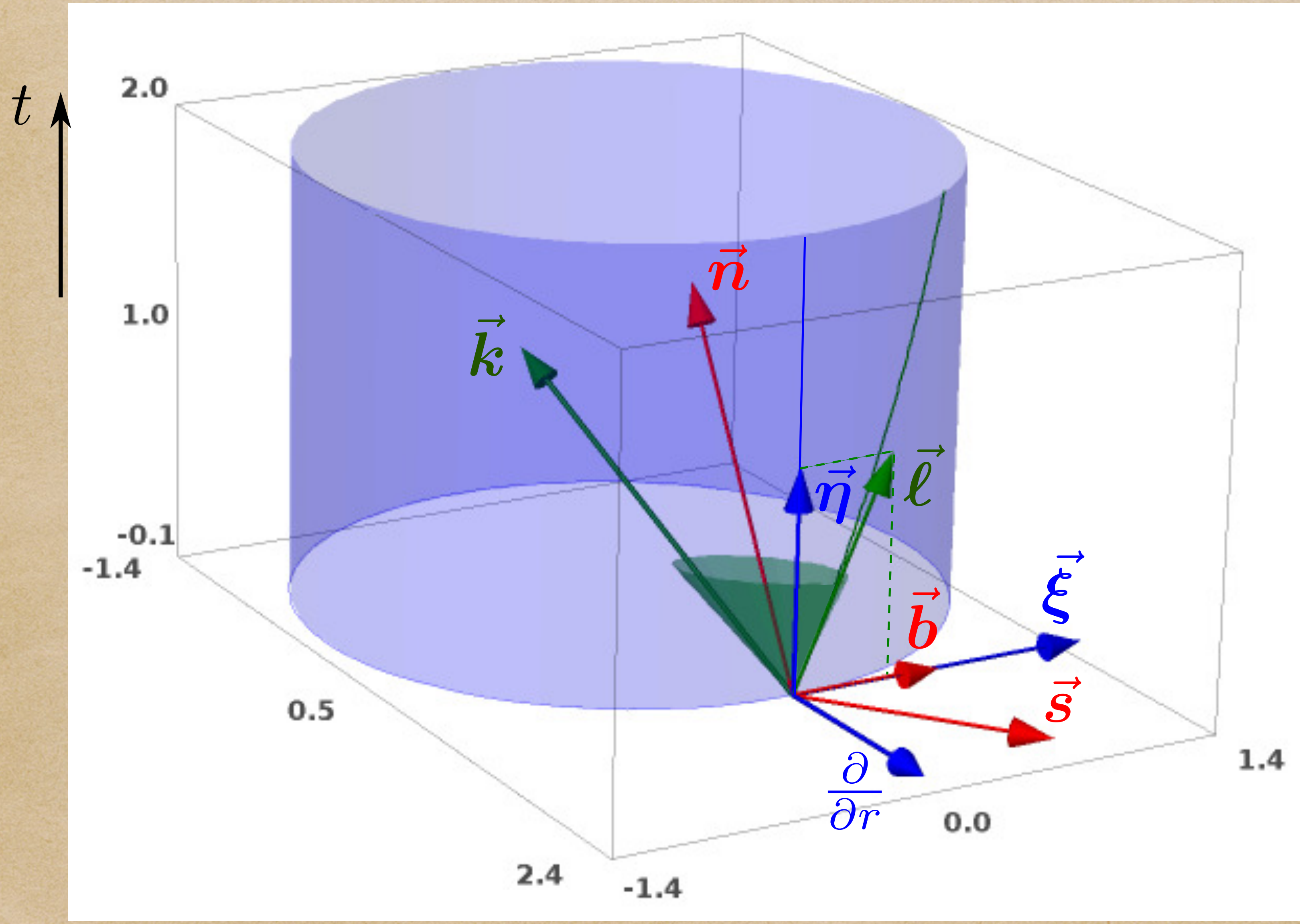
(stationary & axisymmetric)

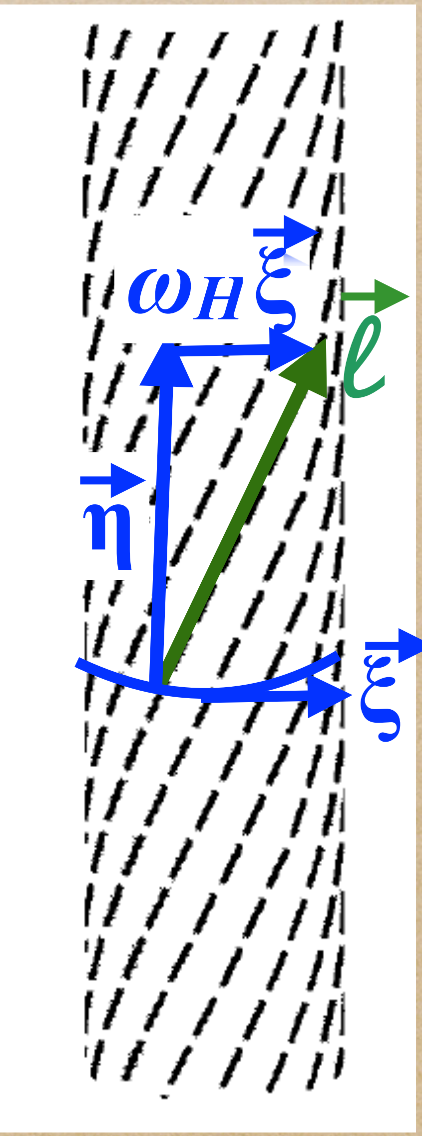
$$\vec{\ell} = \vec{\eta} + \omega_H \vec{\xi},$$

