Surf-Riding in Relativistic Winds

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Crab in X-Rays

Cygnus A in Radio

Electromagnetically (not thermal photon) driven outflows
\[ \sigma_1 \sim 5 \times 10^{-3}, \gamma_1 \sim 3 \times 10^6, m_i c^2 \gamma_1 = 0.15 e \Phi_{\text{magnetosphere}} \]

\[ \dot{N}_i \approx \dot{N}_{\text{GJ}} = 2 \times 10^{34} \text{s}^{-1} \dot{N}_\pm \approx 3 \times 10^{38} \text{s}^{-1} \sim 10^4 \dot{N}_i \Rightarrow \text{MHD} \]
Other (Young) Pulsar Wind Nebulae (PWN)

G320

G21.5

3C 58

Vela
Electromagnetically Driven Relativistic Flow

\[ \sigma \equiv \frac{B^2}{4\pi mn\gamma c^2} \gg 1 \]

Relativistic Wind and Weak Self-Collimated Jet from a rotating Compact object - “jet” really a plume?

(Bogovalov 2001, Lyubarsky 2002)

Development of a Strong Non-relativistic magnetically Dominated jet from a disk

(Ustyugova, Lovelace, Romanova et al 2000)
\[ \sigma \equiv \frac{B^2}{4\pi mn\gamma c^2} \gg 1 \]

\[ rB = \text{constant}, r^2 nc = \text{constant} \]

\[ \Rightarrow \sigma = \frac{\sigma_0}{\gamma} , \sigma_0 \equiv \left( \frac{B^2}{4\pi mnc^2} \right)_L \approx 2 \times 10^6 (\text{stdnrCrabe}^+) \]

\[ \gamma = \text{constant} \Rightarrow \sigma \gg 1 \text{ everywhere} \]

Pulsar Winds:

Energy in equatorial flow (>90% in Crab)

Equatorial shock:

\[ \text{obs/models} \Rightarrow \sigma(R_s) \ll 1 \]

(the “sigma” problem) and

\[ \gamma(R_s) \sim 10^6 \gg \sigma_L^{1/3} \]

(the “gamma” problem)
Existing Relativistic wind theory yields $\gamma_\infty = \sigma_L^{1/3}, \sigma_\infty = \sigma_L^{2/3} \gg 1$

(Electro)Magnetically Dominated Flow Structure

$$\rho \left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) (\gamma \mathbf{v}) = \eta \mathbf{E} + \frac{1}{c} \mathbf{J} \times \mathbf{B} \approx 0 \ ("Force\ Free")$$

Electric Force omitted from 1D MHD wind theory ($\gamma_\infty = \sigma_L^{1/3}$)

Assume: Fields of **Force-Free** Split Monopole represent wind

$$B_r = \pm \frac{M}{r^2}, \quad B_\phi = \pm \frac{M\Omega}{cr}\sin\theta = E_\theta, \quad M = k\frac{\mu\Omega}{c} \quad (Michel\ 1973)$$
Asymptotically Monopolar Field Lines

Radial Current:
\[ J_r = -\frac{|M|\Omega}{2\pi r^2} \cos \theta, \quad I_{\text{wind}} = -2\pi M \]
(M = open field line flux)

Wind = “Transmission Line”

Equatorial Current Sheet
\[ I_{\text{sheet}} = +2\pi M \]

Field Lines of Force-Free Aligned Dipole (Contopoulos, Kazanas and Fendt 1999): \( k = 1.36 \)

Current flows to “infinity” = nebula/ISM = “earth”, closes in outside world

Energy Carried by Poynting Flux
Particle Motion/Acceleration

\[ m \frac{d(\gamma v)}{dt} = q \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \]

\[ \Rightarrow \]

\[ \mathbf{v} = c \frac{\mathbf{E} \times \mathbf{B}}{B^2} + c \beta_\parallel \frac{\mathbf{B}}{B} + \mathbf{B} \times \frac{m}{qB^2} \frac{d(\gamma v)}{dt} = \mathbf{v}_E + \mathbf{v}_\parallel + \mathbf{v}_D \]

Field line velocity \( \mathbf{v}_E \) (particles stick to field lines)

Inertial ("polarization")

