Cosmic curvature in astronomical measurements

Krzysztof Bolejko
The University of Sydney

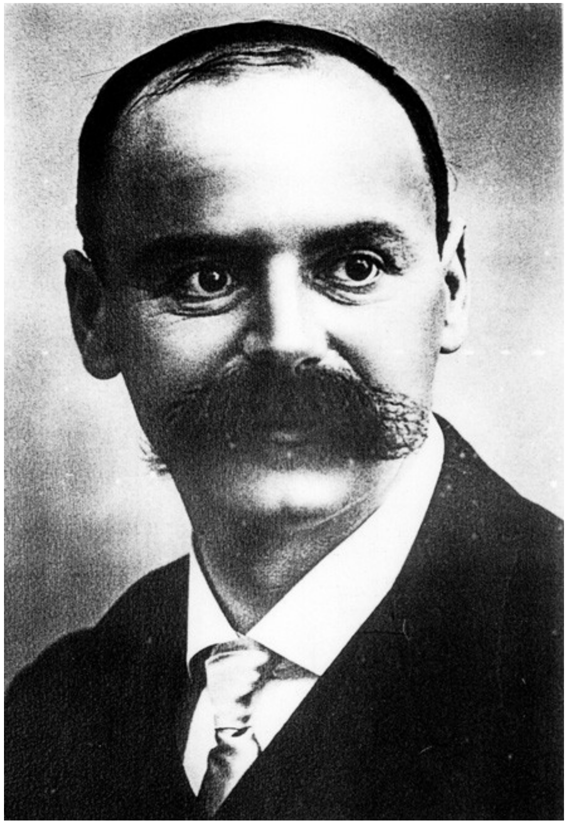
21 September 2017, Astrofizyka Cząstek

Outline

- Historical overview
- Evidence for spatial flatness of our Universe
- Emerging spatial curvature

Über das zulässige Krümmungsmaß des Raumes,
Vierteljahrschrift d. Astronom. Gesellschaft. 35, 337-47 (1900)

"As has recently been shown by Professor Seeliger, the most rational view of the arrangement of stars that one can build on the basis of current observations, is that all visible ***stars****, whose number can be estimated to be no greater than* ***40 million****, can be considered to lie within a space of a* ***few hundred AU****, and that* ***outside this is a relative void****. Even if this view is reassuring, by offering us a significant step in our understanding of the Universe through a complete investigation of this limited stellar system, this reassurance, so satisfying to reason, would be experienced to an even greater degree* ***if we could conceive of space as being closed and finite and filled, more or less completely by this stellar system****."*

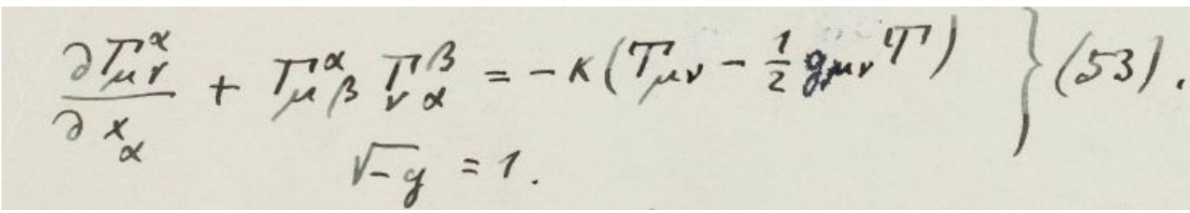


Karl Schwarzschild

Class. Quantum Grav., 15 2539 (1998)

$$G_{ab} = T_{ab}$$

Annalen der Physik, 354, 769 (1916)



$$\left.\begin{array}{l} \frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x_{\alpha}} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} = -\kappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right) \\ \sqrt{-g} = 1 . \end{array}\right\} (53).$$

original manuscript available at *http://new.alberteinstein.info*

Albert Einstein

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

Königlich Preußischen Akademie der Wissenschaften, 142 (1917)

$$G_{\mu\nu} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right) \qquad (13)$$

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right). \qquad (13a)$$

Albert Einstein

Astronomy. — *"On the curvature of space"*. By Prof. W. DE SITTER.

(Communicated in the meeting of 1917, June 30).

1. In order to make possible an entirely relative conception of inertia, EINSTEIN [1]) has replaced the original field equations of his theory by the equations

$$G_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} \lambda = - \varkappa T_{\mu\nu} + \tfrac{1}{2} \varkappa g_{\mu\nu} T \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In my last paper [2]) I have pointed out two different systems of $g_{\mu\nu}$ which satisfy these equations. The system A is EINSTEIN's, in which the whole of space is filled with matter of the average density ϱ_0. In a stationary state, and if all matter is at rest without any stresses or pressure, then we have $T_{\mu\nu} = 0$ with the exception of $T_{44} = g_{44} \varrho_0$. In the system B this "world-matter" does not exist: we have $\varrho_0 = 0$ and consequently all $T_{\mu\nu} = 0$. The line element in the two systems was there found to be

$$ds^2 = - R^2 \{ d\chi^2 + \sin^2 \chi \, [d\psi^2 + \sin^2 \psi \, d\vartheta^2] \} + c^2 \, dt^2, \quad . \quad . \quad (2A)$$

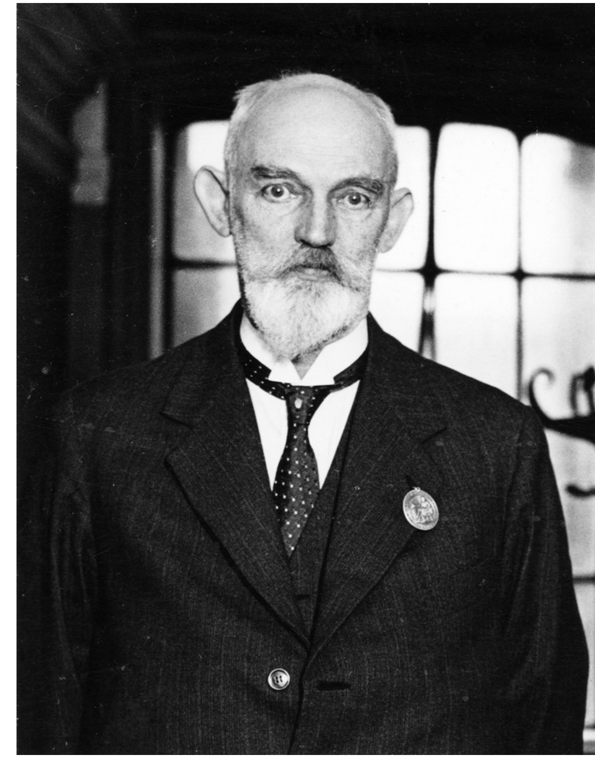
$$ds^2 = - R^2 \{ d\omega^2 + \sin^2 \omega \, [d\chi^2 + \sin^2 \chi \, (d\psi^2 + \sin^2 \psi \, d\vartheta^2)] \} . \quad (2B)$$

In the system A we have:

$$\lambda = \frac{1}{R^2} \quad , \quad \varkappa \varrho_0 = 2\lambda , \quad . \quad . \quad . \quad . \quad (3A)$$

and in B:

$$\lambda = \frac{3}{R^2} \quad , \quad \varrho_0 = 0 . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3B)$$



Willem de Sitter

Proceedings of the Royal Netherlands Academy of Arts and Sciences, 20 I, 229 (1918)

Astronomy. — *"On the curvature of space"*. By Prof. W. DE SITTER.

(Communicated in the meeting of 1917, June 30).

1. In order to make possible an entirely relative conception of inertia, EINSTEIN [1]) has replaced the original field equations of his theory by the equations

$$G_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} \lambda = - \varkappa T_{\mu\nu} + \tfrac{1}{2} \varkappa g_{\mu\nu} T \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In my last paper [2]) I have pointed out two different systems of $g_{\mu\nu}$ which satisfy these equations. The system A is EINSTEIN's, in which the whole of space is filled with matter of the average density ϱ_0. In a stationary state, and if all matter is at rest without any stresses or pressure, then we have $T_{\mu\nu} = 0$ with the exception of $T_{44} = g_{44}\, \varrho_0$. In the system B this "world-matter" does not exist: we have $\varrho_0 = 0$ and consequently all $T_{\mu\nu} = 0$. The line element in the two systems was there found to be

$$ds^2 = - R^2 \left\{ d\chi^2 + \sin^2 \chi \left[d\psi^2 + \sin^2 \psi \, d\vartheta^2 \right] \right\} + c^2 \, dt^2, \quad . \quad . \quad (2A)$$

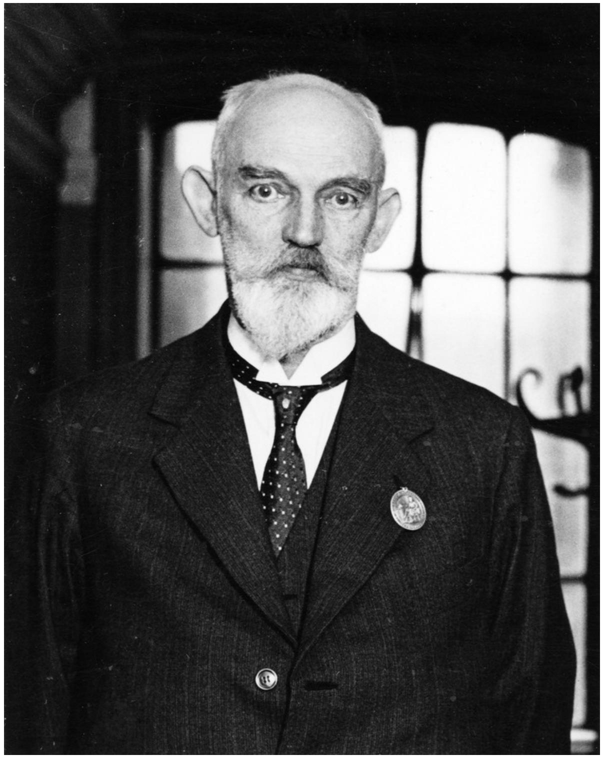
$$ds^2 = - R^2 \left\{ d\omega^2 + \sin^2 \omega \left[d\chi^2 + \sin^2 \chi \left(d\psi^2 + \sin^2 \psi \, d\vartheta^2 \right) \right] \right\} . \quad (2B)$$

In the system A we have

$$\lambda = \frac{1}{R^2} \quad , \quad \varkappa \varrho_0 = 2\lambda, \quad . \quad . \quad . \quad . \quad (3A)$$

and in B:

$$\lambda = \frac{3}{R^2} \quad , \quad \varrho_0 = 0 . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3B)$$



Willem de Sitter

Proceedings of the Royal Netherlands Academy of Arts and Sciences, 20 I, 229 (1918)

Astronomy. — *"On the curvature of space"*. By Prof. W. de Sitter.

(Communicated in the meeting of 1917, June 30).

1. In order to make possible an entirely relative conception of inertia, Einstein [1]) has replaced the original field equations of his theory by the equations

$$G_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} \lambda = -\varkappa T_{\mu\nu} + \tfrac{1}{2} \varkappa g_{\mu\nu} T \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In my last paper [2]) I have pointed out two different systems of $g_{\mu\nu}$ which satisfy these equations. The system A is Einstein's, in which the whole of space is filled with matter of the average density ϱ_0. In a stationary state, and if all matter is at rest without any stresses or pressure, then we have $T_{\mu\nu} = 0$ with the exception of $T_{44} = g_{44}\,\varrho_0$. In the system B this "world-matter" does not exist: we have $\varrho_0 = 0$ and consequently all $T_{\mu\nu} = 0$. The line element in the two systems was there found to be

$$ds^2 = -R^2 \{ d\chi^2 + sin^2 \chi \,[d\psi^2 + sin^2 \psi \, d\vartheta^2] \} + c^2 \, dt^2, \quad . \quad . \quad (2A)$$

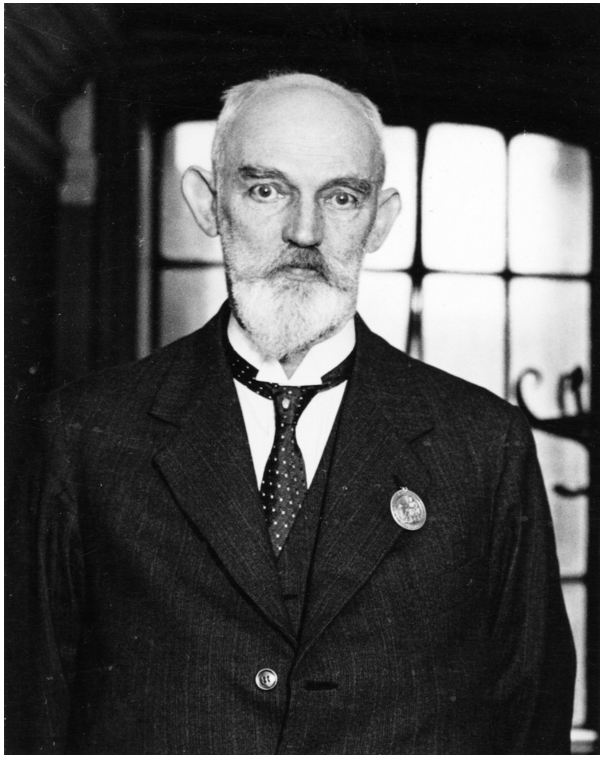
$$ds^2 = -R^2 \{ d\omega^2 + sin^2 \omega \,[d\chi^2 + sin^2 \chi \,(d\psi^2 + sin^2 \psi \, d\vartheta^2)] \} \,. \quad (2B)$$

In the system A we have:

$$\lambda = \frac{1}{R^2}, \qquad \varkappa \varrho_0 = 2\lambda, \quad . \quad . \quad . \quad . \quad (3A)$$

and in B:

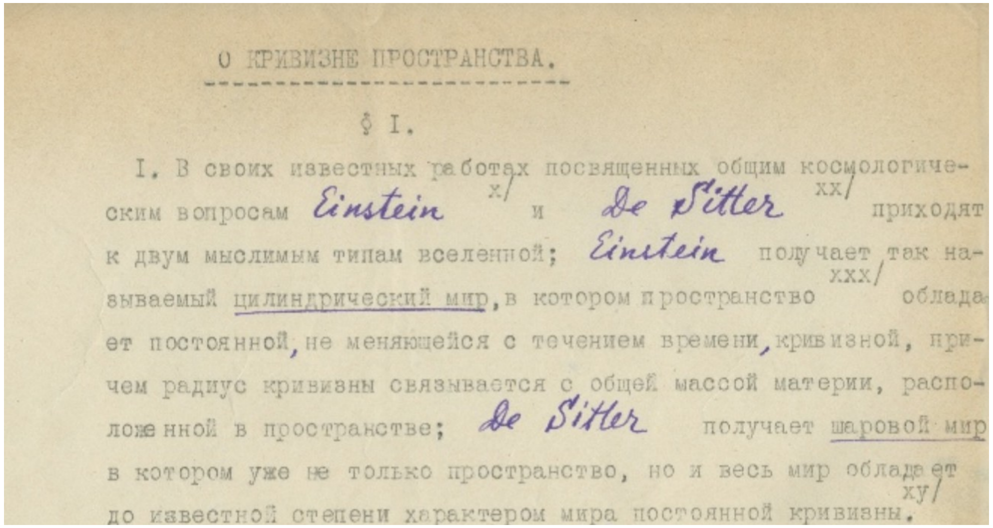
$$\lambda = \frac{3}{R^2}, \qquad \varrho_0 = 0 \,. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3B)$$



Willem de Sitter

Proceedings of the Royal Netherlands Academy of Arts and Sciences, 20 I, 229 (1918)

Über die Krümmung des Raumes
Zeitschrift für Physik, 10, 377 (1922)

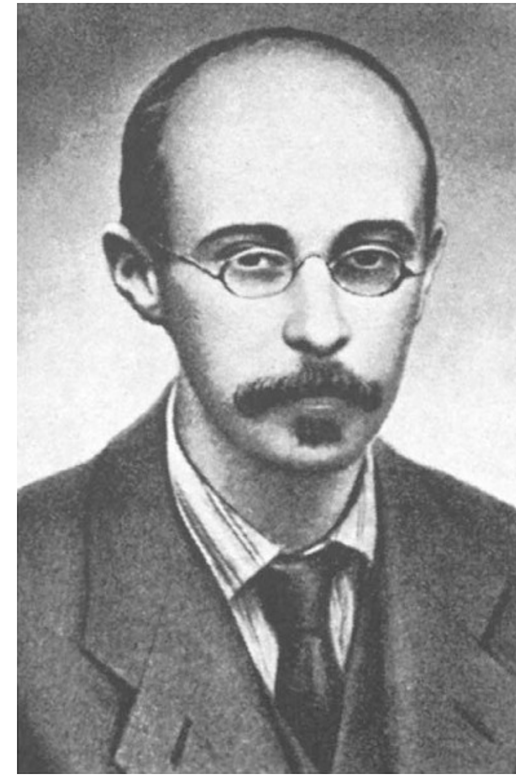


О КРИВИЗНЕ ПРОСТРАНСТВА.

§ I.

I. В своих известных работах посвященных общим космологическим вопросам Einstein x/ и De Sitter xx/ приходят к двум мыслимым типам вселенной; Einstein получает так называемый цилиндрический мир, в котором пространство xxx/ обладает постоянной, не меняющейся с течением времени, кривизной, причем радиус кривизны связывается с общей массой материи, расположенной в пространстве; De Sitter получает шаровой мир в котором уже не только пространство, но и весь мир обладает xy/ до известной степени характером мира постоянной кривизны.

original manuscript available at https://www.lorentz.leidenuniv.nl



Alexander Friedman

On the Curvature of Space†

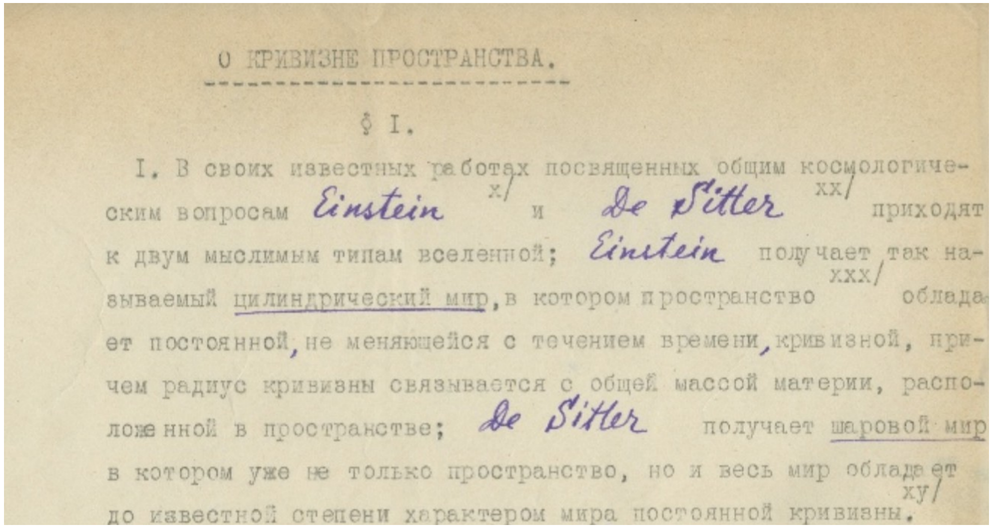
By A. Friedman in Petersburg *

With one figure. Received on 29. June 1922

§1. 1. In their well-known works on general cosmological questions, Einstein[1] and de Sitter[2] arrive at two possible types of the universe; Einstein obtains the so-called cylindrical world, in which space[3] has constant, time-independent curvature, where the curvature radius is connected to the total mass of matter present in space; de Sitter obtains a spherical world in which not only space, but in a certain sense also the world can be addressed as a world of constant curvature.[4] In doing so both Einstein

Gen. Rel. Grav. 31, 1991, 1999

Über die Krümmung des Raumes
Zeitschrift für Physik, 10, 377 (1922)

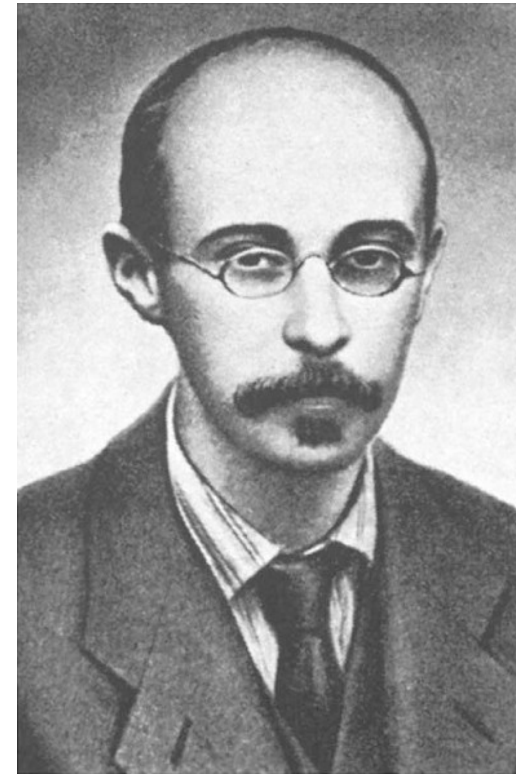


О КРИВИЗНЕ ПРОСТРАНСТВА.