$$\vec{\ell} \cdot \vec{\ell} = 0$$

$$\omega_H = a/[2mr_H],$$

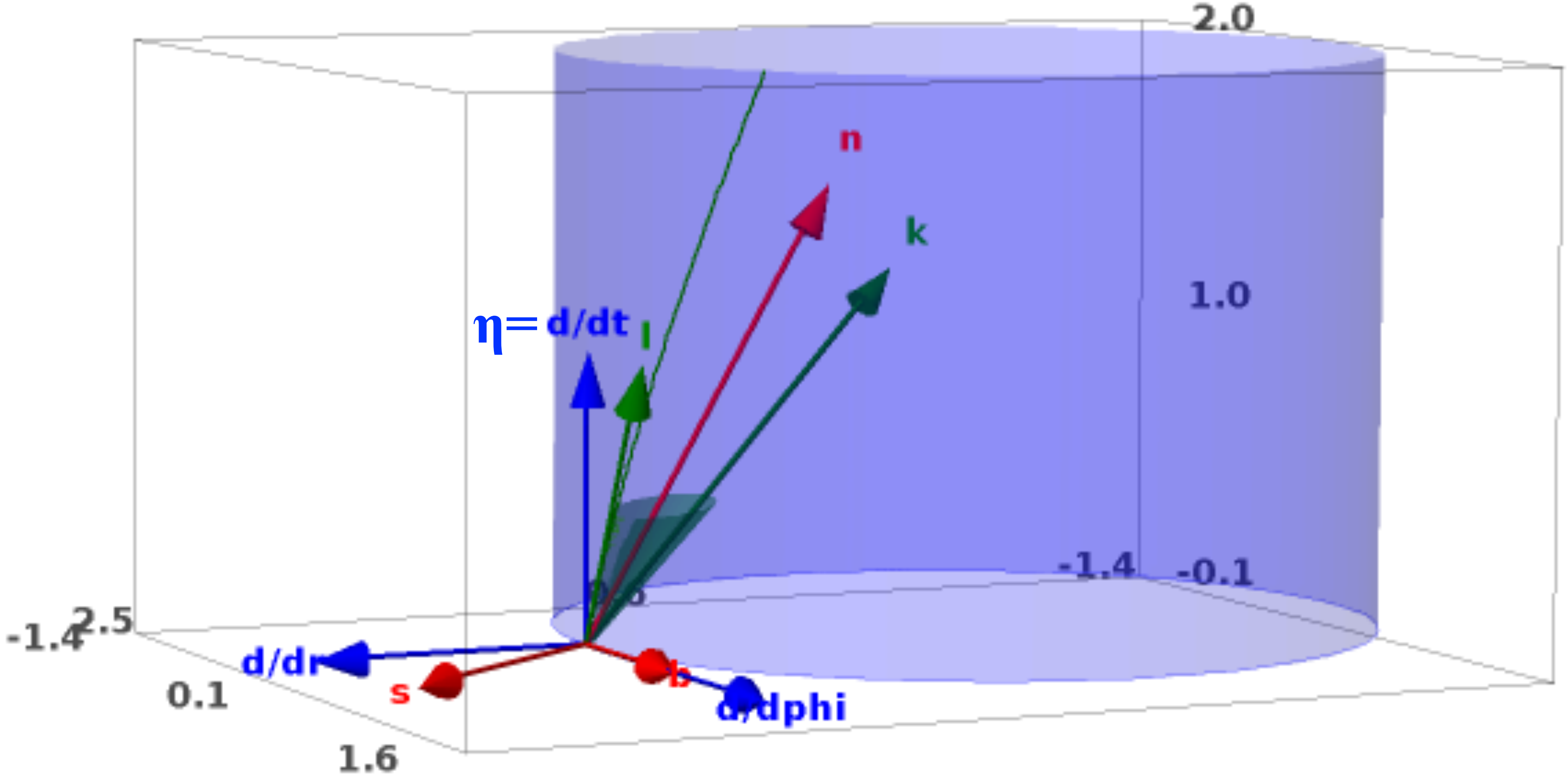
$$r_H = m + \sqrt{m^2 - a^2}$$

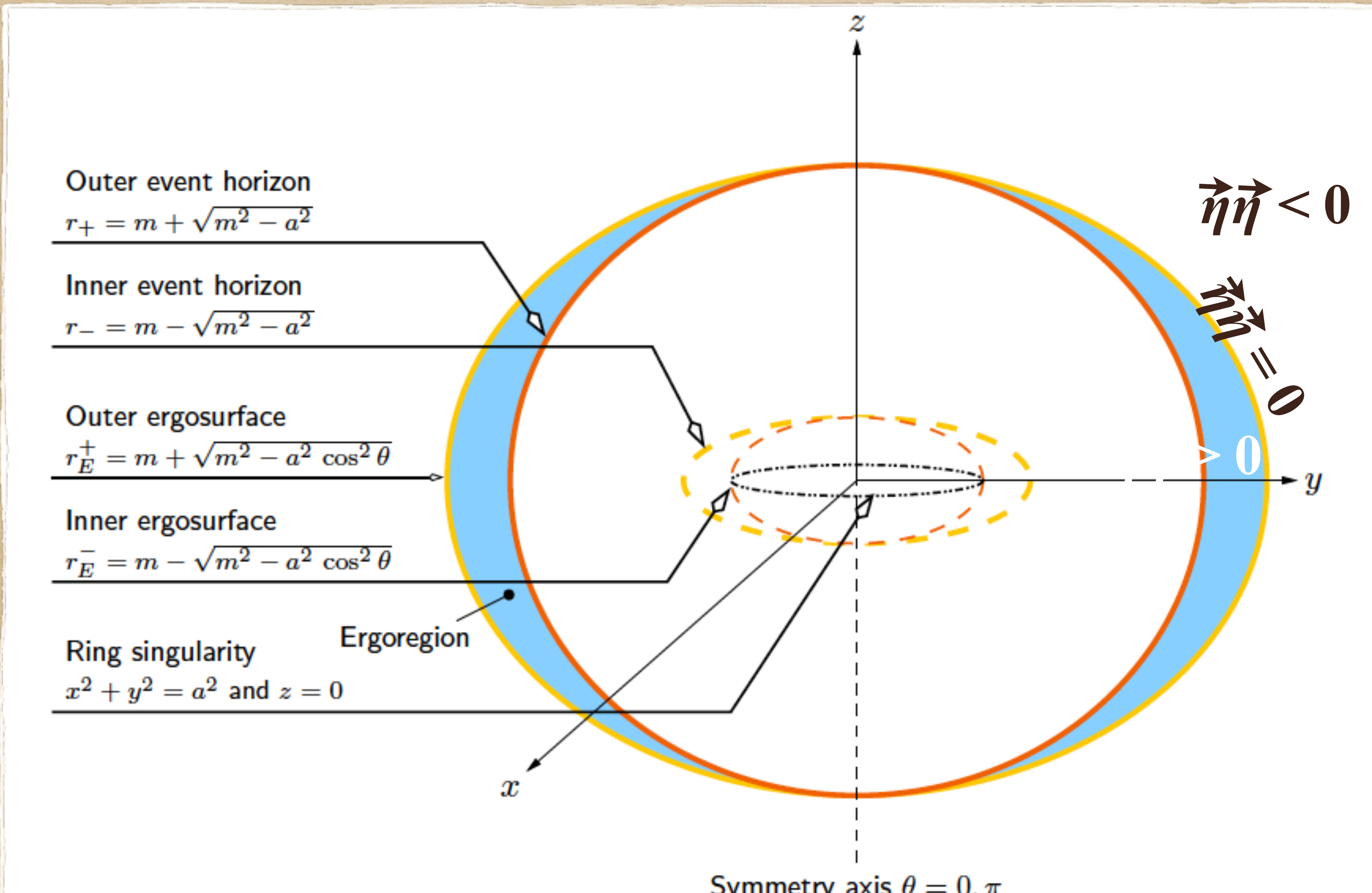




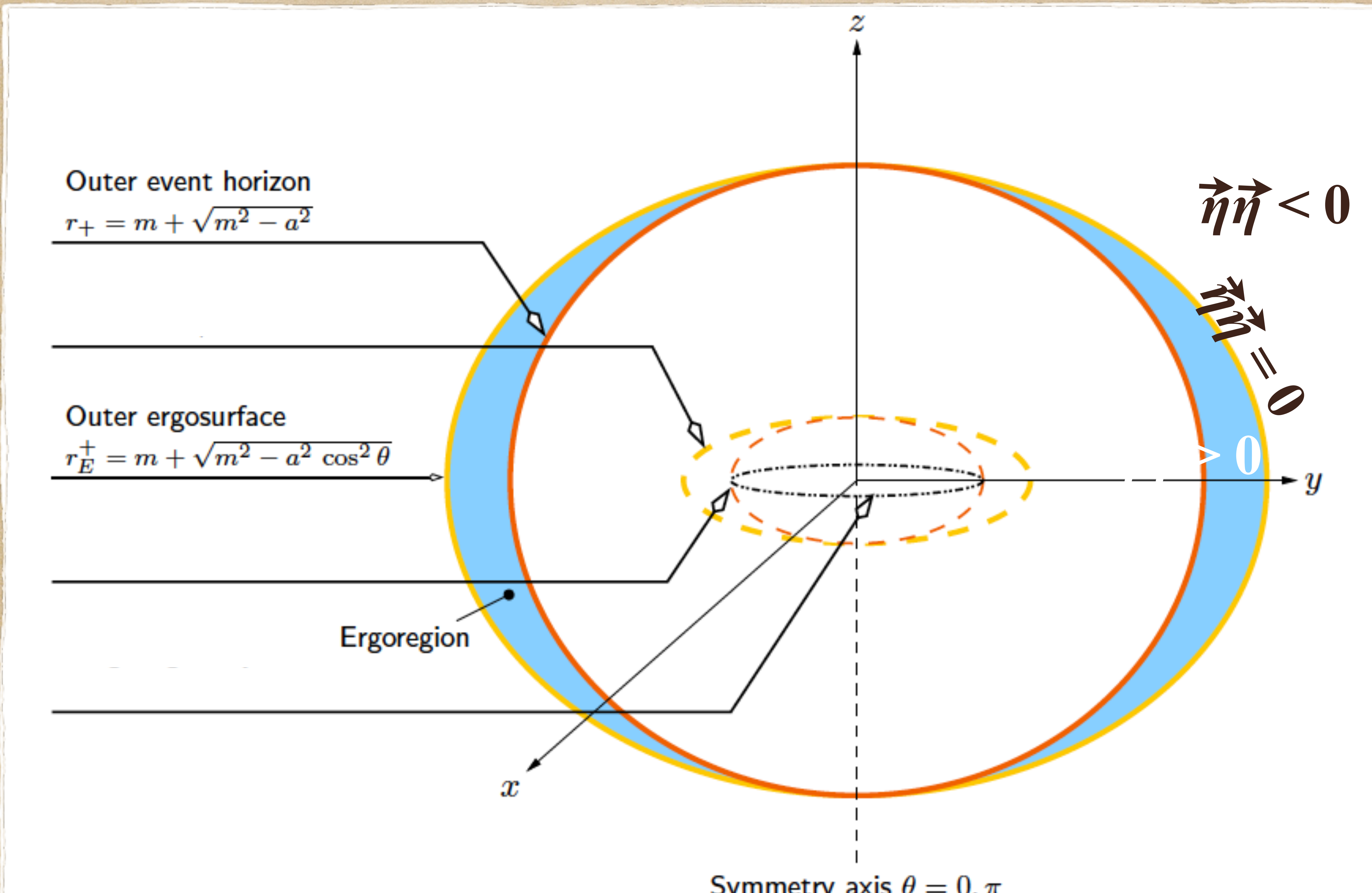
Non-linear relation between BH mass and rotational energy

$$M^2 = M_{\rm irr}^2 + \frac{J^2}{4M_{\rm irr}^2} \qquad M_{\rm irr}^2 = \frac{S}{16\pi} \qquad \Delta M_{\rm irr} \geq 0$$





z
Outer event horizon
$r_+ = m + \sqrt{m^2 - a^2}$
Inner event horizon
$r_- = m - \sqrt{m^2 - a^2}$
Outer ergosurface
$r_E^+ = m + \sqrt{m^2 - a^2 \cos^2\theta}$
Inner ergosurface
$r_E^- = m - \sqrt{m^2 - a^2 \cos^2\theta}$
Ergoregion
Ring singularity
$x^2 + y^2 = a^2$ and $z = 0$
x
y
Symmetry axis $\theta = 0, \pi$
$\vec{\eta}\vec{\eta} < 0$
$\vec{\eta}\vec{\eta} = 0$
> 0



