Effects of small wavenumber Alfvén waves on particle acceleration

H. Ohno (Yamagata Junior College)

ABSTRACT

Non-thermal emission observed in astrophysical sources (e.g. supernova remnants, radio galaxies, clusters of galaxies) is produced by relativistic particles (cosmic rays). Acceleration at astrophysical shocks is believed to be a mechanism for the origin of the relativistic particles. On the other hand, there are sources which are difficult to be explained through the shock acceleration. Radio halo emission in a cluster of galaxies is such a source. The radio halo is diffuse synchrotron radio emission seen in the central region on Mpc scale(e.g., Feretti, 2003). The radio halos are observed in merging clusters. This might suggest that the merger shocks accelerate relativistic electrons responsible for the halo emission. However, X-ray observations show that the shocks in the inner regions of the clusters have low Mach numbers. The acceleration efficiency at these shocks is expected to be low (Blasi et al. 2004). It

has been shown that acceleration of the relativistic electrons by MHD turbulence is suitable to explain the radio halo emission (e.g., Petrosian 2001, Ohno, Takizawa & Shibata 2002, Brunetti et al. 2004).

One of the acceleration mechanisms by MHD turbulence is acceleration by Alfvén waves. The resonant interaction of particles with the Alfvén waves results in diffusion in energy space, which can be described by a diffusion coefficient (Schlickeiser 2002). In the quasi-linear theory, the particles with a Lorentz factor γ resonate with the Alfvén waves having the wavenumbers larger than the resonant wavenumber k_{res}, where $k_{res} = \Omega_0/(c\gamma)$. In addition, it has been suggested that non-resonant waves enhance the diffusion in energy space(e.g., Terasawa 1989). The aim of this paper is to investigate effects of Alfvén waves having wavenumbers smaller than k_{res} on the particle acceleration by using test particle simulations.

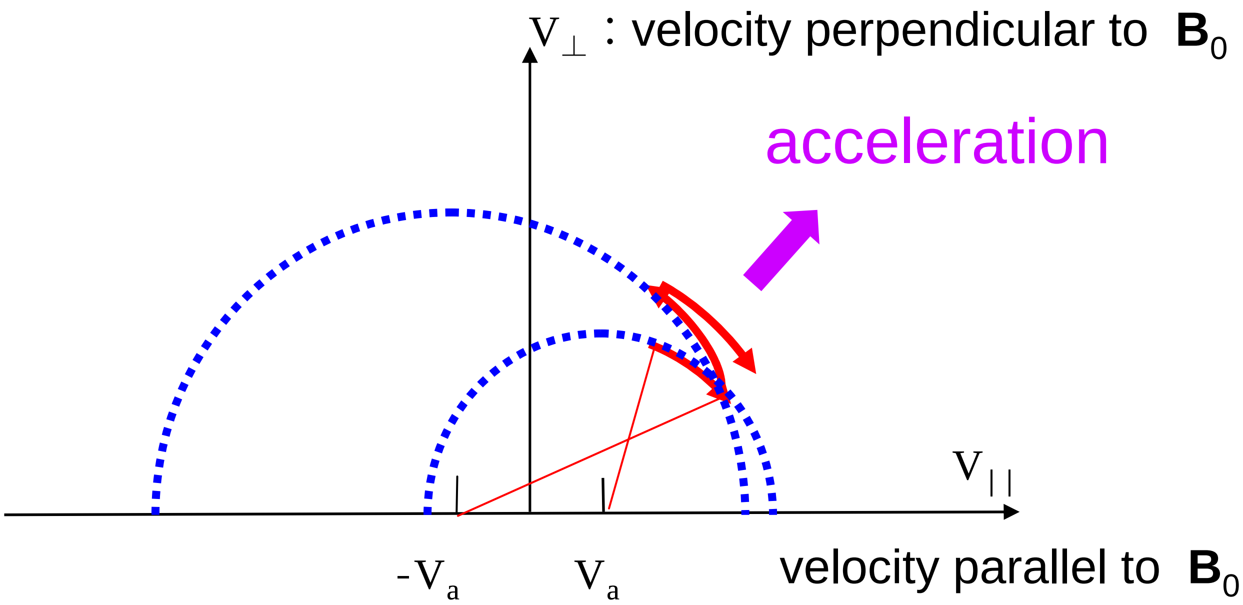
When only resonant Alfvén waves with small amplitude exist, simulated diffusion coefficients are similar to that by the quasi-linear theory. If the non-resonant Alfvén waves are added in the wavenumbers smaller than k_{res}, it is found that non-linear effects by the non-resonant Alfvén waves enhance the energy diffusion. The non-linear enhancement becomes stronger with increasing the energy density of the non-resonant Alfvén waves.