§ I.

I. В своих известных работах посвященных общим космологическим вопросам Einstein x/ и De Sitter xx/ приходят к двум мыслимым типам вселенной; Einstein получает так называемый цилиндрический мир, в котором пространство xxx/ обладает постоянной, не меняющейся с течением времени, кривизной, причем радиус кривизны связывается с общей массой материи, расположенной в пространстве; De Sitter получает шаровой мир в котором уже не только пространство, но и весь мир обладает xy/ до известной степени характером мира постоянной кривизны.

original manuscript available at https://www.lorentz.leidenuniv.nl



Alexander Friedman

On the Curvature of Space†

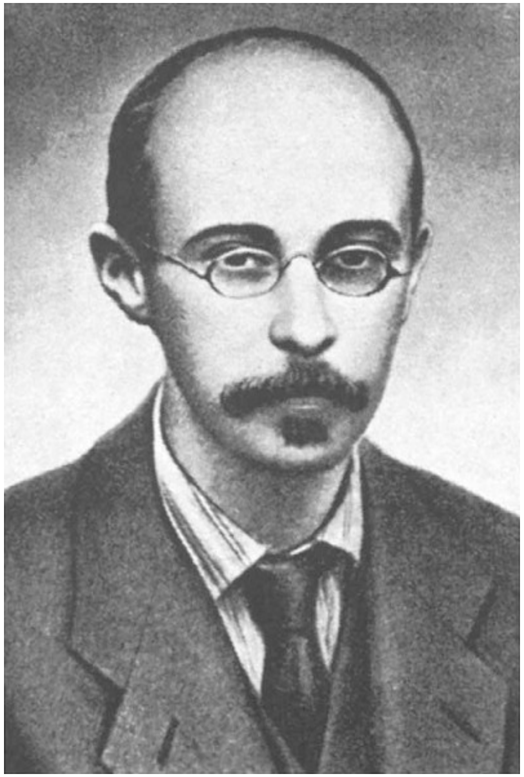
By A. Friedman in Petersburg *

With one figure. Received on 29. June 1922

§1. 1. In their well-known works on general cosmological questions, Einstein[1] and de Sitter[2] arrive at two possible types of the universe; Einstein obtains the so-called cylindrical world, in which space[3] has constant, time-independent curvature, where the curvature radius is connected to the total mass of matter present in space; de Sitter obtains a spherical world in which not only space, but in a certain sense also the world can be addressed as a world of constant curvature.[4] In doing so both Einstein

Gen. Rel. Grav. 31, 1991, 1999

Über die Moglichkeit einer Welt mit konstanter negativer Krümmung des Raumes
Zeitschrift für Physik, 21, 326 (1924)


Alexander Friedmann

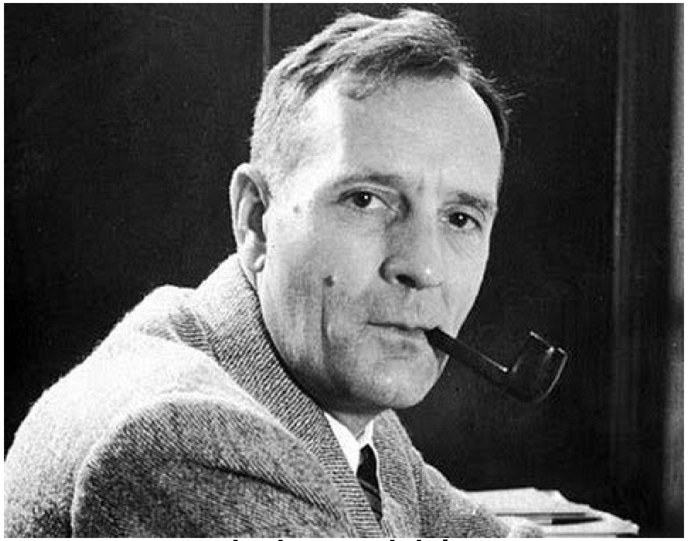
On the Possibility of a World with Constant Negative Curvature of Space†

By A. Friedmann in Petersburg *

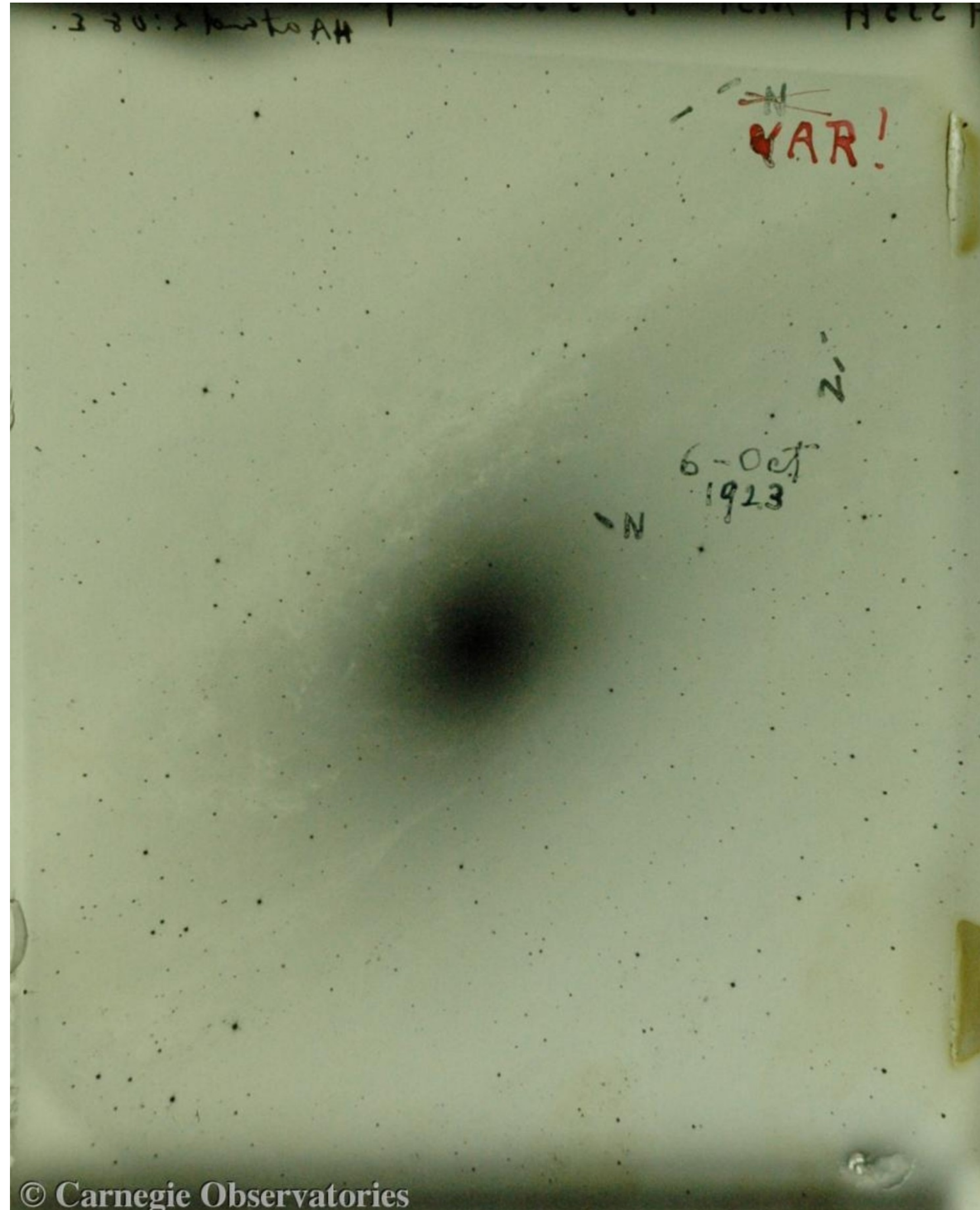
Received on 7. January 1924

§1. 1. In our Notice "On the curvature of space"[1] we have considered those solutions of the *Einstein* world equations, which lead to world types that possess a positive constant curvature as a common feature; we have discussed all such possible cases. The possibility of deriving from the world equations a world of constant positive spatial curvature stands, however, in close relation with the question of the finiteness of space. For this reason it may be of interest to investigate whether one can obtain from the same world equations a world of constant negative curvature, the finiteness of which (even under some supplementary assumptions) can hardly be argued for.

Gen. Rel. Grav. 31, 2001, 1999



Edwin Hubble



UN UNIVERS HOMOGÈNE DE MASSE CONSTANTE ET DE RAYON CROISSANT, RENDANT COMPTE DE LA VITESSE RADIALE DES NÉBULEUSES EXTRA-GALACTIQUES

Note de M. l'Abbé G. LEMAÎTRE

1. GÉNÉRALITÉS.

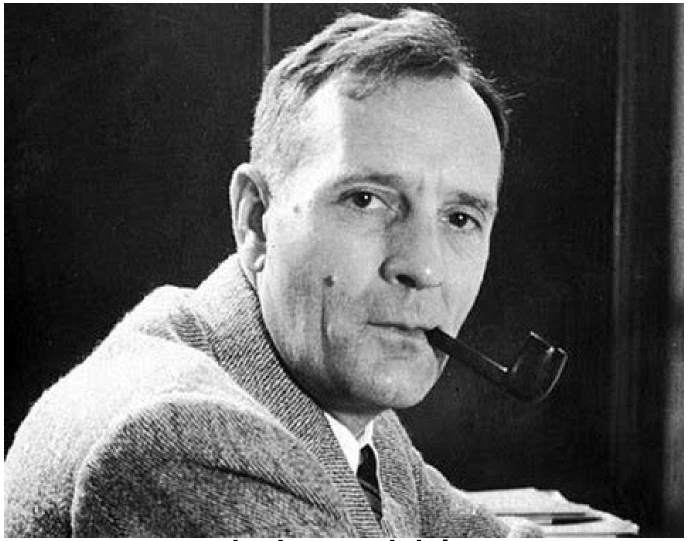
La théorie de la relativité fait prévoir l'existence d'un univers homogène où non seulement la répartition de la matière est uniforme, mais où toutes les positions de l'espace sont équivalentes, il n'y a pas de centre de gravité. Le rayon R de l'espace est constant, l'espace est elliptique de courbure positive uniforme $1/R^2$, les droites issues d'un même point repassent à leur point de départ après un parcours égal à πR, le volume total de l'espace est fini et égal à $\pi^2 R^3$, les droites sont des lignes fermées parcourant tout l'espace sans rencontrer de frontière (1).

Deux solutions ont été proposées. Celle de DE SITTER ignore la présence de la matière et suppose sa densité nulle. Elle conduit à certaines difficultés d'interprétation sur lesquelles nous aurons l'occasion de revenir, mais son grand intérêt est d'expliquer le fait que les nébuleuses extra-galactiques semblent nous fuir avec une énorme vitesse, comme une simple conséquence des propriétés du champ de gravitation, sans supposer que nous nous trouvons en un point de l'univers doué de propriétés spéciales.

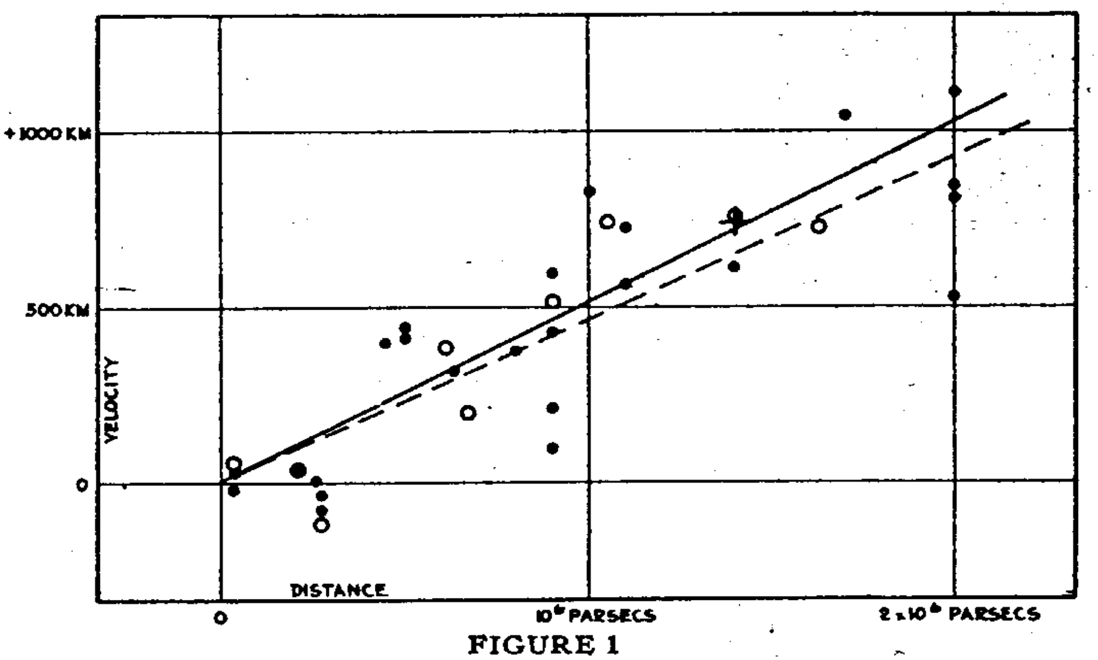
Georges Lemaître

$$K > 0$$

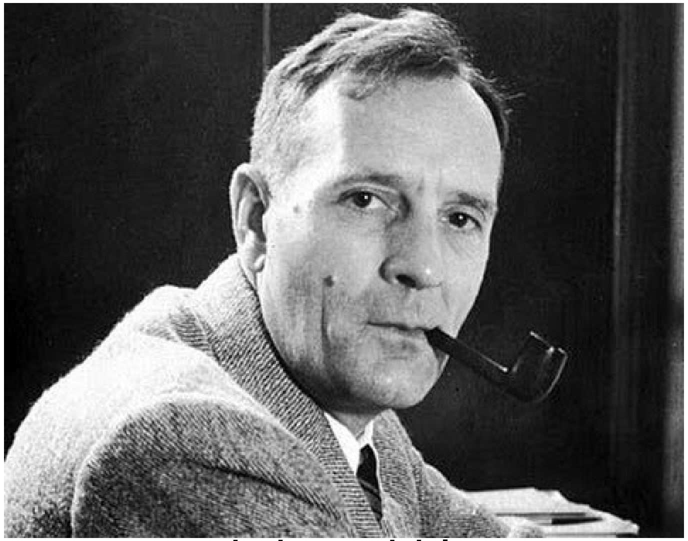
Annals of the Scientific Society of Brussels, 47A, 41 (1927)



Edwin Hubble



Hubble, *PNAS*, 15, 168 (1929)



Edwin Hubble

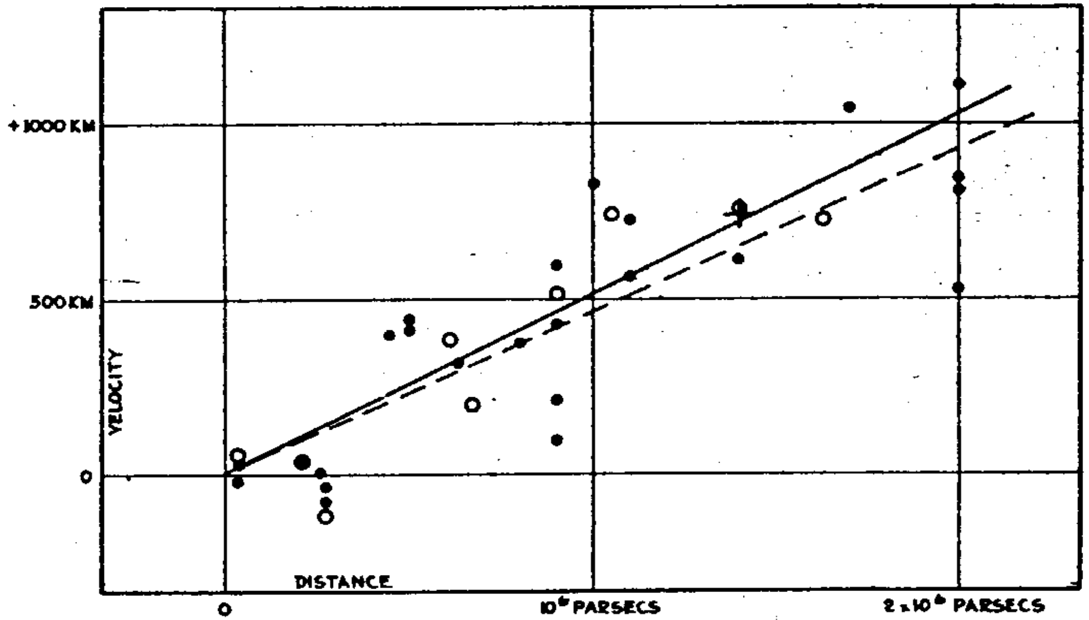


FIGURE 1

Hubble, *PNAS*, 15, 168 (1929)

THE LAW OF RED-SHIFTS

George Darwin Lecture, delivered by Dr Edwin Hubble on* 1953 *May* 8

* *Editorial Note.*—This paper comprises the text of the George Darwin Lecture, which the late author had intended to revise before publication. His notes, together with the manuscript from which he spoke, made it clear what form he wished the published material to take. A reorganization of the original manuscript according to his marginal notes, with the addition of a few connecting sentences, was the extent of the editing required.—A. R. Sandage.

The term " apparent velocity " will be discarded, and replaced by " velocity " signifying $c \,.\, d\lambda/\lambda$, or red-shifts expressed on a scale of velocities. The procedure is not formally correct but it is convenient.