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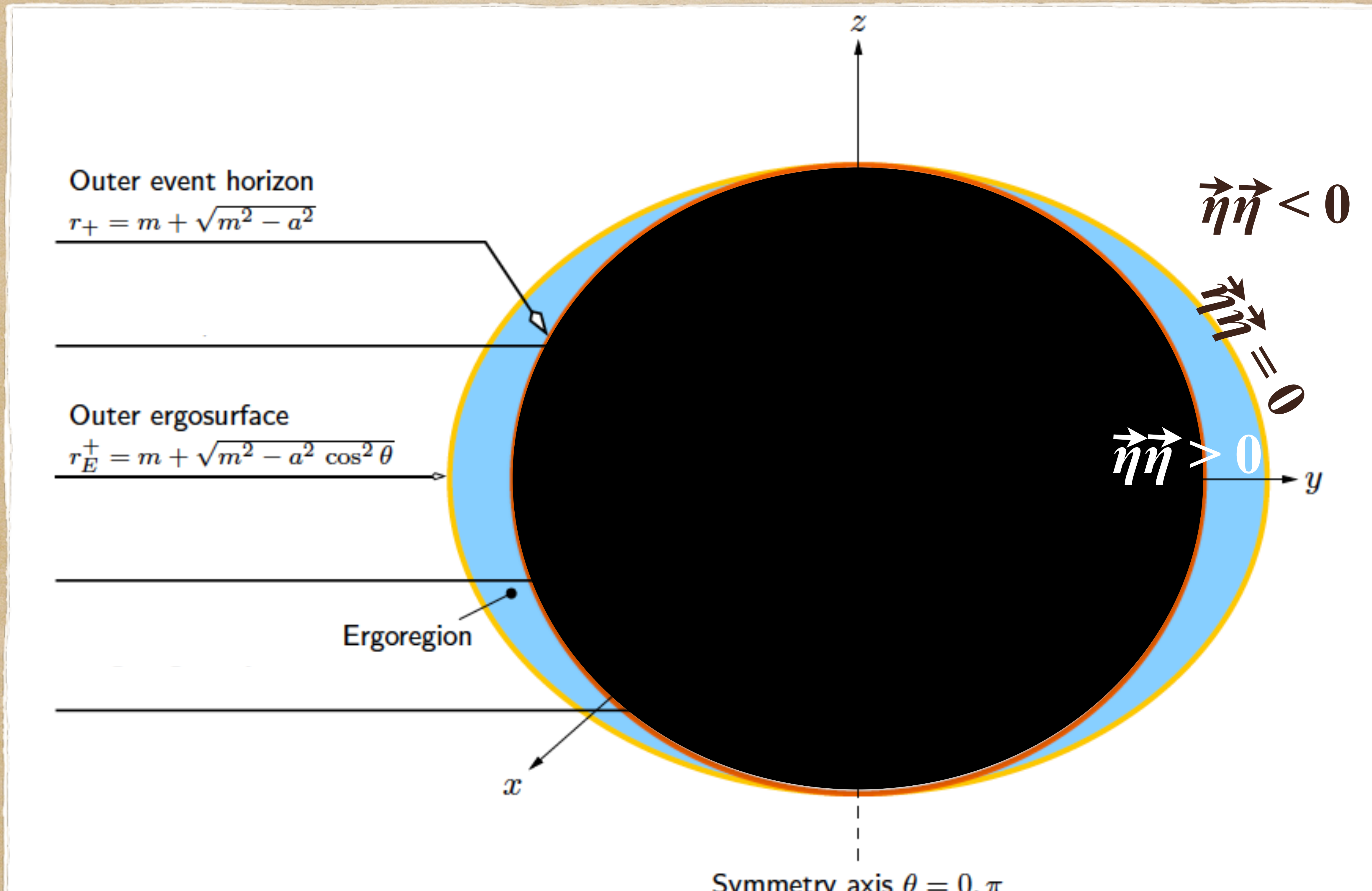

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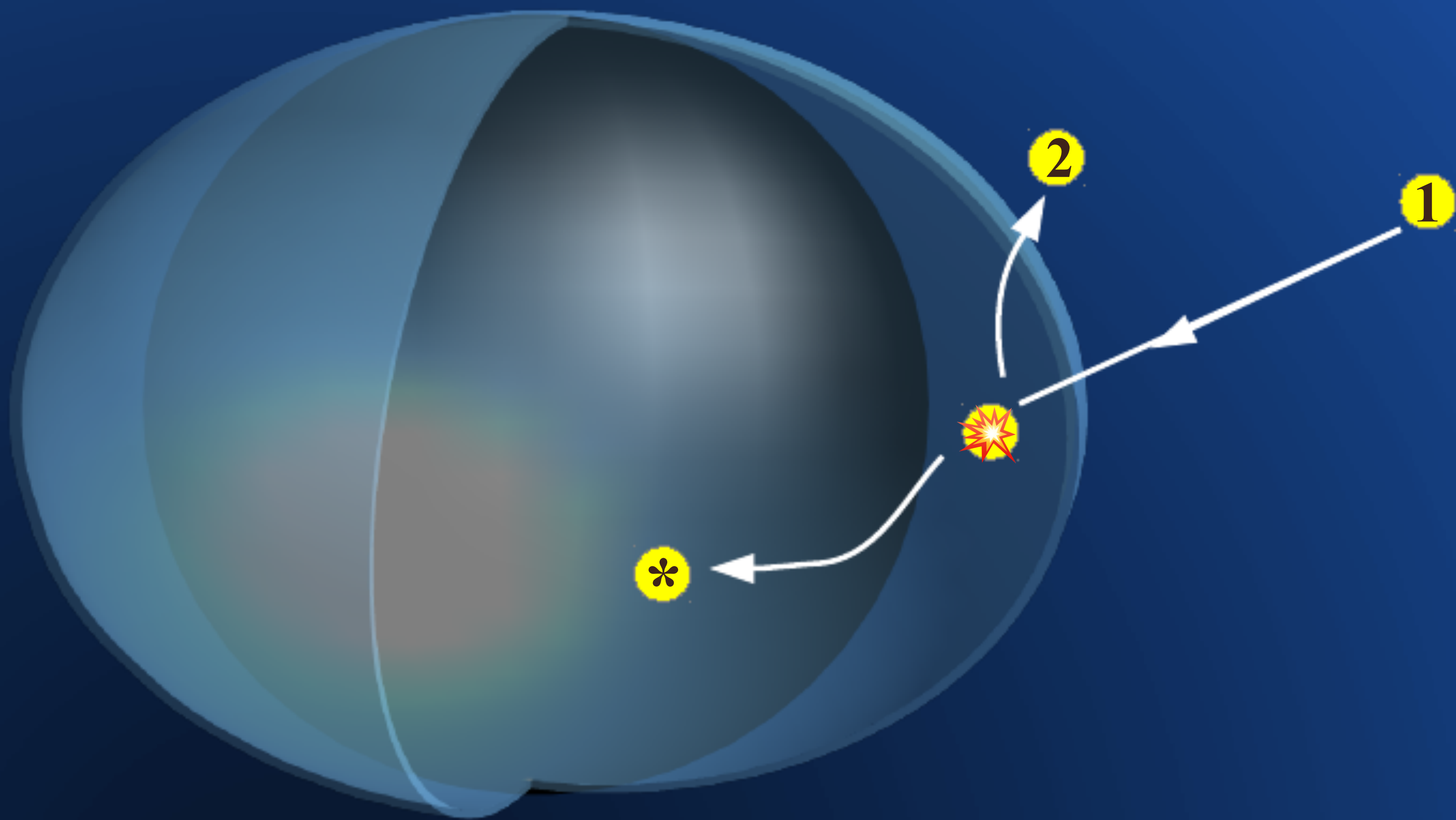
Penrose process

$\vec{p}_1 = \vec{p}_2 + \vec{p}_*$ → $E_1 = E_2 + \Delta E_H$



$\vec{\eta}$ - timelike (at ∞) stationarity Killing vector

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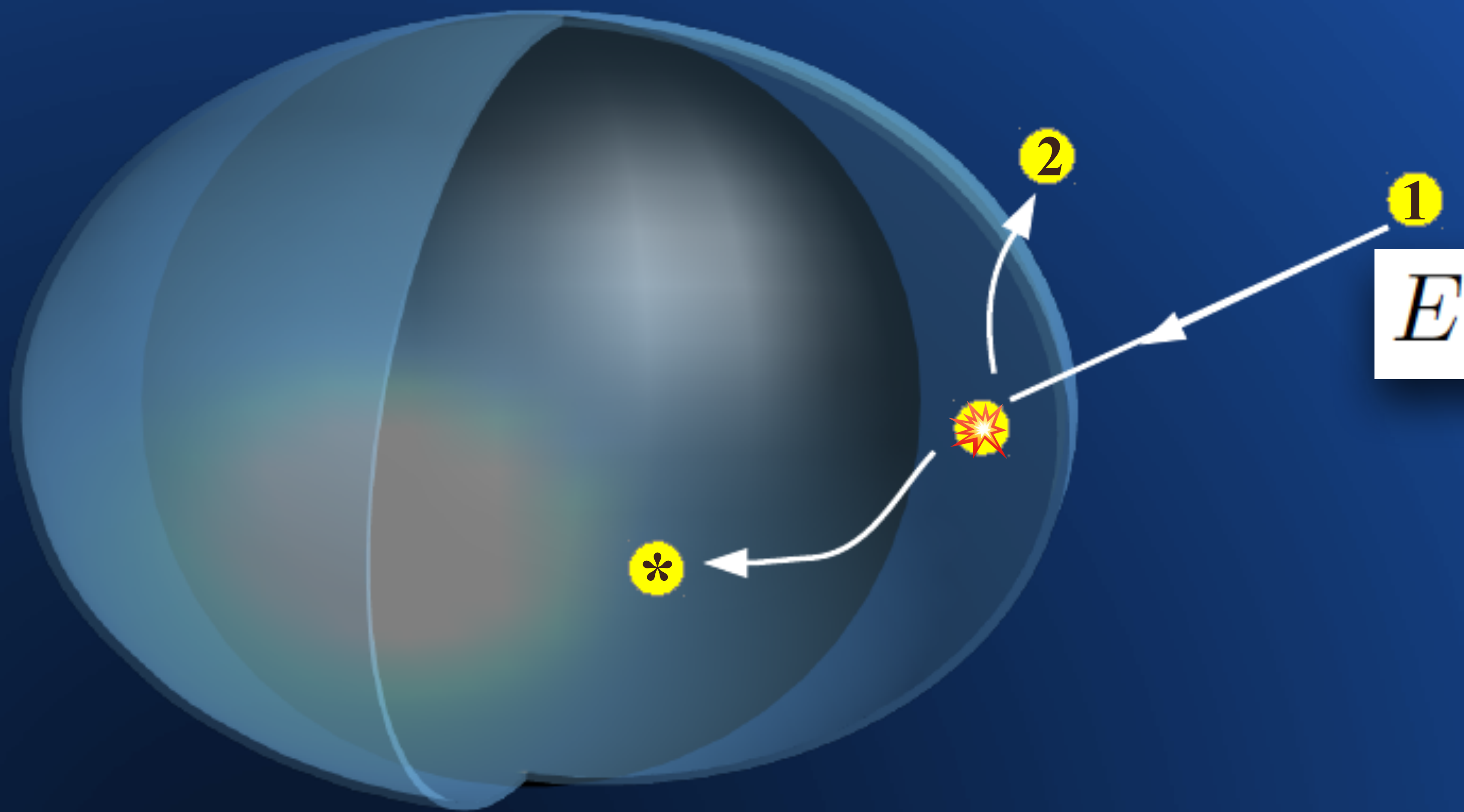