1 Resonant Acceleration by Alfvén waves

One of acceleration mechanisms by MHD turbulence is acceleration by Alfvén waves.

Resonant Acceleration

The resonant interaction of particles with the Alfvén waves propagating in both directions results in diffusion in energy space.



The particle energy is conserved in the wave rest frame.

A constant of motion H ⇨ $\Delta\mu$

The energy diffusion is described by a diffusion coefficient.

(Schlickeiser 2002)

The quasi-linear theory

Assumption :

- wave amplitude is small.
- wave phases are random.

When the assumptions are invalid, nonlinear effects were found (Terasawa 1989, Michałek and Ostrowski 1996, Kuramitsu and Hada 2000).

The pitch angle averaged diffusion coefficient

(e.g. Eilek & Henriksen 1984)

$$D_p = \frac{2\pi^2 q^2 v_A^{\ 2}}{c^3} \int_{k_{res}}^{k_{\max}} dk \left[1 - \left(\frac{v_A}{c} \mp \frac{\Omega m}{pk} \right)^2 \right] \frac{P(k)}{k}$$

q :charge

v_A : Alfvén velocity

m :mass

p : momentum

$P(k)$: wave power spectrum

$$k_{res} = \frac{\Omega_0}{c\gamma}$$

γ : Lorentz factor

The particles with a Lorentz factor γ resonate with the Alfvén waves having $k \geq k_{res}$.

The resonant wavenumber

$$k_{res} = \frac{\Omega_0}{c\gamma}$$

In addition, the waves having $k < k_{res}$ can scatter the particles.

The power spectrum of the turbulent Alfvén waves

$$P(k) \propto k^{-w} \qquad w > 1 \qquad k_{\min} \leq k \leq k_{\max}$$

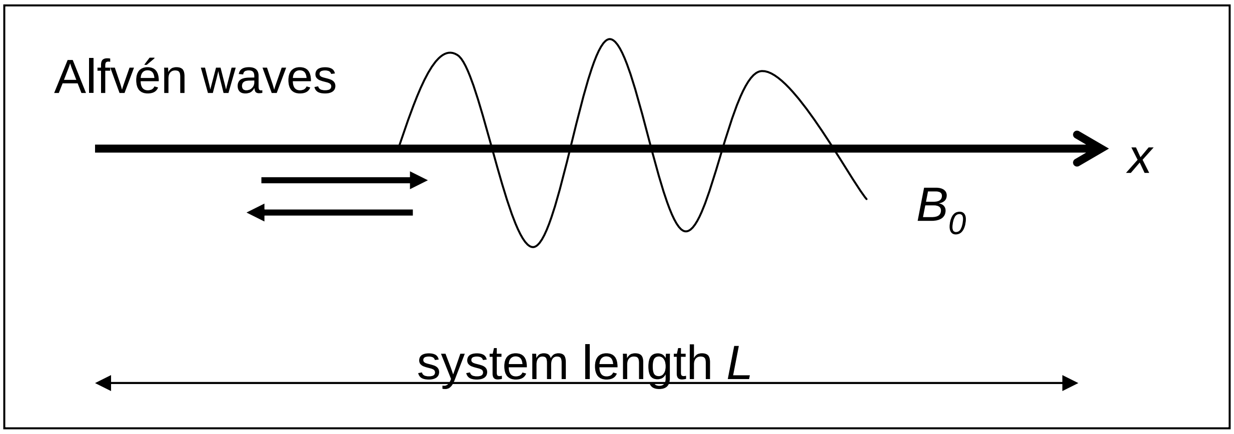
Wave amplitude becomes larger as k is smaller.

The aim of this paper

We investigate efffects of Alfvén waves having $k < k_{res}$ on the energy diffusion by using test particle simulations.

2 Model

We integrate the equation of motion for electrons in a given turbulent field and evaluate the diffusion coefficient in energy space (Terasawa 1989; Michałek and Ostrowski 1996).