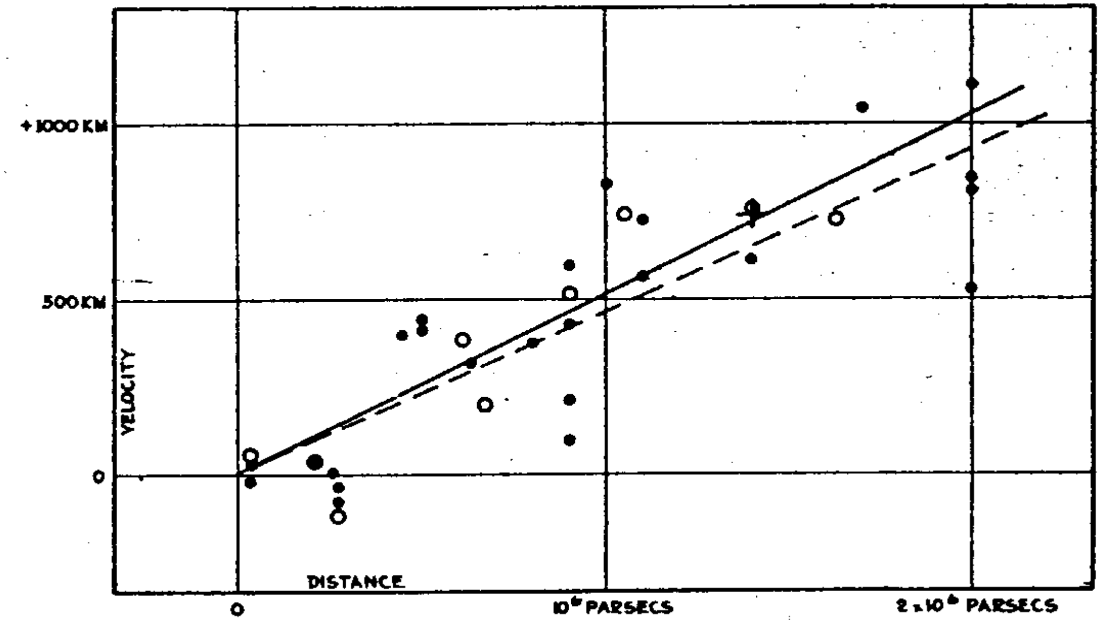
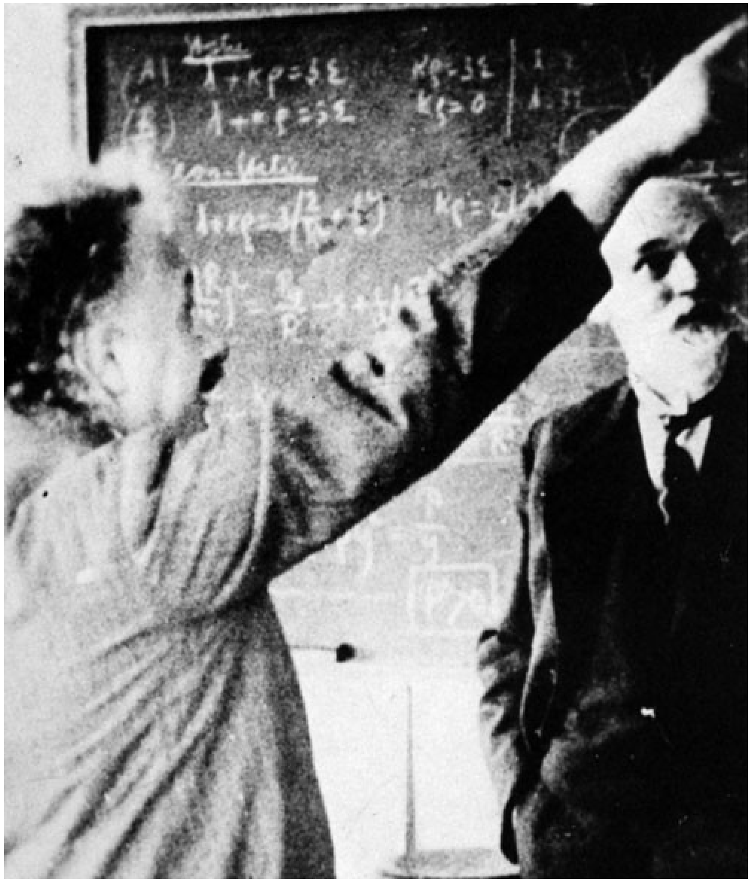
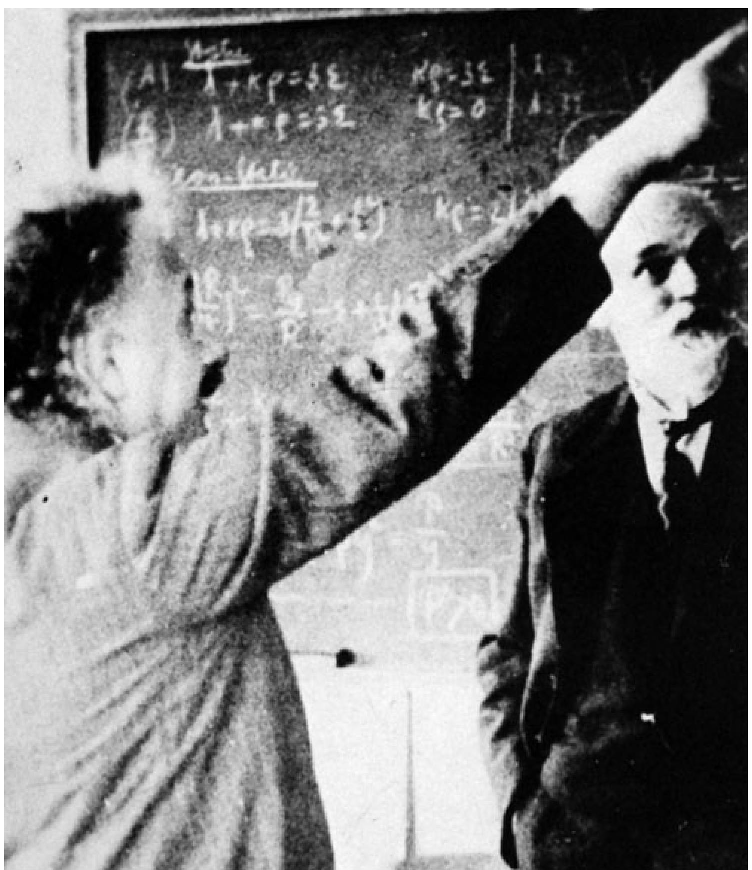


FIGURE 1

Hubble, *PNAS*, 15, 168 (1929)





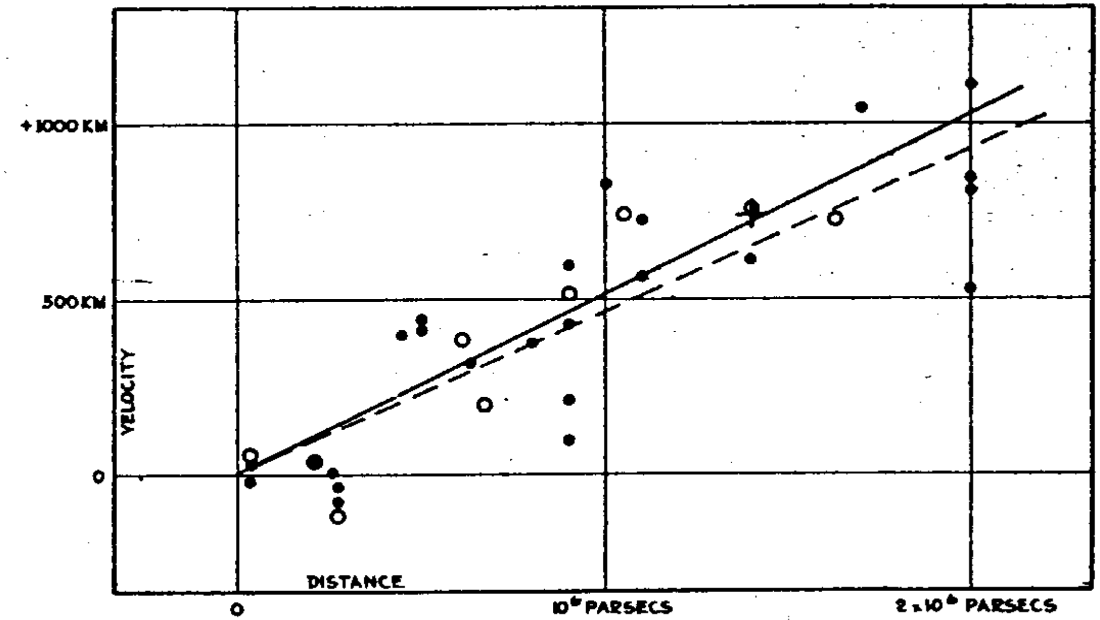


FIGURE 1

Hubble, *PNAS*, 15, 168 (1929)

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 18 March 15, 1932 Number 3

ON THE RELATION BETWEEN THE EXPANSION AND THE MEAN DENSITY OF THE UNIVERSE

By A. Einstein and W. de Sitter

Communicated by the Mount Wilson Observatory, January 25, 1932

$\Lambda = 0 \qquad K = 0$

A NEW MODEL FOR THE EXPANDING UNIVERSE

F. Hoyle

(Received 1948 August 5)

Summary

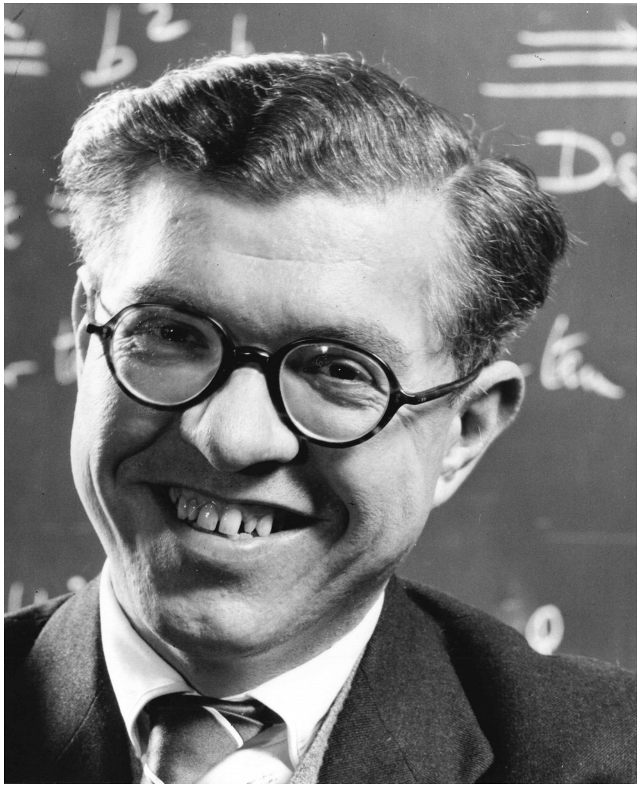
By introducing continuous creation of matter into the field equations of general relativity a stationary universe showing expansion properties is obtained without recourse to a cosmical constant.

---

1. *Introduction.*—Creation of matter was mentioned about twenty years ago by Jeans (1) who remarked:

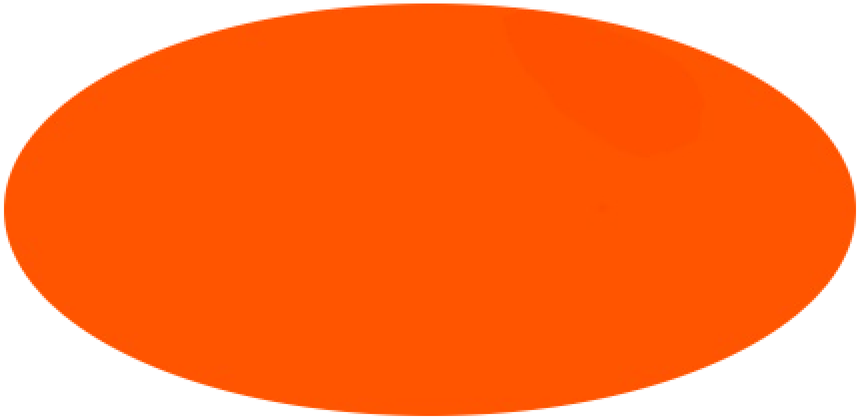
"The type of conjecture which presents itself, somewhat insistently, is that the centres of the nebulae (galaxies) are of the nature of singular points, at which matter is poured into our universe from some other and entirely extraneous spatial dimension, so that, to a denizen of our universe, they appear as points at which matter is being continually created". Subsequent astrophysical developments have, however, shown little support for this particular form of creation.

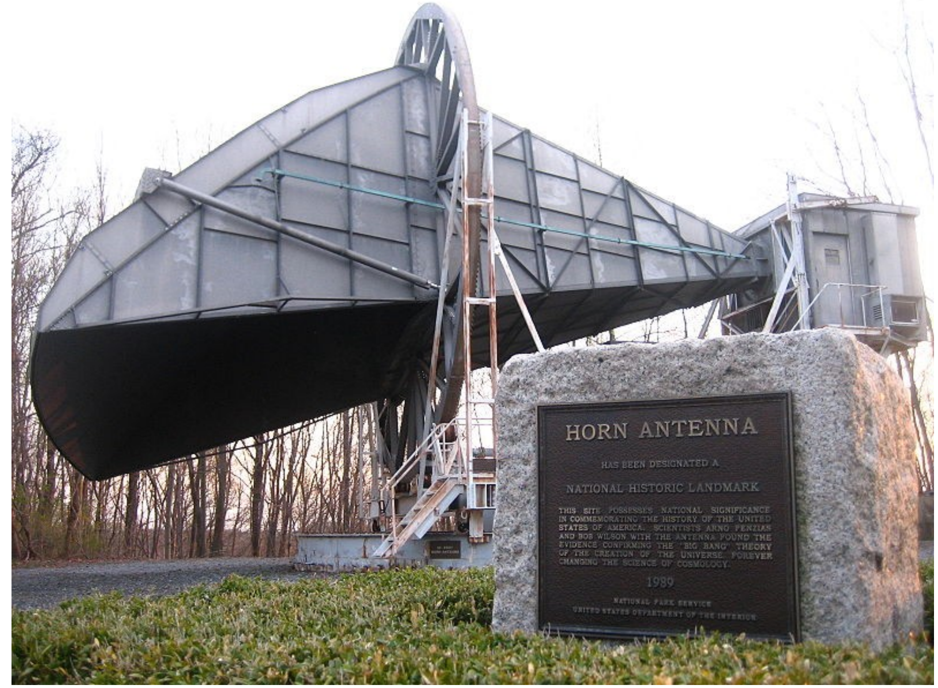
More recently Dirac (2) has pointed out that continuous creation of matter can be related to the wider questions of cosmology. The following work is concerned with this aspect of the matter and arose from a discussion with Mr T. Gold who remarked that through continuous creation of matter it might be possible to obtain an expanding universe in which the proper density of matter remained constant. This possibility seemed attractive, especially when taken in conjunction with aesthetic objections to the creation of the universe in the remote past. For it is against the spirit of scientific enquiry to regard observable effects as arising from "causes unknown to science", and this in principle is what creation-in-the-past implies.

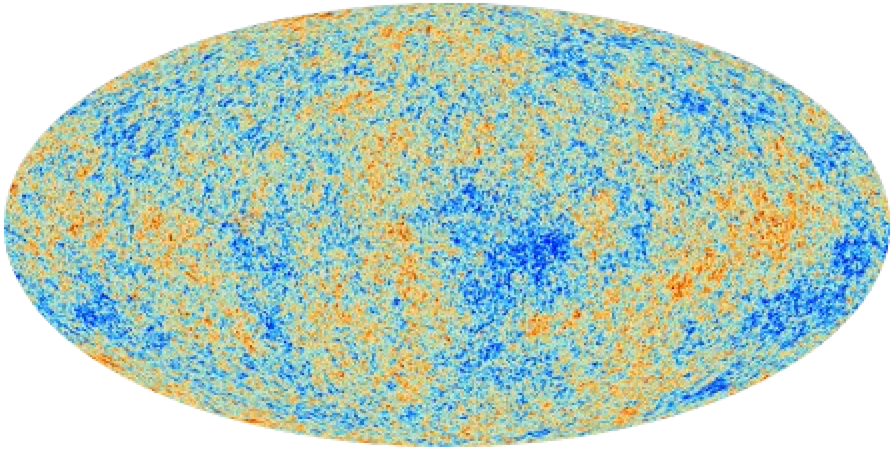


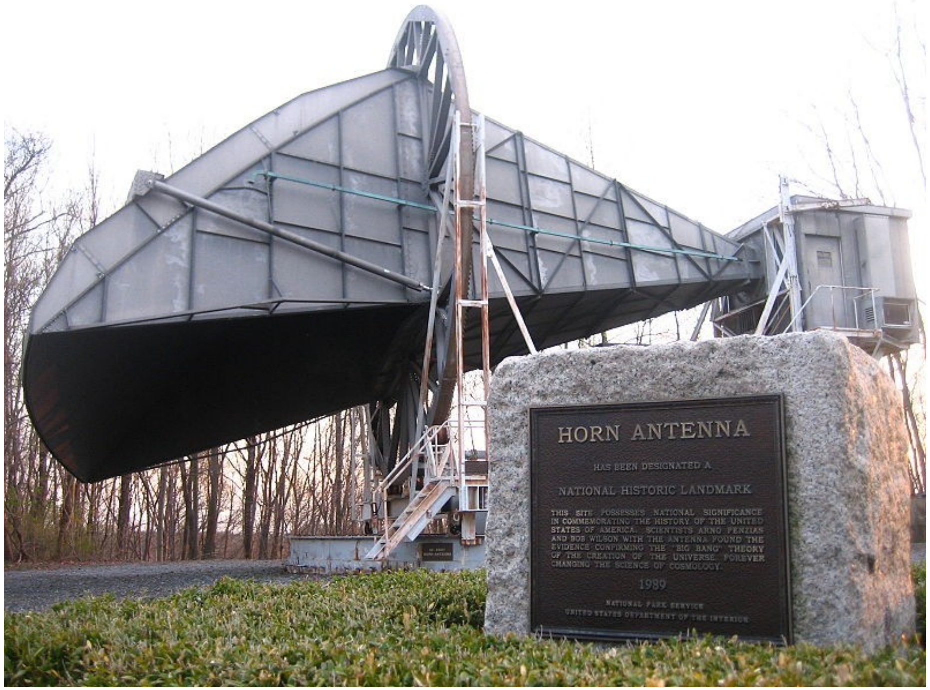
Fred Hoyle

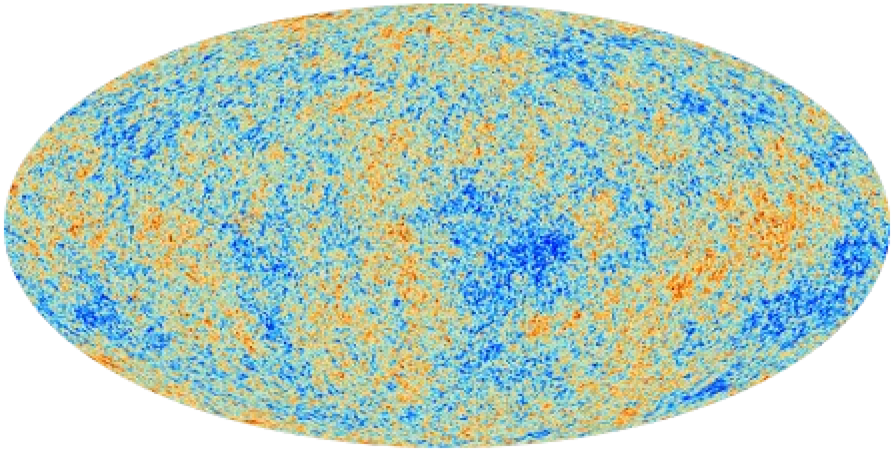
$$K = 0$$

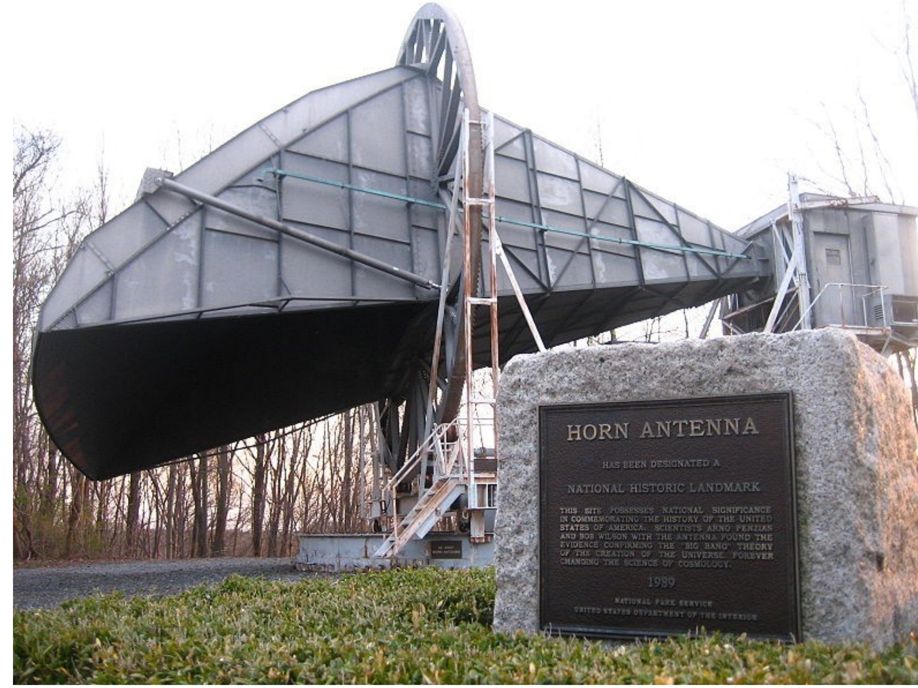



HORN ANTENNA
HAS BEEN DESIGNATED A
NATIONAL HISTORIC LANDMARK
THIS SITE POSSESSES NATIONAL SIGNIFICANCE IN COMMEMORATING THE HISTORY OF THE UNITED STATES OF AMERICA. SCIENTISTS ARNO PENZIAS AND BOB WILSON WITH THE ANTENNA FOUND THE EVIDENCE CONFIRMING THE "BIG BANG" THEORY OF THE CREATION OF THE UNIVERSE, FOREVER CHANGING THE SCIENCE OF COSMOLOGY.
1989
NATIONAL PARK SERVICE
UNITED STATES DEPARTMENT OF THE INTERIOR




