Dynamo Action in Spiral Galaxies

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Presenting my thesis work done at the
Inter-University Centre for Astronomy and Astrophysics, Pune, India

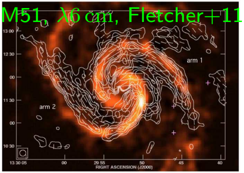
Supervised by: Kandaswamy Subramanian (IUCAA)

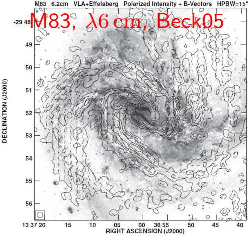
Collaborators: Anvar Shukurov (Newcastle), Katherine Stoker (Newcastle),
Alice Quillen (Rochester)

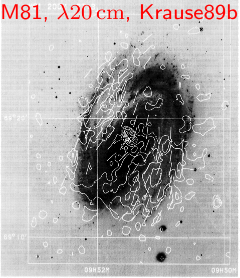
Cosmic Magnetic Fields, Krakow

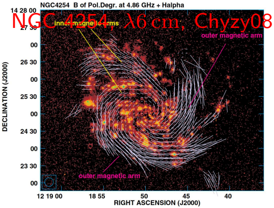
October 20, 2014

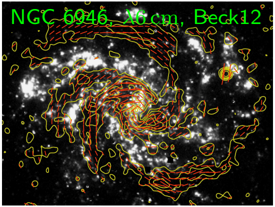
Observational Motivation

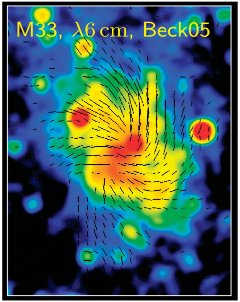

M51, $\lambda 6$ cm, Fletcher+11

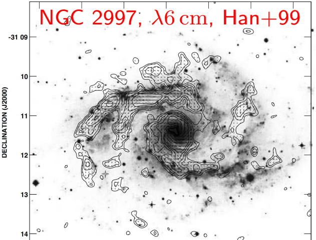

M83, $\lambda 6$ cm, Beck05

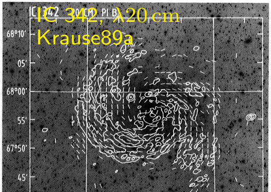

M81, $\lambda 20$ cm, Krause89b


NGC 4254, $\lambda 6$ cm, Chyzy08


NGC 6946, $\lambda 6$ cm, Beck12


M33, $\lambda 6$ cm, Beck05


NGC 2997, $\lambda 6$ cm, Han+99


IC 342, $\lambda 20$ cm, Krause89a

Mean-Field Dynamo Theory

The mean-field formalism **Ideal MHD limit $R_m = lu/\eta \gg 1$**

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B}) \qquad \boldsymbol{U} = \overline{\boldsymbol{U}} + \boldsymbol{u} \qquad \boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b} \qquad \boldsymbol{\mathcal{E}} = \overline{\boldsymbol{u} \times \boldsymbol{b}}$$

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \boldsymbol{\mathcal{E}}\right) \qquad \frac{\partial \boldsymbol{b}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \boldsymbol{b} + \boldsymbol{u} \times \overline{\boldsymbol{B}}\right) + \boldsymbol{\nabla} \times (\boldsymbol{u} \times \boldsymbol{b} - \boldsymbol{\mathcal{E}})$$

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The mean emf $\mathcal{E}$ **First Order Smoothing Approximation**

$$\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)\boldsymbol{\mathcal{E}} = \left(\alpha \overline{\boldsymbol{B}} - \eta_{\mathrm{t}} \boldsymbol{\nabla} \times \overline{\boldsymbol{B}}\right)\frac{1}{\tau_{\mathrm{c}}} \qquad \tau_{\mathrm{c}} \simeq \tau \simeq l/u$$

$$\alpha_{\mathrm{k}} \simeq -\frac{1}{3}\tau_{\mathrm{c}}\overline{\mathbf{u} \cdot (\boldsymbol{\nabla} \times \mathbf{u})} \qquad \eta_{\mathrm{t}} \simeq \frac{1}{3}\tau_{\mathrm{c}}\overline{\boldsymbol{u}^2} \qquad \alpha = \alpha_{\mathrm{k}} + \alpha_{\mathrm{m}} \qquad \alpha_{\mathrm{m}} \simeq \frac{1}{3}\tau_{\mathrm{c}}\frac{\overline{\mathbf{b} \cdot (\boldsymbol{\nabla} \times \mathbf{b})}}{4\pi\rho}$$