The equation of motion

$$\frac{d\boldsymbol{u}}{dt} = \frac{q}{m_0}\left(\boldsymbol{E} + \frac{1}{c}\frac{\boldsymbol{u}}{\gamma}\times\boldsymbol{B}\right)$$

$$\frac{dx}{dt} = v_x$$

γ : Lorentz factor

$$\boldsymbol{u} = \gamma\boldsymbol{v}$$

$$\boldsymbol{B} = (B_0, B_y, B_z)$$

$$\boldsymbol{E} = (0, E_y, E_z)$$

Normalization constant

the speed of light *c* for velocity, Ω_0^{-1} for time, B_0 for fields.

Turbulent fields are given by superposing circularly polarized Alfvén waves.

$$B_y = \sum_j \left[\sum_k b^j{}_k \cos\left(kx - \omega^j{}_k t + \alpha^j{}_k\right) \right]$$

$$B_z = \sum_j \left[\sum_k b^j{}_k \sin\left(kx - \omega^j{}_k t + \alpha^j{}_k\right) \right]$$

$$E_y = \sum_j \left[\frac{\omega^j{}_k}{ck} B^j{}_z \right]$$

$$E_z = \sum_j \left[-\frac{\omega^j{}_k}{ck} B^j{}_y \right]$$

wave frequency $\omega^j{}_k = \pm k v_A$

j : propagation direction (+/- x direction)

polarization direction (left / right)

$\alpha^j{}_k$: wave initial phase (random)

The wavenumber

$$|k| = m \times k_1$$

$$k_1 = 2\pi / L$$

The wave amplitude

$$\left(b^j{}_k\right)^2 = \left(\delta B^j\right)^2 \frac{m^{-w}}{\sum_{m_{\min}}^{m_{\max}} m^{-w}}$$

w : the power law index

Parameters

- initial Lorentz factor γ_0=10
- 1000 electrons
- V_A/c=0.03
- w=2
- the system size L is chosen so that the resonant wavenumber k_{res} corresponds to m_{res}=100.

Free Parameters

- m_{min}, m_{max} : m corresponding to k_{min}, k_{max}.
- $\delta\tilde{B}^2$: The total energy density of the waves in the range $m_{min} \le m \le m_{max}$

Diffusion coefficient

The energy diffusion coefficient is estimated by

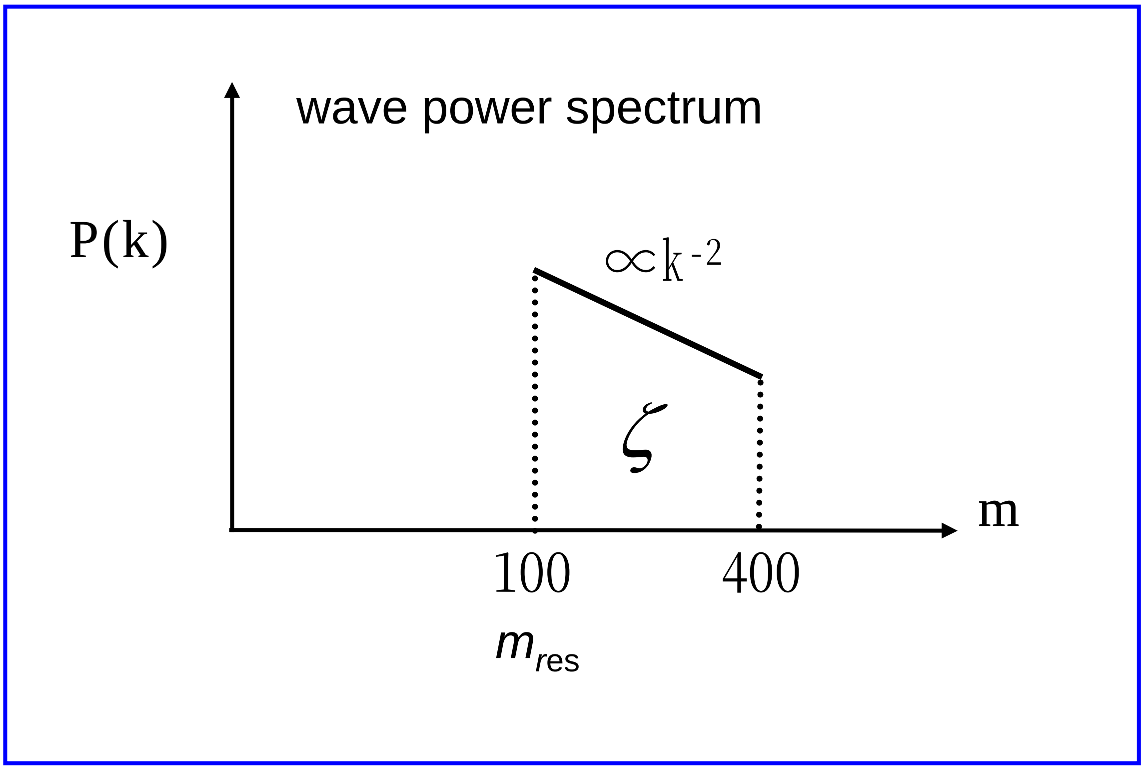
$$D^{sim} = \frac{\langle(\gamma - \gamma_0)^2\rangle}{2\tilde{t}}$$

<> : ensemble average

3. Results

1) The case when $m_{min}=m_{res}$.