HORN ANTENNA
HAS BEEN DESIGNATED A
NATIONAL HISTORIC LANDMARK
THIS SITE POSSESSES NATIONAL SIGNIFICANCE IN COMMEMORATING THE HISTORY OF THE UNITED STATES OF AMERICA. SCIENTISTS ARNO PENZIAS AND BOB WILSON WITH THE ANTENNA FOUND THE EVIDENCE CONFIRMING THE "BIG BANG" THEORY OF THE CREATION OF THE UNIVERSE, FOREVER CHANGING THE SCIENCE OF COSMOLOGY.
1989
NATIONAL PARK SERVICE
UNITED STATES DEPARTMENT OF THE INTERIOR





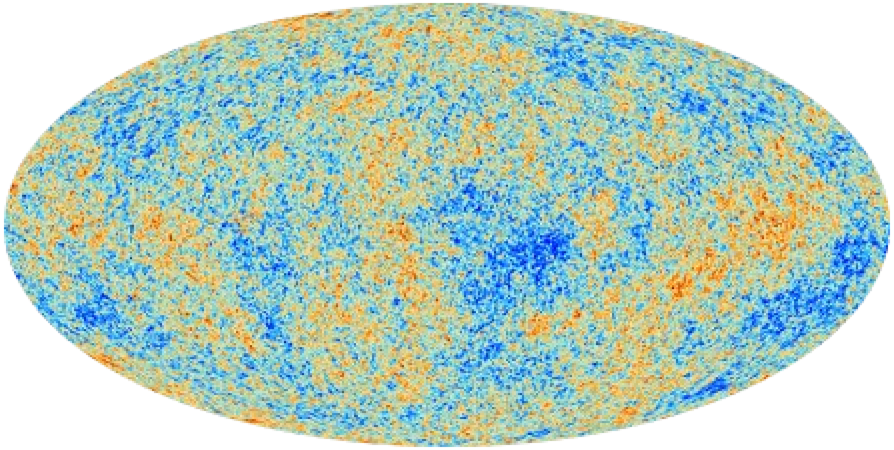
PERTURBATIONS OF A COSMOLOGICAL MODEL AND ANGULAR VARIATIONS OF THE MICROWAVE BACKGROUND

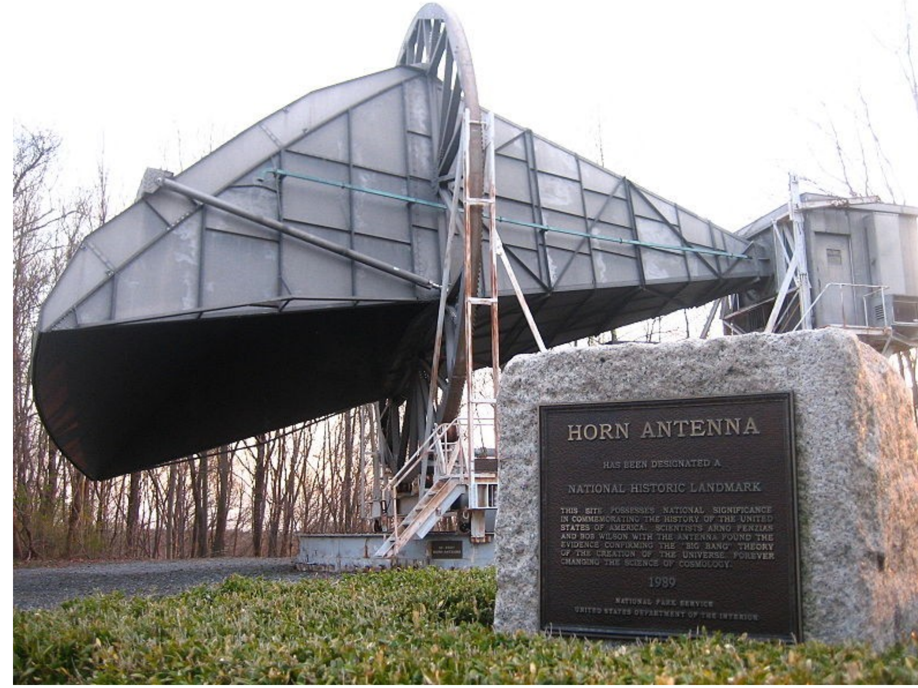
R. K. Sachs and A. M. Wolfe
Relativity Center, The University of Texas, Austin, Texas
Received May 13, 1966

ABSTRACT

We consider general-relativistic, spatially homogeneous, and isotropic $k = 0$ cosmological models with either pressure zero or pressure one-third the energy density. The equations for general linearized perturbations away from these models are explicitly integrated to obtain density fluctuations, rotational perturbations, and gravitational waves. The equations for light rays in the perturbed models are integrated. The models are used to estimate the anisotropy of the microwave radiation, assuming this radiation is cosmological It is estimated that density fluctuations now of order 10 per cent with characteristic lengths now of order 1000 Mpc would cause anisotropies of order 1 per cent in the observed microwave temperature due to the gravitational redshift and other general-relativistic effects. The $p = 0$ models are compared in detail with corresponding Newtonian models The perturbed Newtonian models do not contain gravitational waves, but the density perturbations and rotational perturbations are surprisingly similar.

I. INTRODUCTION





1967ApJ...147...73S

PERTURBATIONS OF A COSMOLOGICAL MODEL AND ANGULAR VARIATIONS OF THE MICROWAVE BACKGROUND

R. K. Sachs and A. M. Wolfe
Relativity Center, The University of Texas, Austin, Texas
Received May 13, 1966

ABSTRACT

We consider general-relativistic, spatially homogeneous, and isotropic $k = 0$ cosmological models with either pressure zero or pressure one-third the energy density. The equations for general linearized perturbations away from these models are explicitly integrated to obtain density fluctuations, rotational perturbations, and gravitational waves. The equations for light rays in the perturbed models are integrated. The models are used to estimate the anisotropy of the microwave radiation, assuming this radiation is cosmological It is estimated that density fluctuations now of order 10 per cent with characteristic lengths now of order 1000 Mpc would cause anisotropies of order 1 per cent in the observed microwave temperature due to the gravitational redshift and other general-relativistic effects. The $p = 0$ models are compared in detail with corresponding Newtonian models The perturbed Newtonian models do not contain gravitational waves, but the density perturbations and rotational perturbations are surprisingly similar.

I. INTRODUCTION

Future observations may exclude the homogeneous, isotropic, general-relativistic $k = 0$ models, even as zero-order approximations. At present they are as acceptable as any other models and considerably simpler than most models.

$K \to 0$

PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

Inflationary universe: A possible solution to the horizon and flatness problems

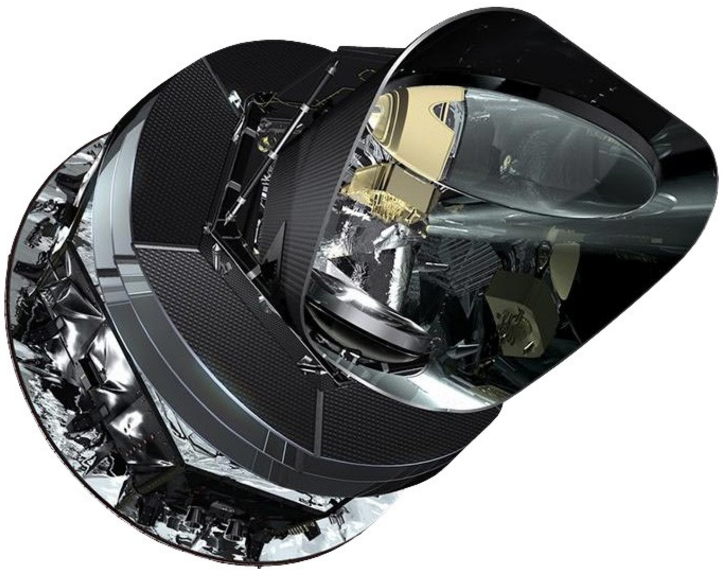
Alan H. Guth*
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 11 August 1980)

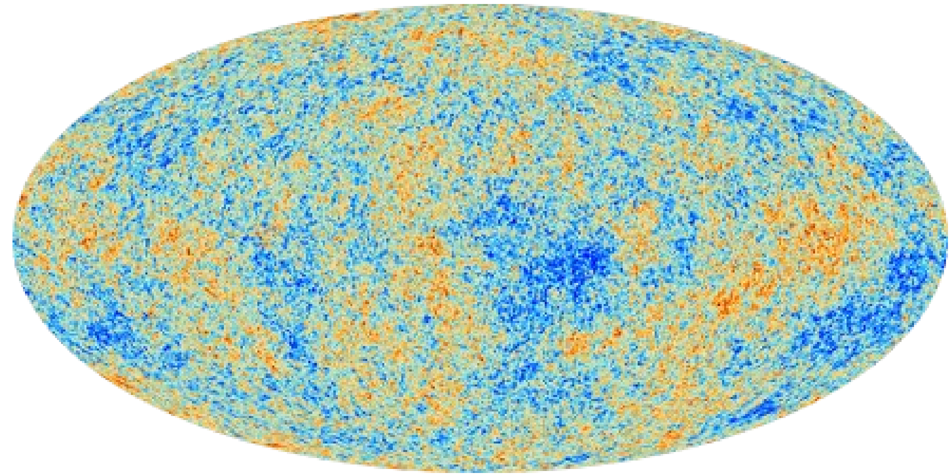
The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