Drift velocity \( \mathbf{v}_D \ll \mathbf{v}_E \) (particles cross field lines)

\[ \mathbf{v}_E = c \beta_E = c \frac{x}{\sqrt{1 + x^2}}, \gamma_E = (1 - \beta_E^2)^{-1/2} = \sqrt{1 + x^2} \rightarrow x = \frac{r \sin \theta}{R_L} \]

Particles ride field lines in transmission line = Linear Accelerator (Buckley 1977) - "SLAC in the Sky" (Buckley 1977; C&K 2002)
Linear Accelerator (continued)

Proper treatment of $v_\parallel$ (with centrifugal force) and plasma dense (quasi-neutral, not charge separated):

$$\gamma = \sqrt{\gamma_L^2 + \gamma_E^2} \rightarrow \frac{r\sin\theta}{R_L}, \ r\sin\theta >> \gamma_L R_L \sim \text{few hundred} R_L \text{ (PSR)}$$

$$\Rightarrow$$

$$\sigma = \frac{\sigma_0}{\gamma} \rightarrow (\?) l, \ r\sin\theta \rightarrow R_L \sigma_0, \ \gamma \rightarrow (\?) \sigma_0 = \frac{1}{2} \frac{q\Phi}{mc^2} (= \gamma_\infty ?),$$

$$m = m_\pm \left( \frac{n_\pm}{n_{GJ}} \right) (+m_i, \text{if ions present})$$

PSR:  $m \sim 10^3 m_\pm \sim m_p$ (equator), $\gamma \sim 10^6$ (Crab, PSR1509)

$$m \sim 10^5 m_\pm, \ \gamma \sim 10^4$ (Crab, higher latitudes)???

Acceleration completed within Inner Wind (PSR), $r<<R_s$
Linear Accelerator Mechanism

\[ \mathbf{v} = \mathbf{v}_E \Rightarrow \gamma \rightarrow \gamma_E \rightarrow \frac{r \sin \theta}{R_L} \]

but

\[ q \mathbf{v}_E \cdot \mathbf{E} = 0 \]

But, Polarization drift carries particles across field lines, parallel to \( \mathbf{E} \) and always down the potential gradient

\[ v_D = \frac{mc}{qB^2} \mathbf{B} \times \frac{d(\gamma \mathbf{v})}{cdt} \propto \frac{1}{qR_L} \frac{\mathbf{E}}{B_\phi}, B_\phi = E \]

\[ \Rightarrow \]

\[ \frac{d \mathcal{E}}{dt} = mc^2 \frac{d \gamma}{dt} = mc^2 \left( \frac{d \gamma}{dr} \right) = q \mathbf{v} \cdot \mathbf{E} = q v_D \cdot \mathbf{E} = \frac{mc^2}{R_L} \]

\[ \Rightarrow \gamma = \frac{r}{R_L}. \]
Surf-riding in oblique rotator: same as aligned rotator (?)

Oblique Split Monopole (Bogovalov 1999)

Field magnitudes and drifts \((E \times B, \text{polarization})\) same as aligned rotator

Surf-riding also works within the current sheet
But, flow is MHD, not Force Free:

Radial Field Accelerator stops for \( r > R_F = \sigma_0^{1/3} R_L \) (Beskin et al 1998)

\[
\rho c^2 \beta_r \frac{\partial \gamma}{\partial r} = - \frac{\partial}{\partial r} \left( \frac{B^2}{8\pi} \right) - \frac{B^2}{4\pi} + \text{centrifugal force}
\]

\[
= + \frac{B^2}{4\pi} \frac{\partial \beta_r}{\partial r} + \text{centrifugal force} = \frac{B^2}{4\pi \gamma^3} \frac{\partial \gamma}{\partial r} + \text{centrifugal force}, \quad \text{Outward B pressure and inward B tension almost balanced}
\]

\[
B_\phi = - \frac{M \sin \theta}{r R_L} \frac{1}{\beta_r} \neq - \frac{M \sin \theta}{r R_L} = B_\phi \text{(force free)}
\]

\[
\Rightarrow \left( 1 - \frac{\sigma_0}{\gamma^3} \right) \frac{\partial \gamma}{\partial r} = \text{centrifugal acceleration} \rightarrow 0, \quad r > R_L \sigma_0^{1/3} \equiv R_F \quad \text{and} \quad \gamma > \gamma_F = \sigma_0^{1/3}, \quad \text{Effective mass density} = \rho \gamma^3
\]

\[
M_F^2 = \frac{c^2 \gamma^2}{B^2 / 4\pi \rho \gamma} = \frac{\gamma^3}{\sigma_0} > 1 \Rightarrow \quad \text{Inertia dominates EM stress, accel stops}
\]

\[
\text{Force free} \quad \gamma = r / R_L \Rightarrow M_F^2 < 1 \quad \text{for} \quad r < R_F = R_L \sigma_0^{1/3}. \quad \text{Force free ends when} \quad \sigma >> 1
\]
Linear accelerator exists, but only interior to magnetosonic radius $R_F$ (and appears in practice only if $\gamma_L < (\sigma_0)^{1/3}$: Beskin et al 1998

