Kinetic Instabilities in Relativistic Plasmas: The Harris Instability Revisited

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Kinetic Modeling of Astrophysical Plasmas
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Outline

What we needed

- Mechanism to produce aperiodic fluctuations
- Applicable to relativistic, magnetized, and anisotropic media

What we had

- Weibel & Harris instabilities
- General description of linear instabilities

What we did

- Relativistic generalization
- Inclusion of perturbed magnetic fields & ion motion
- Coupling of electrostatic/electromagnetic modes

What we obtained

- Powerful tool to describe instabilities
- Gamma-ray bursts: prediction for particle composition

Motivation

Aperiodic fluctuations

- Purely growing modes
- Not propagating!
- Finite systems

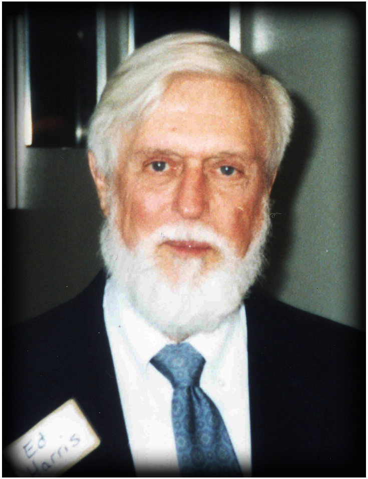
Astrophysical jets

- Gamma-ray bursts:
  - Turbulent fields
  - Dissipation
  - Shock wave
  - Radiation
- Magnetized systems
- Rest frame: perpendicular motion only
- Hot gas: relativistic temperatures

☞ Harris instability

Harris Instability I

Harris, Ed



- 1948 Bachelor of Science in Electrical Engineering from UT (University of Tennessee)
- 1959 Masters in Physics from UT
- 1953 Ph. D. in Physics from UT in Theoretical Atomic and Nuclear Physics
- 1953 – 1957 Naval Research Laboratory, Washington, D. C.
- 1957 – 1994 Professor at the Department of Physics
- 1975 – 1994 Benford Foundation Distinguished Professor in Physics
- 1994 – 2003 The Department's most active emeritus professor

Harris Instability II

Key Features[1]

- Electrostatic instability
- Aperiodic fluctuations
- Magnetic background field included
- Ion motion neglected
- Fluctuating magnetic fields neglected
- Limited to non-relativistic regime

Instability Condition

- Temperature anisotropy $T_\perp > T_{||}$
- Velocity anisotropy $v_\perp \neq v_{||}$
- Original distribution $f \propto \delta(v_{||})\,\delta(v_\perp - v_0)$
- Instability if $k_\perp v_0 \gtrsim 2\Omega$

[1] Harris, Phys. Rev. Lett. *2*, 34 (1959)

Harris Instability III

Terrestrial Applications

- Intense charged particle beams[1,2]
- Beams with high temperature anisotropy[3]
- Microinstabilities in Tokamak reactors[4]

Astrophysical Appliations

- Key ingredient for plasma radiation[5]
- Target for inverse Compton scattering[6]
- Shock front between AGN accretion disk and jets[7]
- AGN and GRB jets with relativistic velocity dispersion

[1] Startsev, Davidson, & Qin, Phys Rev. ST Accel. Beams *6*, 084401 (2003)

[2] Davidson et al., Phys. Rev. ST Accel. Beams *7*, 114801 (2004)

[3] Wang, Phys. Rev. ST Accel. Beams *7*, 024201 (2004)

[4] Seki et al., Phys. Rev. Lett. *62*, 1989 (1989)

[5] Benz, *Plasma Astrophysics: Kinetic Processes in Solar and Stellar Coronae* (1993)

[6] Tautz, Lerche, & Schlickeiser, Phys. Plasmas *13*, 052112 (2006)

[7] Gvaramadze & Machabeli, ASP Conf. Series *54*, 85 (1994)

Harris Instability IV

Neutral Points

- Definition: $\omega(\hat{k}) = 0$
- *Always* unstable modes for $k \gtrless \hat{k}$
- No neutral points: no information