A&A 594, A13 (2016)
DOI: 10.1051/0004-6361/201525830

Planck 2015 results

XIII. Cosmological parameters

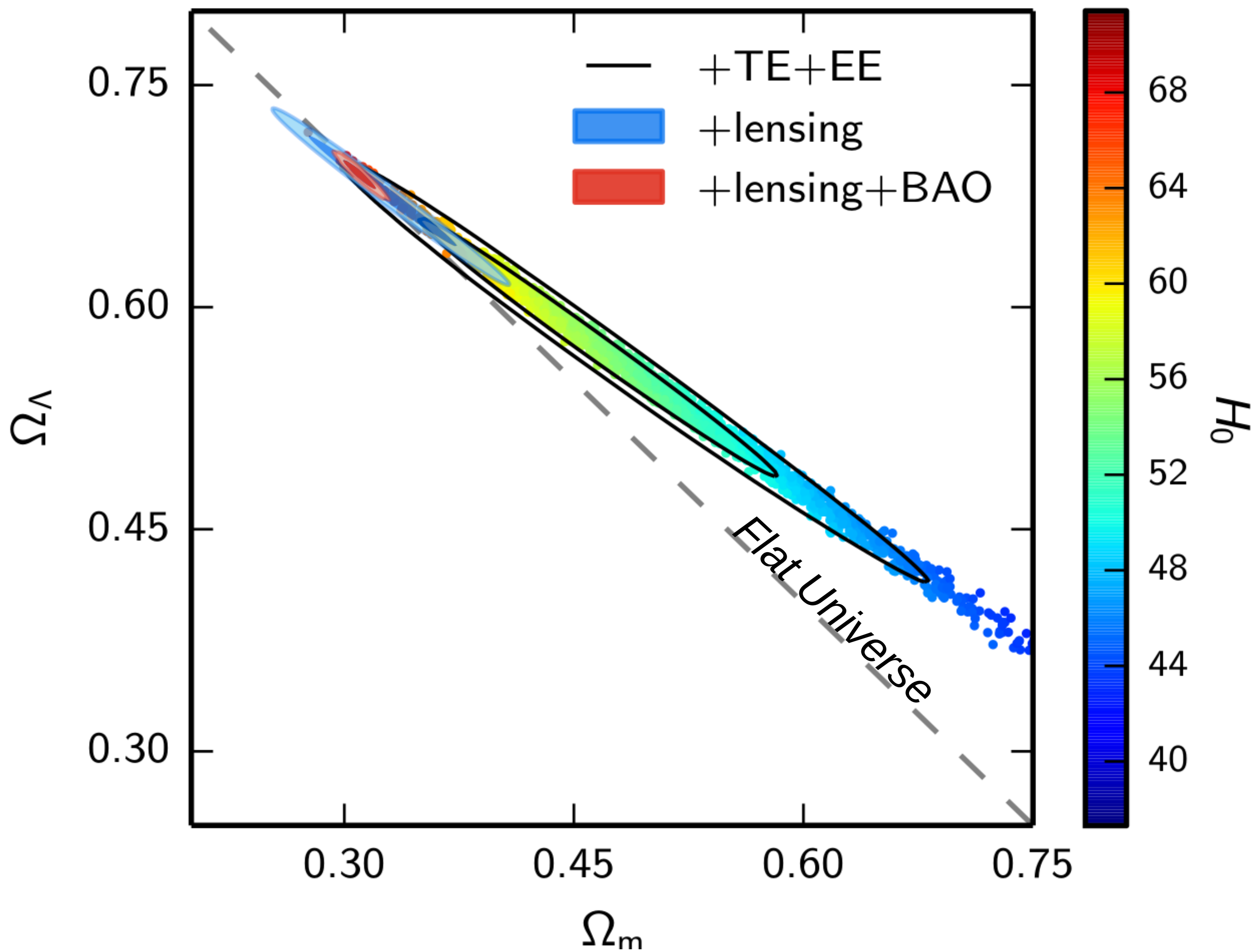




A&A 594, A13 (2016)
DOI: 10.1051/0004-6361/201525830

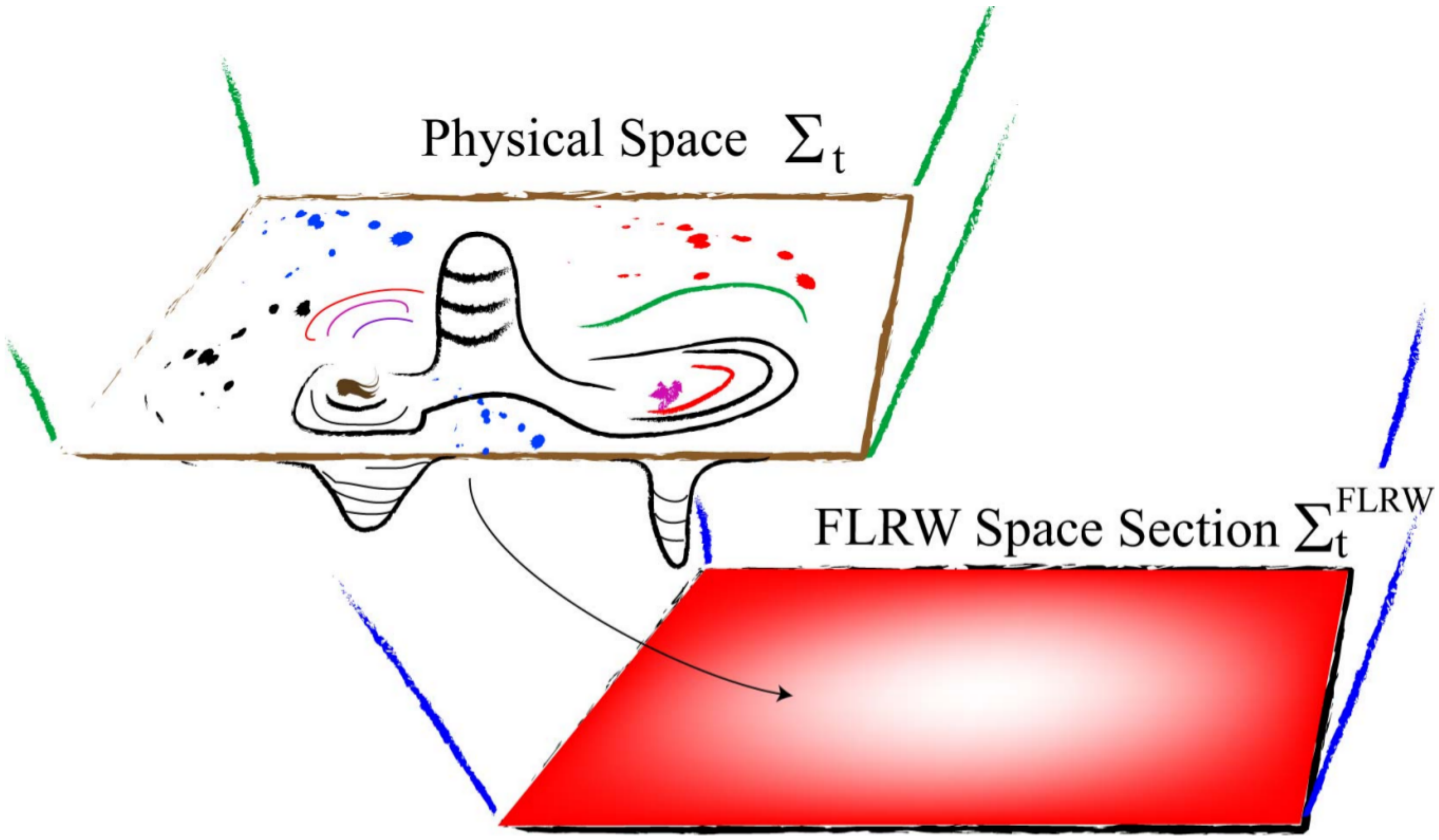
Planck 2015 results

XIII. Cosmological parameters



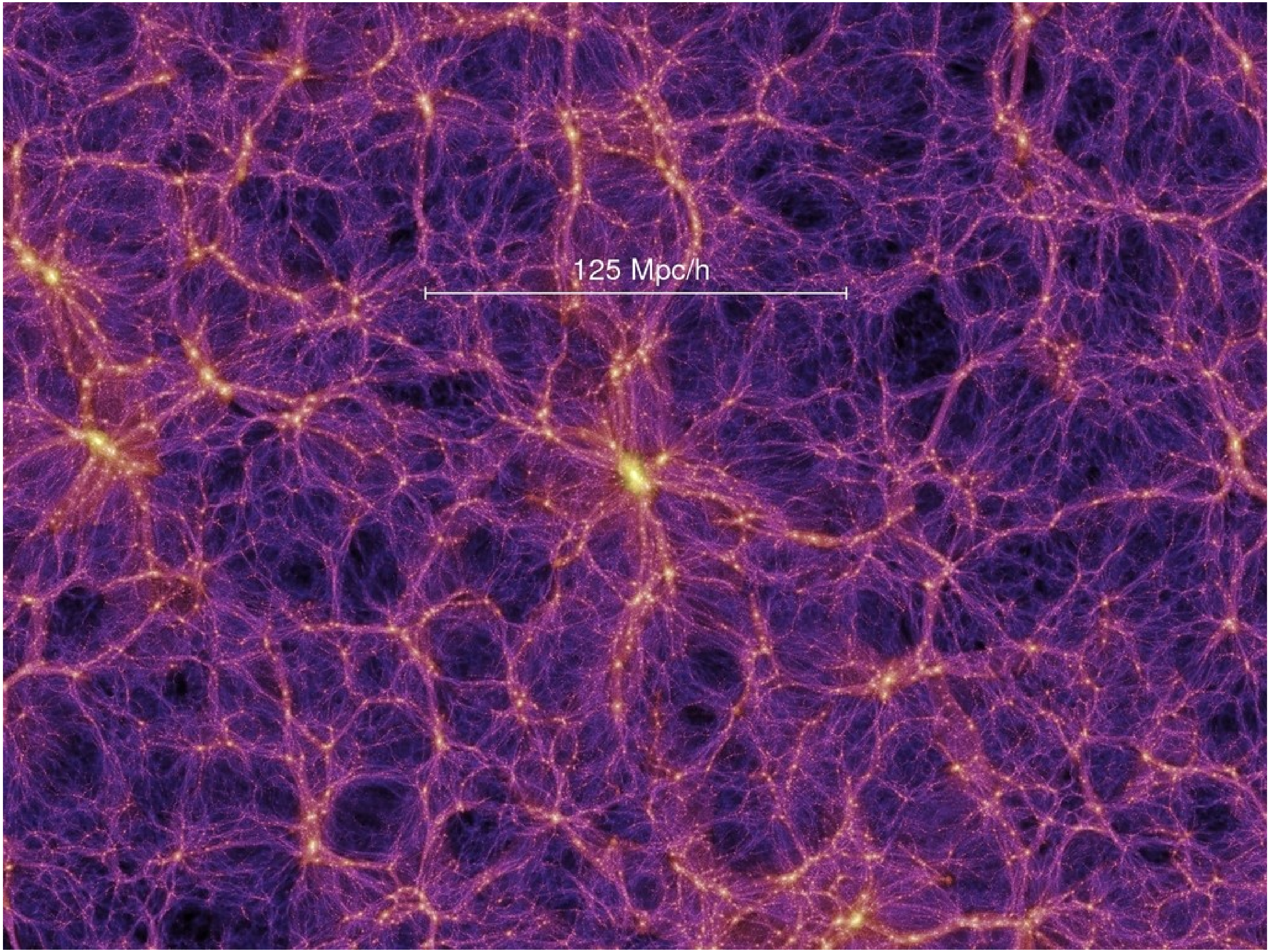
$$\Omega_K = -0.005^{+0.016}_{-0.017} \quad (95\%, \textit{Planck}\ \text{TT+lowP+lensing})$$

Emerging spatial curvature



Buchert, Carfora, *Class. Quant. Grav.* 25 , 195001 (2008)

Millennium universe



Springel, Frenk & White, Nature, 440, 1137 (2006)

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

$$T_{ab} = \rho u_a u_b + p h_{ab} + \pi_{ab} + q_a u_b + u_a q_b$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

conservation equation

$$T^{ab}{}_{;b} = 0$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

conservation equation

$$\dot{\rho} + \Theta(\rho + p) + \sigma^{ab}\pi_{ab} + q^{a}{}_{;a} + q^{a}A_{a} = 0$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

conservation equation

$$\dot{\rho} + \Theta(\rho + p) + \sigma^{ab}\pi_{ab} + q^{a}{}_{;a} + q^{a} A_{a} = 0$$

Ricci identities

$$u_{a;d;c} - u_{a;c;d} = R_{abcd} u^{b}$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

conservation equation

$$\dot{\rho} + \Theta(\rho + p) + \sigma^{ab}\pi_{ab} + q^{a}{}_{;a} + q^{a} A_{a} = 0$$

Ricci identities

$$\dot{\Theta} = -\tfrac{1}{3}\Theta^2 - \tfrac{1}{2}(\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda$$

$$\dot{\sigma}_{\langle ab \rangle} = -\tfrac{2}{3}\Theta\sigma_{ab} - \sigma_{c\langle a}\sigma^{c}{}_{b\rangle} - \omega_{\langle a}\omega_{b\rangle} + D_{\langle a}A_{b\rangle} + A_{\langle a}A_{b\rangle} - E_{ab} + \tfrac{1}{2}\pi_{ab}$$

$$\dot{\omega}_{\langle a \rangle} = -\tfrac{2}{3}\Theta\omega_a - \tfrac{1}{2}\,curl\,A_a + \sigma_{ab}\omega^b$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

conservation equation

$$\dot{\rho} + \Theta(\rho + p) + \sigma^{ab}\pi_{ab} + q^a{}_{;a} + q^a A_a = 0$$

Ricci identities

$$\dot{\Theta} = -\tfrac{1}{3}\Theta^2 - \tfrac{1}{2}(\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda$$

$$\dot{\sigma}_{\langle ab\rangle} = -\tfrac{2}{3}\Theta\sigma_{ab} - \sigma_{c\langle a}\sigma^c{}_{b\rangle} - \omega_{\langle a}\omega_{b\rangle} + D_{\langle a}A_{b\rangle} + A_{\langle a}A_{b\rangle} - E_{ab} + \tfrac{1}{2}\pi_{ab}$$

$$\dot{\omega}_{\langle a\rangle} = -\tfrac{2}{3}\Theta\omega_a - \tfrac{1}{2}\,curl\,A_a + \sigma_{ab}\omega^b$$

$$R_{ab[cd;e]} = 0$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

conservation equation

$$\dot{\rho} + \Theta(\rho + p) + \sigma^{ab}\pi_{ab} + q^{a}{}_{;a} + q^{a} A_{a} = 0$$

Ricci identities

$$\dot{\Theta} = -\tfrac{1}{3}\Theta^{2} - \tfrac{1}{2}(\rho + 3p) - 2(\sigma^{2} - \omega^{2}) + D^{a} A_{a} + A_{a} A^{a} + \Lambda$$

$$\dot{\sigma}_{\langle ab\rangle} = -\tfrac{2}{3}\Theta\sigma_{ab} - \sigma_{c\langle a}\sigma^{c}{}_{b\rangle} - \omega_{\langle a}\omega_{b\rangle} + D_{\langle a} A_{b\rangle} + A_{\langle a} A_{b\rangle} - E_{ab} + \tfrac{1}{2}\pi_{ab}$$

$$\dot{\omega}_{\langle a\rangle} = -\tfrac{2}{3}\Theta\omega_{a} - \tfrac{1}{2}\, curl\, A_{a} + \sigma_{ab}\omega^{b}$$

$$\dot{E}_{\langle ab\rangle} = -\Theta E_{ab} - \tfrac{1}{2}(\rho + p)\sigma_{ab} + curl\, H_{ab} - \tfrac{1}{2}\dot{\pi}_{ab} - \tfrac{1}{6}\Theta\pi_{ab}$$

$$+ \ 3\sigma^{c}_{\langle a}\left(E_{b\rangle c} - \tfrac{1}{6}\pi_{b\rangle c}\right) + \epsilon_{cd\langle a}\left[2A^{c} H^{d}_{b\rangle} - \omega^{c}\left(E_{b\rangle}{}^{d} + \tfrac{1}{2}\pi^{d}_{b\rangle}\right)\right]$$

$$\dot{H}_{\langle ab\rangle} = -\Theta H_{ab} - curl\, E_{ab} + \tfrac{1}{2}\, curl\, \pi_{ab} + 3\sigma^{c}_{\langle a} H_{b\rangle c} - \epsilon_{cd\langle a}\left(2A^{c} E^{d}_{b\rangle} + \omega^{c} H^{d}_{b\rangle}\right)$$

$$G_{ab} - \Lambda g_{ab} = T_{ab}$$

conservation equation

$$\dot\rho + \Theta(\rho + p) + \sigma^{ab}\pi_{ab} + q^a{}_{;a} + q^a A_a = 0$$

Ricci identities

$$\dot\Theta = -\tfrac{1}{3}\Theta^2 - \tfrac{1}{2}(\rho + 3p) - 2(\sigma^2 - \omega^2) + D^a A_a + A_a A^a + \Lambda$$

$$\dot\sigma_{\langle ab\rangle} = -\tfrac{2}{3}\Theta\sigma_{ab} - \sigma_{c\langle a}\sigma^c{}_{b\rangle} - \omega_{\langle a}\omega_{b\rangle} + D_{\langle a}A_{b\rangle} + A_{\langle a}A_{b\rangle} - E_{ab} + \tfrac{1}{2}\pi_{ab}$$

$$\dot\omega_{\langle a\rangle} = -\tfrac{2}{3}\Theta\omega_a - \tfrac{1}{2}\,curl\,A_a + \sigma_{ab}\omega^b$$

$$\boldsymbol{\omega_{ab} = 0 \qquad A_a = 0 \qquad q_a = 0}$$

$$\boldsymbol{p = 0 \qquad \pi_{ab} = 0 \qquad H_{ab} = 0}$$

$$\dot E_{\langle ab\rangle} = -\Theta E_{ab} - \tfrac{1}{2}(\rho + p)\sigma_{ab} + curl\,H_{ab} - \tfrac{1}{2}\dot\pi_{ab} - \tfrac{1}{6}\Theta\pi_{ab}$$
$$+\ 3\sigma^c{}_{\langle a}\left(E_{b\rangle c} - \tfrac{1}{6}\pi_{b\rangle c}\right) + \epsilon_{cd\langle a}\left[2A^c H^d_{b\rangle} - \omega^c\left(E_{b\rangle}{}^d + \tfrac{1}{2}\pi^d_{b\rangle}\right)\right]$$

$$\dot H_{\langle ab\rangle} = -\Theta H_{ab} - curl\,E_{ab} + \tfrac{1}{2}\,curl\,\pi_{ab} + 3\sigma^c{}_{\langle a}H_{b\rangle c} - \epsilon_{cd\langle a}\left(2A^c E^d_{b\rangle} + \omega^c H^d_{b\rangle}\right)$$

Silent Cosmology

$$\dot{\rho} = -\Theta\rho$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}\rho - 6\Sigma^2 + \Lambda$$

$$\dot{\Sigma} = -\frac{2}{3}\Theta\Sigma + \Sigma^2 - W$$

$$\dot{W} = -\Theta W - \frac{1}{2}\rho\Sigma - 3\Sigma W$$

FLRW Cosmology

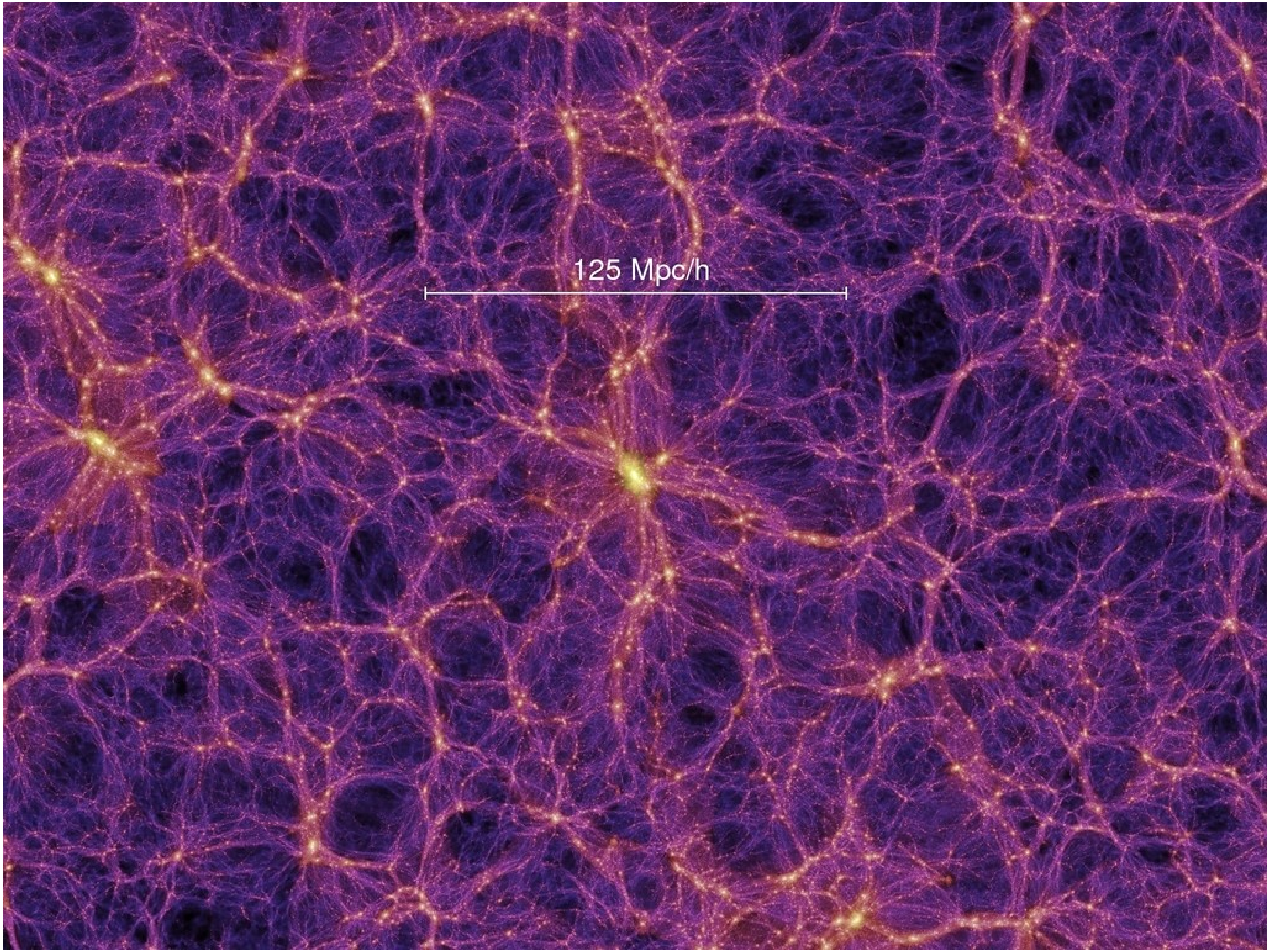
$$\dot{\rho} = -\Theta\,\rho$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}\rho + \Lambda$$

$\Sigma \equiv 0\,,\quad$ *shear free*

$W \equiv 0\,,\quad$ *conformally flat*

Millennium universe



Springel, Frenk & White, Nature, 440, 1137 (2006)

Silent Cosmology

$$\dot{\rho} = -\Theta\rho$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}\rho - 6\Sigma^2 + \Lambda$$

$$\dot{\Sigma} = -\frac{2}{3}\Theta\Sigma + \Sigma^2 - W$$

$$\dot{W} = -\Theta W - \frac{1}{2}\rho\Sigma - 3\Sigma W$$

Silent Cosmology

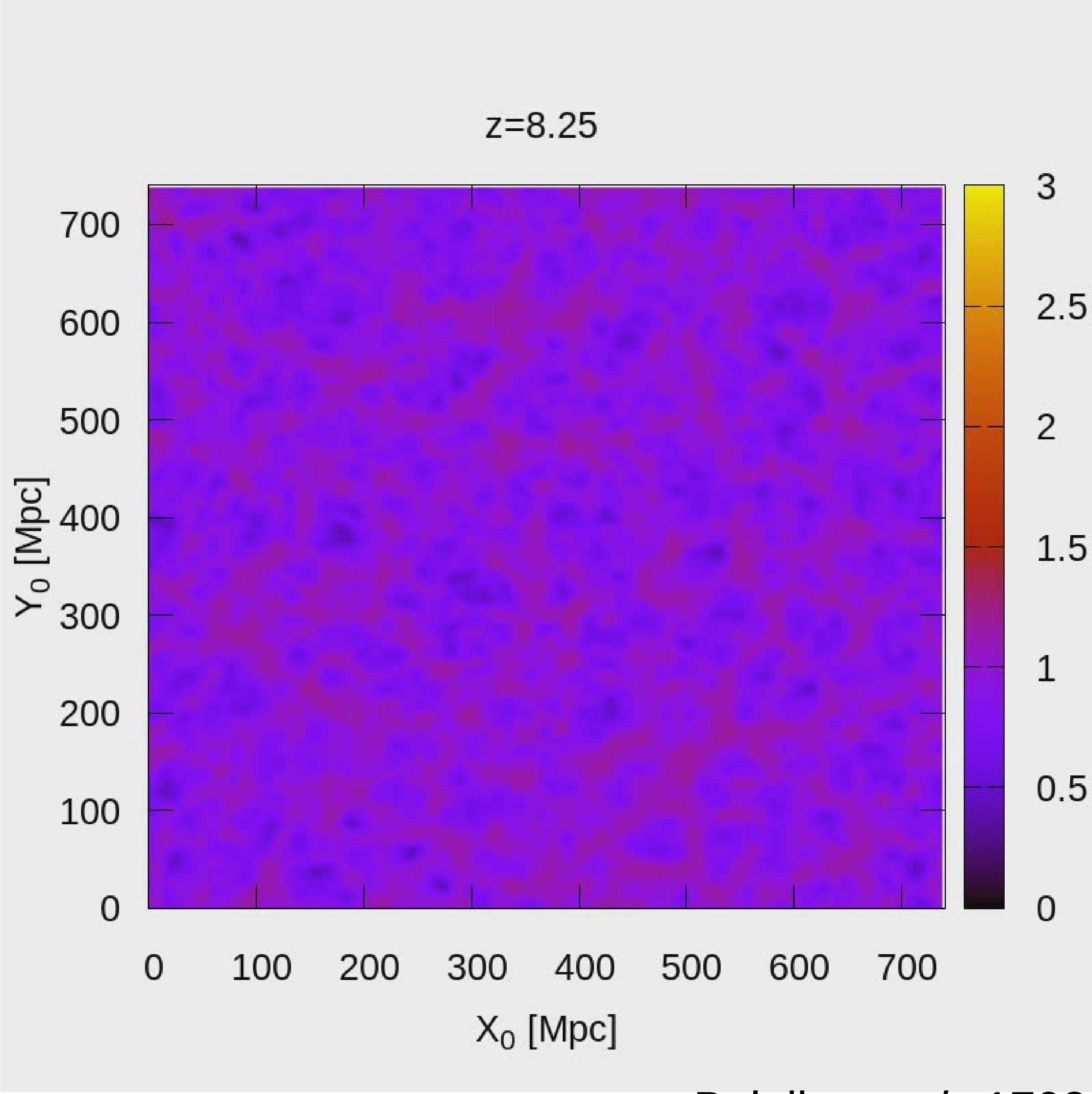
$$\rho_i = \bar{\rho} + \Delta\rho = \bar{\rho}\left(1 + \delta_i\right)$$

$$\Theta_i = \bar{\Theta} + \Delta\Theta = \bar{\Theta}\left(1 - \frac{1}{3}\delta_i\right)$$