$E_1 = -\vec{\eta} \cdot \vec{p}_1$

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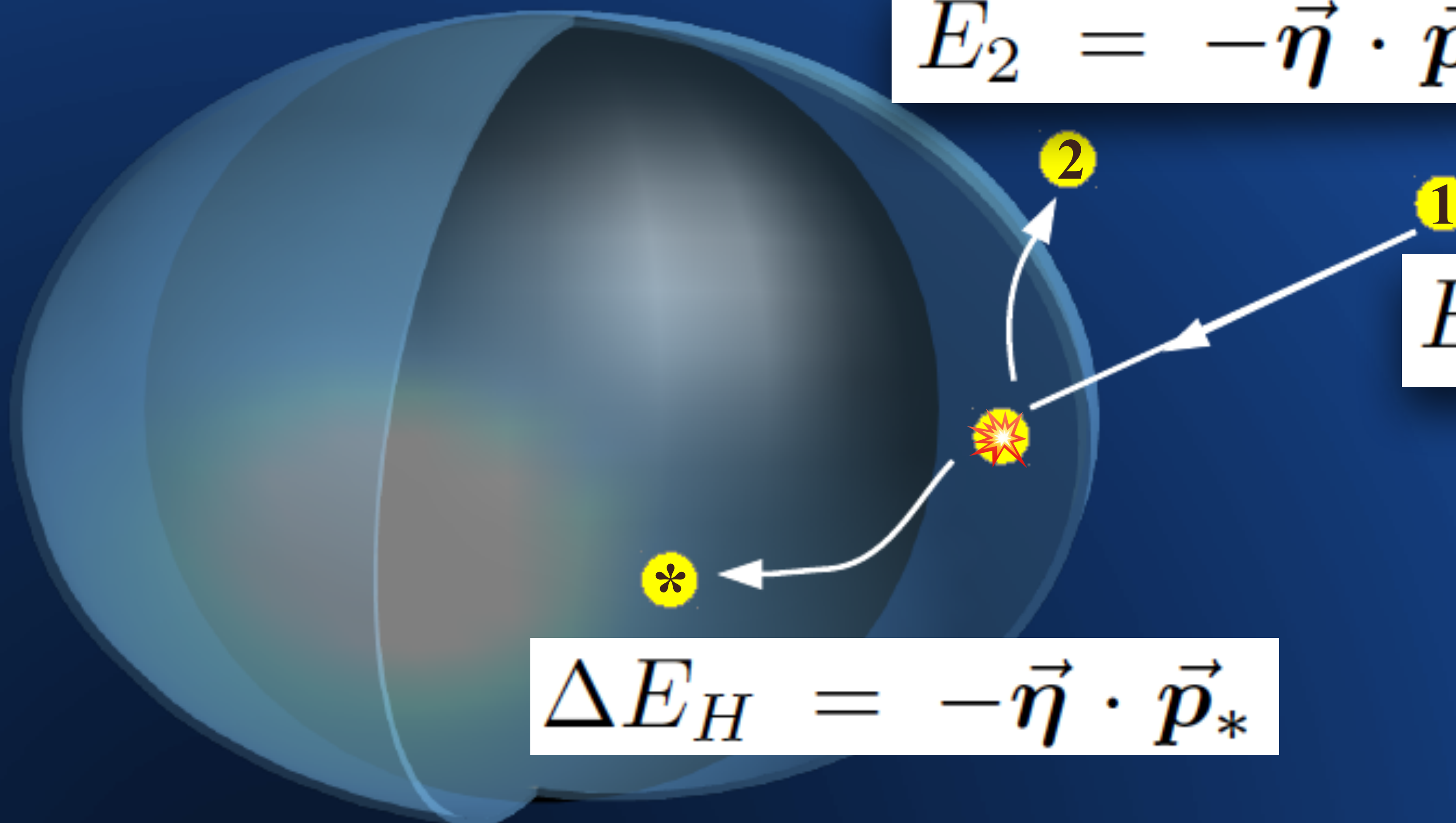
$$E_2 = -\vec{\eta} \cdot \vec{p}_2$$



$$E_1 = -\vec{\eta} \cdot \vec{p}_1$$

$$\Delta E_H = -\vec{\eta} \cdot \vec{p}_*$$

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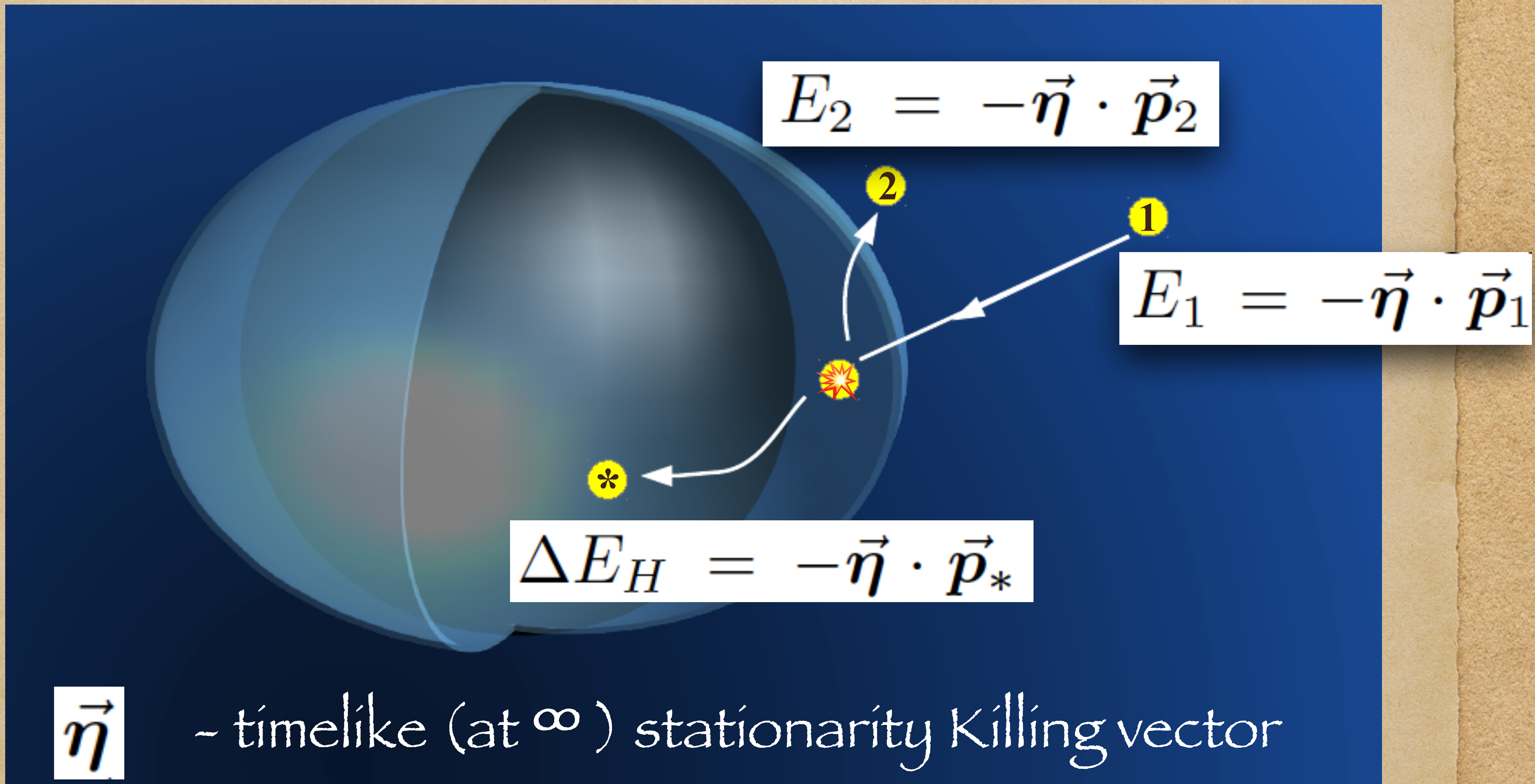

$E_2 > E_1$ if, and only if $\Delta E_H < 0$

$\vec{\eta}$ - timelike (at ∞) stationarity Killing 4-vector

$\vec{\xi}$ - spacelike axisymmetry Killing 4-vector

There are no stationary ($\vec{u} \sim \vec{\eta}$) observers in the ergoregion ($\vec{\eta}\cdot\vec{\eta} > 0$) but there are observers rotating with spacetime:

<u>ZAMO</u> - Zero Angular-Momentum Observers

$$\vec{u} = q\left(\vec{\eta} + \omega\vec{\xi}\right) \qquad \omega = -\frac{\vec{\eta}\cdot\vec{\xi}}{\vec{\xi}\cdot\vec{\xi}} \, .$$

for ZAMOs $\quad \vec{u}\cdot\vec{u} \leq 0$

Therefore energy measured by ZAMOs is always non-negative:

$$-\left(\vec{\boldsymbol{\eta}} + \omega \vec{\boldsymbol{\xi}}\right) \vec{\boldsymbol{p}}_* = (\Delta E_H - \omega_H \Delta J_H) \geq 0$$

