THE MULTIFREQUENCY VARIABILITY OF Mrk 421

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We present a model of the multifrequency variability of the blazar Mrk 421. The model explains correlated variability observed from Very High Energy (VHE) gamma rays to radio frequencies. We assume that the dominant part of the stationary emission from the radio frequencies to the X-rays is generated by the synchrotron radiation of relativistic electrons ejected from the central engine. The particles move from the center of the source with relativistic velocities and form an inhomogeneous jet. We perform detailed calculations of the radiation transfer and calculate evolution of the electron energy spectrum along the jet. We explain the observed variability by the evolving synchrotron and Inverse-Compton (IC) radiation of a compact component (a blob) which travels along the jet. We assume that the high energy electrons are generated in situ by acceleration, for example by diffusive shock waves or a localized turbulence inside the blob. The particles evolve along the jet. They are cooled by the radiative processes and by the adiabatic expansion which compete with the acceleration process. We apply the model to a new observations we obtained in the radio domain for Mrk 421. The radio data gathered in February-April 2001 show a well defined radio outburst which corresponds to an X-ray outburst observed by RXTE-ASM and a gamma-ray flare detected by HEGRA in the TeV range. The best of our knowledge, this is the first direct observational evidence for a flare observed simultaneously in the radio range and at the very high energies.
The draft shows structure of a blazar, assumed for our multifrequency modeling and spectral energy distributions generated by the model. We apply the model to the observations of Mrk 421 gathered in the NED database and to simultaneous, high energy observations performed by RXTE (3-20 keV) and HEGRA (above 1 TeV) experiments (Krawczynski et al. ApJ, 559 - 2001).
THE MODEL: GEOMETRY OF THE JET

We approximate geometry of the jet by a system of homogeneous cylindrical elements – slices. The slices travel from the central engine with constant velocity \( V_{jet} \) and expand adiabatically from conical \( r_{jet} = 1 \) or paraboloid \( (r_{jet} < 1) \) geometry. The length \( L_{jet} \) and the radius \( R_{jet} \) of each element rise with time according to following formulae:

\[
L_{jet} (t_{jet}^i) = L_{jet}^i = V_{exp} t_{jet}^i \\
R_{jet} (t_{jet}^i) = R_{jet}^i = R_{jet}^1 \left( \frac{t_{jet}^i}{t_{jet}^1} \right)^{r_{jet}}
\]

where \( V_{exp} \) [cm/s] is expansion velocity of the cylinder length, \( R_{jet}^1 \) is the radius of the first cylinder, \( r_{jet} \) describes geometry of the jet and \( t_{jet}^i \) are the time values defined by:

\[
t_{jet}^1 = \frac{L_{jet}^1}{V_{exp}} \\
t_{jet}^i = \frac{L_{jet}^{i-1} + 2V_{jet} t_{jet}^{i-1}}{2V_{jet} - V_{exp}}, \quad \text{for} \quad i > 1.
\]

This parameterization provides correlation between expansion rate of the slices and their velocity in space. It means that shift of the slice in space equals half of length increment. By analogy to the jet we also describe the blob by several slices. The length of blob’s slice evolves exactly like the length of jet’s slice \( L_{blob} (t_{blob}^i) = L_{jet} (t_{jet}^i) \). The radius of the blob’s slice \( R_{blob} \) can be smaller than the radius of the jet however, it evolves proportional to the radius of jet’s slice. A position of the current blob related to the jet shape is given by \( a = 1...n_j \) which indicates position where first blob’s slice is placed. The position of the last blob’s slice is given by \( b = b - n_b + 1 \), where \( n_b \) is the number of blob’s slices.
THE MODEL: RADIATION TRANSFER ALONG THE JET

Calculating radiation transfer along the jet we consider a case where the angle between the jet’s symmetry axis and the line of sight is relatively small (about a few degrees). Therefore we assume that the solution of radiation transfer along path parallel to the jet axis is enough precise for the modeling. To obtain total observed flux density \( F_{\text{tot}}^a = F_{\text{in}}^a + F_{\text{mid}}^a + F_{\text{out}}^a \) from the jet and the blob, placed inside the jet at position indicated by \( a \), is necessary to divide the jet for three different regions.