This is the case when only the resonant waves exist.



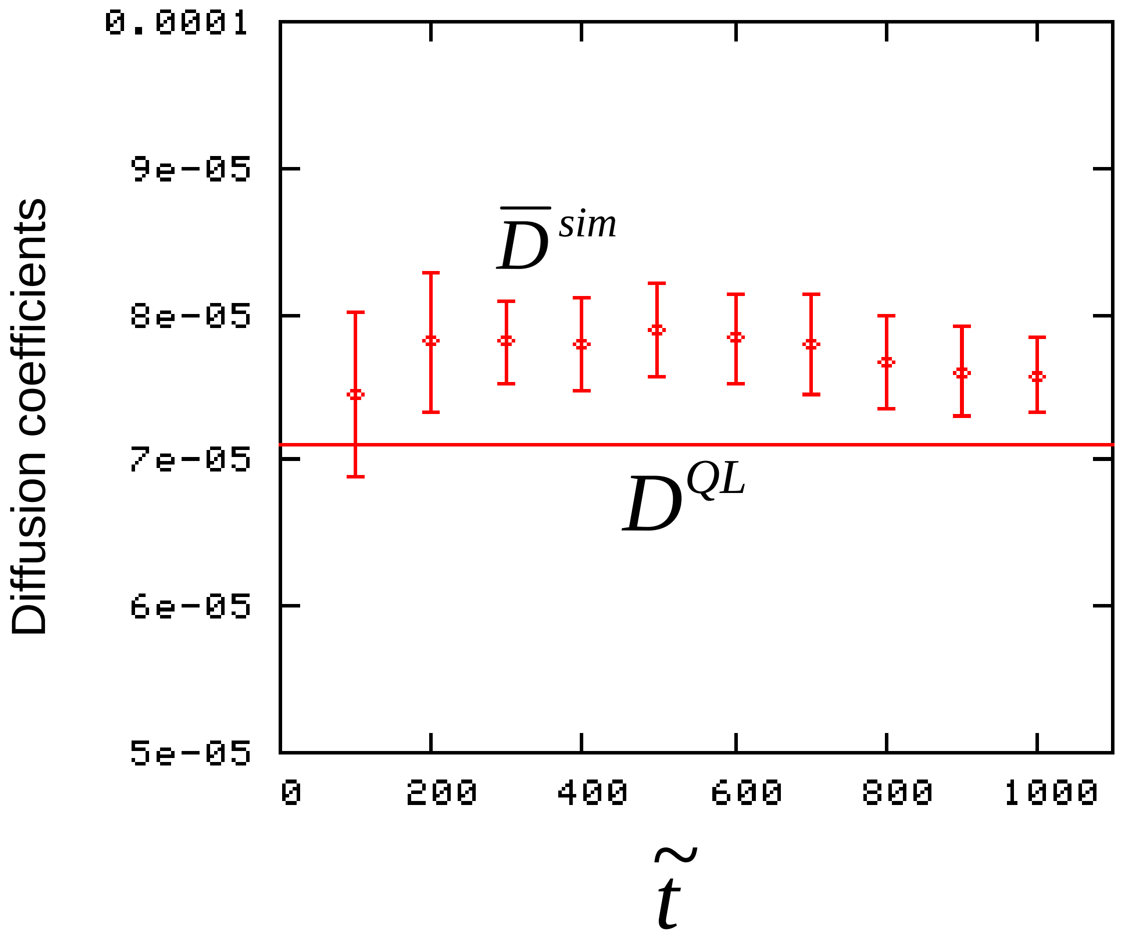
The quasi-linear diffusion coefficient

$$D^{QL} \approx \frac{\pi}{2}\frac{(w-1)}{2(w+2)}\left(\frac{v_a}{c}\right)^2 \gamma\zeta$$

ζ : the energy density of the resonant waves.

