Magnetoluminescence

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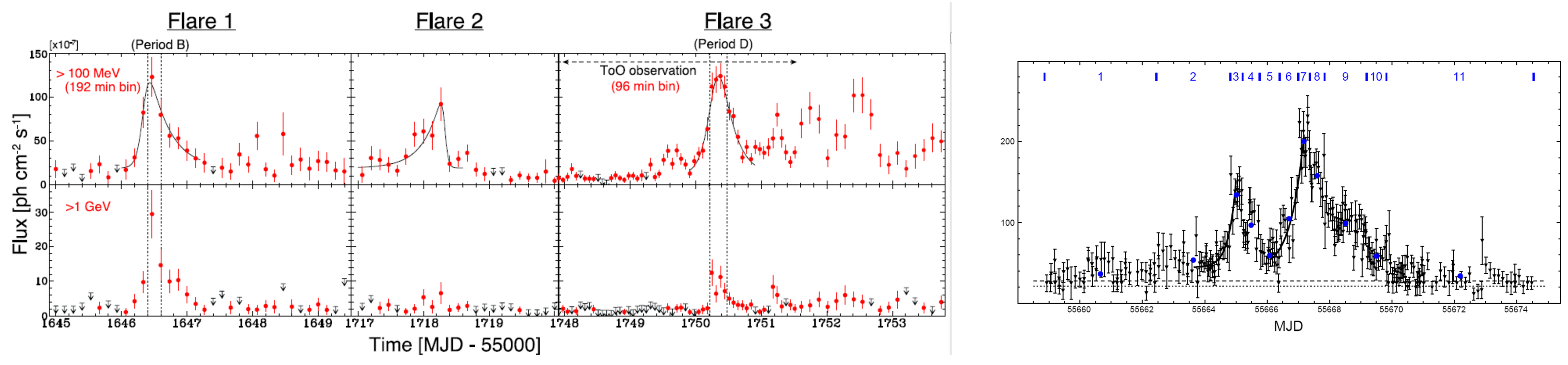
Abstract

We describe a new mechanism – magnetoluminescence – that promotes rapid conversion of electromagnetic energy to high energy pairs and radiation in relativistic plasmas, to account for highly efficient and rapid flaring in pulsar wind nebulae, AGN jets, GRBs and magnetars. In this mechanism, magnetically dominant equilibria become unstable and transition to lower energy states while preserving, at least initially, the 'tangling' (helicity) of the field. The Ohmic dissipation is associated with radiation reaction on the highest energy particles which carry the electrical current. The unstable region eventually evolves at light speed with E>~B and ultimately implodes releasing more energy.

Motivation

- Blazars and quasars, e.g. PKS 2155-304, 3C 279, exhibit flares on time scales as short as hours to a few minutes, while the lower bound on the jet width determined by the pair production opacity is comparatively much larger.
- The Crab Nebula produces ~10 hr gamma-ray flares peaking at ~400 MeV, requiring synchrotron radiation from ~3 PeV electrons in 1 mG magnetic field. The total (isotropic) energy release is $\sim 10^{41}$ erg for a typical flare, equivalent to energy contained in a region with length scale $\sim 10^{16}$ cm ~100 light hours. Either there's strong beaming, or energy is concentrated in a small volume, or both.



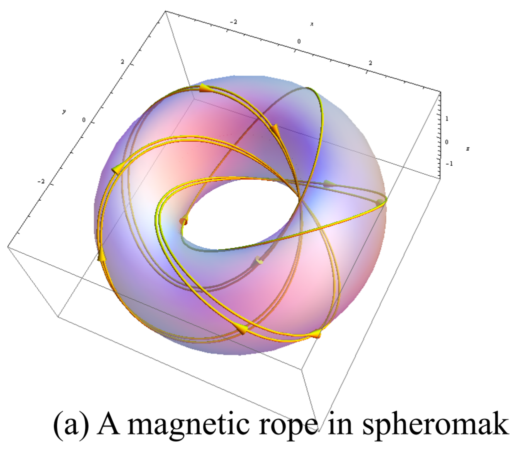
Left: Gamma-ray light curves of 3C 279 around three largest flares, from [6]; right:Gamma-ray light curve of the Crab during the April 2011 flare, from [4].