- first we calculate contribution, to the observed flux density, generated within a part of the source smaller or equal to \( R_{\text{blob}}^a \):

\[
F_{\text{in}}^a = \frac{\pi \delta_{\text{jet}}^3 (1 + z)}{d_i^2} \sum_{i=1}^{a} f^i, \quad f^i = \left\{ \begin{array}{ll}
\left( (R_{\text{blob}}^i)^2 - (R_{\text{blob}}^{i-1})^2 \right) I_{\text{tot}}^i, & l^i = h^i = 1 \\
\left( (R_{\text{jet}}^i)^2 - (R_{\text{jet}}^{i-1})^2 \right) I_{\text{tot}}^i + \left( (R_{\text{blob}}^i)^2 - (R_{\text{blob}}^{i-1})^2 \right) I_{\text{tot}}^{h^i}, & h^i = l^i + 1
\end{array} \right.
\]

where \( l^i \) is the index number of the jet slice with maximal radius just smaller than \( R_{\text{blob}}^a \) and \( h^i \) is the number of jet’s slice with minimal radius just bigger than \( R_{\text{blob}}^a \). Please note that for some cases \( R_{\text{blob}}^a < R_{\text{jet}}^1 \) therefore \( l^i = 1 \).

- second contribution, to the total radiation, comes from medium part of source larger than \( R_{\text{blob}}^a \) but smallest than \( R_{\text{jet}}^h \) given by:

\[
F_{\text{mid}}^a = \frac{\pi \delta_{\text{jet}}^3 (1 + z)}{d_i^2} \left( (R_{\text{jet}}^h)^2 - (R_{\text{blob}}^a)^2 \right) I_{\text{tot}}^h
\]

- third part of radiation \( F_{\text{out}}^a \) comes from the outer part of the source, biggest than \( R_{\text{jet}}^h \) and is calculated according to formula:

\[
F_{\text{out}}^a = \frac{\pi \delta_{\text{jet}}^3 (1 + z)}{d_i^2} \sum_{i=h^a+1}^{n^j} \left( (R_{\text{jet}}^i)^2 - (R_{\text{jet}}^{i-1})^2 \right) I_{\text{tot}}^i
\]

The intensity \( I_{\text{tot}} \), in all above equations, is defined by:

\[
I_{\text{tot}}^i = \sum_{j=i}^{l^i} I_{\text{tot}}^{i-1} \exp \left( -\tau_x^i \right) + I_x^i,
\]

\[
I_x^i = \left\{ \begin{array}{ll}
I_{\text{blob}}^i, & b \leq i \leq a \\
I_{\text{jet}}^i, & i < b \text{ or } i > a
\end{array} \right., \quad \tau_x^i = \left\{ \begin{array}{ll}
L_{\text{blob}}^i k_{\text{blob}}^i, & b \leq i \leq a \\
L_{\text{jet}}^i k_{\text{jet}}^i, & i < b \text{ or } i > a
\end{array} \right.
\]

where \( I_{\text{jet/ blob}}^i = (j_{\text{jet/ blob}}^i/L_{\text{jet/ blob}}^i) [1 - \exp \left( -k_{\text{jet/ blob}}^i L_{\text{jet/ blob}}^i \right)] \), \( j_{\text{jet/ blob}} \) and \( k_{\text{jet/ blob}} \) are respectively emission and absorption coefficients. The intensity is calculated under assumption that \( I_{\text{tot}}^i = 0 \) for \( i - 1 < j \). In all above formulas, \( \delta_{\text{jet}} \) is the Doppler factor of the jet and \( d_i \) is the luminosity distance.
THE MODEL: EVOLUTION OF ELECTRONS ENERGY SPECTRUM