Mean-Field Dynamo Theory

The mean-field formalism — Ideal MHD limit $R_m = lu/\eta \gg 1$

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The mean emf $\mathcal{E}$ — Minimal τ Approximation

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Mean-Field Dynamo Theory

The mean-field formalism

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Dynamo equation for $\overline{B}$

Limit of $\tau \to 0$

$$\tau \frac{\partial^2 \overline{B}}{\partial t^2} + \frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \alpha \overline{\boldsymbol{B}} - \eta_{\mathrm{t}} \boldsymbol{\nabla} \times \overline{\boldsymbol{B}}\right) + \tau \boldsymbol{\nabla} \times \left(\overline{U} \times \frac{\partial \overline{B}}{\partial t}\right)$$

Mean-Field Dynamo Theory

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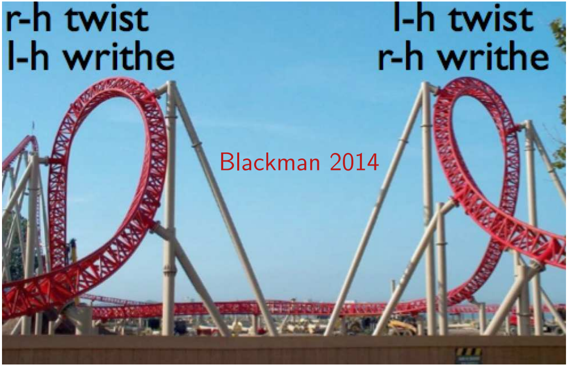
Dynamo equation for $\overline{B}$ — General case of finite τ

$$\tau \frac{\partial^2 \overline{\boldsymbol{B}}}{\partial t^2} + \frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \alpha \overline{\boldsymbol{B}} - \eta_{\rm t} \boldsymbol{\nabla} \times \overline{\boldsymbol{B}}) + \tau \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{U}} \times \frac{\partial \overline{\boldsymbol{B}}}{\partial t}\right)$$

- Effects of order $\Gamma\tau \ll 1$.
- Effects of order $\Omega_{\rm p}\tau \sim 0.2 - 0.4 \sim (10 - 25)^{\circ}$.

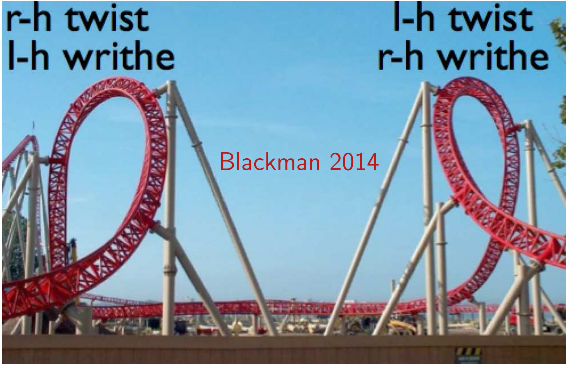
The Dynamical α-Quenching Non-Linearity



Blackman 2014

- Magnetic helicity: Measure of twist & writhe or, equivalently, linkage in the field.
- Magnetic helicity is very well conserved in a closed system.
- If the initial helicity$= 0$, then growth of large-scale helicity $\Rightarrow$ growth of oppositely signed small-scale helicity.
- This small-scale helicity back-reacts via the Lorentz force to quench the dynamo.

The Dynamical α-Quenching Non-Linearity



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Evolution equation for α_{m} (Subramanian & Brandenburg 2006, Shukurov et al. 2006)

$$\alpha = \alpha_{\mathrm{k}} + \alpha_{\mathrm{m}}, \qquad \frac{\partial \alpha_{\mathrm{m}}}{\partial t} = -\frac{2\eta_{\mathrm{t}}}{l^2}\left(\underbrace{\frac{\boldsymbol{\mathcal{E}}\cdot\overline{\boldsymbol{B}}}{B_{\mathrm{eq}}^2}}_{\text{source term}} + \underbrace{\frac{\alpha_{\mathrm{m}}}{R_{\mathrm{m}}}}_{\text{Ohmic dissipation}}\right) \underbrace{-\,\nabla\cdot\boldsymbol{\mathcal{F}}}_{\text{flux term}}, \qquad \boldsymbol{\mathcal{F}} = \underbrace{\overline{\boldsymbol{U}}\alpha_{\mathrm{m}}}_{\text{advective flux}} \underbrace{-\,\kappa_{\mathrm{t}}\boldsymbol{\nabla}\alpha_{\mathrm{m}}}_{\text{diffusive flux}} \underbrace{+\ \ldots}_{\text{other fluxes?}}$$