Magnetoluminescence

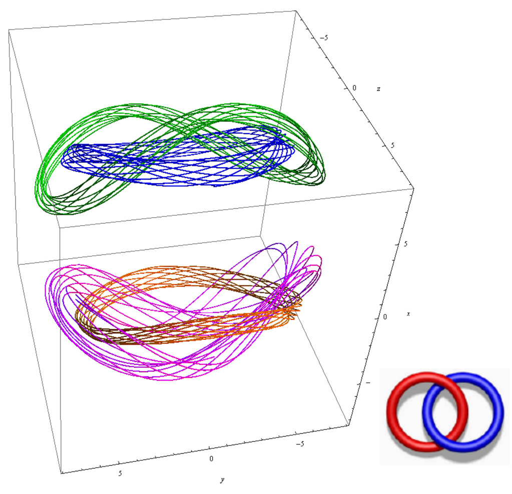
We propose the following sequence:

- The prime mover (a magnetized neutron star or black hole) "winds up" the magnetic field and produces a highly magnetized wind/jet with largely toroidal field.
- The outflow is decelerated by external stresses and the freshly- equilibrated magnetic field is either unstable or evolves to instability through relatively slow reconnection which changes the linkage/ helicity of current-carrying flux ropes; these small scale reconnection events may account for the steady acceleration of intermediate energy electrons.
- When these flux ropes reach a catastrophe point they suddenly untangle without serious change of topology. Large electric field is induced volumetrically; particles are accelerated efficiently toward the radiation reaction limit with efficient conversion of electromagnetic energy to radiation.
- The pressure of the configuration drops and an implosion may be produced. Surrounding medium pushes in and further enhances energy release.

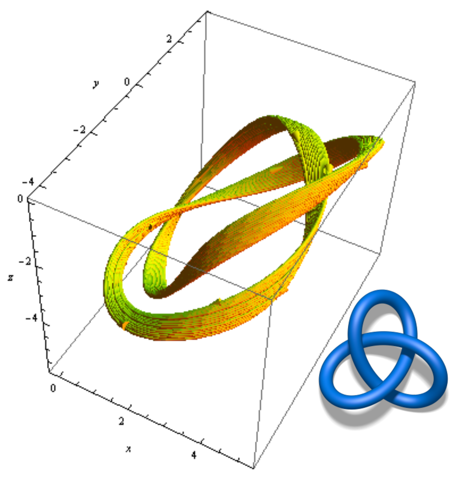
Topology of magnetic configurations



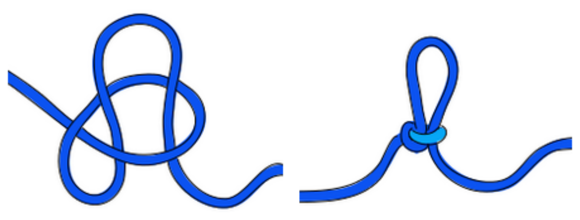
(a) A magnetic rope in spheromak



(b) Linked magnetic ropes



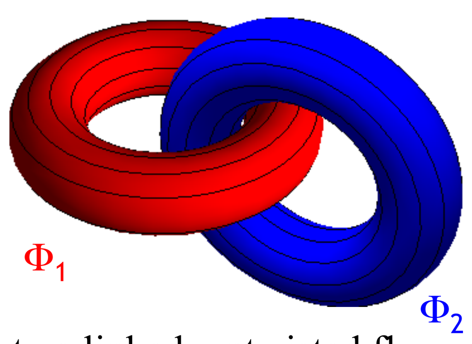
(c) A knotted magnetic rope



(d) Hitch: equivalent to a straight line

Magnetic field lines usually lie on nested toroidal surfaces or wander chaotically. We designate the 2D flux surfaces as magnetic ropes. They can be hairy as surrounded by chaotic field lines. Possible topological configurations of closed magnetic ropes include: knot, link, unknot (equivalent to a circle); for open magnetic ropes, there can be braid and unbraid.

Helicity – a topological invariant



The two linked, untwisted flux tubes have helicity $H = 2\Phi_1\Phi_2$

Helicity evolution:

$$\frac{dH}{dt} = -2\int \mathbf{E}\cdot\mathbf{B}\,dV$$

Helicity $H = \int \mathbf{A}\cdot\mathbf{B}\,dV$ is well defined (independent of gauge) if the system is confined by a flux preserving surface (if there are open field lines, relative helicity has to be used). It characterizes the knottedness and linkedness of magnetic field lines. Helicity is conserved when flux freezing is true or $\mathbf{E}\cdot\mathbf{B}\neq 0$ region has negligible volume [1].

