Scattering length of relativistic particles in aperiodic fluctuations

Anne Stockem

Ruhr-University Bochum, Theoretical Space and Astrophysics

October 2008

Outline

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- AF: Magnetic field generation

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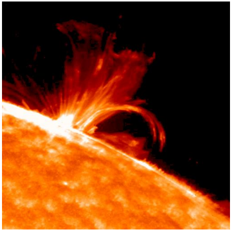

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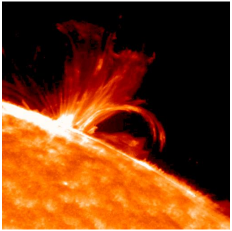

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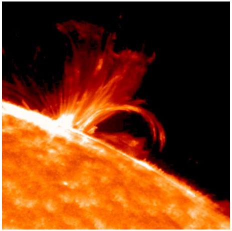

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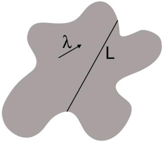

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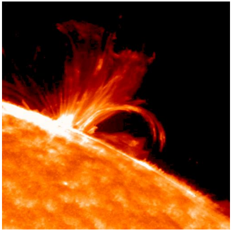

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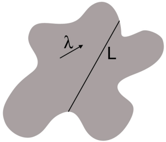

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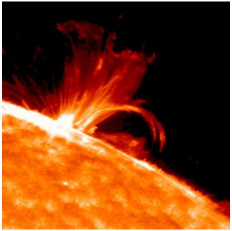

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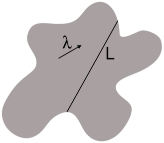

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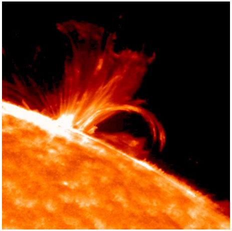

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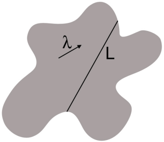

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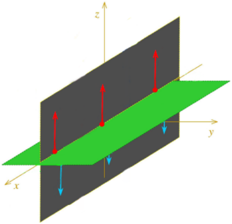
The Physical Principle

Linear phase:



- Counterstreaming particles

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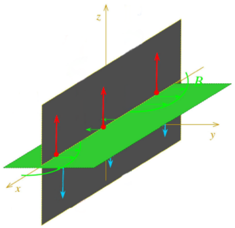

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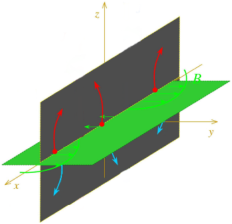

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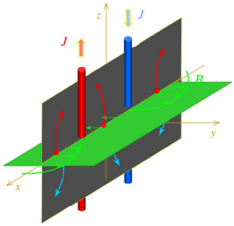

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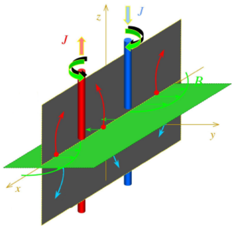

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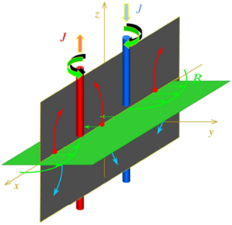

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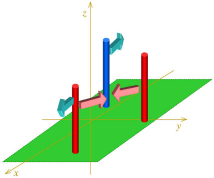

- Biot-Savart interaction

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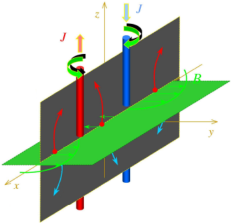

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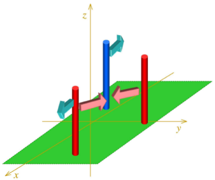

- Biot-Savart interaction
- Merging of the current filaments

Interaction between Particles and Field

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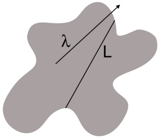

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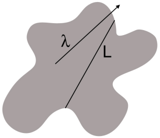

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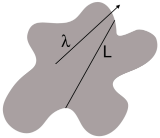