Relativistic Generalization[1]

- For $k_{||} = 0$: similar equation as that of Harris

$$\mathcal{H}(k_\perp) = 1 - \sum_a \frac{\omega_{p,a}^2}{k_\perp^2} \int \mathrm{d}^3 p\, \gamma \left[1 - J_0^2\left(\frac{k_\perp v_\perp}{\Omega_a}\right)\right] \frac{\partial f_a}{\partial p_\perp} = 0$$

- For $f \propto \delta(v_\perp - v_0)$ one has $J_0(v_0)\, J_1(v_0) \overset{!}{\gtrless} 0$
- For $f \propto e^{-v^2}$ there are no neutral points (and no instabilities)
- General case ($k_{||} \neq 0$):
  resonant wave-particle effects (more complicated)

[1] Tautz, Lerche, & Schlickeiser, Phys. Plasmas *13*, 052112 (2006)

TLS Method I

Well-known Approach

- Linearize Vlasov-Maxwell equations
- Define Maxwell operator

$$\Lambda_{lm} = \frac{k^2c^2}{\omega^2}\left(\frac{k_l k_m}{k^2} - \delta_{lm}\right) + \delta_{lm} + \frac{4\pi i}{\omega}\,\sigma_{lm}$$

  with σ_{lm}: relativistic conductivity tensor
- Zeros of $\det \Lambda_{lm}$ correspond to dispersion relations $\omega(k)$

Small Frequencies[1]

- Expand Maxwell operator for $\omega \ll 1$
- For perpendicular wave propagation:

$$F(k_\perp) + E(k_\perp)\,\omega^2 + \cdots = 0$$

- General case ($k_{\|} \neq 0$): more complicated

[1] Tautz, Lerche, & Schlickeiser, J. Math. Phys. *48*, 013302 (2007)

TLS Method II

Effect of the Ions

- Consider case of $k_{\|} = 0$
- Conductivity tensor given by

$$\sigma_{lm} = -i\sum_a e_a n_a^2 \int \mathrm{d}^3p \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{T_{lm,a}}{\omega - n\Omega_a/\gamma_a}$$

- Neutral points require $\omega \to 0$, leading to

$$\sigma_{lm} \propto \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left(1 + \frac{\gamma^2}{n^2}\frac{\omega^2}{\Omega^2}\right) \frac{T_{lm}}{n}$$

Different limiting cases

1. Harris' original approach
   Set first $m_i \to \infty$: no ion contribution to neutral points
2. TLS new approach
   Set first $\omega \to 0$: large ion contribution to neutral points

TLS Method III

Neutral Points

- From dispersion relation $F(k_\perp) + E(k_\perp)\,\omega^2 = 0$:

$$\text{Neutral points} \quad \Leftrightarrow \quad F(k_\perp) = 0$$

- Neutral points equation

$$\mathcal{H}(k_\perp) \cdot T(k_\perp) = \mathcal{C}(k_\perp)$$

$$\swarrow \qquad \downarrow \qquad \searrow$$

Harris — Transverse — Coupling

Mode Coupling

- Now coupled electrostatic/electromagnetic mode: "quasi-longitudinal" or "quasi-transverse"
- Mode coupling sensitive to mass ratio:

$$\mathcal{C} \propto \sum_a \omega_{p,a}^2 \Omega_a^{-1}$$

- Therefore: $\mathcal{C} = 0$ for electron-positron plasma

Simple Examples

Distribution function

- Mono-energetic particles[1]

$$f_a = \frac{1}{2\pi v_\perp} \delta(v_{||}) \delta(v_\perp - V_a)$$

- Simplest case to sort out specific effects
- Application: jet rest-frame[2]
- Two fundamentally different cases
  1. Immobile ions, $V_i = 0$
     (allowed because $m_i \to \infty$ after $\omega \to 0$!)
  2. Pair plasma, $V_e = V_p$
     (no mode coupling!)
- Future extensions:
  - Power-law distributions
  - Thermal Maxwellian plasmas