Topology preserving instabilities

We consider linear force-free equilibria (Beltrami fields) in spherical or Cartesian geometry as our prototypical systems.

In a spherical domain with fixed, perfectly conducting wall at r=R, we have

$$\mathbf{B} = \lambda\mathbf{r}\times\nabla\chi + \nabla\times(\mathbf{r}\times\nabla\chi)$$

where χ is the solution to $\nabla^2\chi + \lambda^2\chi = 0$. χ can be written as superposition of $\chi_{lm} = j_l(\lambda r)Y_{lm}(\theta,\phi)$ but only a single l can be present and λR should be the n-th zero of j_l. We call them spheromaks in general [1].

In Cartesian geometry, similarly $\mathbf{B} = \alpha\mathbf{\Psi} + \nabla\times\mathbf{\Psi}$, where $\nabla\cdot\mathbf{\Psi} = 0$ and $\nabla^2\mathbf{\Psi} + \alpha^2\mathbf{\Psi} = 0$. We use periodic boundary condition in this case.

We studied the stability of these force-free equilibria using both analytical approach and numerical simulations. We find some of them to be unstable to ideal modes; in force-free and MHD simulations, these instabilities grow exponentially and quickly give rise to regions where electric energy density is comparable to magnetic energy density. The system becomes turbulent on dynamic time scales and eventually relaxes to the longest wavelength configuration.

Linear stability analysis

Force-free evolution equations can be written as

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E},\quad \frac{\partial\mathbf{E}}{\partial t} = \nabla\times\mathbf{B} - \mathbf{j},$$

$$\mathbf{j} = \left[(\mathbf{B}\cdot(\nabla\times\mathbf{B}) - \mathbf{E}\cdot(\nabla\times\mathbf{E}))\,\mathbf{B} + (\nabla\cdot\mathbf{E})\mathbf{E}\times\mathbf{B}\right]/B^2,$$

Let $\mathbf{v} = \mathbf{E}\times\mathbf{B}/B^2$, define $\vec{\xi}$ such that $\mathbf{v} = \partial\vec{\xi}/\partial t$, the perturbation on a static equilibrium evolves in linear regime according to

$$\mathbf{B}_1 = \nabla\times(\vec{\xi}\times\mathbf{B}_0),$$

$$B^2\partial^2\vec{\xi}/\partial t^2 = (\nabla\times\mathbf{B}_0)\times(\nabla\times(\vec{\xi}\times\mathbf{B}_0)) + [\nabla\times(\nabla\times(\vec{\xi}\times\mathbf{B}_0))]\times\mathbf{B}_0$$

Using the energy principle of Bernstein et al [2], we can define a potential energy

$$V = \int d^3r(\nabla\times(\vec{\xi}\times\mathbf{B}_0))\cdot[(\nabla\times(\vec{\xi}\times\mathbf{B}_0)) - \vec{\xi}\times(\nabla\times\mathbf{B}_0)]$$

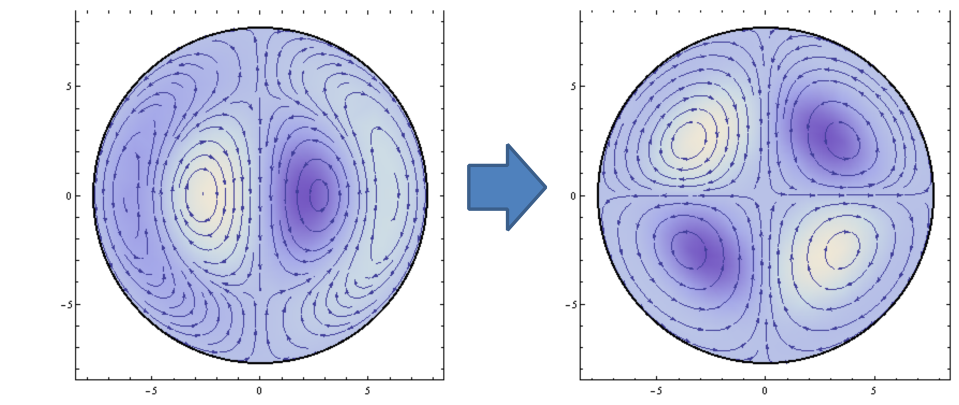
And the dispersion relation can be written as $\omega^2 = \dfrac{V}{\frac{1}{2}\int d^3r B^2(\vec{\xi}_\perp)^2}$

This allows a variational approach to the stability problem.