$$\Sigma_i = -\frac{1}{3}\Delta\Theta = \frac{1}{9}\bar{\Theta}\,\delta_i$$

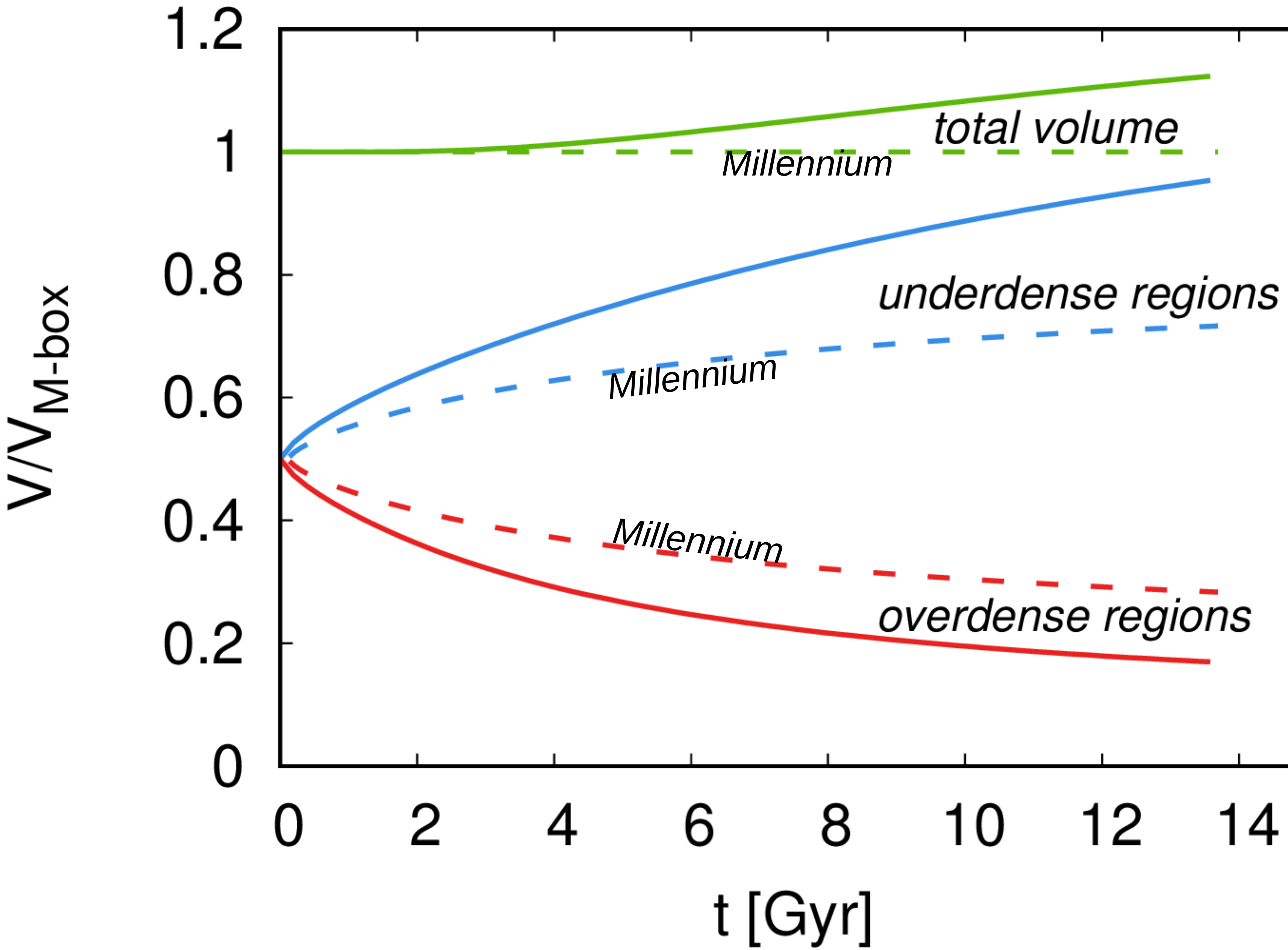
$$W_i = -\frac{1}{6}\bar{\rho}\,\delta_i$$

Silent Universe Simulation



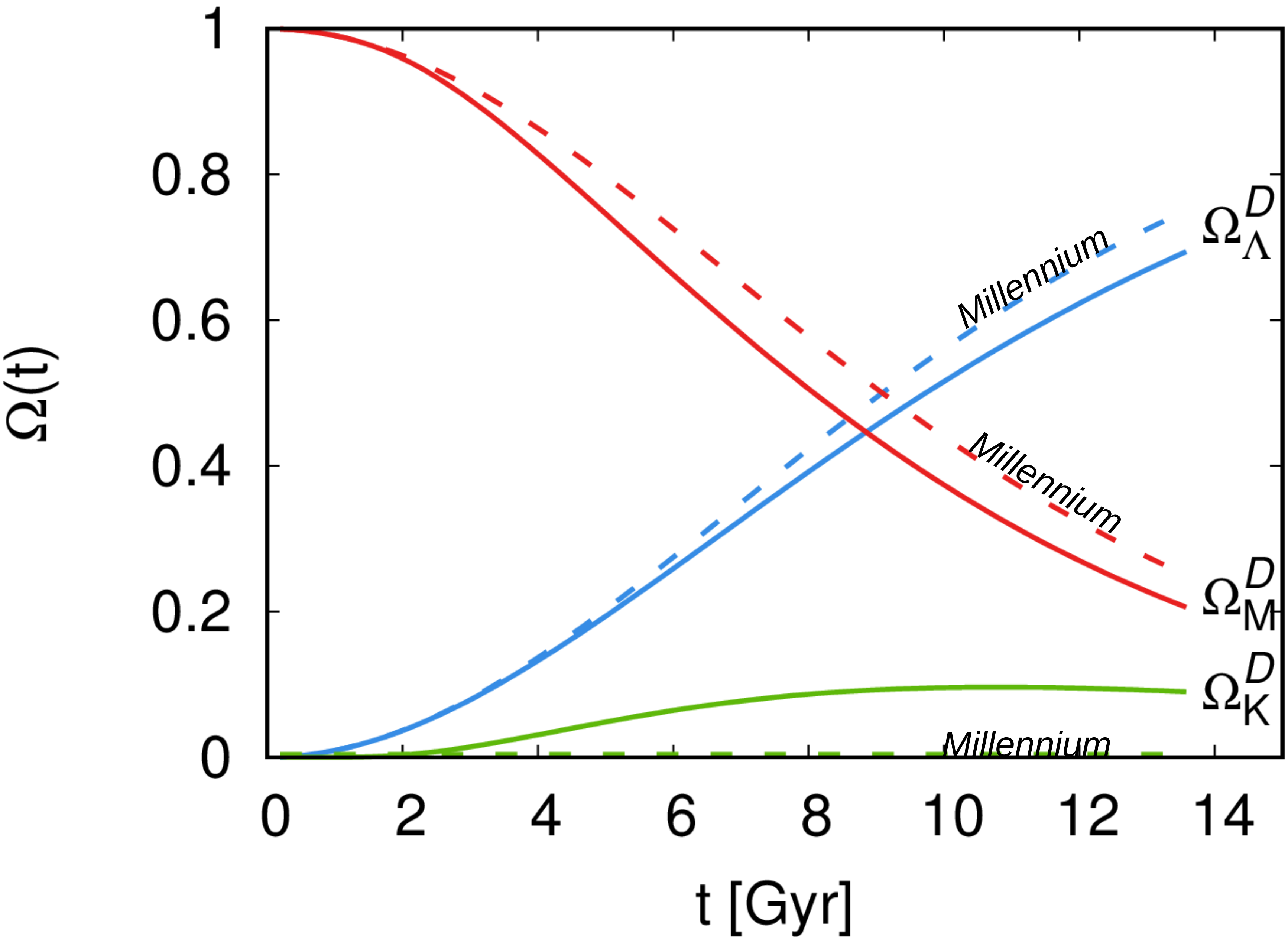
Bolejko, *arxiv*:1708.0940 (2017)

Evolution of the volume



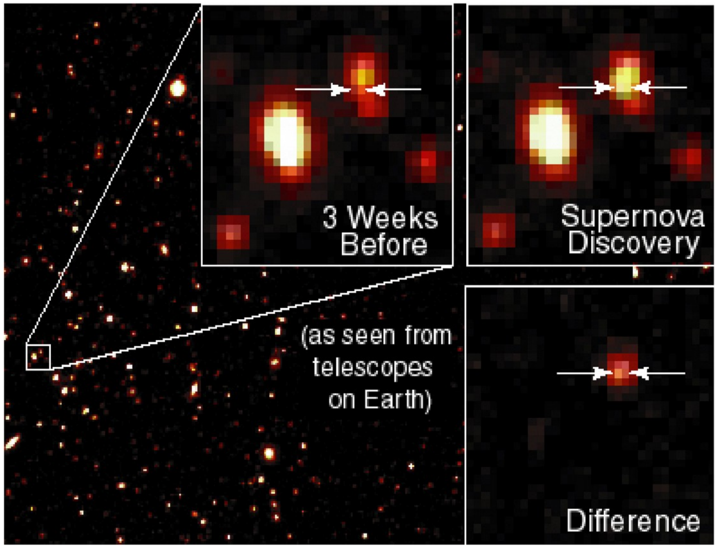
Bolejko, *arxiv*:1708.0940 (2017)

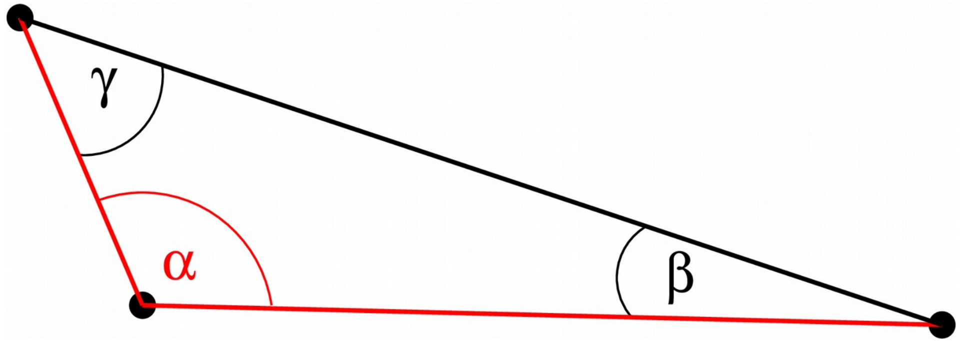
Evolution of the cosmic parameters



Bolejko, *arxiv*:1708.0940 (2017)

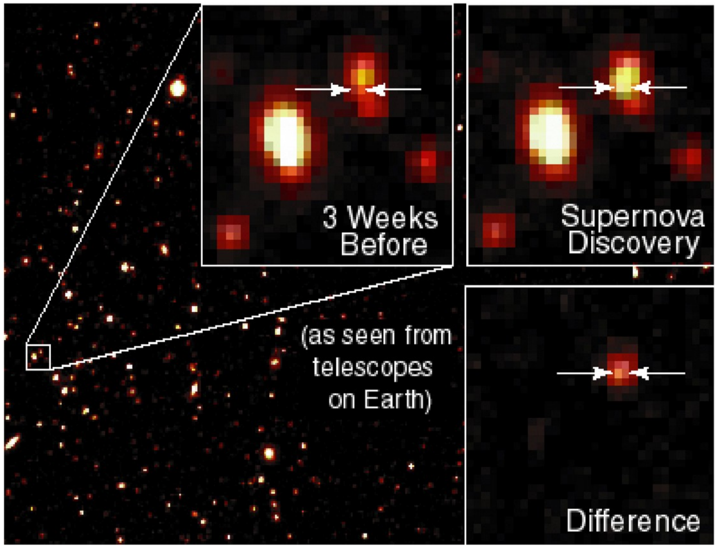
Low-redshift constraints on curvature

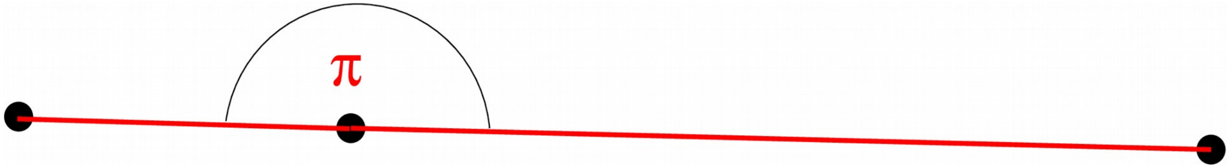




Rasanen, Bolejko, Finoguenov, *Phys. Rev. Lett.* 115, 101301 (2015)

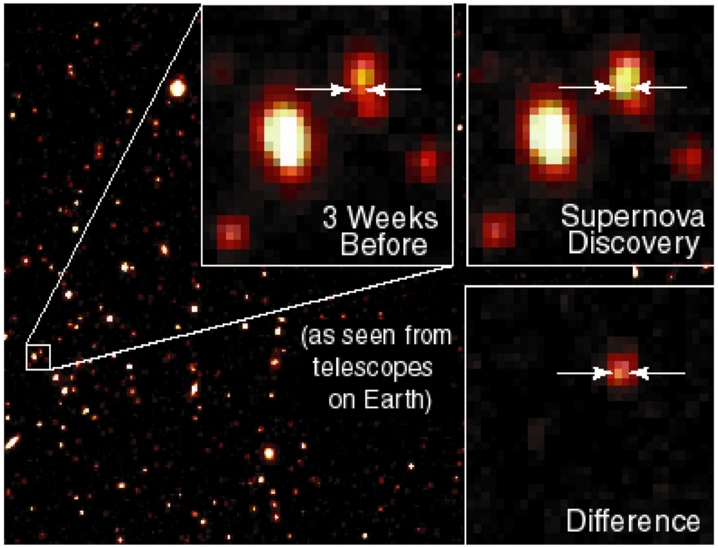
Low-redshift constraints on curvature

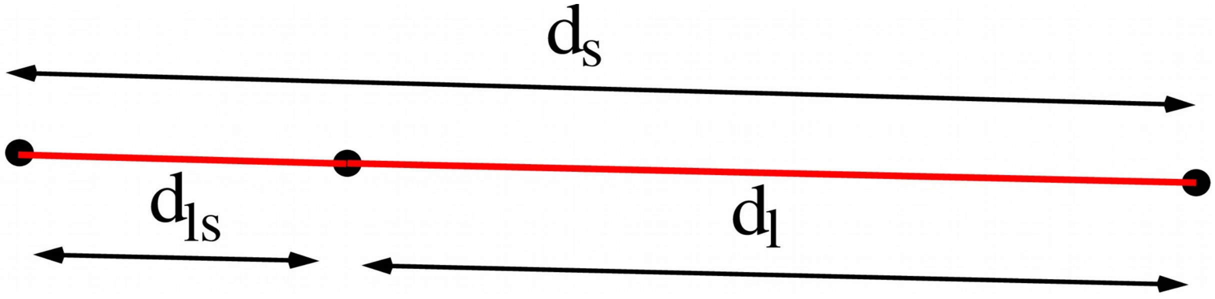




Rasanen, Bolejko, Finoguenov, *Phys. Rev. Lett.* 115, 101301 (2015)