$(\omega \longrightarrow \omega_H)$

Hence if $\Delta E_H < 0$ then $\omega_H \Delta J_H \leq \Delta E_H$

Since $\omega_H \geq 0$ when $\omega_H \neq 0$

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$$\Delta J_H < 0.$$

T - energy moment tensor

$$T_{\mu\nu}\ell^{\mu}\ell^{\nu}|_{\mathcal{H}} \geq 0.$$

- null energy condition

- **Energy conservation**

$$P^{\alpha} = -T^{\alpha}_{\ \mu}\eta^{\mu}$$

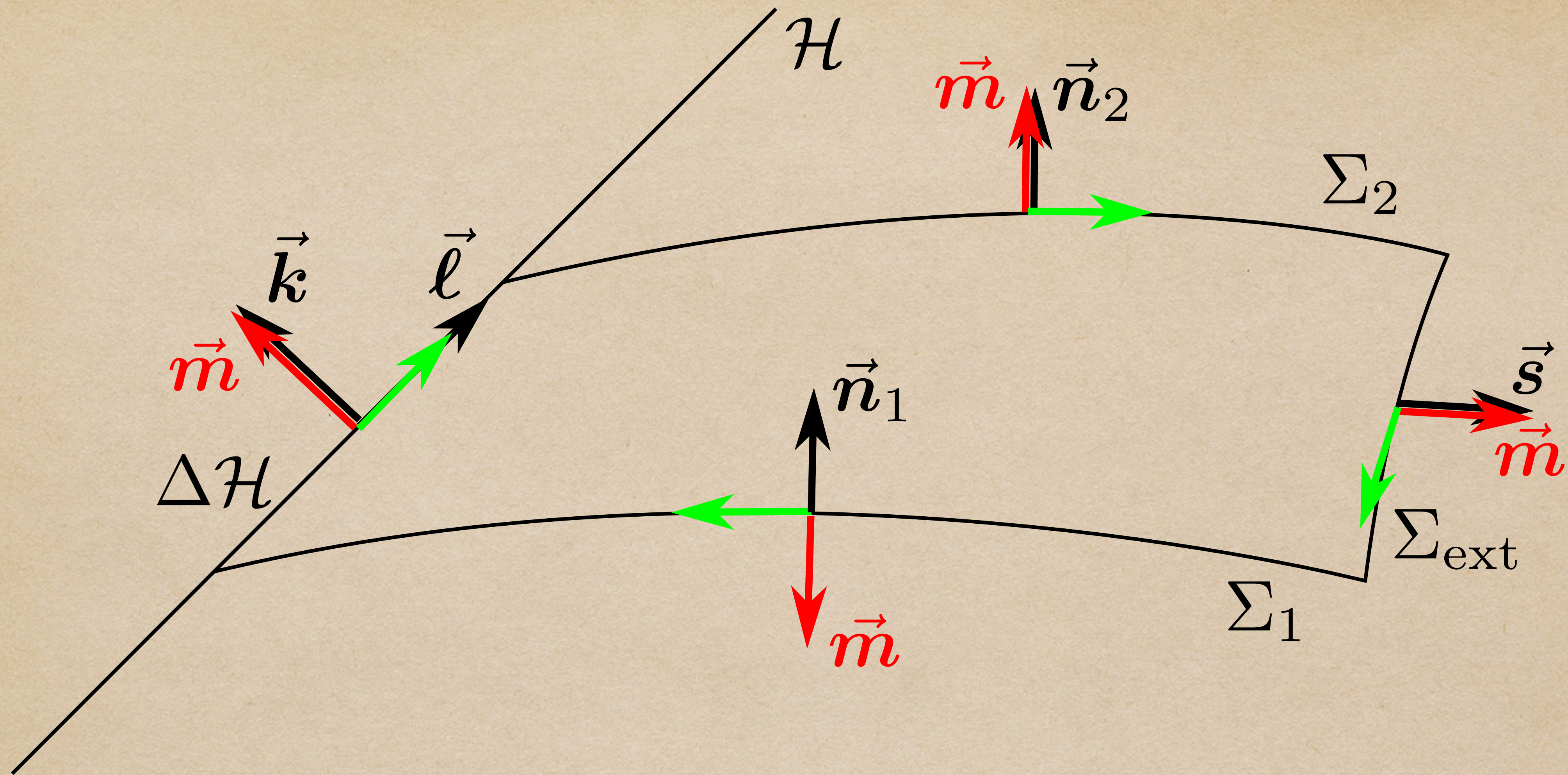
- Noether current (« energy momentum density vector »)

$$\nabla_{\mu}P^{\mu} = 0$$

so by Stoke's theorem:

$$\oint_{\mathcal{V}} \boldsymbol{\epsilon}(\vec{\boldsymbol{P}}) = 0,$$

$$\epsilon(\vec{\boldsymbol{P}})_{\alpha\beta\gamma} = P^{\mu}\epsilon_{\mu\alpha\beta\gamma}$$



$$\int_{\Sigma_1\downarrow} \epsilon(\vec{P}) + \int_{\underleftarrow{\Delta\mathcal{H}}} \epsilon(\vec{P}) + \int_{\Sigma_2\uparrow} \epsilon(\vec{P}) + \int_{\underrightarrow{\Sigma_{\rm ext}}} \epsilon(\vec{P}) = 0$$

$$E_1 := \int_{\Sigma_1 \uparrow} \epsilon(\vec{\boldsymbol{P}}) = -\int_{\Sigma_1} P_\mu n_1^\mu \, \mathrm{dV}$$

$$E_2 := \int_{\Sigma_2 \uparrow} \epsilon(\vec{\boldsymbol{P}}) = -\int_{\Sigma_2} P_\mu n_2^\mu \, \mathrm{dV}$$

$$\Delta E_{\mathrm{ext}} := \int_{\underrightarrow{\Sigma_{\mathrm{ext}}}} \epsilon(\vec{\boldsymbol{P}}) = \int_{\Sigma_{\mathrm{ext}}} P_\mu s^\mu \, \mathrm{dV}$$

$$\Delta E_H := \int_{\underleftarrow{\Delta\mathcal{H}}} \epsilon(\vec{\boldsymbol{P}}) = -\int_{\Delta\mathcal{H}} P_\mu \ell^\mu \, \mathrm{dV}$$

$$E_2 + \Delta E_{\mathrm{ext}} - E_1 = -\Delta E_H$$

****

$$M^\alpha = T^\alpha_{\ \mu} \xi^\mu$$

- angular-momentum density vector

$$J_2 + J_{\mathrm{ext}} - J_1 = -\Delta J_H$$

Energy « gain »: $\Delta E := E_2 + \Delta E_{\text{ext}} - E_1$

can be positive, if and only if $\Delta E_H < 0$

We refer to any such process as a Penrose process.

$$T_{\mu\nu}\ell^\mu\ell^\nu = T_{\mu\nu}(\eta^\nu + \omega_H \xi^\nu)\ell^\mu = -P_\mu \ell^\mu + \omega_H M_\mu \ell^\mu$$

$$-\int_{\Delta\mathcal{H}} P_\mu \ell^\mu \, \mathrm{dV} + \omega_H \int_{\Delta\mathcal{H}} M_\mu \ell^\mu \, \mathrm{dV} \geq 0$$ $$\omega_H \Delta J_H \leq \Delta E_H$$ $$\Delta J_H < 0$$

For a matter distribution or a nongravitational field obeying the null energy condition, a necessary and sufficient condition for energy extraction from a rotating black hole is that it absorbs negative energy ΔE_H and negative angular momentum ΔJ_H .

Penrose process in terms of the Noether current

$\Delta E_H < 0$ implies $P_\mu \ell^\mu > 0$

but since $\vec{\boldsymbol{\ell}}$ is a future-directed null vector

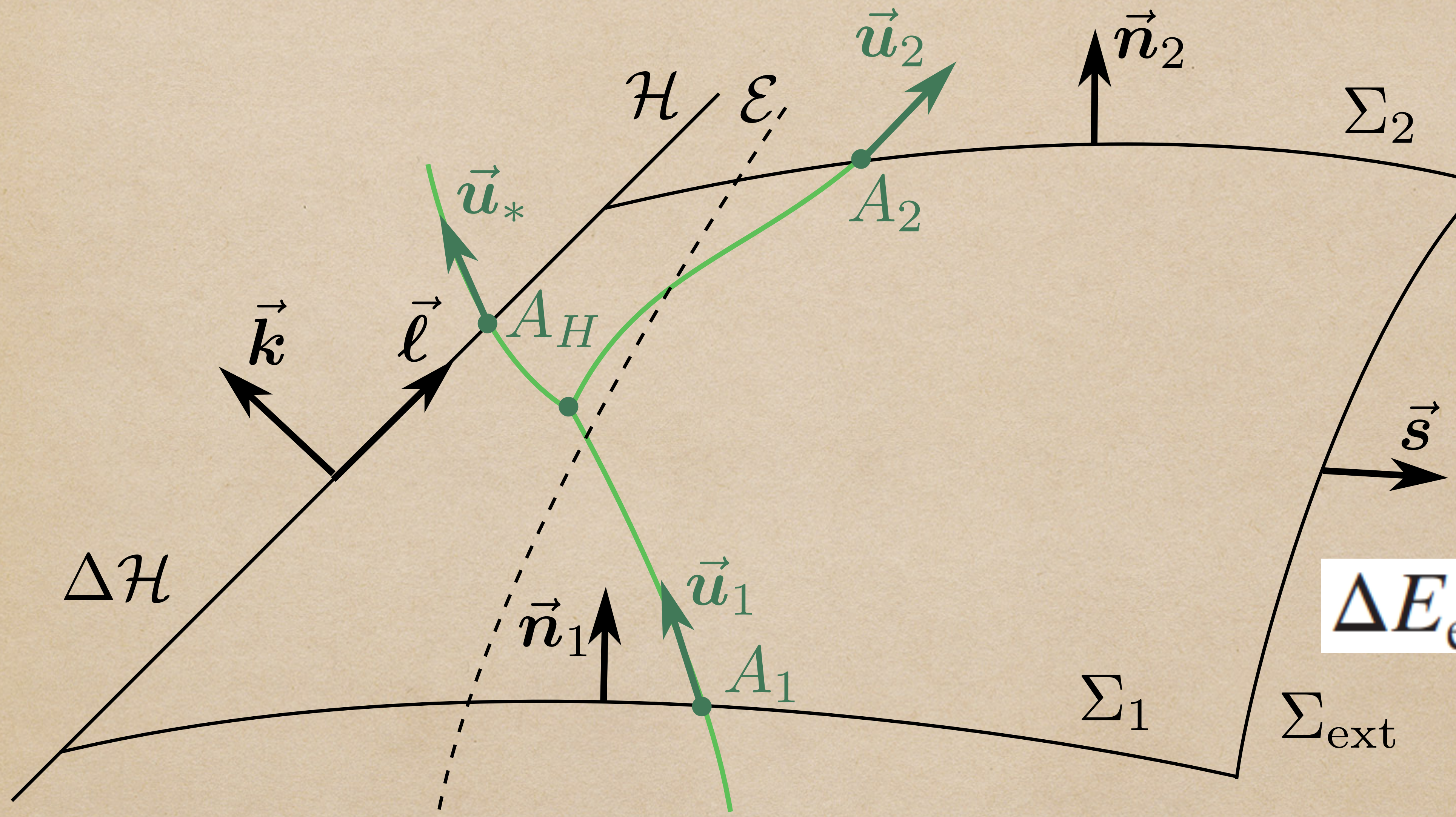
this is possible if, and only if $\vec{\boldsymbol{P}}$ is either

(i) spacelike, or

(ii) or past-directed timeline or past-directed null

A necessary condition for a Penrose process to occur is to have the Noether current $\vec{\boldsymbol{P}}$ be spacelike or past directed (timelike or null) on some part of $\Delta\mathcal{H}$.