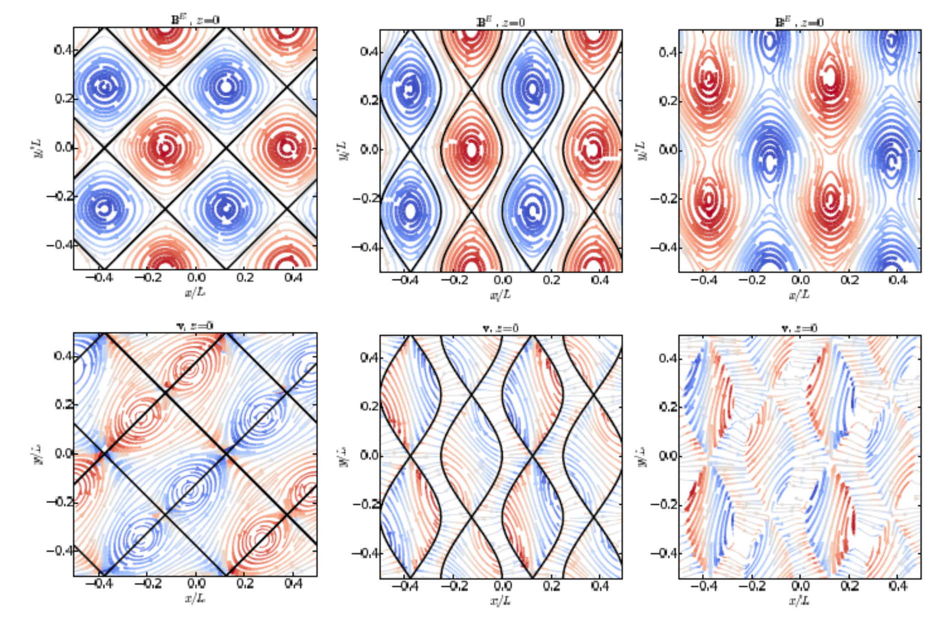
In Cartesian coordinates, the linearized perturbation equation can be put into Sturm-Liouville form:

$$\frac{\partial}{\partial x_l}\left(Y_{iljk}\frac{\partial}{\partial x_j}\xi_k\right) + \omega^2B^2\xi_i = 0$$

where $Y_{iljk} = B_lB_j\delta_{ik} - B_lB_i\delta_{jk} - B_kB_j\delta_{il} + \frac{1}{2}B^2(\delta_{il}\delta_{jk} + \delta_{ij}\delta_{lk})$.



Using the variational principle with analytical trial functions, we find that spheromak solutions with n>1 (axisymmetric or non-axisymmetric) are generally unstable to ideal modes. The above figure (magnetic stream plot on meridional plane) shows that l=1, n=2, m=0 solution can evolve into a state similar to l=2, n=1, m=0. This has been verified by force-free simulations. This represents slipping of two magnetic ropes.



Here we show the ideal instabilities rising from the Cartesian Beltrami fields with α=2:

$$\mathbf{B}^E = (B_3\cos\alpha z - B_2\sin\alpha y,\, B_1\cos\alpha x - B_3\sin\alpha z,\, B_2\cos\alpha y - B_1\sin\alpha x)$$

Top panel shows stream lines of equilibrium field configurations for $(B_1,B_2,B_3) = (1,1,0)$, $(1,1/2,0)$ and $\approx(-0.814,0.533,0.232)$ on the z=0 plane. Bottom panel shows the corresponding velocity field $\mathbf{v} = \mathbf{E}\times\mathbf{B}^E/|\mathbf{B}^E|^2$ of the unstable ideal mode arising from the simulations. The color indicates the perpendicular vector component. The thickness of the streamline is proportional to the vector magnitude. The black lines indicate the location of the separatrices in the equilibrium solutions.

The instability, in particular the first case, has been verified using variational principle with analytical trial functions.

These represent merging of magnetic ropes with similar polarities.

See also discussion in Jonathan Zrake's talk and [5].

Conclusions and outlook

- Our studies on prototypical force-free equilibria, using both analytical variational principle and force-free/MHD simulations, have found configurations that are unstable to ideal modes; they develop regions with E~B during nonlinear evolution. These could be potential setup for testing our magnetoluminesence model.
- To understand the acceleration process we are also performing kinetic simulations. Preliminary results are promising and show the formation of regions for efficient particle acceleration and emergence of suprathermal population.

Acknowledgements

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