Blandford-Znajek Mechanism. Horizon or Ergoregion?

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Plan of the talk

1. Equations
2. Equations
3. More equations
4. Few pictures
5. Some discussion
6. Apologies for too many equations

Space-time equations of electrodynamics

$$\nabla_\beta \, {}^*F^{\alpha\beta} = 0$$

$$\nabla_\beta F^{\alpha\beta} = I^\alpha$$

${}^*F^{\alpha\beta}$, $F^{\alpha\beta}$ - Faraday and Maxwell tensors

I^α - electric current 4-vector

3+1 split of Special Relativity. Vacuum case.

$$ds^2 = -dt^2 + \gamma_{ij} dx^i dx^j$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$

$$\partial_t \boldsymbol{B} + \boldsymbol{\nabla} \times \boldsymbol{E} = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho$$

$$-\partial_t \boldsymbol{E} + \boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{J}$$

Black Hole 3+1 split. Vacuum case.

$$x^\nu \to (t, x^i)$$

$$ds^2 = (\beta^2 - \alpha^2)dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

α - lapse function β - shift vector

A fiducial observes “at rest” in the space (FIDO) has 4-velocity

$$n^\mu = \frac{1}{\alpha}(1, -\beta^i)$$

The space itself “flows” through the coordinate grid.

Black Hole 3+1 split. Vacuum case.

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\partial_t \boldsymbol{B} + \nabla \times \boldsymbol{E} = 0$$

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$-\partial_t \boldsymbol{D} + \nabla \times \boldsymbol{H} = \boldsymbol{J}$$

Constitutive equations:

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B}$$

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D}$$

The same appearance as of the Maxwell equations in matter.

Space acts as an electromagnetically active medium with exotic properties. In its lab frame, FIDO sees B as the magnetic and D as the electric fields.

This works both for the BL and KS foliations of space-time.

Gravitationally-induced electric field.

Even in the absence of charges in the BH magnetosphere, non-vanishing stationary axisymmetric magnetic field *B* implies non-vanishing electric field *D*.

From now on we use BL-coordinates

Steady-state, $\rho = 0$, $J = 0$:

$$\nabla \times \boldsymbol{H} = 0 \longrightarrow B^{\phi} = 0$$

$$\nabla \times \boldsymbol{E} = 0 \longrightarrow \nabla \times \boldsymbol{\alpha D} = -\nabla \times (\boldsymbol{\beta} \times \boldsymbol{B})$$

If $D = 0$ then

$$\nabla \times (\boldsymbol{\beta} \times \boldsymbol{B}) = 0$$

If this leads to B=0, then the above statement is true.

Gravitationally-induced electric field.

In Boyer-Lindquist coordinates $\boldsymbol{\beta} = (-\Omega_f, 0, 0)$

where Ω_f is the FIDO's angular velocity.

$$\boldsymbol{\nabla} \times (\boldsymbol{\beta} \times \boldsymbol{B}) = 0 \longrightarrow B^m \partial_m \Omega_f = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \longrightarrow B = \boldsymbol{\nabla} \times \boldsymbol{A}$$

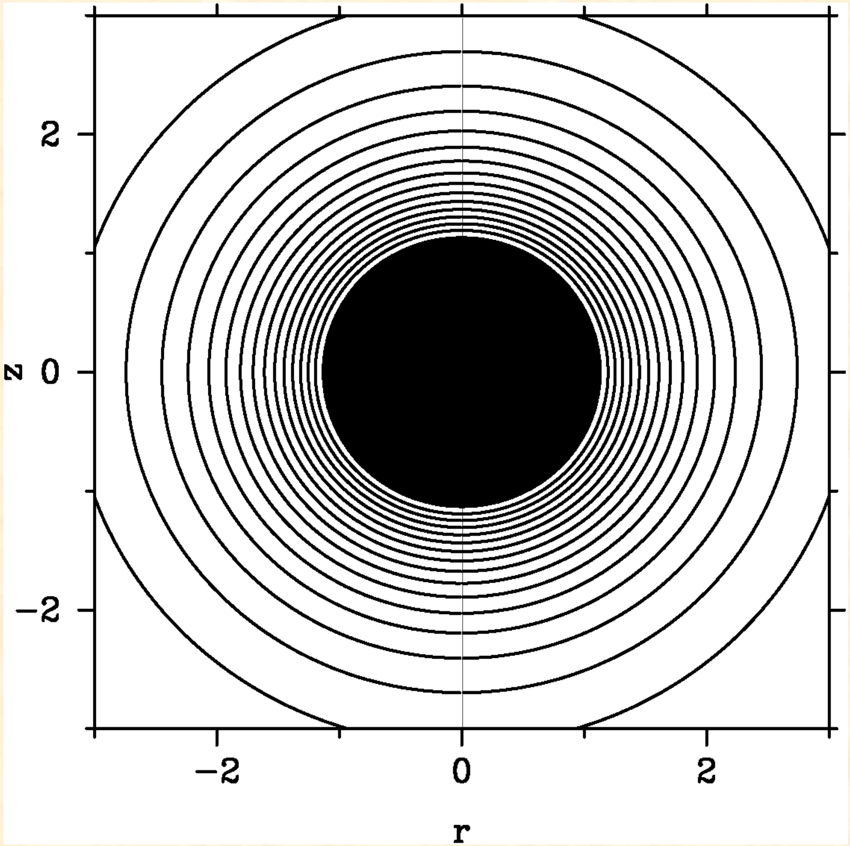
Combining these two equations, $\boxed{A_\phi = f(\Omega_f)}$