Mechanical Penrose process



$$\Delta E_{\rm ext} = 0$$

$$T_{\alpha\beta}(M) = \mathfrak{m}\int_{-\infty}^{+\infty} \delta_{A(\tau)}(M)\; g_\alpha{}^\mu(M, A(\tau))u_\mu(\tau) \times g_\beta{}^\nu(M, A(\tau))u_\nu(\tau)\, \mathrm{d}\tau$$

$$\delta_A(M) = \frac{1}{\sqrt{-g}}\,\delta(x^0 - z^0)\,\delta(x^1 - z^1)\,\delta(x^2 - z^2)\,\delta(x^3 - z^3),$$

$$P_\alpha(M) = \mathfrak{m}\int_{-\infty}^{+\infty} \delta_{A(\tau)}(M)\,[-\,g_\sigma{}^\nu(M, A(\tau))u_\nu(\tau)\eta^\sigma(M)] \times g_\alpha{}^\mu(M, A(\tau))u_\mu(\tau)\,\mathrm{d}\tau.$$

$E_1 = -\mathfrak{m}_1\,(\eta_\mu u_1^\mu)|_{A_1} = -\mathfrak{m}_1\,\eta_\mu u_1^\mu$, $\Delta E_H = -\mathfrak{m}_*\,(\eta_\mu u_*^\mu)|_{A_H} = -\mathfrak{m}_*\,\eta_\mu u_*^\mu$

$E_2 = -\mathfrak{m}_2\,\eta_\mu u_2^\mu$ $E_2 + \Delta E_{\mathrm{ext}} - E_1 = -\Delta E_H$ so $E_2 > E_1$ if and only if

$\Delta E_H < 0,$ if and only if $\eta_\mu u_*^\mu > 0$

(which is possible in the ergosphere only)

$\vec{P}_*$ is collinear to $\vec{u}_*$ so it is timelike and past-directed

because [ ] is negative.

$$\delta_A(M) = \frac{1}{\sqrt{-g}}\,\delta(x^0 - z^0)\,\delta(x^1 - z^1)\,\delta(x^2 - z^2)\,\delta(x^3 - z^3),$$

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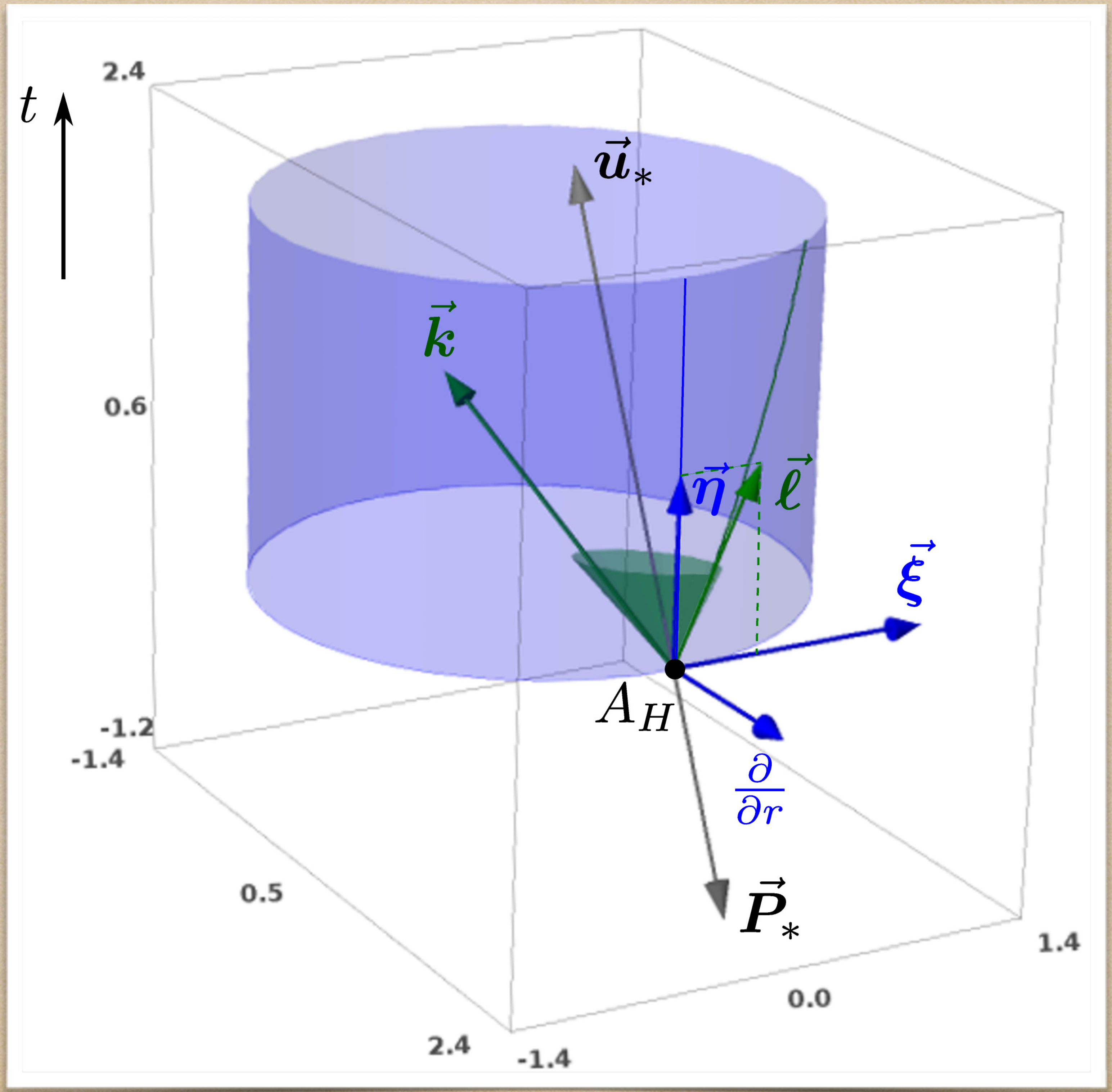
Mechanical Penrose process



General electromagnetic field

$$T_{\alpha\beta} = \frac{1}{\mu_0}\left(F_{\mu\alpha}F^{\mu}{}_{\beta} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\, g_{\alpha\beta}\right)$$

Therefore the integrand in $\Delta E_H = -\int_{\Delta\mathcal{H}} P_\mu \ell^\mu \, \mathrm{dV}$ is:

$$\boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \frac{1}{\mu_0}\left(F_{\mu\rho}\eta^\rho F^{\mu}{}_{\sigma}\ell^\sigma - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\, \vec{\boldsymbol{\eta}}\cdot\vec{\boldsymbol{\ell}}\right)$$

since $\vec{\boldsymbol{\eta}}\cdot\vec{\boldsymbol{\ell}} = 0$

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = F_{\mu\rho}\eta^\rho F^{\mu}{}_{\sigma}\ell^\sigma$$

- pseudoelectric field 1-form on $\mathscr{H}$

$$\boldsymbol{E} := \boldsymbol{F}(., \vec{\boldsymbol{\ell}})$$

Hence $\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \boldsymbol{F}(\vec{\boldsymbol{E}}, \vec{\boldsymbol{\eta}})$

or $\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{E}} - \omega_H \boldsymbol{F}(\vec{\boldsymbol{E}}, \vec{\boldsymbol{\xi}})$

therefore $\Delta E_H < 0,$

if $\omega_H \boldsymbol{F}(\vec{\boldsymbol{E}}, \vec{\boldsymbol{\xi}}) > \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{E}}$ in some part of $\Delta\mathcal{H}$.

This is the most general condition on <u>any electromagnetic field</u> configuration allowing black-hole energy extraction through a Penrose process

(Since $\vec{\boldsymbol{E}}$ is tangent to $\mathscr{H}$ $\vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{E}} \geq 0$)

- Stationary and axisymmetric electromagnetic field

$$\mathcal{L}_{\vec{\eta}} \boldsymbol{F} = 0 \qquad \text{and} \qquad \mathcal{L}_{\vec{\xi}} \boldsymbol{F} = 0$$

therefore

$$\boldsymbol{F}(., \vec{\eta}) = \mathbf{d}\Phi$$
$$\boldsymbol{F}(., \vec{\xi}) = \mathbf{d}\Psi$$
$$^\star\boldsymbol{F}(\vec{\eta}, \vec{\xi}) = I,$$

Φ, Ψ and *I* are gauge-invariant. Introducing a 1-form ***A*** such that ***F***=d**A** one can choose A so that

$$\Phi = \langle \boldsymbol{A}, \vec{\eta} \rangle = A_t \qquad \Psi = \langle \boldsymbol{A}, \vec{\xi} \rangle = A_\varphi.$$

and $\boldsymbol{E} = \mathbf{d}(\Phi + \omega_H \Psi)$ is a pure gradient.