We describe the evolution of electrons energy spectrum which refer to the entire source taken as entity by the kinetic equation:

\[
\frac{\partial N^*_{\text{jet/blob}}(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ \left[ C^\text{cool}_{\text{jet/blob}}(t) \gamma^2 - (C^\text{acc}_{\text{jet/blob}}(t) - C^\text{adia}_{\text{jet/blob}}(t)) \right] N^*_{\text{jet/blob}}(\gamma, t) \right\},
\]

where \( C^\text{cool} \) describes losses of the energy due to synchrotron and IC radiation:

\[
C^\text{cool}_{\text{jet}} = 4 \sigma_T U^\text{jet}_B / (3m_e c), \quad C^\text{cool}_{\text{blob}} = 4 \sigma_T (U^\text{blob}_B + U^\text{blob}_r) / (3m_e c),
\]

in above formulas \( U^\text{jet/blob}_B = B^2_{\text{jet/blob}} / (8 \pi) \) and \( U^\text{blob}_r = U^\text{blob}_B / \eta \), where parameter \( \eta \) can be estimated from high energy emission of a blazar. Distribution of the magnetic field intensity along the jet is given by:

\[
B_{\text{jet}} = B^i_{\text{jet}} = B^1_{\text{jet}} (t^1_{\text{jet}}) / (t^i_{\text{jet}})^{m_{\text{jet}}}, \quad B_{\text{blob}} = c_B B_{\text{jet}}
\]

We apply for the modeling an acceleration process described by following coefficients:

\[
C^\text{acc}_{\text{jet}} = A^1_{\text{jet}} (t^1_{\text{jet}} / t)^{a_{\text{jet}}}, \quad C^\text{acc}_{\text{blob}} = A^1_{\text{blob}} (t^1_{\text{jet}} / t)^{a_{\text{blob}}},
\]

Please note that we do not specify which acceleration scenario, among variety of models discussed in literature, could provide best physical conditions required by the model. Detailed description of the acceleration may introduce, into modeling, too many additional parameters difficult to constrain from observational data. Thus we selected very simple, however arbitrary description of the acceleration, with a minimal number of free parameters. Taking into account our parameterization of geometry evolution we can write coefficients, which characterize losses of the electrons energy due to adiabatic expansion by:

\[
C^\text{adia}_{\text{jet/blob}} = \nabla v(t) / 3 = d_{\text{jet/blob}} / t_{\text{jet}},
\]

where \( v(t) \) is a velocity field which determines expansion of a slice volume, \( d_{\text{jet}} = (2r_{\text{jet}} + 1) / 3 \) and \( d_{\text{blob}} \) is a free parameter. We succeeded to find analytical solution of the kinetic equation for the initial distribution described by a power law function:

\[
N^*_{\text{jet/blob}}(\gamma, t^1_{\text{jet}}) = K^1_{\text{jet/blob}} \left[ \gamma^{-n_{\text{jet}}} - \gamma^2 (\gamma^\text{cut}_{\text{jet/blob}})^{-n_{\text{jet/blob}} - 2} \right],
\]

with sharp cutoff at the highest energies \( (\gamma^\text{cut}_{\text{jet/blob}}) \), where \( K^1 \) is initial particle density and \( n_{\text{jet/blob}} \) is a slope of the spectrum. To apply the solution for calculations of the emission and the absorption coefficients we have to convert it to unit volume. In case where \( V^s_{\text{jet/blob}} \sim t^2_{\text{jet}} \), the conversion is given by:

\[
N_{\text{jet/blob}}(\gamma, t) = N^*_{\text{jet/blob}}(\gamma, t) \left( t^1_{\text{jet}} / t \right)^{-3d_{\text{jet}}}. 
\]
THE MODEL: HOST GALAXY AND EXTENDED RADIO STRUCTURES

One motivation of our present analysis is to describe simultaneously all parts of the blazar spectrum. We look for correlation at different wavelengths and try to interpret multifrequency light curves from the radio bands to the gamma rays. Assumptions which we selected for the jet model, developed above, provides good reproduction of the spectrum from GHz radio frequencies to the X-rays. The jet model in not able to explain flux observed at the lower radio frequencies due to the electrons self-absorption. In that range, contribution of more extended component is needed to be taken into account. Such contribution could be described as a cloud at the end of the jet, possibly corresponding to a radio lobe fed by the jet itself. Stationary state is then ensured by losses of particles energy related mainly to the synchrotron radiation. The observed flux density for such source is calculated according to formula:

\[ F_{\text{ext}} = \frac{\pi \delta_{\text{ext}}^3 (1 + z)}{d_l^2} \frac{j_{\text{ext}}}{k_{\text{ext}}} \left( 1 - \frac{2}{\tau_{\text{ext}}} e^{-\tau_{\text{ext}}[\tau_{\text{ext}} + 1]} \right) \]

where \( \tau_{\text{ext}} = 2R_{\text{ext}} k_{\text{ext}} \), \( j_{\text{ext}} \) and \( k_{\text{ext}} \) are the synchrotron emission and the electrons self-absorption coefficients respectively, \( \delta_{\text{ext}} \) is the Doppler factor of the source. The electrons energy distribution, for such simple cloud, is described by the spectrum calculated for the last jet’s slice. However, the particle density inside the cloud is much higher due to accumulation of particles during long term evolution of the source. Therefore we define the particles energy distribution by:

\[ N_{\text{ext}}(\gamma) = c_{\text{ext}} N_{\text{jet}}(\gamma, t_{\text{nj}}) \]

where \( c_{\text{ext}} \) is a free parameter, which describes difference in the particles density. We assume also that the magnetic field intensity inside the cloud equals to the magnetic field intensity inside the last jet’s slice (\( B_{\text{ext}} = B_{\text{nj}} \)). This simple approach reproduces reasonably well the emission of extended radio structure. The real situation probably involves more complex geometry and distribution of particles. However, we introduced this part of the model only to reach a correct level of permanent low frequency radio radiation. The thermal radiation of the host galaxy, quite well visible at the infrared and optical wave lengths, is simple reproduced by the black body radiation:

\[ F_{\text{th}} = F_{\text{th}}^{\text{max}} P(\nu, T_{\text{th}}) / P(3k_B T_{\text{th}} / h, T_{\text{th}}) \]

where \( P(\nu, T) \) is the Planck’s function, \( F_{\text{th}}^{\text{max}} \) is observed flux density for the maximum of thermal radiation, \( T_{\text{th}} \) is the temperature, \( k_B \) is the Boltzmann’s constant and \( h \) is the Planck’s constant. This basic approach gives good reproduction of the thermal emission with only two free parameters \( (F_{\text{th}}^{\text{max}}, T_{\text{th}}) \). We introduced it to explain observed level of radiation at the optical wave lengths, where we also calculate the light curves.
We apply the model to the observations gathered during campaign of radio observations performed in January-March 2001 by Toruń Centre for Astronomy (TCfA) – Department of Radio Astronomy in Poland. The observations were carried out by 32 m radio telescope using cooled ($T_{sys} \approx 30$ [K]) beam switching receiving system at frequency 5 GHz. In addition to observations at 5 GHz in Torun, we obtained also data at 1.4 & 2.7 GHz from the Decimetric Radiotelescope of the Nancay Observatory in France. We compared the results obtained from the radio campaign with the X-ray light curves obtained from RXTE-ASM experiment (2-10 keV) and with the VHE gamma-ray observations performed by HEGRA experiment above 1 TeV (Aharonian et al. A&A, 393, 2002).

Looking at the light curves obtained at the radio frequencies we could see significant increment of the emission at 5 GHz (of about 300 mJy) and 2.7 GHz (of about 500 mJy) but no activity at 1.4 GHz. This agrees qualitatively with our theoretical investigations performed in frame of acceleration scenario. Moreover, simultaneously to radio activities also at the X-rays and the gamma-rays was observed activity. No time delay between activity at the X-rays and highest radio frequencies suggests that emitting region was from beginning of its evolution partially thin for radio radiation. Also absorption of blob radiation by jet’s electrons was not efficient. To simulate such emitting region we placed it initially in the middle part of the jet ($a = 50$). To explain observed activity we assume acceleration of the electrons inside the blob. Initially acceleration process is very efficient and causes fast increment of the source brightness. However, efficiency of the acceleration decreases rapidly during evolution of the blob. Therefore cooling of the particles due to synchrotron and IC radiation and cooling due to adiabatic expansion of the source appear quickly more efficient than the acceleration process. Brightness of the source starts to decrease rapidly. The model is able reasonably well explain the light curves except the radio light curve obtained at 2.7 GHz. This may indicate that evolution of the blob was slightly more complex at the radio frequencies.