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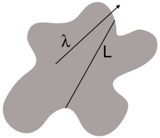

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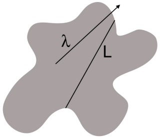

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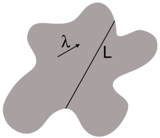

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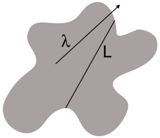

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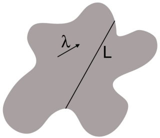

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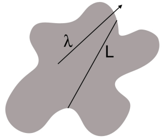

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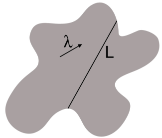

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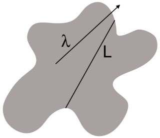

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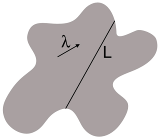

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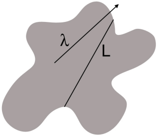

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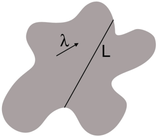

Consequences:

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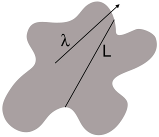

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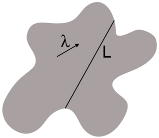

Consequences:

- Calculation of λ
- Appropriate model is necessary!

The Scattering Length

Parallel mean free path length:

$$\lambda_{\parallel} = \frac{3v}{8} \int_{-1}^{1} d\mu \, \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)}$$

Jokipii (1966); Hasselmann and Wibberenz (1968); Earl (1974)

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Diffusion coefficient:

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Damping rate:

Assumption: $\Gamma(k_\perp) = c\, l_e^{r-1}\, k_\perp^r$ Chang et al. (2008)

Model Dependency

Thermal particles:

- $r = 1$:

$$\frac{\lambda_\parallel}{L} \approx \frac{8.18 \cdot 10^{-20}}{n_{-3} L_1^2} \left(\frac{m}{m_e}\right)^{\frac{3}{2}} \theta^{-\frac{1}{2}}$$

- $r = 3$:

$$\frac{\lambda_\parallel}{L} \approx \frac{3.84 \cdot 10^{-3}}{n_{-3}^{1/6} L_1^{1/3}} \left(\frac{m}{m_e}\right)^{\frac{7}{12}} \theta^{\frac{5}{12}}$$

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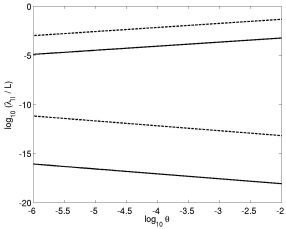

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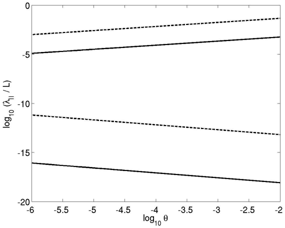

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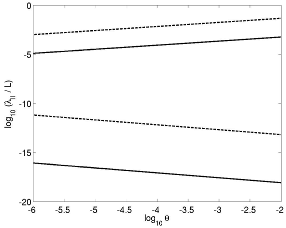

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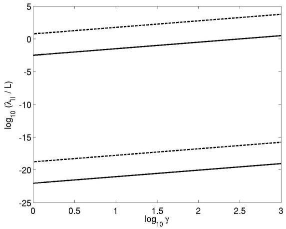
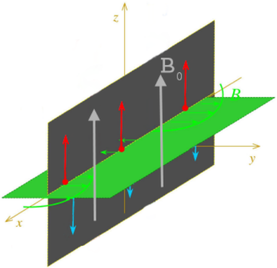


Influence of an Ambient Magnetic Field

Geometry:



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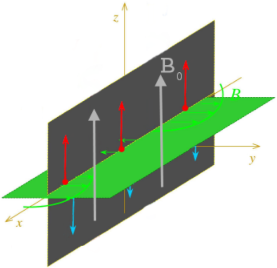

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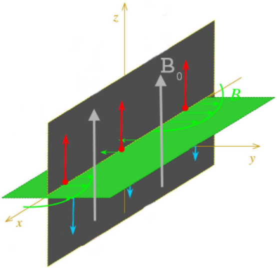