Either Ω_f is constant on magnetic flux surfaces

or A_ϕ is constant throughout the whole space (B=0).

Gravitationally-induced electric field.

Levels of Ω_f for a Kerr black hole with a=0.99



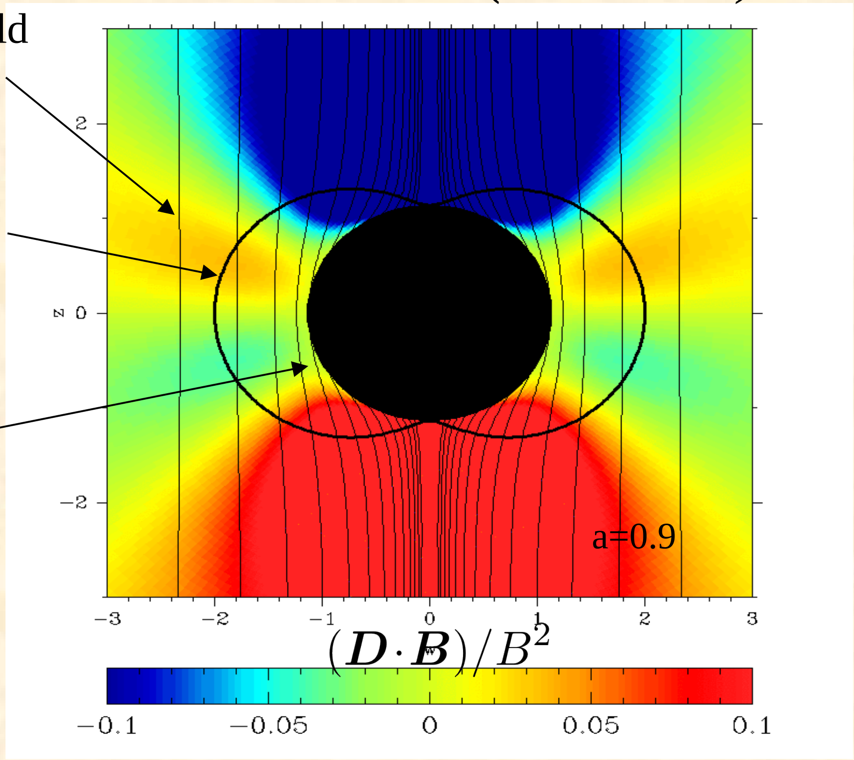
Magnetic field cannot have such flux surfaces as they imply non-uniqueness of *B* at the symmetry axis.

Thus *D* = 0 only if *B* = 0 as well.

The Wald (1974) solution is one example of such gravitationally induced electric field.

Gravitationally-induced electric field.

Wald's solution (Wald 1974)



Vacuum solution for a Kerr BH in uniform magnetic field.

Gravitationally induced electric field with

$$\boldsymbol{D}\cdot\boldsymbol{B} \neq 0$$

Blandford & Znajek (1977) - copious pair production; force-free solution; Poynting flux.

Steady-state magnetosphere

When plenty of *charged particles* are introduced into the space around BH, they move to screen the electric field. Total screening means there is a local inertial frame where electric field vanishes. The Lorentz-invariant conditions are

$$\boldsymbol{D}\cdot\boldsymbol{B} = 0 \quad \text{and} \quad B^2 > D^2$$

The electric field is weaker than the magnetic one and perpendicular to it.