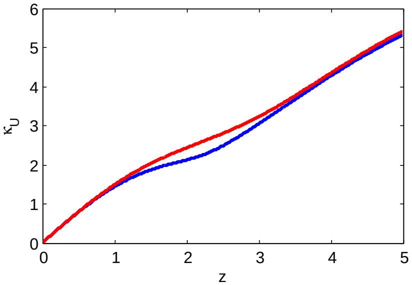
[1] Tautz, Lerche, & Schlickeiser, J. Math. Phys. *48*, 013302 (2007)

[2] Tautz & Lerche, Astrophys. J. *653*, 447 (2006)

Neutral Points I

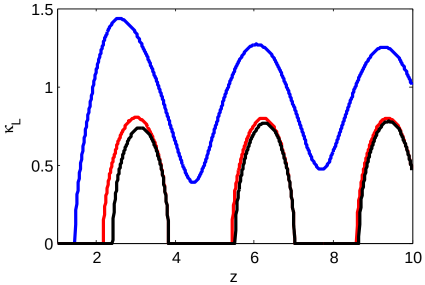
Electron-Proton Plasma

- Longitudinal neutral points



- Small coupling effect
- Always one neutral point, regardless of parameters
- No relativistic correction

- Transverse neutral points

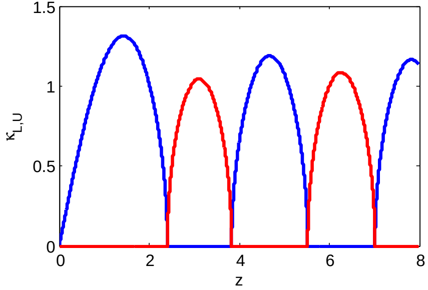


- Huge coupling effect
- No neutral point for too high background B field
- Small relativistic correction

Neutral Points II

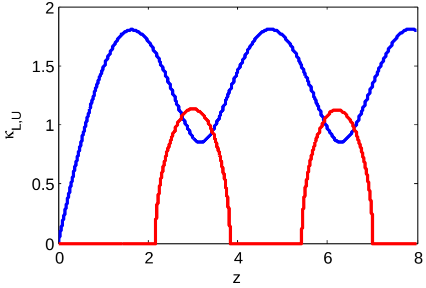
Electron-Positron Plasma

- Non-relativistic case



- No coupling
- No transverse neutral points for large *B*
- Only one neutral point

- Relativistic case

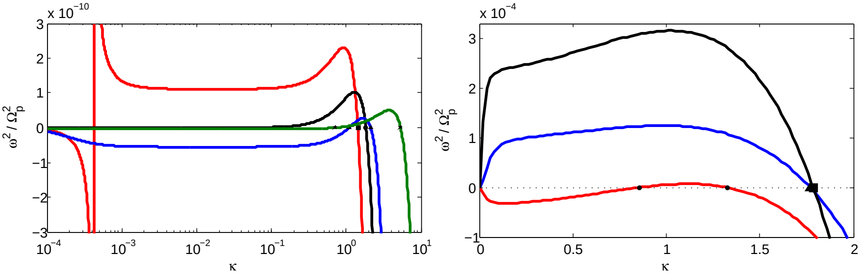


- No coupling
- No transverse neutral points for large *B*
- One/two neutral points

Instability Rates

Frequency Determination

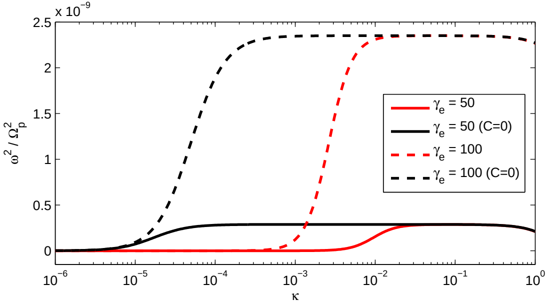
- From dispersion relation: $\omega^2(k_\perp) = -\dfrac{F(k_\perp)}{E(k_\perp)}$
- Therefore: Instability if
  - neutral points ($F = 0$),
  - $F > 0 \ \wedge \ E > 0$, or
  - $F < 0 \ \wedge \ E < 0$
- Zeros in E: Higher order in ω needed