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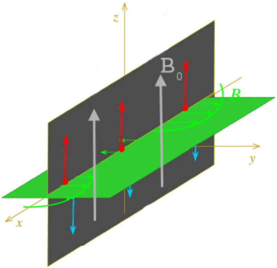

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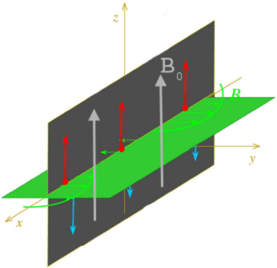

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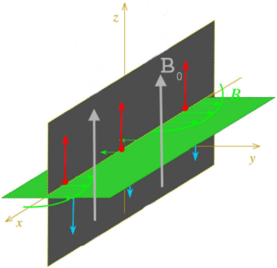

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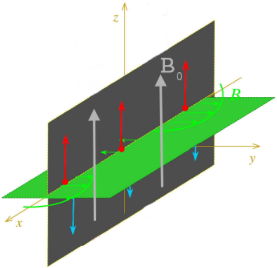
Consequences:

- Growth rate is reduced

Influence of an Ambient Magnetic Field

Geometry:



Interpretation:

- Magnetic field fluctuations $\delta\mathbf{B} \perp \mathbf{e_z}$
- Ambient field $\mathbf{B_0} \parallel \mathbf{e_z}$
- Charge separation slows down
- Particle confinement for $B_0 \geq B_c$

Consequences:

- Growth rate is reduced
- No amplification for $B_0 \geq B_c$

Analytical Results of the FI

Additional assumption:

Cold plasma approach: $T = 0$

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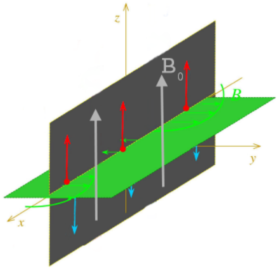
Analytics:

- Aperiodic fluctuations: $\delta B_\perp \propto \mathrm{e}^{\imath \mathbf{k}\cdot\mathbf{x}+\sigma t}$

Magnetic field:



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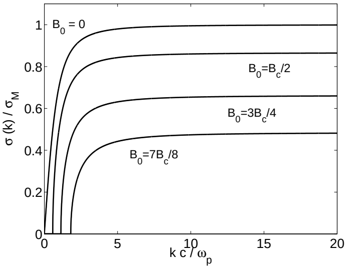
- Aperiodic fluctuations: $\delta B_\perp \propto \mathrm{e}^{\imath \mathbf{k}\cdot\mathbf{x}+\sigma t}$
- Maximum growth rate: $\sigma_{max} = \sqrt{\Omega_{max}^2 - \Omega^2}$

Linear growth rate:



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$\Omega_{max} = \omega_p U/\sqrt{\gamma}c$
$\Omega = eB_0/m$

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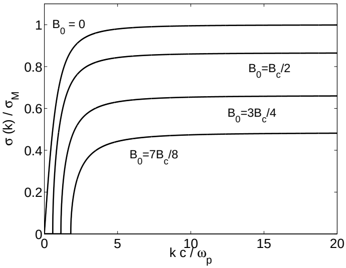

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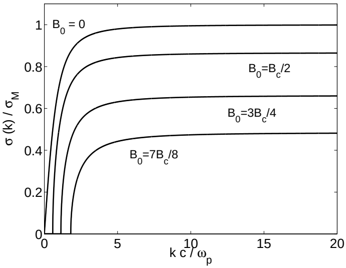
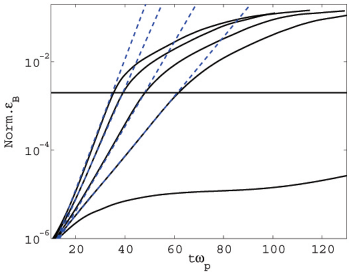


The Energy Densities

Magnetic energy density:



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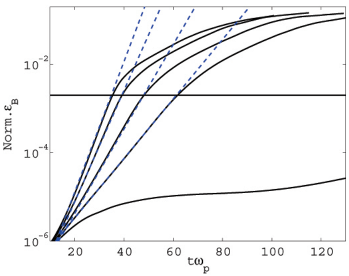

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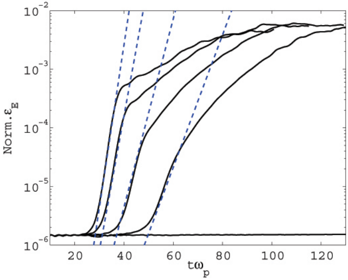

Scattering length

Motivation

Particle/Field

The Filamentation Instability

Conclusions

The Energy Densities

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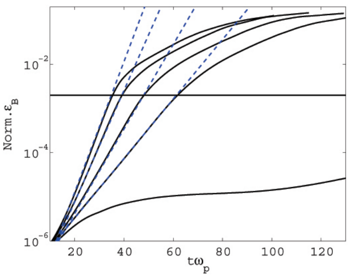

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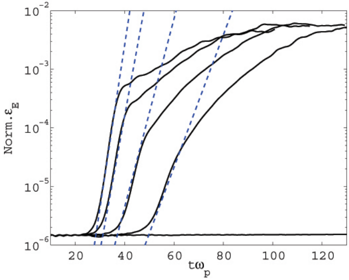

Results:

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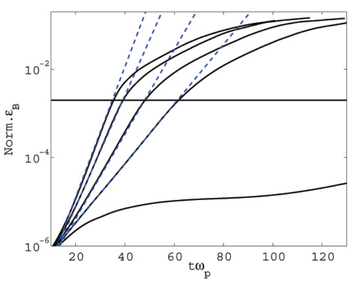

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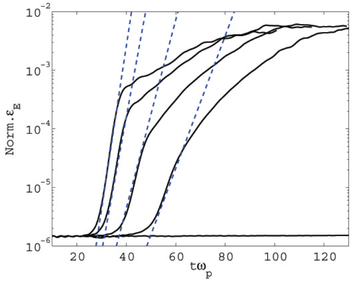

Results:

- Same amplification level independent of B_0

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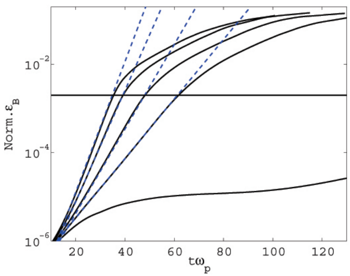

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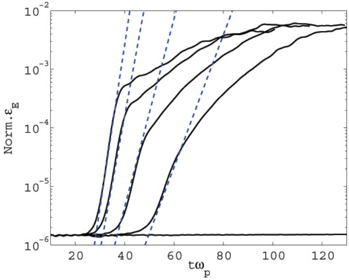

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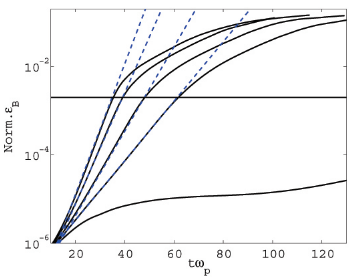

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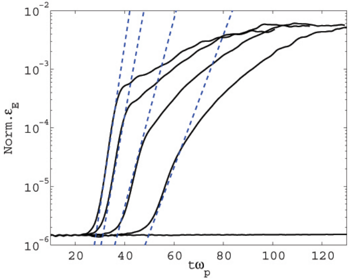

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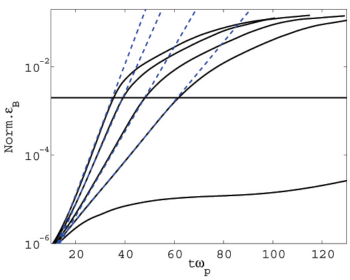