Force free case (Blandford-Znajek)

$$\boldsymbol{F}(\vec{\boldsymbol{j}}, .) = 0$$

$\vec{\boldsymbol{j}}$ - electric 4-current. From stationarity

$$\vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{\nabla}}\Phi = 0 \qquad \text{and} \qquad \vec{\boldsymbol{j}} \cdot \vec{\boldsymbol{\nabla}}\Psi = 0$$

so there exists a function ω(Ψ) such that

$$\mathbf{d}\Phi = -\omega(\Psi)\mathbf{d}\Psi$$

One gets

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \vec{\boldsymbol{\nabla}}\Phi \cdot \vec{\boldsymbol{\nabla}}(\Phi + \omega_H \Psi)$$

so $\vec{\boldsymbol{\nabla}}\Psi$ is tangent to $\mathscr{H}$

(Blandford & Znajek 1977)

One gets

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \omega(\Psi)\,(\omega(\Psi) - \omega_H)\; \vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi.$$

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$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \omega(\Psi)\,(\omega(\Psi) - \omega_H)\ \vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi.$$

$$\vec{\boldsymbol{\ell}} \cdot \vec{\boldsymbol{\nabla}}\Psi = \vec{\boldsymbol{\eta}} \cdot \vec{\boldsymbol{\nabla}}\Psi + \omega_H \vec{\boldsymbol{\xi}} \cdot \vec{\boldsymbol{\nabla}}\Psi = \underbrace{\mathcal{L}_{\vec{\boldsymbol{\eta}}}\Psi}_{0} + \omega_H \underbrace{\mathcal{L}_{\vec{\boldsymbol{\xi}}}\Psi}_{0} = 0$$

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so $\vec{\boldsymbol{\nabla}}\Psi$ is tangent to $\mathscr{H}$

- therefore on $\mathscr{H}$ $\vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi \geq 0$

(Blandford & Znajek 1977)

One gets

$$\mu_0 \boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) = \omega(\Psi)\left(\omega(\Psi) - \omega_H\right) \vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi .$$

$$\vec{\boldsymbol{\ell}} \cdot \vec{\boldsymbol{\nabla}}\Psi = \vec{\boldsymbol{\eta}} \cdot \vec{\boldsymbol{\nabla}}\Psi + \omega_H \vec{\boldsymbol{\xi}} \cdot \vec{\boldsymbol{\nabla}}\Psi = \underbrace{\mathcal{L}_{\vec{\boldsymbol{\eta}}}\Psi}_{0} + \omega_H \underbrace{\mathcal{L}_{\vec{\boldsymbol{\xi}}}\Psi}_{0} = 0$$

so $\vec{\boldsymbol{\nabla}}\Psi$ is tangent to $\mathscr{H}$

- therefore on $\mathscr{H}$ $\vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi \geq 0$ and

$$\boldsymbol{T}(\vec{\boldsymbol{\eta}}, \vec{\boldsymbol{\ell}}) < 0 \iff \begin{cases} 0 < \omega(\Psi) < \omega_H \\ \vec{\boldsymbol{\nabla}}\Psi \cdot \vec{\boldsymbol{\nabla}}\Psi \neq 0 \end{cases}$$

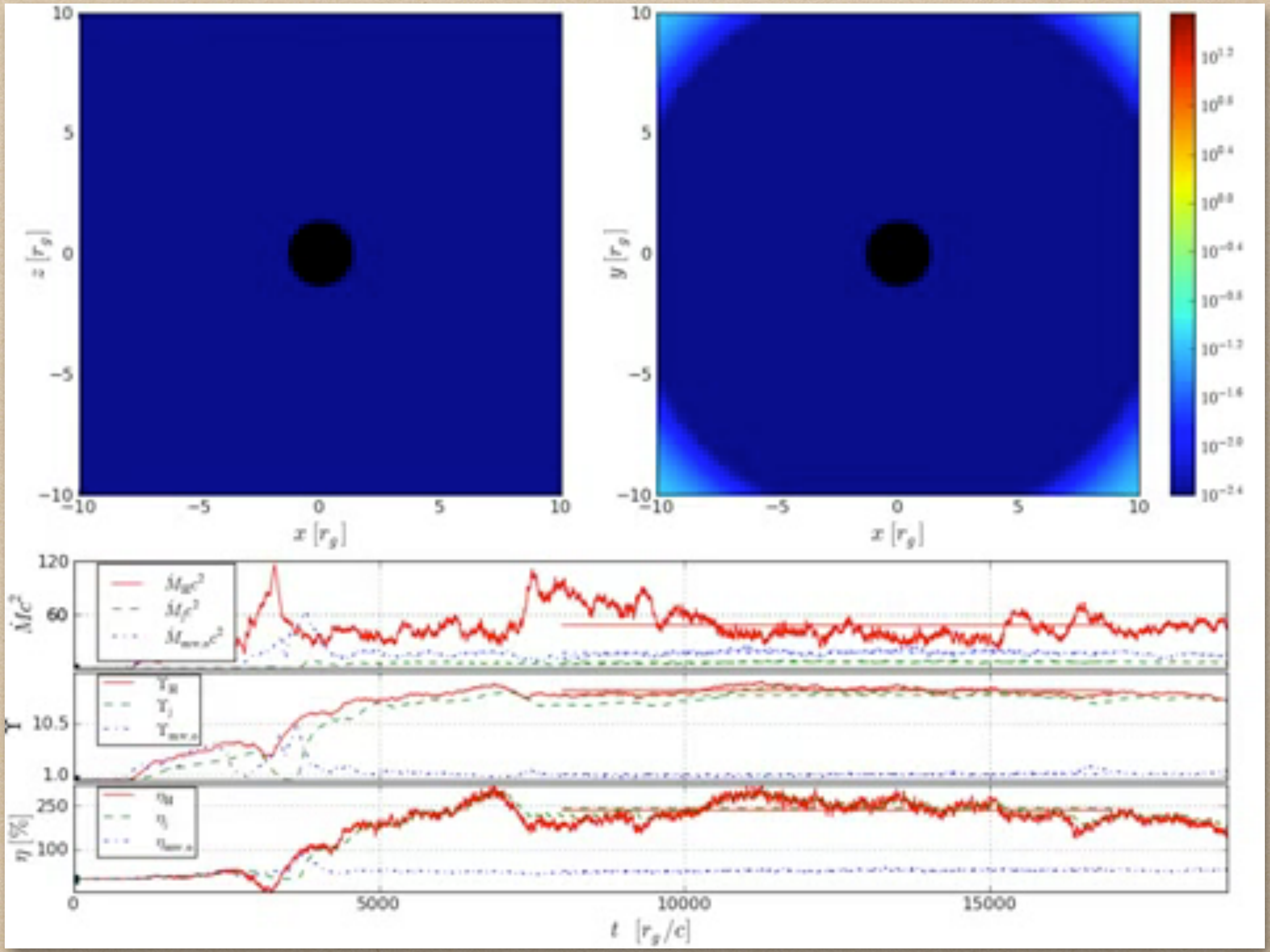
(Blandford & Znajek 1977)

Blandford-Znajek = Penrose

For a stationary and axisymmetric force-free electromagnetic field, a necessary condition for the Penrose process to occur is

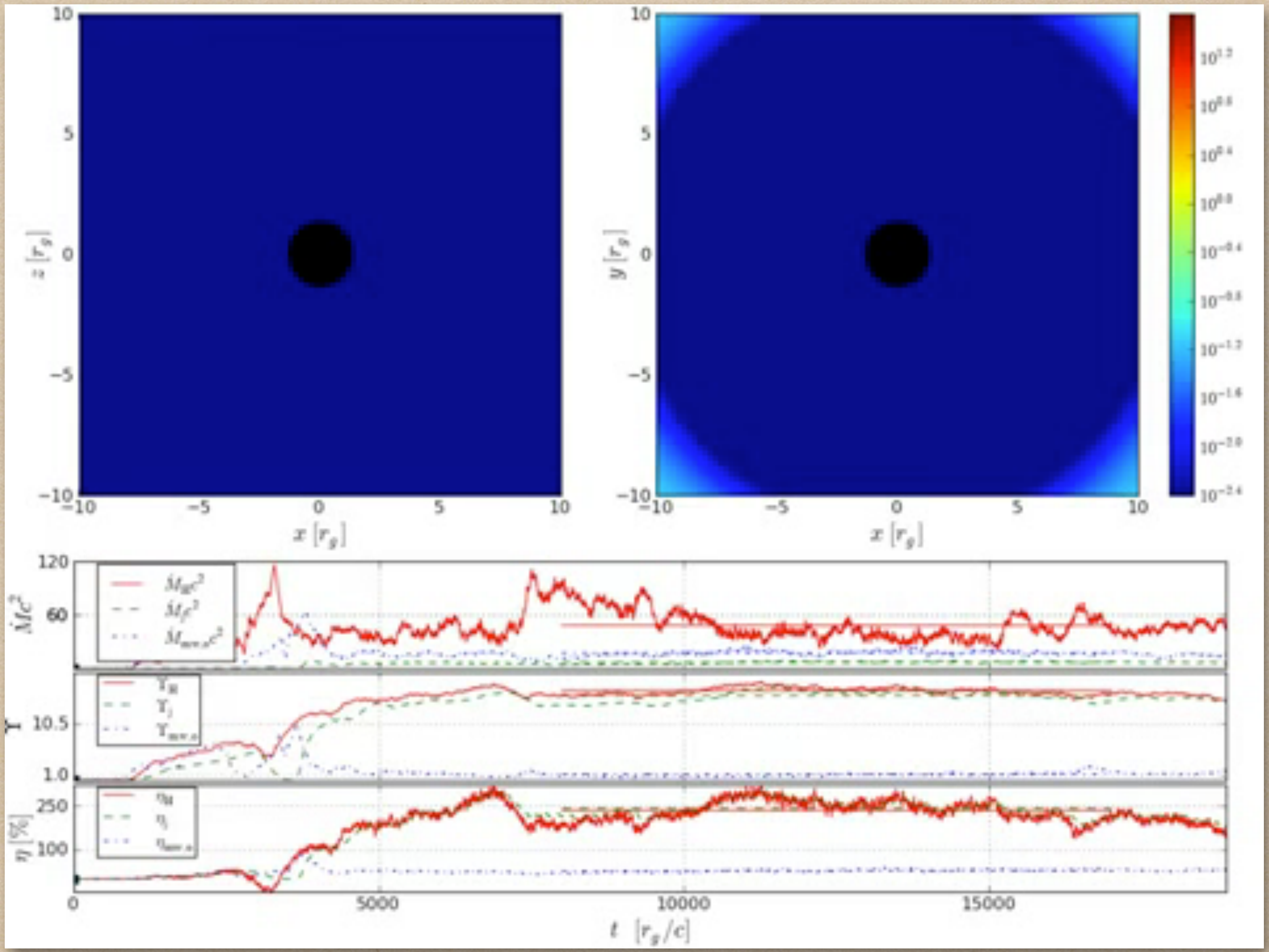
$$0 < \omega(\Psi) < \omega_H \quad \text{in some part of } \Delta\mathcal{H}.$$