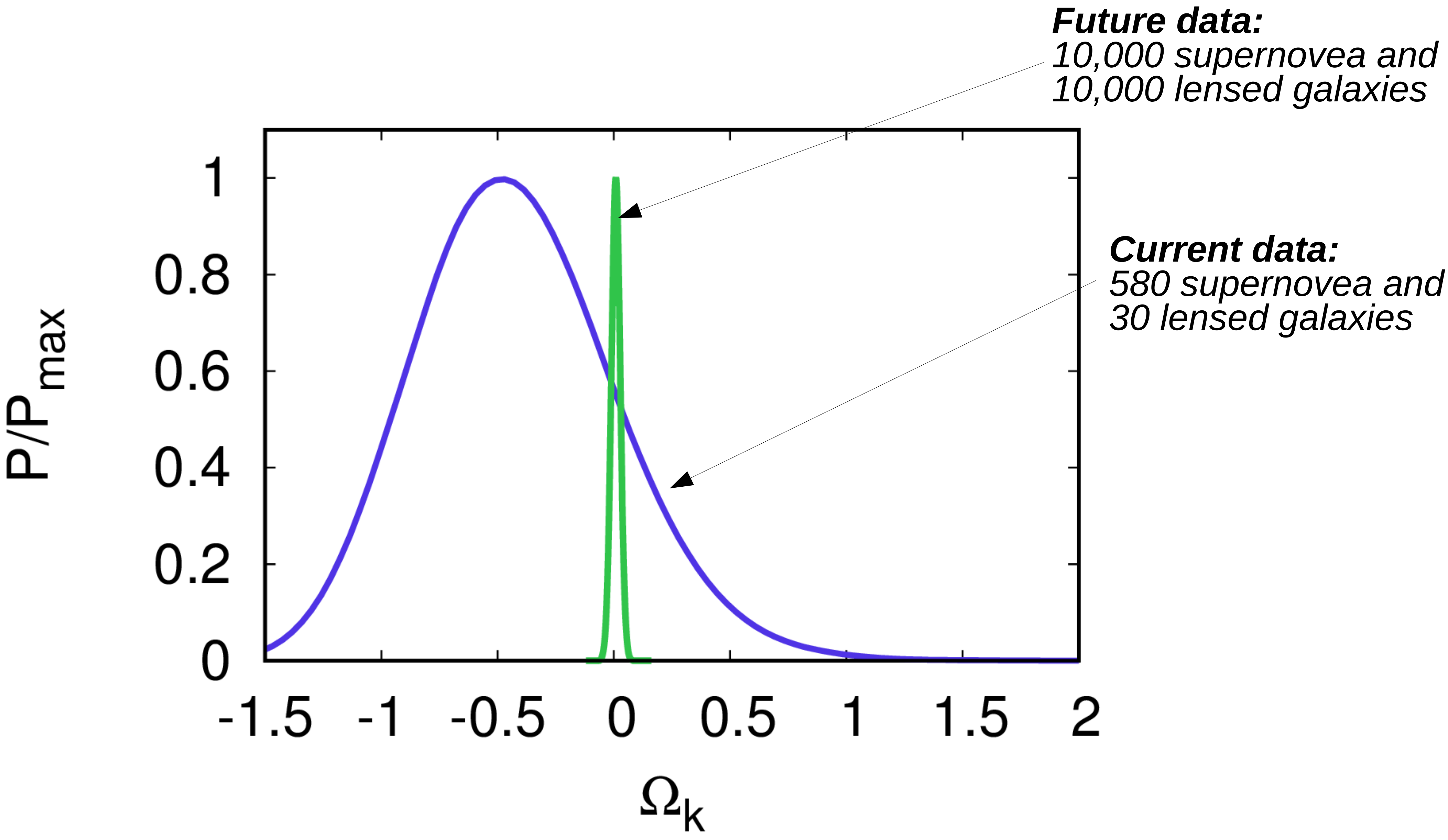
Low-redshift constraints on curvature





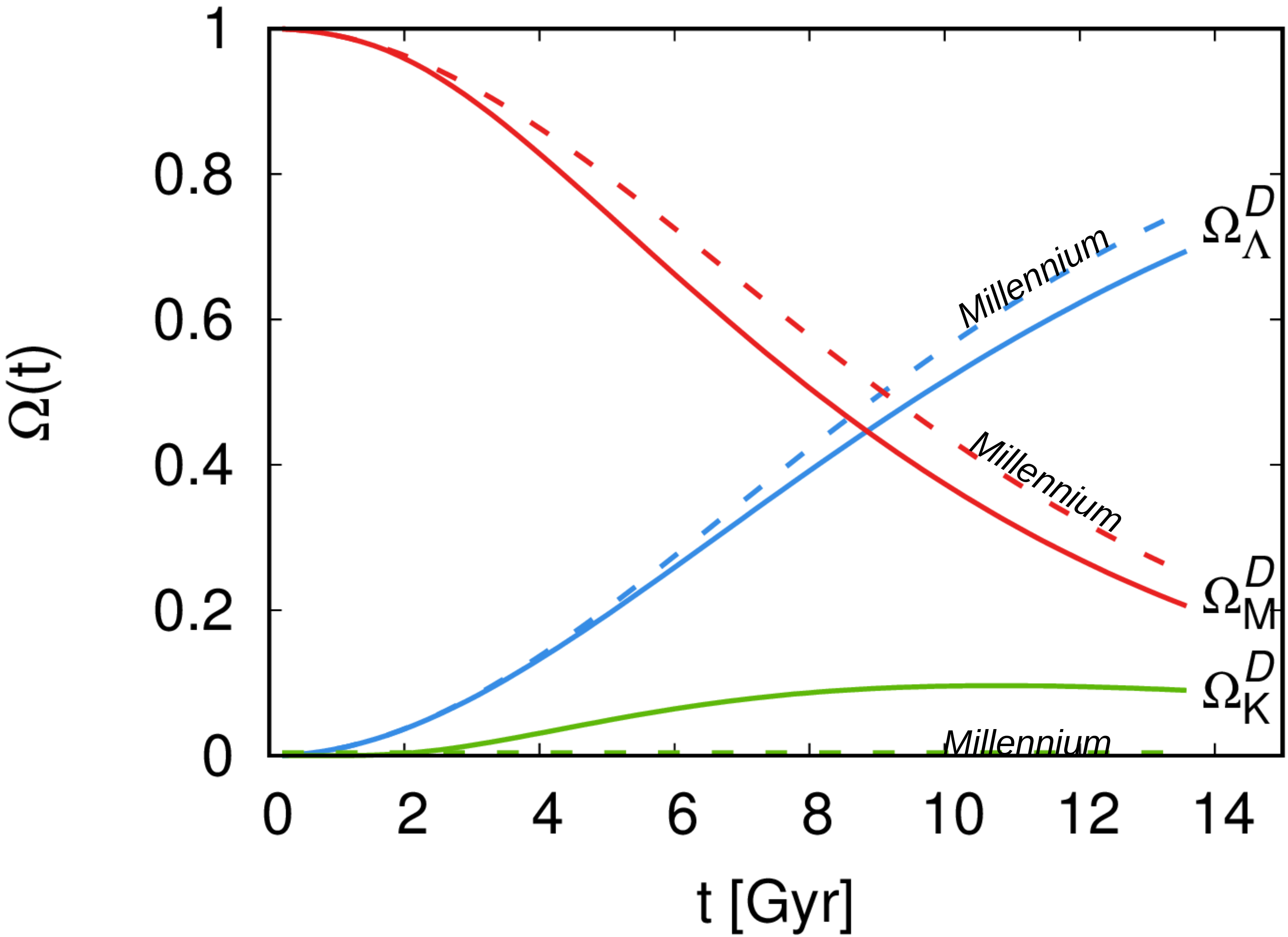
Rasanen, Bolejko, Finoguenov, *Phys. Rev. Lett.* 115, 101301 (2015)

Low-redshift constraints on curvature



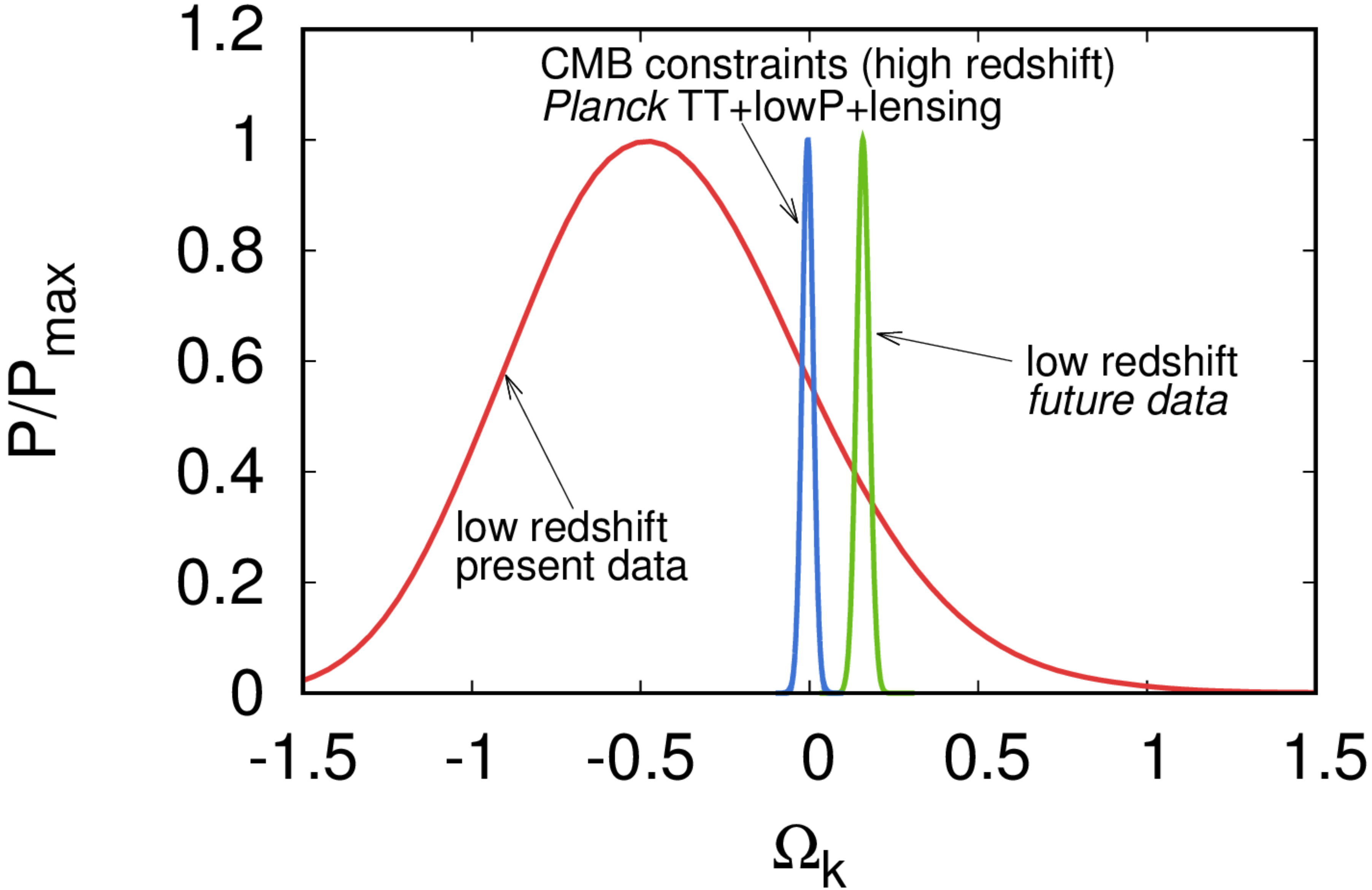
Rasanen, Bolejko, Finoguenov, *Phys. Rev. Lett.* 115, 101301 (2015)

Evolution of the cosmic parameters



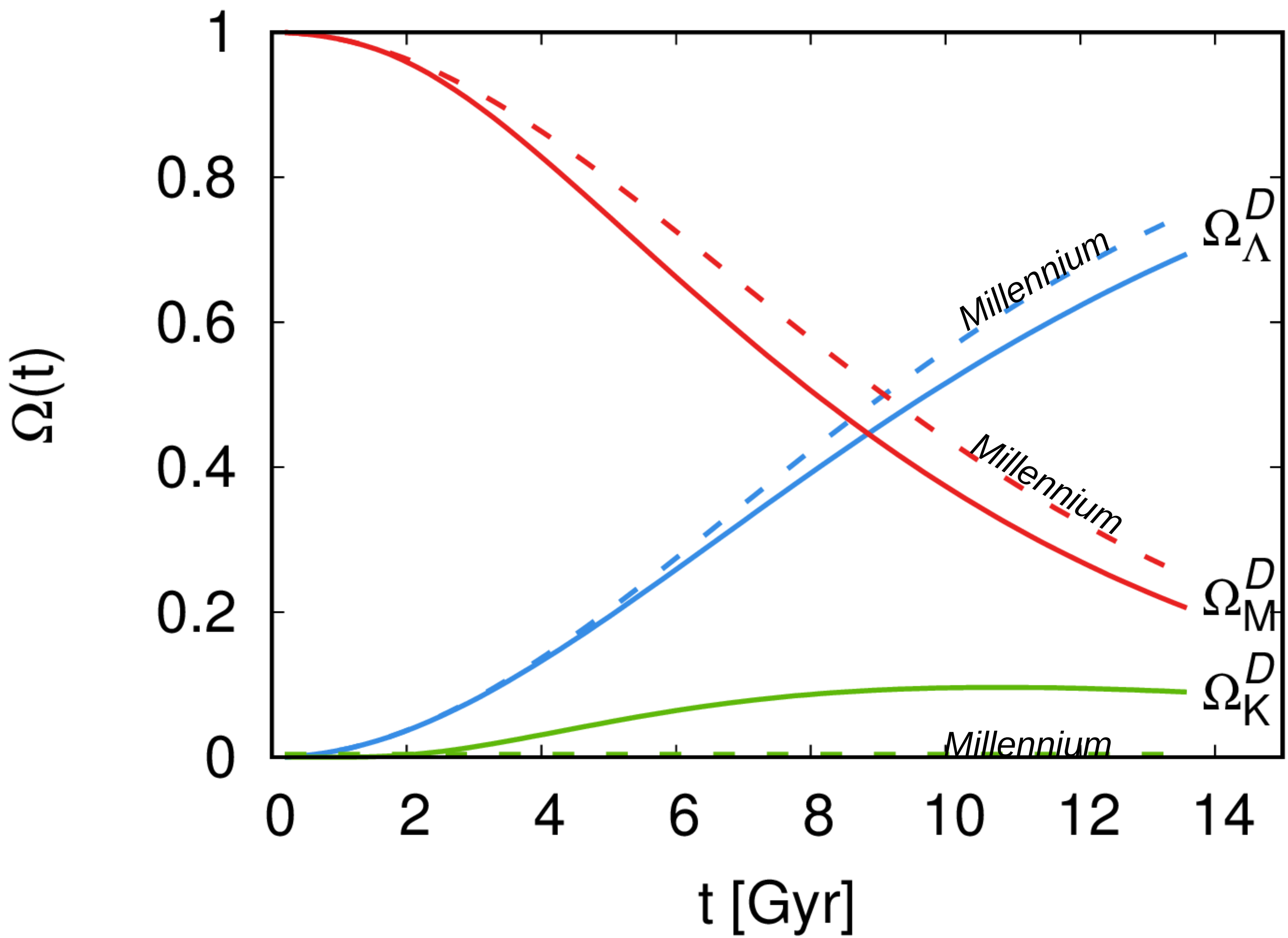
Bolejko, *arxiv*:1708.0940 (2017)

Measuring the curvature



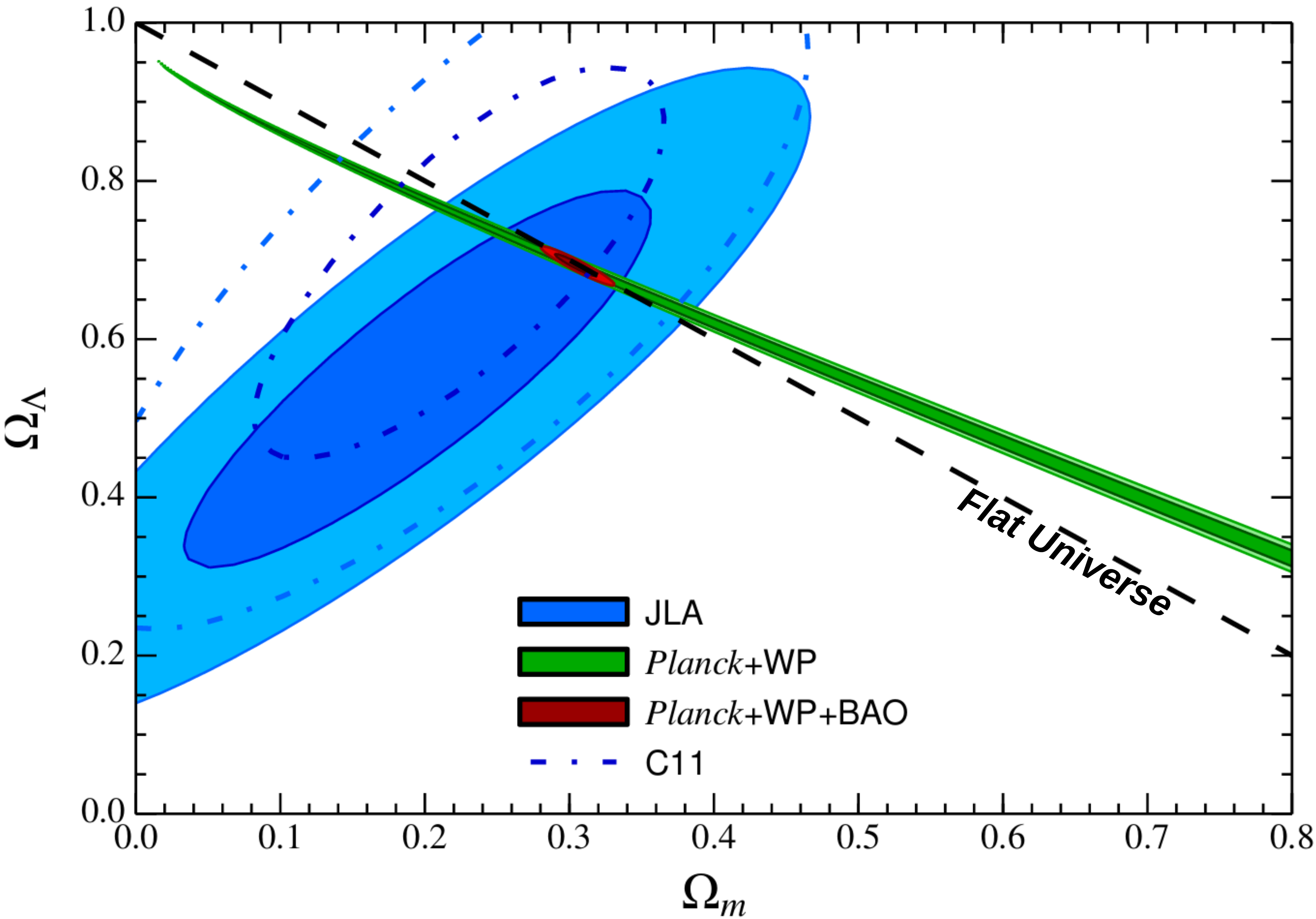
Bolejko, *arxiv*:1707.01800 (2017)

Evolution of the cosmic parameters



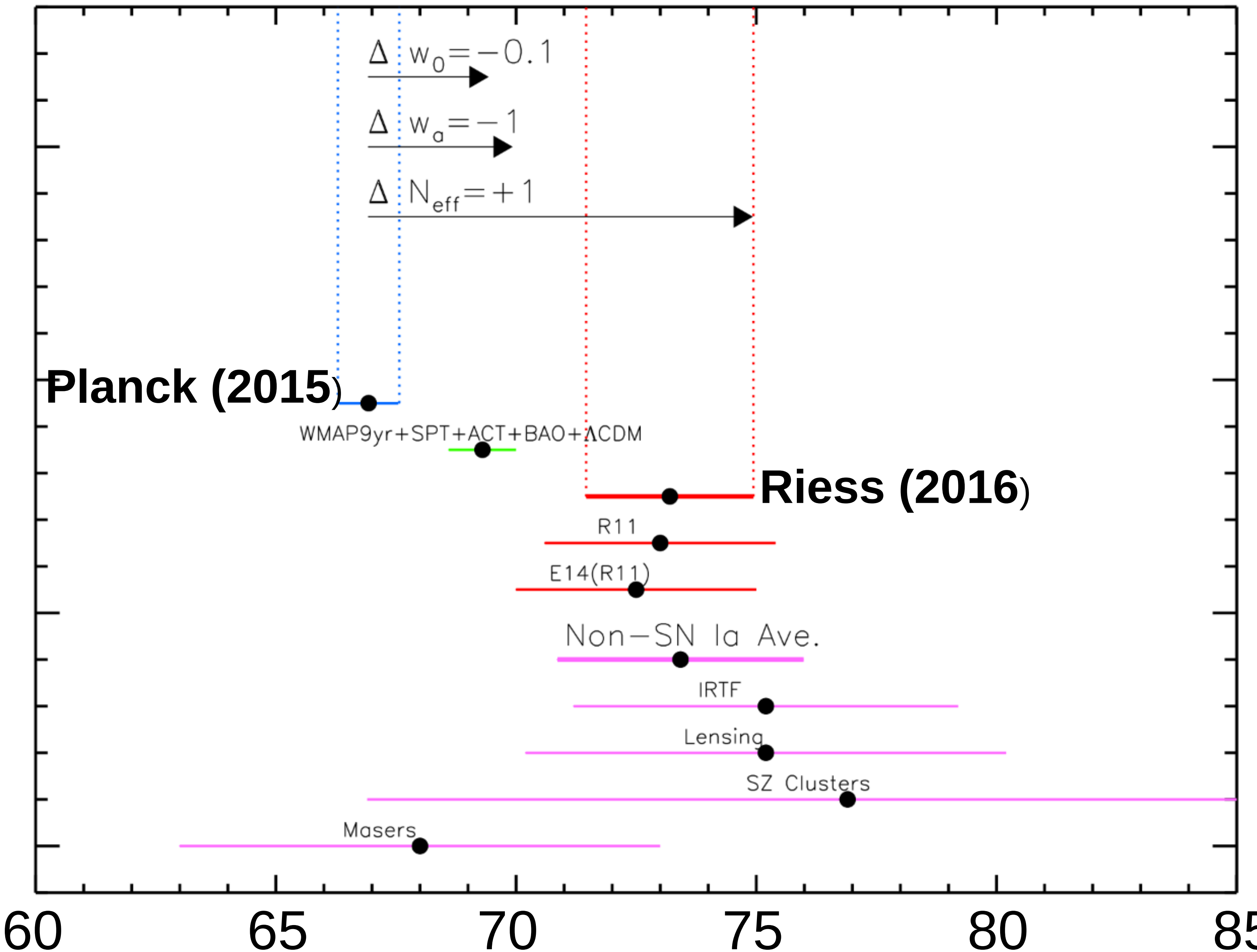
Bolejko, *arxiv*:1708.0940 (2017)