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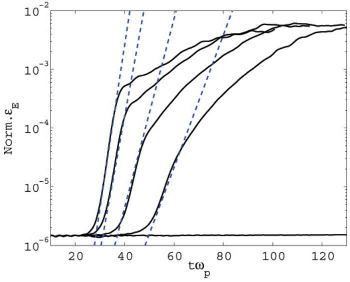
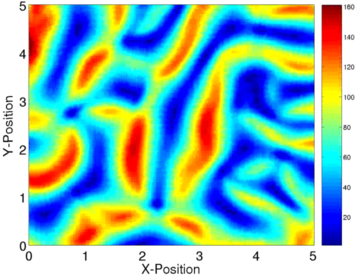


Results:

- Same amplification level independent of B_0
- ϵ_B: Growth for $B_0 = B_c$
- Non-linear effect
- Amplification of ϵ_E starts later

Magnetic Field Evolution

Perpendicular component:



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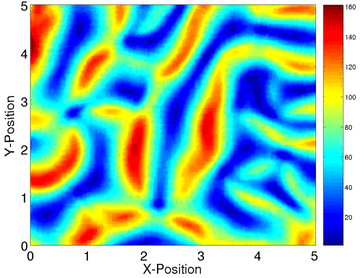

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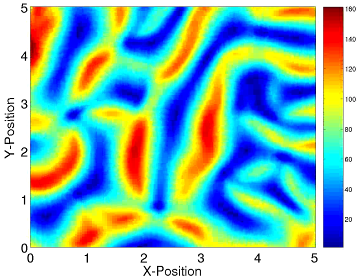
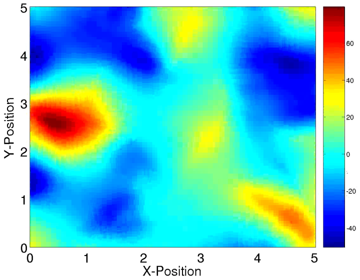


Parallel component:



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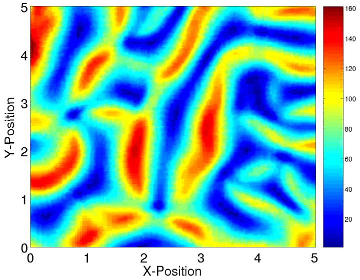

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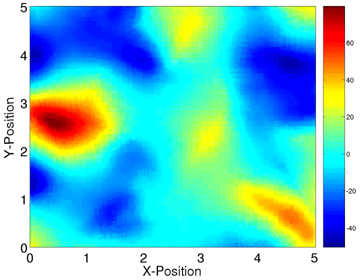

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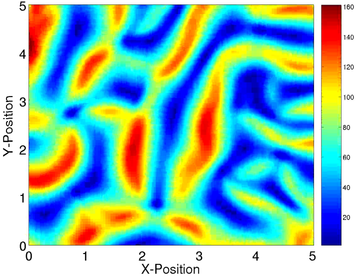

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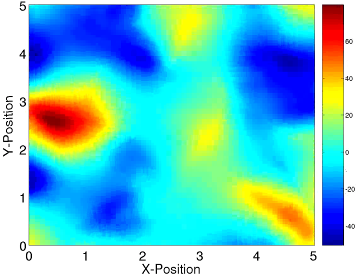
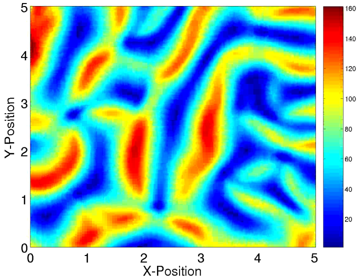


Comparison:

- Strong perpendicular component has developed

Magnetic Field Evolution

Perpendicular component:



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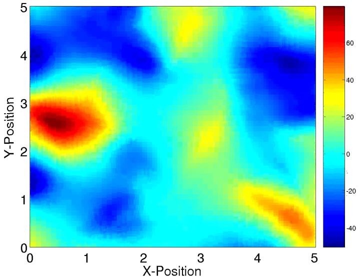

Comparison:

- Strong perpendicular component has developed
- Amplification of $B_{||}$: temperature effect

Conclusions

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Thanks for your attention!