Steady-state magnetosphere

$$\left.\begin{array}{l}\mathbf{D}\cdot\mathbf{B}=0\\ \boldsymbol{E}=\alpha\boldsymbol{D}+\boldsymbol{\beta}\times\boldsymbol{B}\end{array}\right\}\longrightarrow\quad \mathbf{E}\cdot\mathbf{B}=0$$

$$\left.\begin{array}{r}\boldsymbol{E}\cdot\boldsymbol{B}=0\\ \boldsymbol{\nabla}\times\boldsymbol{E}=0\end{array}\right\}\longrightarrow\quad \begin{array}{l}\boldsymbol{E}=-\boldsymbol{\omega}\times\boldsymbol{B}\\ \boldsymbol{\omega}=\Omega\boldsymbol{\partial}_\phi\\ \boldsymbol{\nabla}\boldsymbol{\Omega}\cdot\boldsymbol{B}=0\end{array}$$

If a magnetic field line rotates it does so with the same angular velocity Ω .

Steady-state magnetosphere

$$\boldsymbol{L}_p = -H_\phi \boldsymbol{B}_p$$ - angular momentum flux

$$\boldsymbol{S}_p = -(H_\phi \Omega) \boldsymbol{B}_p$$ - flux of “red-shifted energy”

$$\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J} \longrightarrow H_\phi = \frac{I}{2\pi}$$

Only when currents flow the magnetosphere is “alive”.

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D} \longrightarrow H_\phi = \alpha B_\phi$$

The importance of the ergoregion.

Can a BH magnetosphere be dead?

For a non-rotating dead magnetosphere

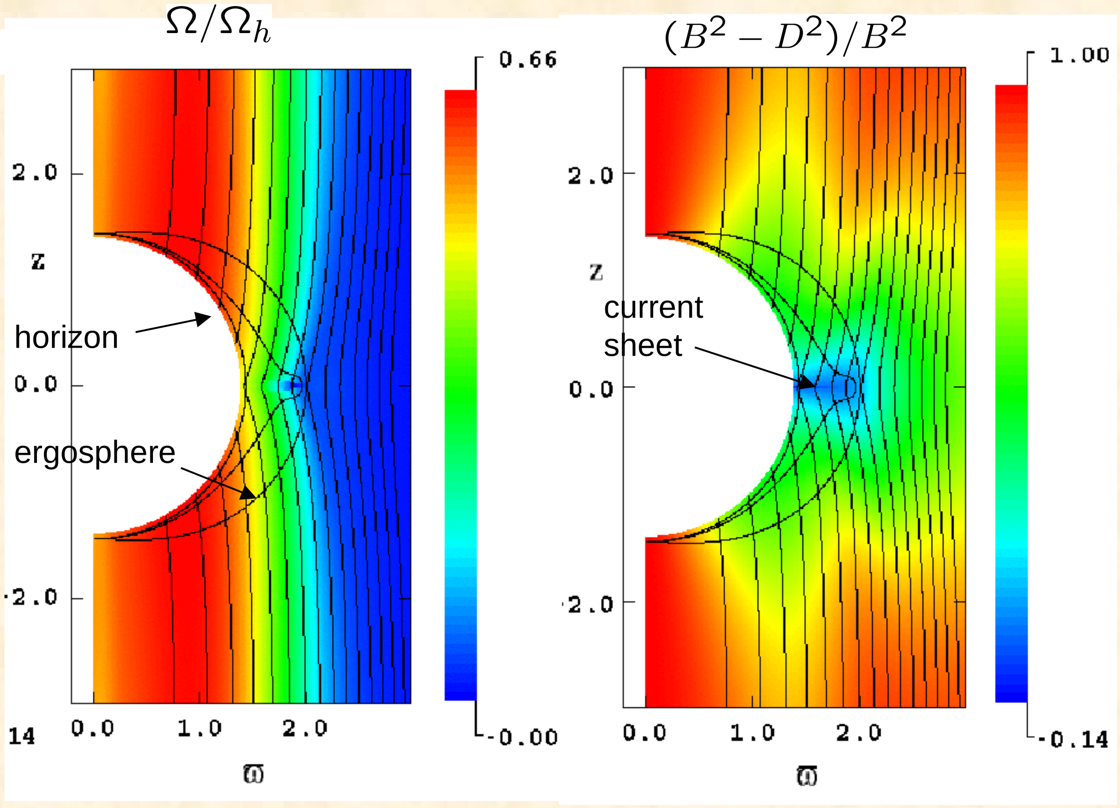
$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B} = 0 \longrightarrow D^2 = \frac{\beta^2}{\alpha^2} B^2$$

Outside the ergosphere, $\alpha > \beta$ and the screening condition B > D is satisfied. Thus, the field lines that never enter the ergoregion can make the **dead zone**.

Inside the ergosphere $\alpha < \beta$ and the field lines that enter the ergosphere cannot be part of the dead zone.

