Towards general-relativistic pulsar magnetospheres

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Abstract

Pulsars are believed to lose their rotational kinetic energy primarily by a large amplitude low frequency electromagnetic wave which is eventually converted into particle creation, acceleration and followed by a broad band radiation spectrum [1]. To date, there exist no detailed calculation of the exact spin-down luminosity with respect to the neutron star magnetic moment and spin frequency, including general-relativistic effects. Estimates are usually given according to the flat space-time magnetodipole formula [2]. I present accurate solutions of the general-relativistic electromagnetic field around a slowly rotating magnetized neutron star. The full set of time-dependent Maxwell equations are solved in a curved space-time following the 3+1 formalism [3]. The numerical code is based on a pseudo-spectral method [4] and adapted to an arbitrary background metric. Stationary solutions are readily obtained and compared to semi-analytical calculations [3]. Some new results about its extension to force-free solutions in general relativity are also presented.

1. The space-time metric

$$ds^2 = \alpha^2\, c^2\, dt^2 - \gamma_{ab}\,(dx^a + \beta^a\, c\, dt)\,(dx^b + \beta^b\, c\, dt)$$

- α lapse function
- β shift vector
- γ_{ab} spatial metric

2. Maxwell equations in 3+1 formalism

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{1}{\sqrt{\gamma}}\,\partial_t(\sqrt{\gamma}\,\mathbf{B})$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{1}{\sqrt{\gamma}}\,\partial_t(\sqrt{\gamma}\,\mathbf{D})$$

3. Two important constitutive relations

$$\varepsilon_0\, \boldsymbol{E} = \alpha\, \boldsymbol{D} + \varepsilon_0\, c\, \boldsymbol{\beta} \times \boldsymbol{B}$$

$$\mu_0\, \boldsymbol{H} = \alpha\, \boldsymbol{B} - \frac{\boldsymbol{\beta} \times \boldsymbol{D}}{\varepsilon_0\, c}$$

4. The force-free current

$$\mathbf{J} = \rho\, \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\mathbf{B} \cdot \nabla \times \mathbf{H} - \mathbf{D} \cdot \nabla \times \mathbf{E}}{B^2}\, \mathbf{B}$$

5. Spin-down luminosity as a diagnostic

Poynting flux through a sphere of radius r

$$L = \int_\Omega \boldsymbol{E} \wedge \boldsymbol{H}\, r^2\, d\Omega$$

6. Numerical algorithm

Pseudo-spectral discontinuous Galerkin method

- finite volume formulation in radius.
- high-order interpolation with Legendre polynomials.
- non uniform radial grid.
- spectral interpolation in longitude/latitude.
- vector spherical harmonic decomposition.
- 4th order Runge-Kutta time integration.
- Lax-Friedrich flux.
- stabilization by filtering and limiting.
- exact boundary conditions on the neutron star surface.
- outgoing waves at the outer boundary.