Local Analytical Steady-State Solutions

Inputs: l, u, $\Omega(r)$, $\overline{U}_z(z)$, h, $\rho(r)$, κ_t

Sur, Shukurov & Subramanian 2007
Chamandy, Shukurov, Subramanian & Stoker 2014

Local Analytical Steady-State Solutions

Inputs: l, u, $\Omega(r)$, $\overline{U}_z(z)$, h, $\rho(r)$, κ_t

Sur, Shukurov & Subramanian 2007
Chamandy, Shukurov, Subramanian & Stoker 2014

Useful: $D = \frac{\alpha_\mathrm{k} h^3}{\eta_\mathrm{t}^2}\frac{d\Omega}{d\ln r}, \quad \alpha_\mathrm{k} = \frac{l^2}{h}\Omega, \quad \eta_\mathrm{t} = \frac{1}{3}lu, \quad R_U \equiv \frac{\overline{U}_z h}{\eta_\mathrm{t}}, \quad R_\kappa \equiv \frac{\kappa_\mathrm{t}}{\eta_\mathrm{t}}, \quad t_\mathrm{d} \equiv \frac{h^2}{\eta_\mathrm{t}}.$

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Outputs: $\frac{\langle \overline{B} \rangle}{\sqrt{4\pi\rho u^2}} \simeq \frac{l}{h}\sqrt{R_U + \pi^2 R_\kappa}\sqrt{\frac{D}{D_\mathrm{c}} - 1}, \qquad D_\mathrm{c} = -\frac{\pi^5}{32}\left(1 + \frac{R_U}{\pi^2}\right)^2 \qquad (\text{no}-z\ \text{approx.}),$

Local Analytical Steady-State Solutions

Sur, Shukurov & Subramanian 2007
Chamandy, Shukurov, Subramanian & Stoker 2014

Inputs: $l,\ u,\ \Omega(r),\ \overline{U}_z(z),\ h,\ \rho(r),\ \kappa_\mathrm{t}$

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$$\overline{B}_r\simeq C_0\frac{|D_\mathrm{c}|}{\Omega t_\mathrm{d}}\left\{\cos\frac{\pi z}{2h}+\frac{3}{4\pi^2}\left(\sqrt{\pi|D_\mathrm{c}|}-\frac{R_U}{2}\right)\cos\frac{3\pi z}{2h}+\frac{R_U}{2\pi^2}\sum_{n=2}^{\infty}\frac{(-1)^n(2n+1)}{n^2(n+1)^2}\cos\left[\left(n+\tfrac{1}{2}\right)\frac{\pi z}{h}\right]\right\},$$

$$\overline{B}_\phi\simeq -C_0\sqrt{\frac{4|D_\mathrm{c}|}{\pi}}\left\{\cos\frac{\pi z}{2h}-\frac{3R_U}{8\pi^2}\cos\frac{3\pi z}{2h}+\frac{R_U}{2\pi^2}\sum_{n=2}^{\infty}\frac{(-1)^n(2n+1)}{n^2(n+1)^2}\cos\left[\left(n+\tfrac{1}{2}\right)\frac{\pi z}{h}\right]\right\}\quad(\text{pert. theory}).$$

Local Analytical Steady-State Solutions

Sur, Shukurov & Subramanian 2007
Chamandy, Shukurov, Subramanian & Stoker 2014

Inputs: $l,\ u,\ \Omega(r),\ \overline{U}_z(z),\ h,\ \rho(r),\ \kappa_\mathrm{t}$