$$m_{\min} = m_{res} \ (= 100), m_{\max} = 400, \zeta = 0.04$$

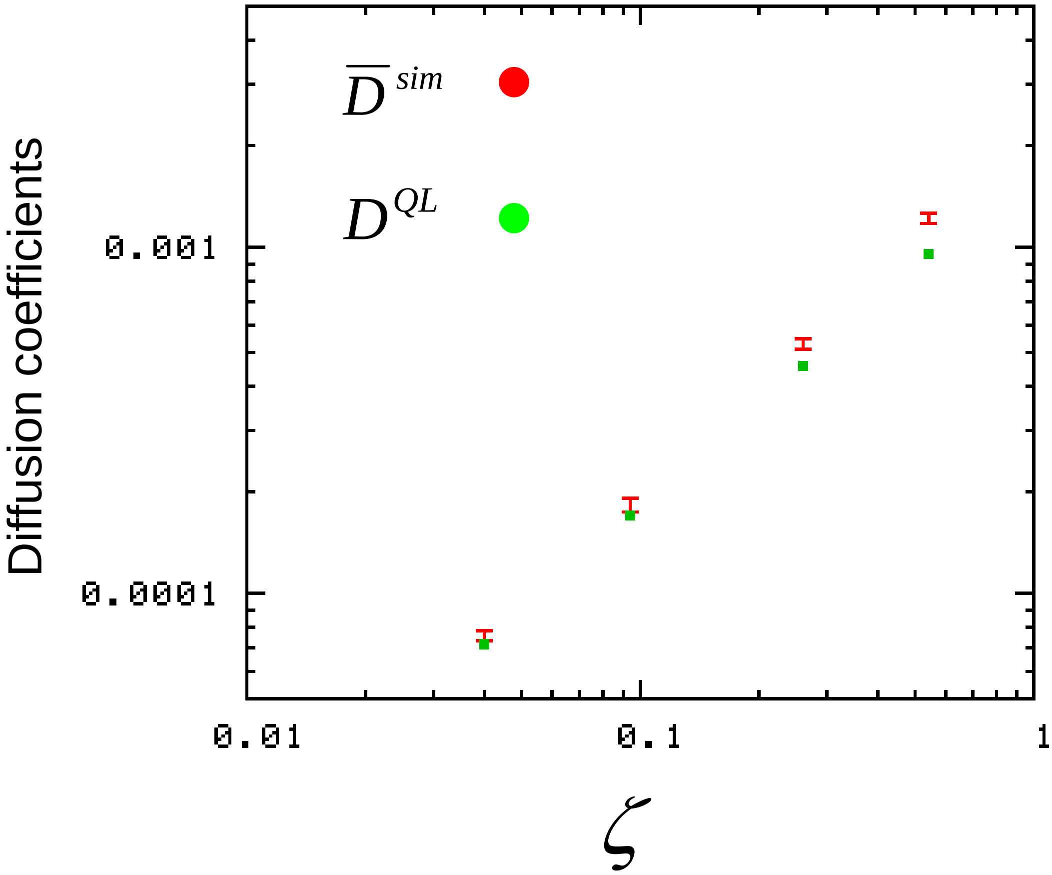
Fig.1 Simulated diffusion coefficient versus t.



We carry out ten simulation runs with different initial wave phase sets α^j_k in each model. We average the simulated diffusion coefficients at each time. The error bars represent the standard deviation.

The simulated diffusion coefficients at t=1000 versus ζ.