JLA Supernova constraints



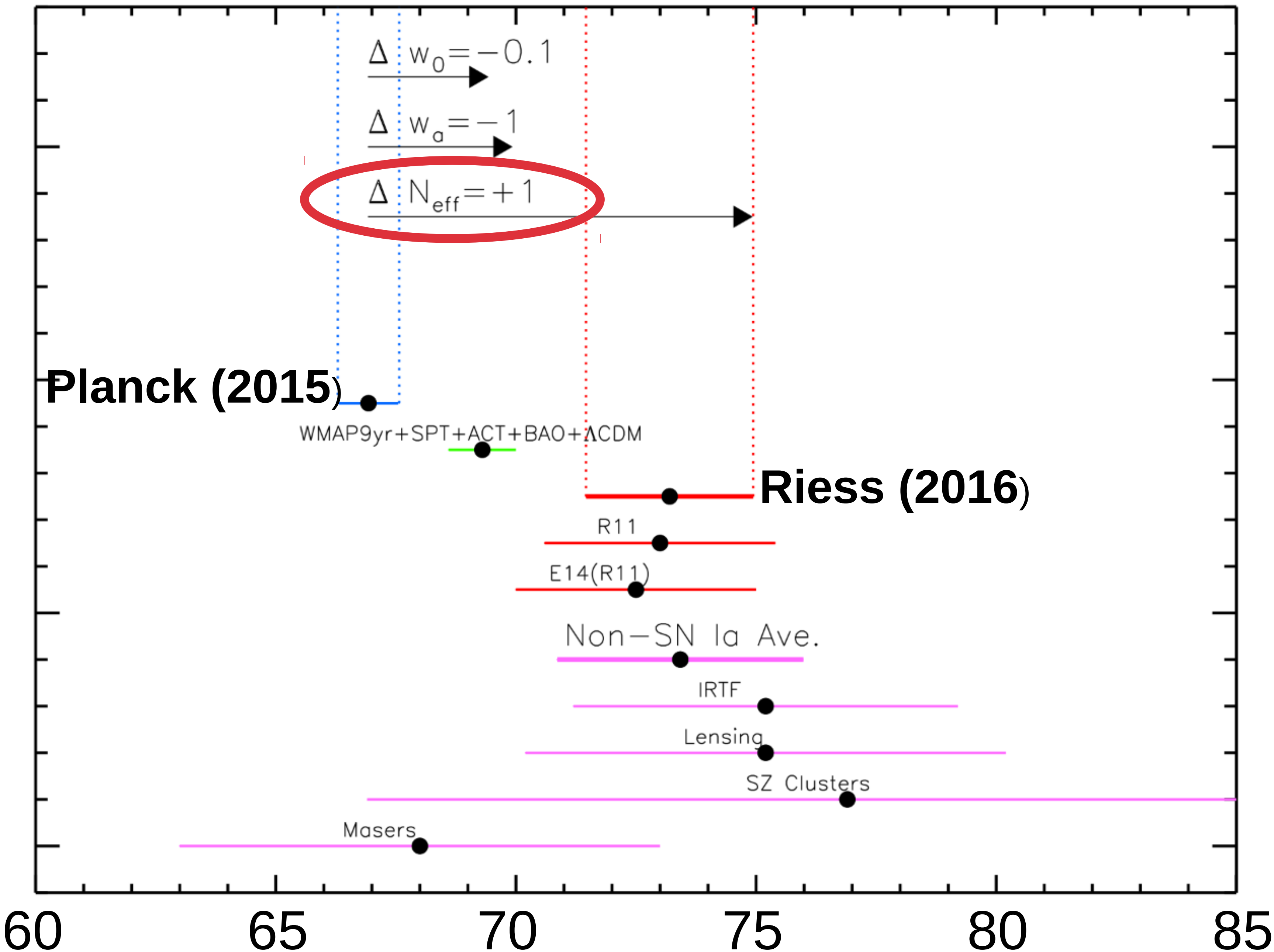
Betoule, Kessler, Guy, et al. *Astron. & Astroph.* 568, A22, (2014)

Hubble constant



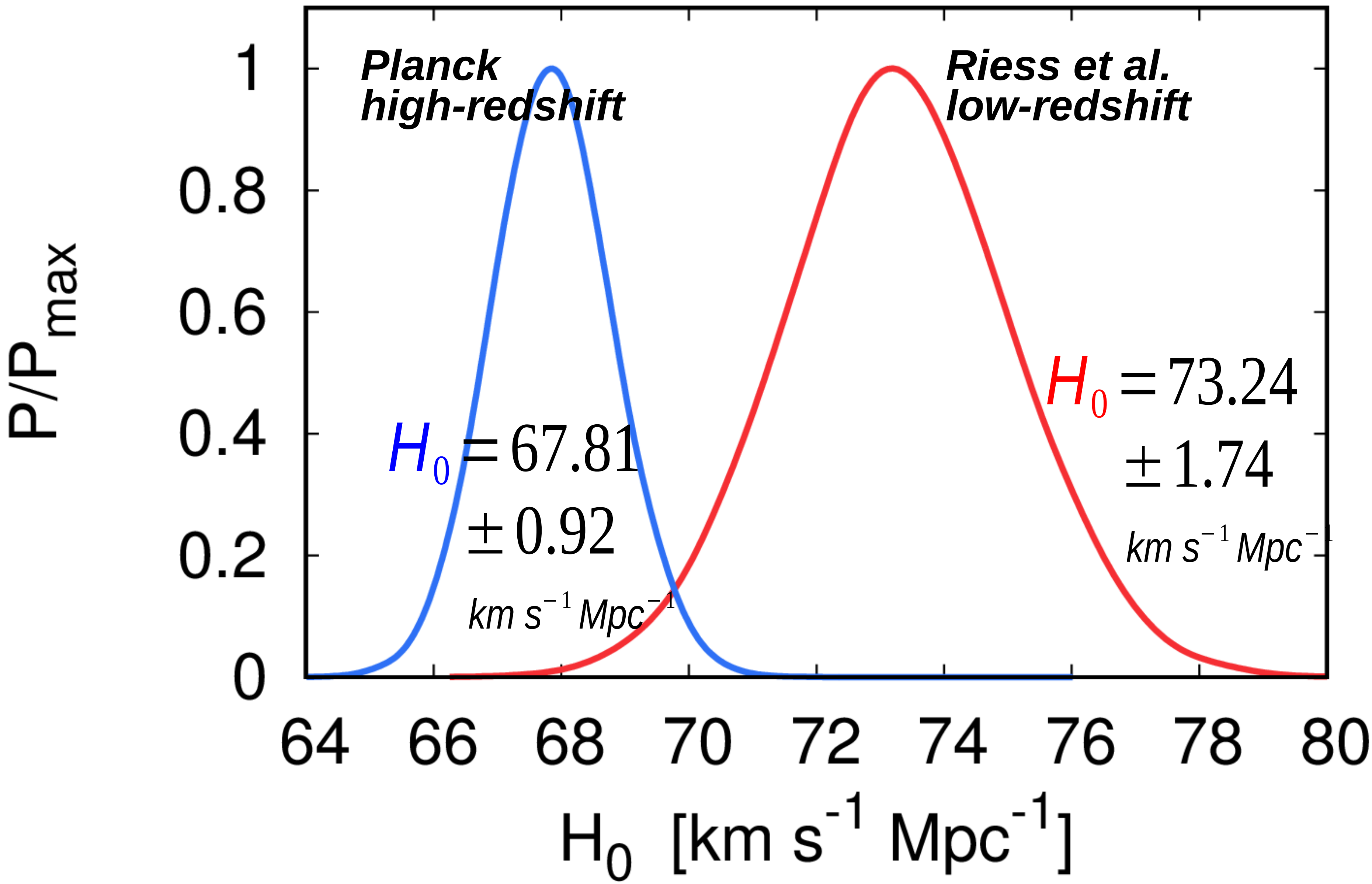
Riess, Macri, Hoffmann et al, *Astrophys. J.* 826, 56 (2016)

Hubble constant

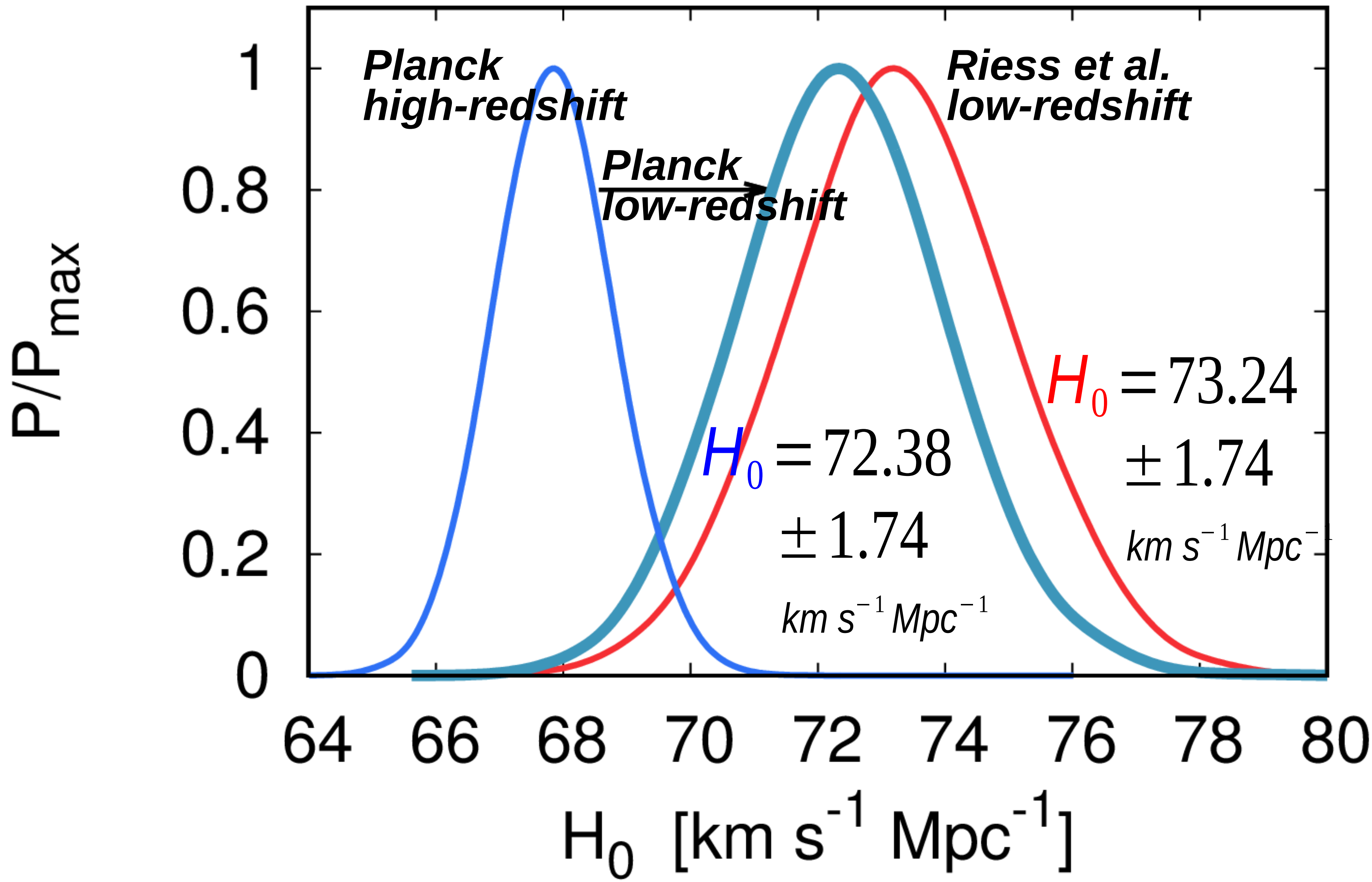


Riess, Macri, Hoffmann et al, *Astrophys. J.* 826, 56 (2016)

H_0 tension resolved



H_0 tension resolved



Summary

Silent Cosmology

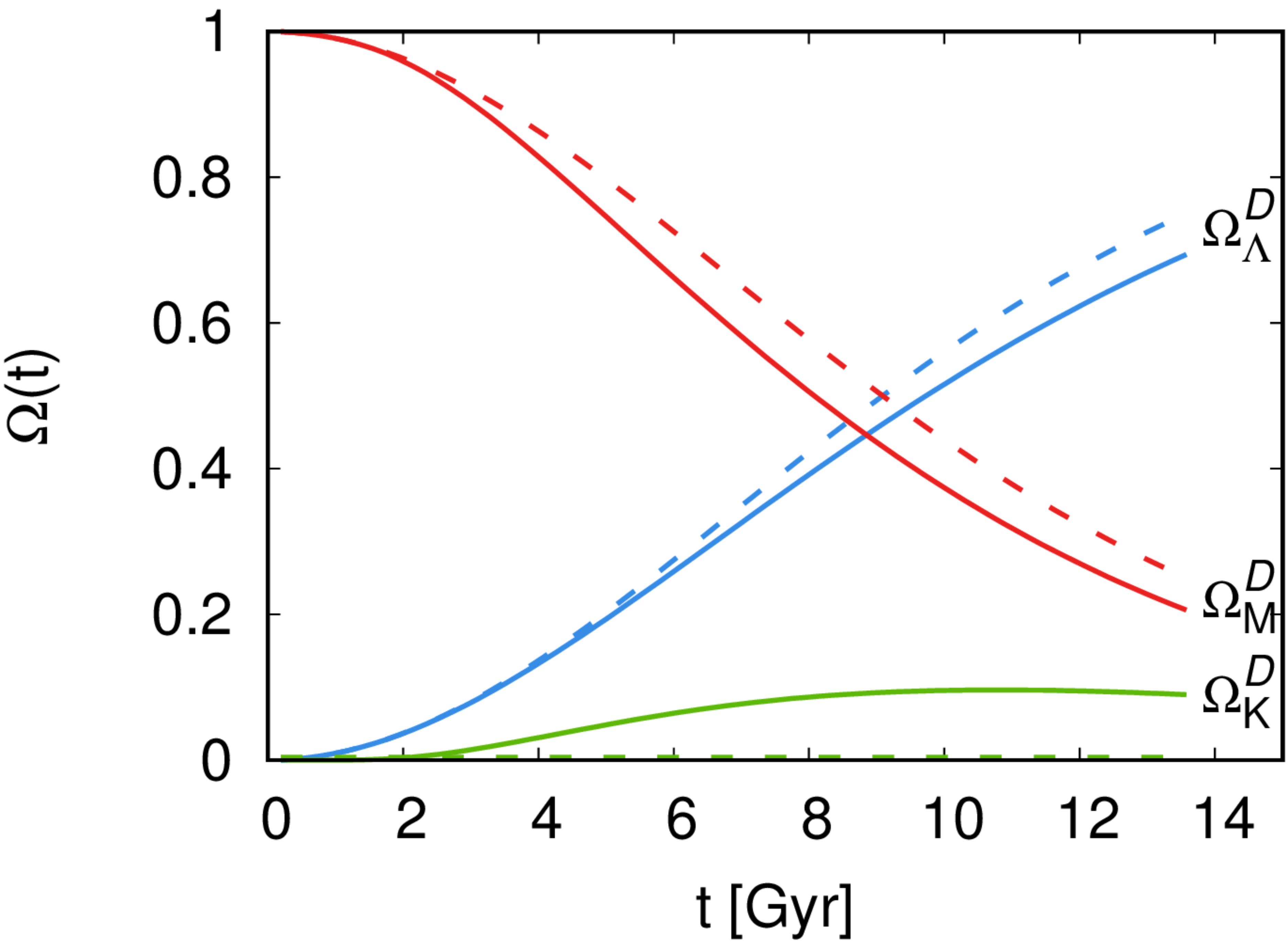
$$\dot{\rho} = -\Theta\,\rho$$

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}\rho - 6\,\Sigma^2 + \Lambda$$

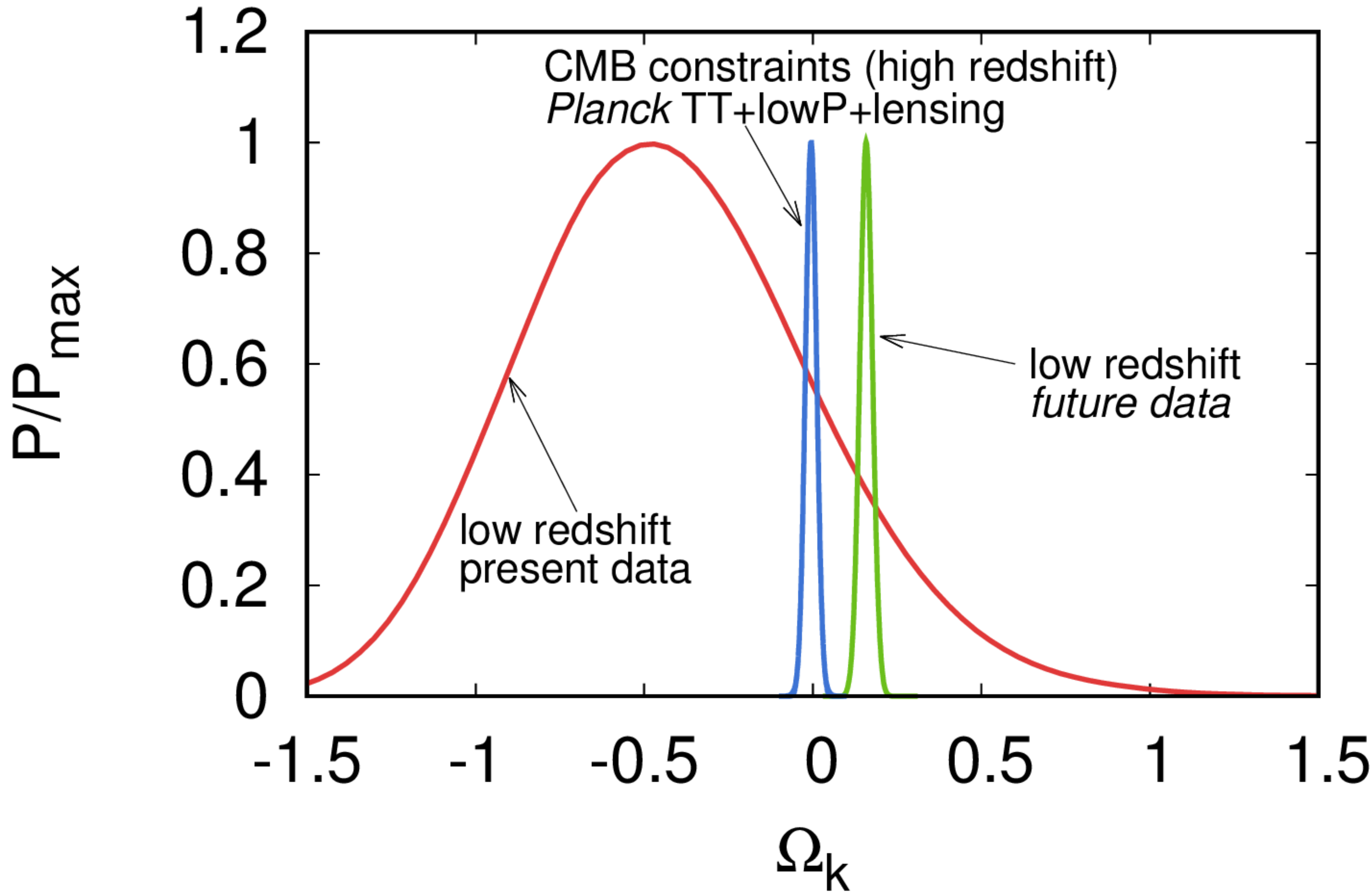
$$\dot{\Sigma} = -\frac{2}{3}\Theta\,\Sigma + \Sigma^2 - W$$

$$\dot{W} = -\Theta\,W - \frac{1}{2}\rho\,\Sigma - 3\,\Sigma\,W$$

$$\Sigma \equiv 0\,, \qquad W \equiv 0$$



Bolejko, *arxiv*:1708.0940 (2017)


CMB constraints (high redshift)
Planck TT+lowP+lensing
low redshift
future data
low redshift
present data
P/P$_{max}$
Ω_k
1.2
1
0.8
0.6
0.4
0.2
0
-1.5
-1
-0.5
0
0.5
1
1.5

Brief history of spatial curvature

- -300: Euclid's *Elements*
- 1813: beginning of non-Euclidean geometry
- 1900: astronomical constraints on spatial curvature
- 1917: cosmological models with $\mathbf{K > 0}$
- 1923: extragalactic astronomy
- 1929: expanding Universe
- 1932-1990s: $\boldsymbol{\Lambda = 0}$, $\boldsymbol{K = 0}$
- 1997-2010s: $\boldsymbol{\Lambda > 0}$, $\boldsymbol{K = 0}$
- 2008-2017: theoretical suggestions for $\mathbf{\Lambda > 0}$, $\mathbf{K < 0}$

- 2022-2025: Euclid measures $\boldsymbol{K}$**,**
  $\boldsymbol{K < 0}$ enters standard cosmology