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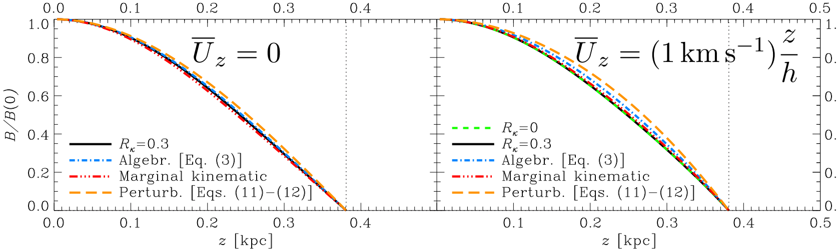
$\overline{B}(4\,\mathrm{kpc},z)/\overline{B}(4\,\mathrm{kpc},0)$

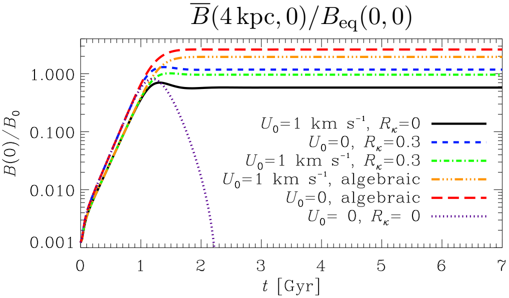


Dynamical vs. Algebraic Quenching

Algebraic quenching heuristic formula:

$$\alpha = \frac{\alpha_\mathrm{k}}{1+a(\overline{B}/B_\mathrm{eq})^q}$$

with $q = 2$ and $a = 1$



$\overline{B}(4\,\mathrm{kpc},0)/B_\mathrm{eq}(0,0)$

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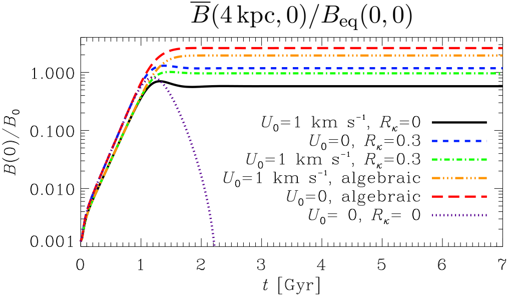

No-z steady-state solution
with
$\tan p_B \equiv \overline{B}_r/\overline{B}_\phi \ll 1$

$$\Rightarrow$$

$$\alpha \simeq \alpha_\mathrm{k}/\left[1 + a(\overline{B}/B_\mathrm{eq})^2\right]$$

with

$$a \simeq \frac{(h/l)^2}{R_U+\pi^2 R_\kappa} \sim 3 - 14$$

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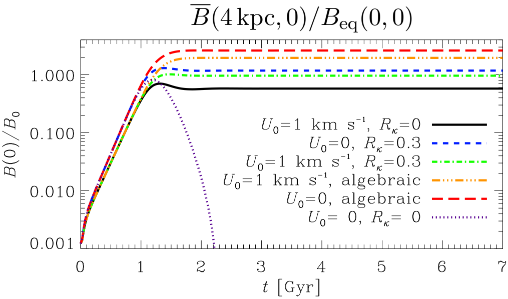


$\overline{B}(4\,\mathrm{kpc},0)/B_\mathrm{eq}(0,0)$

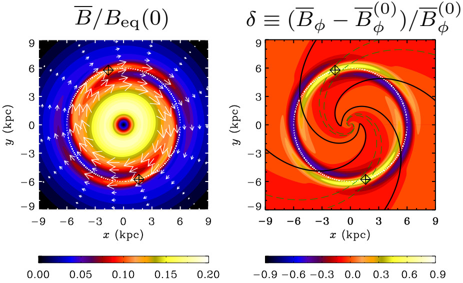
No-z steady-state solution with $\tan p_B \equiv \overline{B}_r/\overline{B}_\phi \ll 1$ $\Bigg\} \Rightarrow$ $\alpha \simeq \alpha_\mathrm{k}/\left[1+a(\overline{B}/B_\mathrm{eq})^2\right]$ with $a \simeq \frac{(h/l)^2}{R_U+\pi^2 R_\kappa} \sim 3 - 14$

- Direct numerical simulation (Gressel, Bendre & Elstner 2013): $\alpha \simeq \alpha_\mathrm{k}/\left[1+a(\overline{B}/B_\mathrm{eq})^2\right]$ with $q = 2.0 \pm 0.3$ and $a = 27 \pm 14$

$\longrightarrow$ Support for dynamical quenching theory.