Stabilization

Mechanisms to Stop/Suppress the Instability

- High background magnetic field strength[1,2]
- Saturation at the end of the linear phase[3]
- Mode coupling[1]



[1] Tautz, Lerche, & Schlickeiser, J. Math. Phys. *48*, 013302 (2007)

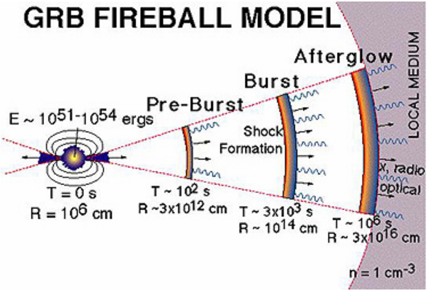
[2] Stockem, Dieckmann, & Schlickeiser, Plasma Phys. Contr. Fusion *50*, 025002 (2008)

[3] Kato, Phys. Plasmas *12*, 080705 (2005)

Gamma-Ray Bursts I

Basics

- Short and intense pulses of soft gamma radiation
- Luminosity: $\sim 10^{51}$ ergs
- Details of the radiation mechanisms not yet resolved[1]
- Line of arguments:[2]
  1. Aperiodic fluctuations
  2. Energy dissipation
  3. Shock waves
  4. Radiation bursts
- Need for instabilities
- Constraints for parameters: particle density, B field

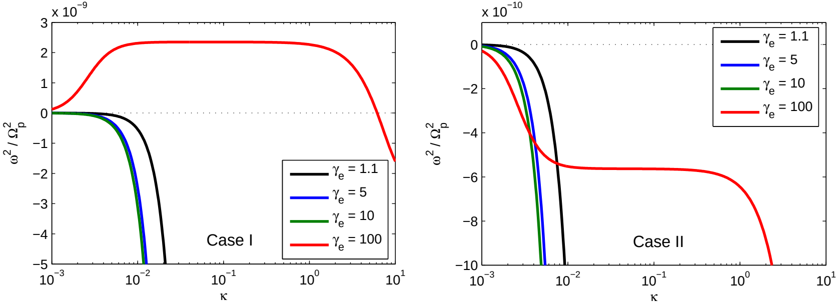


- Check differences for electron-proton vs. electron-positron plasma

[1] Schlickeiser, Vainio, Böttcher, Lerche, Pohl, & Schuster, Astron. Astrophys. *393*, 69 (2002)
[2] Tautz & Lerche, Astrophys. J. *653*, 447 (2006)

Gamma-Ray Bursts II

Electron-proton plasma



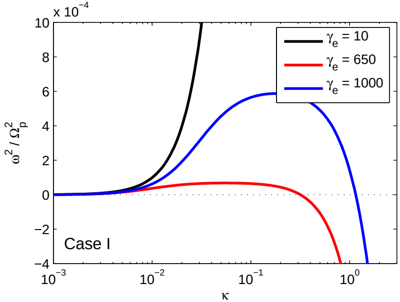
Interpretation

- Negative values for ω^2 correspond to instabilities
- Instability even for moderate electron Lorentz factors
- Higher wavenumbers: frequency approaches constant value;
  - ☞ requires higher orders in ω

Gamma-Ray Bursts III

Electron-positron plasma

- No instability, except for very high Lorentz factors ($\gamma_e \gtrsim 650$)



Conclusion[1]

- Aperiodic fluctuations in the jet of a Gamma-Ray Burst are most likely generated through an electron-proton plasma

[1] Tautz & Lerche, ApJ *653*, 447 (2006)

Summary & Outlook

Harris Instability

- Magnetized plasma
- Aperiodic electrostatic fluctuations

TLS Method

- Generalization of the Harris instability
- Coupled electrostatic/electromagnetic modes
- Suppressed for high B fields

Applications

- Applicable to all distribution functions
- GRB jet rest frame: prediction for particle composition

Future

- Maxwellian, power-law plasmas
- Parallel wave propagation