7. Vacuum solutions

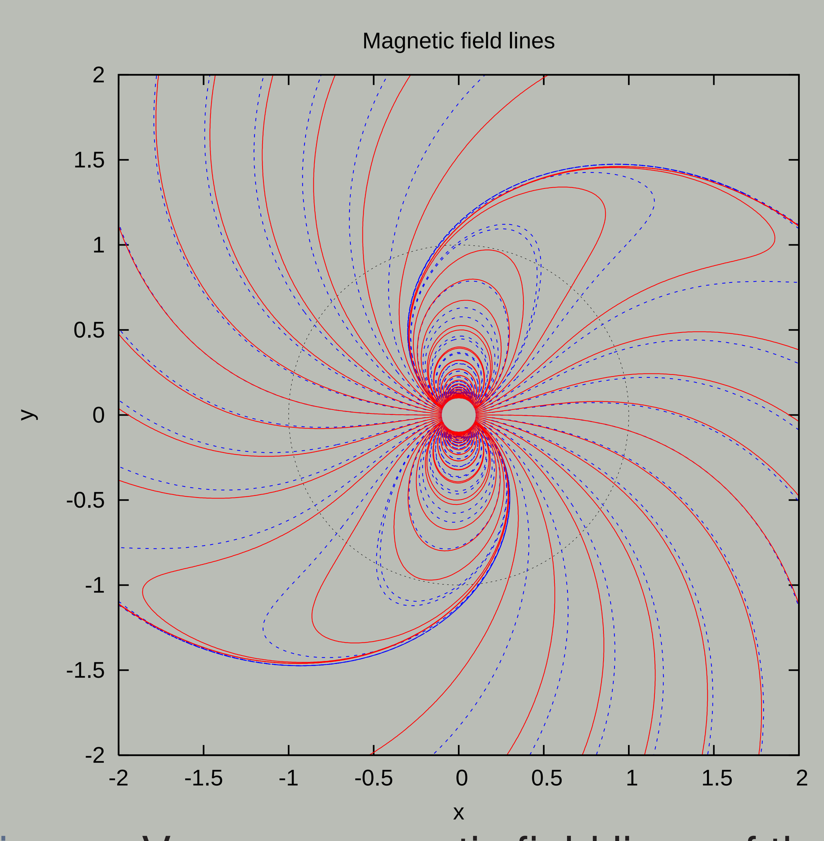

Figure : Vacuum magnetic field lines of the perpendicular rotator in the equatorial plane for $r_L/R = 10$ (flat space-time in blue).

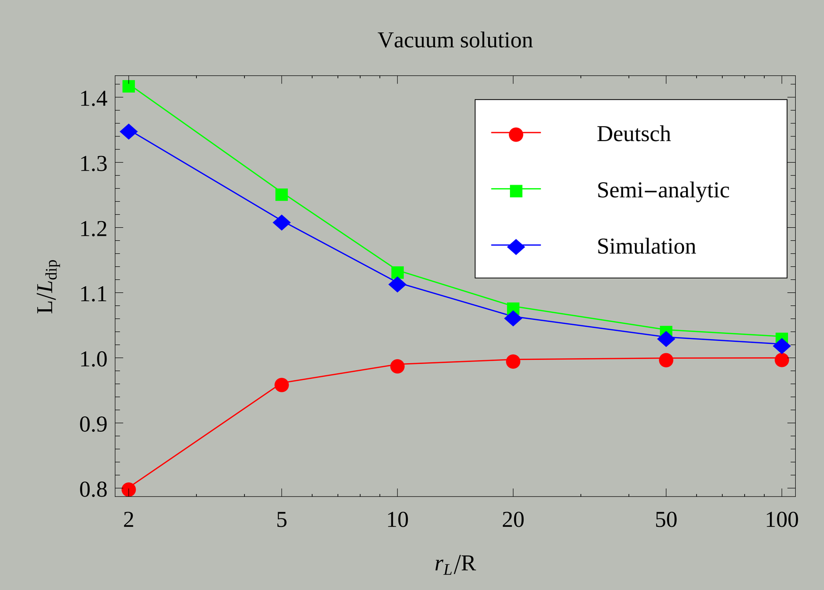

Figure : Normalized Poynting flux L/L_0 for the perpendicular rotator compared to the Deutsch solution [5].