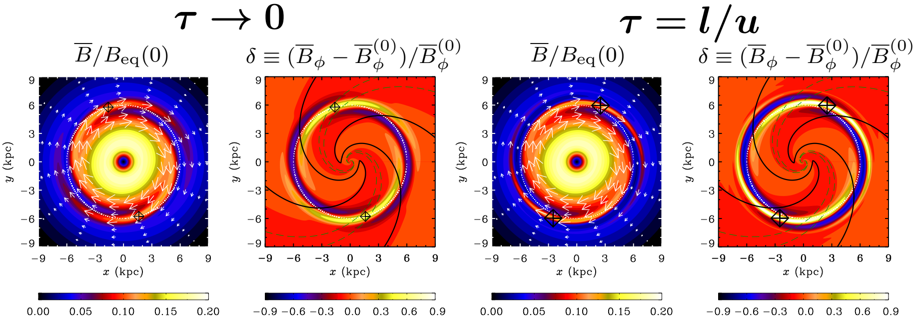
Magnetic arms and the τ effect

$$\boldsymbol{\tau \to 0}$$



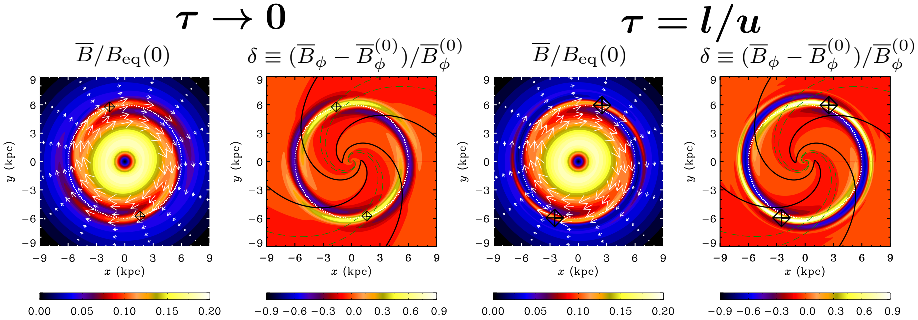
Chamandy, Subramanian & Shukurov 2013a, b

Magnetic arms and the τ effect



Chamandy, Subramanian & Shukurov 2013a, b

Magnetic arms and the τ effect



$\tau \to 0$ $\qquad$ $\tau = l/u$

$\overline{B}/B_{\rm eq}(0)$ $\qquad$ $\delta \equiv (\overline{B}_\phi - \overline{B}_\phi^{(0)})/\overline{B}_\phi^{(0)}$ $\qquad$ $\overline{B}/B_{\rm eq}(0)$ $\qquad$ $\delta \equiv (\overline{B}_\phi - \overline{B}_\phi^{(0)})/\overline{B}_\phi^{(0)}$

$$\begin{pmatrix}\overline{B}_r\\ \overline{B}_\phi\end{pmatrix} = \sum_{m=-\infty}^{\infty}\begin{pmatrix}a_m(r,z)\\ b_m(r,z)\end{pmatrix}\exp\left(im\phi+\Gamma t\right),$$

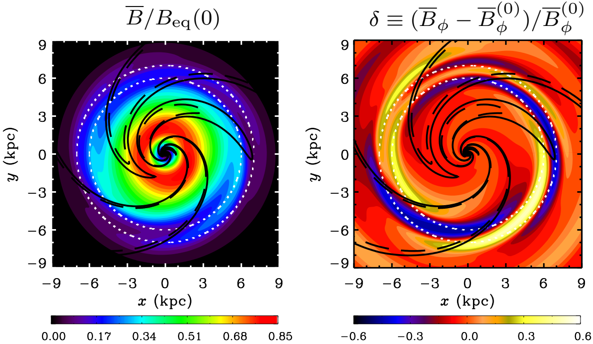
$$\sum_{m=-\infty}^{\infty} \mathrm{e}^{im\phi}\left\{(\Gamma+im\widetilde{\Omega})a_m + \frac{\mathrm{e}^{-i\delta_m}}{C_{|m|}}\left[\frac{\partial}{\partial z}(\alpha b_m) - \eta_{\rm t}\left(\widetilde{\nabla}^2 a_m - m^2\frac{a_m}{r^2} - 2im\frac{b_m}{r^2}\right)\right]\right\} = 0,$$