Fig.2



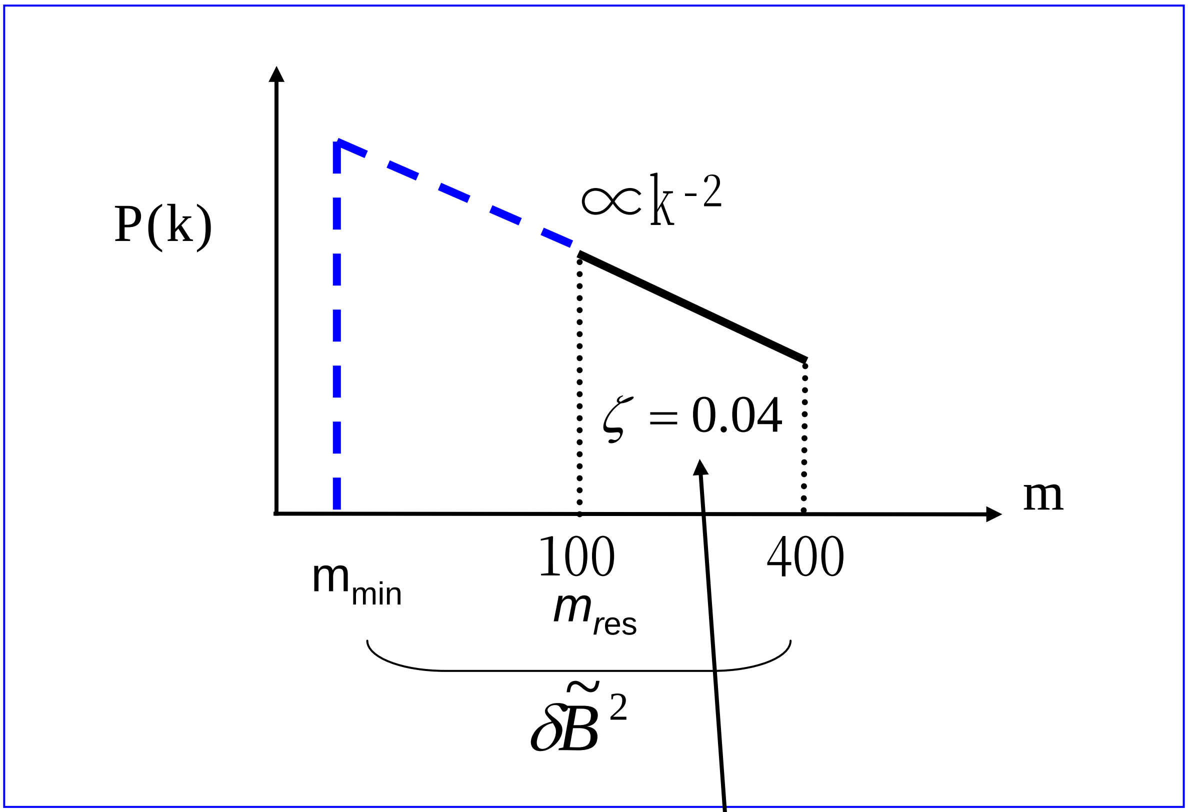
- When $\zeta < 0.1$, $D^{sim} \sim D^{QL}$.
- As ζ increases, D^{sim} rises faster than D^{QL}

(as previously reported by Terasawa(1989), Michałek and Ostrowski (1996)) .

Non-linear enhancement is weak because of the absence of non-resonant Alfvén waves having $m<m_{res}$.

2) The case when $m_{min} < m_{res}$.

How non-linear effects by the non-resonant Alfvén waves modify the energy diffusion?



The energy density of the resonant waves is small.

We consider three models.

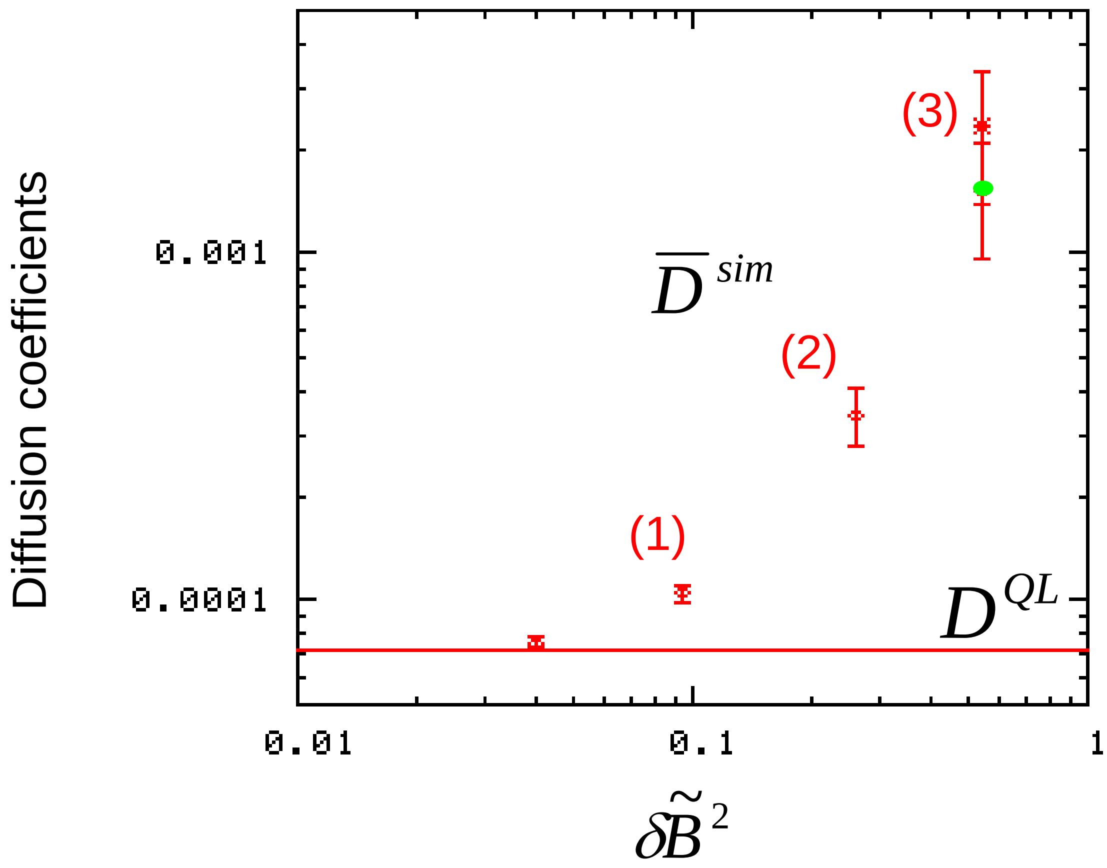
(1) $m_{\min} = 50, \delta\tilde{B}^2 = 0.09$