$$L_0 = \frac{8\,\pi}{3\,\mu_0\,c^3}\,\Omega^4\,B_L^2\,r_L^6 \qquad (3)$$

8. Force-free solutions

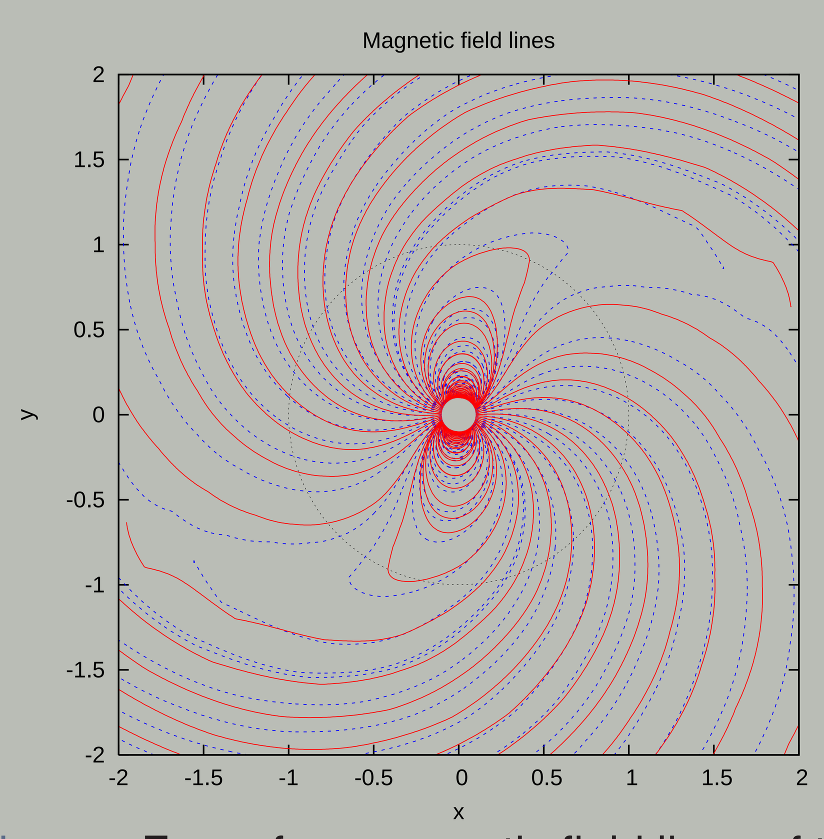

Figure : Force-free magnetic field lines of the perpendicular rotator in the equatorial plane for $r_L/R = 10$ (flat space-time in blue).

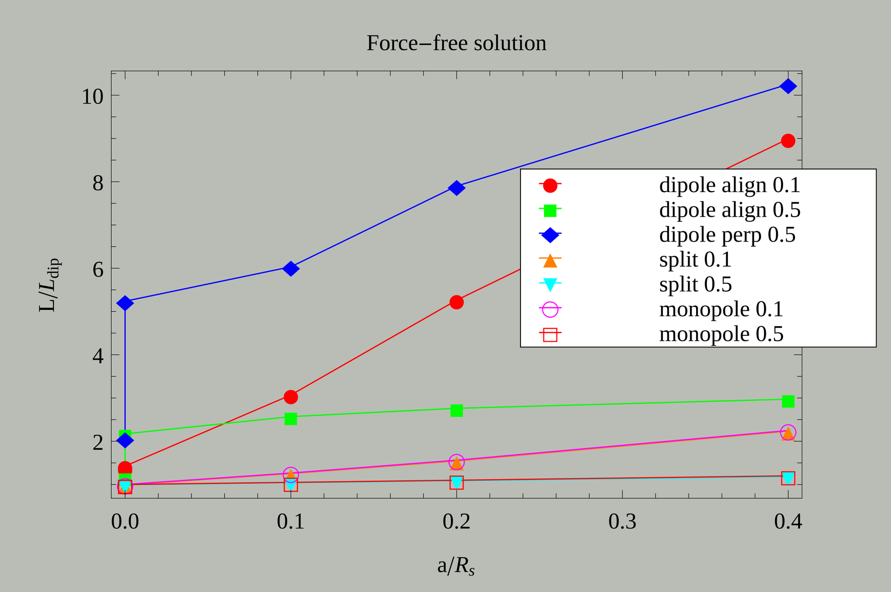

Figure : Normalized Poynting flux L/L_0 for monopole/split monopole and dipole in general relativity.

9. Conclusion & Perspectives

Conclusions

- curved space-time increases the spin-down luminosity.
- seen in vacuum and in force-free simulations.
- code able to handle discontinuities by construction.

Perspectives

- extension to resistive solutions.
- consequences on pulsar light-curves in radio up to high-energy.
- modification of phase-resolved polarization properties (curvature and synchrotron radiation).

Bibliography

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