$$\sum_{m=-\infty}^{\infty} \mathrm{e}^{im\phi}\left\{(\Gamma+im\widetilde{\Omega})b_m - Ga_m - \frac{\mathrm{e}^{-i\delta_m}}{C_{|m|}}\eta_{\rm t}\left(\widetilde{\nabla}^2 b_m - m^2\frac{b_m}{r^2} + 2im\frac{a_m}{r^2}\right)\right\} = 0,$$

where $C_{|m|} \equiv \left[(1+\Gamma\tau)^2 + (m\Omega_{\rm p}\tau)^2\right]^{1/2}$, $\qquad \cos\delta_m = \dfrac{1+\Gamma\tau}{C_{|m|}}$, $\qquad \sin\delta_m = -\dfrac{m\Omega_{\rm p}\tau}{C_{|m|}}$.

Importance of Outflows and of Spiral Evolution

Interfering steady patterns



$\overline{B}/B_{\rm eq}(0)$

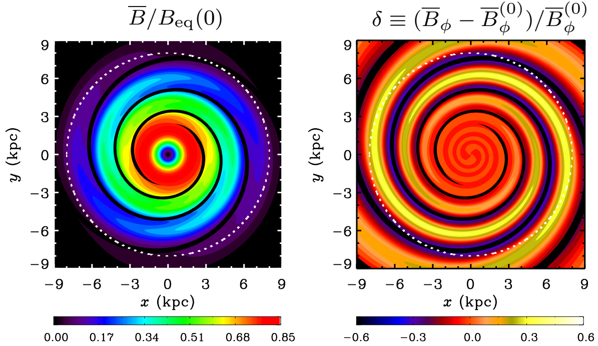
$\delta \equiv (\overline{B}_\phi - \overline{B}_\phi^{(0)})/\overline{B}_\phi^{(0)}$

Chamandy, Shukurov & Subramanian 2014

Chamandy, Subramanian & Quillen 2014

Importance of Outflows and of Spiral Evolution

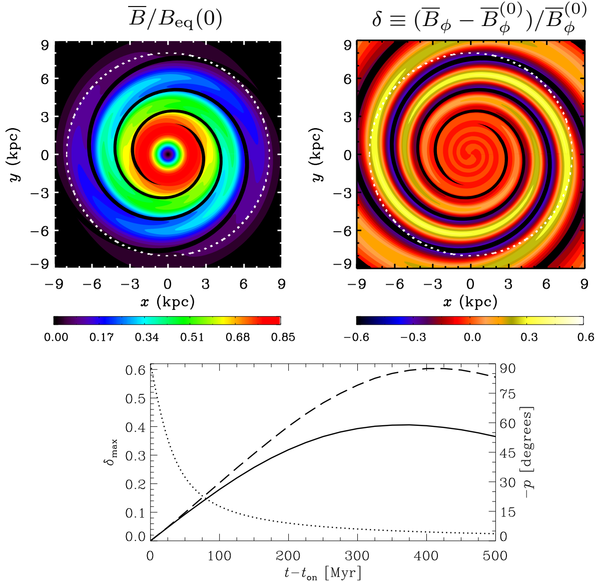
Transient density wave



Chamandy, Shukurov & Subramanian 2014

Importance of Outflows and of Spiral Evolution

Transient density wave



Summary

- Approximate analytical solutions are available and may be useful as a first line of attack or for comparison with observations/simulations.
- The algebraic quenching formula can, in disc galaxies, be understood as an approximation of dynamical quenching theory; this form of quenching seems to be supported by direct simulations.
- The τ effect can lead to phase shifts of magnetic arms from gaseous/stellar arms of $-(1 \text{ to } 2)\Omega_\mathrm{p}\tau \sim -10^\circ$ to -40°. This may help to explain the phase shifts observed in several galaxies.
- Outflows are expected to be concentrated in the gaseous/stellar spiral arms; this can lead naturally to the concentration of regular fields in the interarm regions.
- Characterizing the nature of spiral evolution is crucial for determining the structure and evolution of magnetic arms. Observations showing magnetic arms aligned with gaseous/stellar arms over several kpc seem to favour transient (winding-up) density wave models.