(2) $m_{\min} = 20, \delta\tilde{B}^2 = 0.26$

(3) $m_{\min} = 10, \delta\tilde{B}^2 = 0.54$

$\overline{D}^{sim}$ becomes larger with increasing $\delta\tilde{B}^2$.

Fig.3



Because the non-resonant waves have large amplitude, the non-linear enhancement is strong.

Energy diffusion only by the non-resonant waves

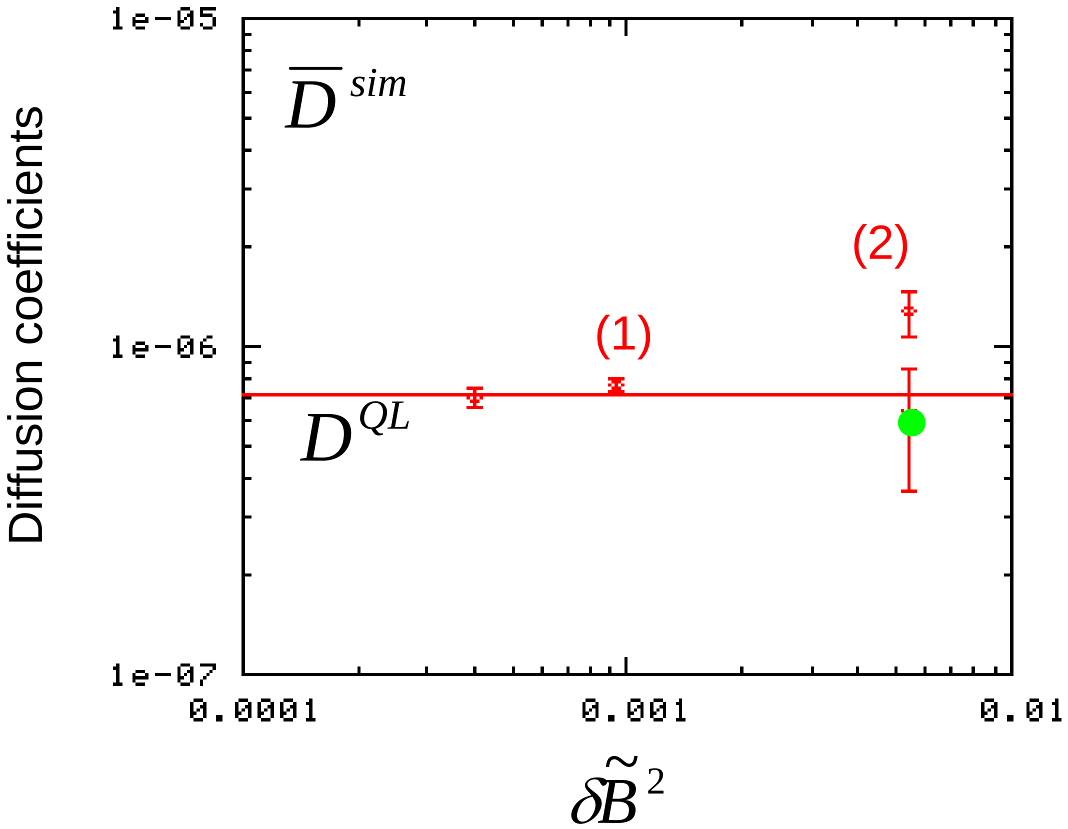
● : model $m_{\min} = 10, m_{\max} = 90, \delta\tilde{B}^2 = 0.54$

The case when ζ =0.0004.