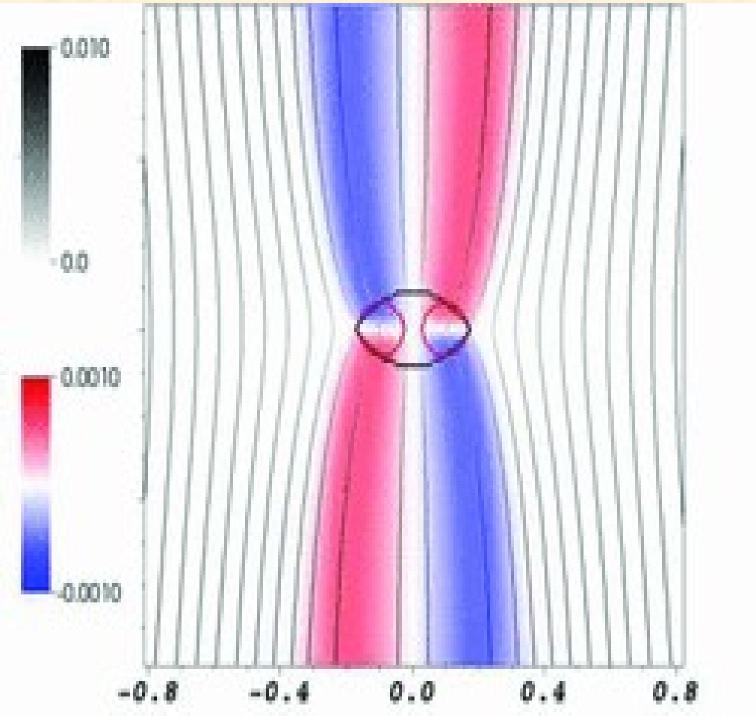
Inclusion of rotation makes no difference, apart from a longer derivation.

Numerical experiments



Kerr BH in a uniform magnetic field (Komissarov 2004)

Numerical experiments



Space-time with ergosphere
but without horizon.
(Ruiz et al., 2012)

Horizon “boundary condition”

Znajek's horizon boundary condition is just a **regularity condition** at the fast critical surface of transonic steady-state solution. In the limit of force-free electrodynamics (magnetodynamics), this surface coincides with the horizon.

In the Kerr-Schild coordinates (non-singular at the horizon),

$$B^{\phi} = \frac{\alpha H_{\phi} - 2rB^{r}\sin^{2}\theta(\Omega - a/2r)}{\Delta\sin^{2}\theta}$$

At the horizon ($r = r_{+}$) $\Delta \to 0$, and for the solution to be finite (physically meaningful)

$$H_{\phi} = \frac{2r_{+}\sin^{2}\theta}{\alpha_{+}}(\Omega - \Omega_{bh})B^{r}$$ - Znajek's condition

Horizon “boundary condition”

In time-dependent problems, Znajek's condition is not required.

In full MHD, the critical surface is outside of the horizon (e.g. Beskin & Kusnetsova 2000) and the Znajek's condition is replaced by another one.

Thus, the **horizon only appears to be important** for the BZ process.

It is not!

BZ mechanism as electromagnetic Penrose process

Local conservation of energy as seen by FIDO in his small lab involves

$$e = \frac{1}{2}(D^2 + B^2)$$ - energy density

$$\boldsymbol{S} = \boldsymbol{D} \times \boldsymbol{B}$$ - energy flux density

FIDO observes energy flow into the black hole - at the horizon, there is only “one-way traffic”.

BZ mechanism as electromagnetic Penrose process

Global conservation of energy involves

$$e_\infty = \frac{1}{2}(\boldsymbol{E}\cdot\boldsymbol{D} + \boldsymbol{B}\cdot\boldsymbol{H})$$ - red-shifted energy

$$\boldsymbol{S}_\infty = \boldsymbol{E}\times\boldsymbol{H}$$ - flux of red-shifted energy

These differ from what FIDO measures!
Unless FIDO is at infinity.

BZ mechanism as electromagnetic Penrose process

For poloidal fluxes of steady-state magnetosphere one has

$$\boldsymbol{S}_\infty = \frac{\Omega - \Omega_f}{\Omega} \hat{\boldsymbol{S}}.$$

These vectors are parallel when $\Omega > \Omega_f$

and anti-parallel when $0 < \Omega < \Omega_f$

In the latter case, we have the phenomenon of “energy counter-flow”. While at the horizon FIDO observes energy flowing into BH, the redshifted energy flows outwards. Like in the Penrose process!

BZ mechanism as electromagnetic Penrose process

In the original Penrose process, the energy counterflow occurs when the redshifted energy is negative.

In the BZ process, the condition for negative redshifted energy is

$$0 < \Omega^2 < \left(\Omega_f^2 - \frac{\alpha^2 B^2}{\gamma_{\phi\phi} B_p^2} \right)^{1/2}$$

which differs from the energy counterflow condition. Only at the horizon they coincide (Koide 2009). Thus, the energy counter-flow is a more general phenomenon.