MAD (magnetically chocked) flows



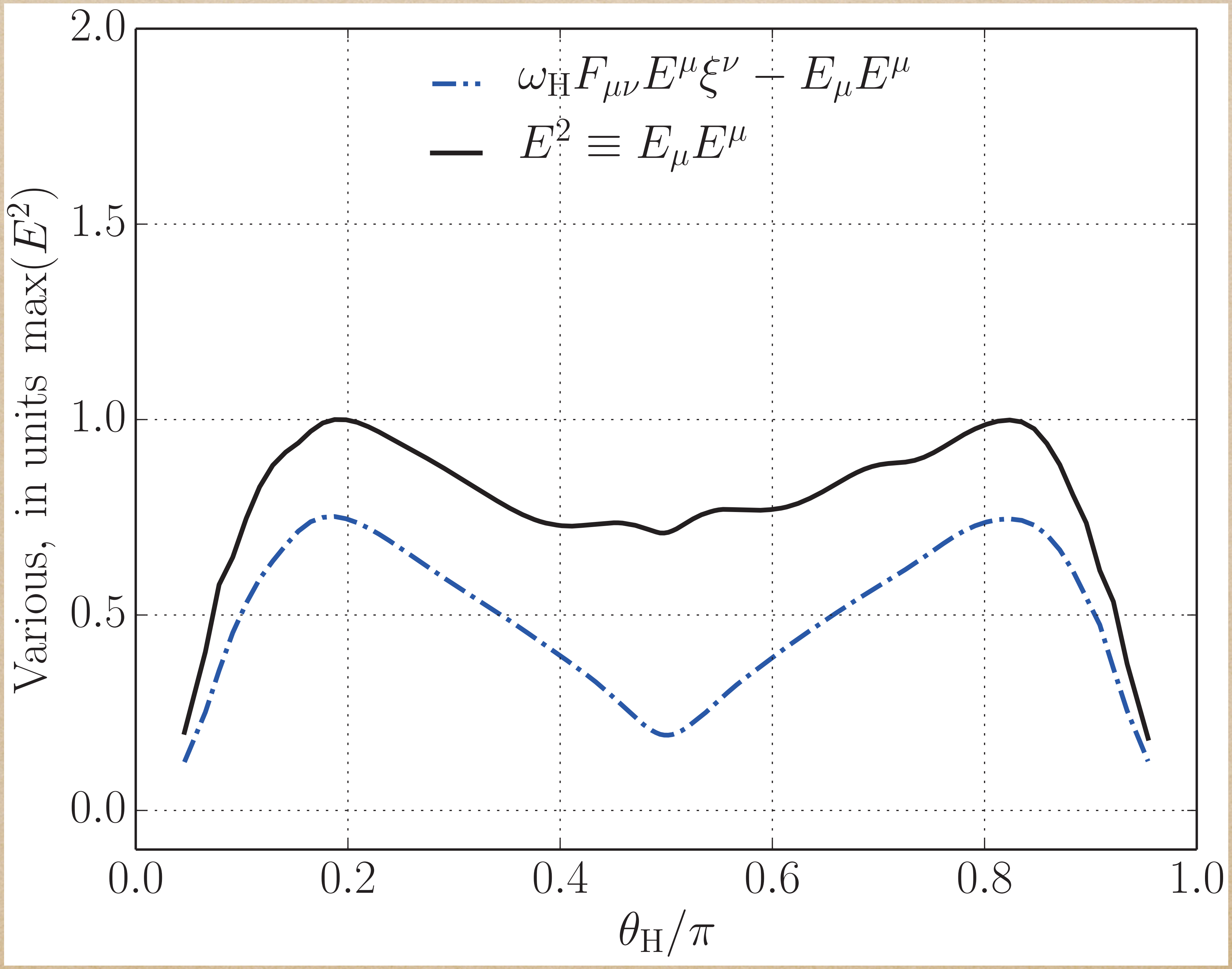
Tchekhovskoy, Narayan, McKinney, Blandford

MAD (magnetically chocked) flows



Tchekhovskoy, Narayan, McKinney, Blandford

MAD at horizon



Noether current in GRMHD

MHD: $u_\mu F^{\mu\nu} = 0$

Magnetic field vector $b^\mu := \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$

$b_\mu u^\mu = 0,$

Hence the energy-momentum tensor

$$T^{(\mathrm{EM})}_{\mu\nu} = b^2 u_\mu u_\nu + \frac{1}{2}b^2 g_{\mu\nu} - b_\mu b_\nu$$

Noether current $P^{(\mathrm{EM})}_\mu = T^{(\mathrm{EM})}_{\mu\nu}\eta^\nu$

$$P^\mu_{(\mathrm{EM})}P^{(\mathrm{EM})}_\mu = P^2_{(\mathrm{EM})} = \frac{1}{4}b^4 g_{tt}$$

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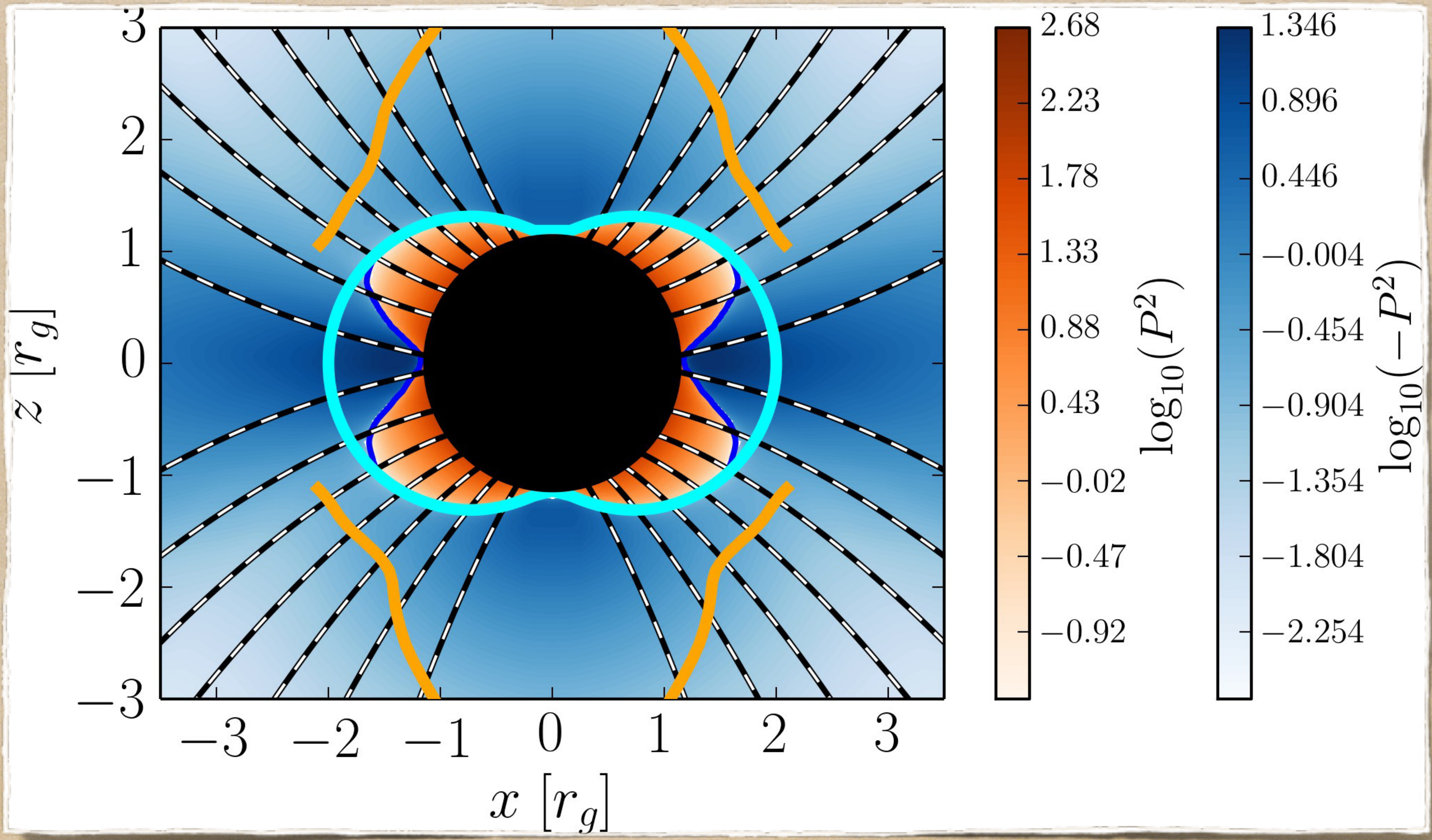
$P^\mu_{(\mathrm{EM})}P^{(\mathrm{EM})}_\mu = P^2_{(\mathrm{EM})} = \frac{1}{4}b^4 g_{tt}$ >0 in the ergosphere

Noether current: MAD

$$T_{\mu\nu} = T^{(\mathrm{MA})}_{\mu\nu} + T^{(\mathrm{EM})}_{\mu\nu}$$

$$P^2 = \left(\frac{1}{2}b^2 + p\right)^2 g_{tt} - A,$$

$$A = 2(\Gamma-1)ub_t^2 + u_t^2(\rho+u+p+b^2)[(2-\Gamma)u+\rho],$$



Conclusions

- The Blandford-Znajek mechanism is rigorously a Penrose process.
- GRMHD simulations of Magnetically Arrested Discs correctly (from the point of view of General Relativity) describe extraction of black-hole rotational energy through a Penrose process.