(1) $m_{\min} = 50, \delta\tilde{B}^2 = 0.0009$

(2) $m_{\min} = 10, \delta\tilde{B}^2 = 0.0054$

Fig.4



The non-linear enhancement is weak.

When δB^2=0.0054, D^{sim}/D^{QL} ∼2.

Energy diffusion only by the non-resonant waves

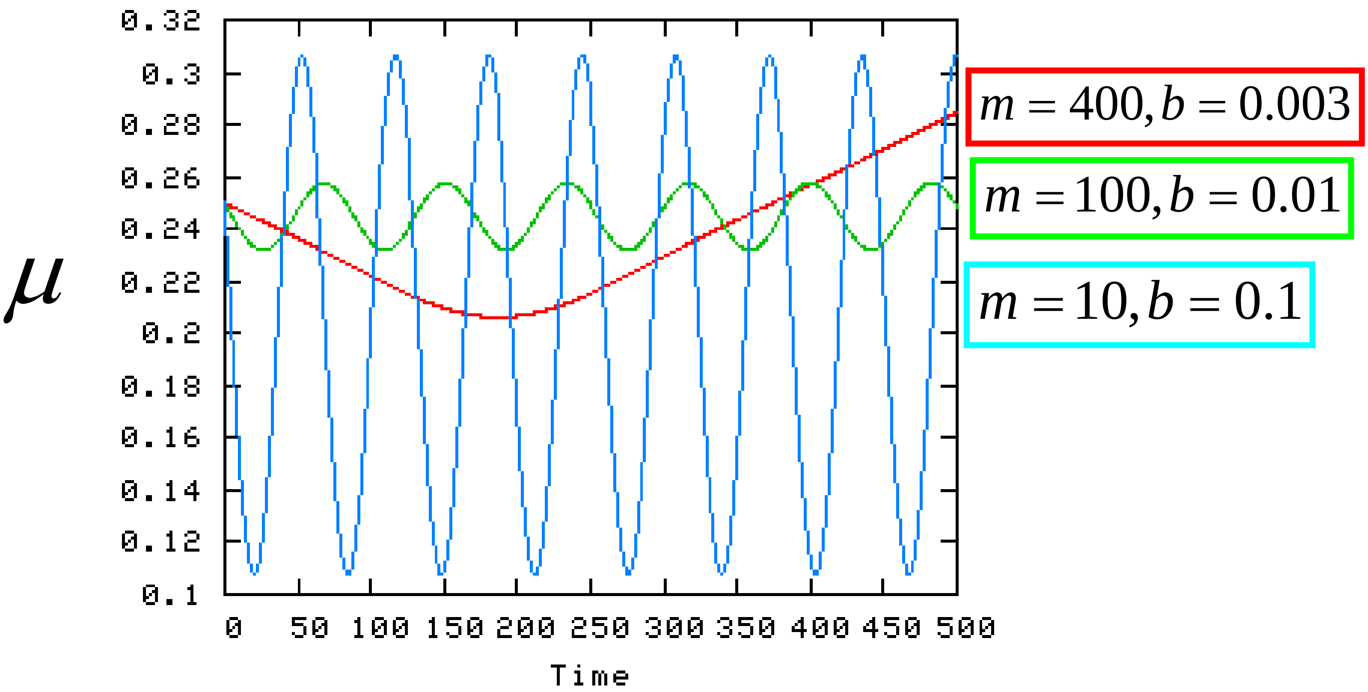
● : model $m_{\min} = 10, m_{\max} = 90, \delta\tilde{B}^2 = 0.0054$

We obtain D_{sim} ∼ D_{QL}.

4. Discussion

1) We compare pitch angle changes by the resonant waves with that by the non-resonant waves.

Model
- Initial pitch angle cosine : μ =0.25
- Monochromatic wave
  - wavenumber $k=m\,k_1$
  - wave amplitude b



The resonant wave case : $m=400, b=0.003$

The pitch angle changes slowly around μ =0.25.

The non-resonant wave case:

The pitch angle changes faster and larger as b is larger.

This enhances the pitch angle scattering and the energy diffusion.

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