Asymptotic monopole conserves $I = 2\pi M$, $rB_\phi$ – weak stress across B drives insignificant cross field current flow

Oblique split monopole with frozen in wavy current sheet the same (formal proof by inertial perturbation of force free result not yet done)
Physics of $M_F = 1$

Nonrelativistic:

\[
\frac{1}{2} \frac{\rho v^2}{B^2} = \frac{\text{KE energy density}}{\text{EM energy density}} = \frac{4\pi \rho v^2}{B^2} = \frac{v^2}{v_A^2} = M_F^2
\]

$M_F < 1$: EM dominated, $v < v_A$; $M_F > 1$: KE dominated, $v > v_A$

Relativistic:

\[
M_F^2 \equiv \frac{\sigma^3}{\gamma_0} = \frac{\gamma^3}{\sigma_0} = \frac{4\pi \rho \gamma}{B^2} c^2 \gamma^2 = \frac{u^2}{u_A^2} = \gamma^2 \frac{\text{KE energy density}}{\text{EM energy density}}
\]

Accelerating EM force parallel to velocity: $m_{\text{eff}} = \gamma^3 m$

\[
\rho c \beta \frac{\partial}{\partial r} (\gamma c \beta) = \rho c^2 \left( \beta \frac{\partial \gamma}{\partial r} + \gamma \frac{\partial \beta}{\partial r} \right) = \rho c^2 \gamma^3 \frac{\partial \beta}{\partial r} \sim -\frac{\partial}{\partial r} \frac{B^2}{8\pi} = \frac{B^2}{4\pi r}
\]

Inertial accel $> EM$ accel when

\[
1 < \frac{\rho c^2 \gamma^3 (\partial \beta / \partial r)}{B^2 / 4\pi r} = \gamma^2 \frac{4\pi \rho c^2 \gamma}{B^2} r \frac{\partial \beta}{\partial r} \approx M_F^2 \Rightarrow
\]

Relativistic EM winds accelerate to $M_F = 1$ (not $\sigma = 1$), $\gamma_\infty \approx \sigma_0^{1/3}$ (not $\gamma_\infty \approx \sigma_0$)
Observations require stronger, non-radiative (equatorial) acceleration for \( r >> R_F \sim 10^2 R_L - 10^3 R_L \)

**Ideal MHD solution does allow acceleration when \( M_F > 1 \)?**

- find poloidal magnetic geometry with curved field lines \( (R_F \sim R_L) \)
- (non-zero \( B_\theta \)), \( d|l|/dr < 0 \) (especially in equator)
- But, radius of curvature necessarily large:
  \[
  \mathcal{R} \sim \gamma^2 r \sim (\sigma_L)^{4/3} r
  \]
- Parabolic field lines (Begelman and Li, asymptotic for \( r > R_F \)):

  \[
  \gamma \approx \gamma_F \ln \left( \frac{r}{R_F} \right) \Rightarrow R_{\text{shock}} \approx R_F \exp \left( \frac{\sigma_F}{\sigma_{\text{shock}}} \right) = R_L \sigma_L^{2/3} \exp \left( \frac{\sigma_L^{2/3}}{\sigma_{\text{shock}}} \right) \sim 1000 \ R_L \exp(10^{5.3}) \gg 10^9 R_L \ (\text{Crab})
  \]

**Substantial need for aligned rotator solution starting with dipole at \( r = 0 \) (not monopole) to make sure asymptotic behavior matches correctly to structure inside \( R_F \), since \( R_F/R_L \) probably \( < (\sigma_L)^{2/3} \) (but > 1) - but unlikely to fix \( \sigma \) problem**

**Full solution would specify latitude distribution of electric current**
Equatorial (Electro)Magnetic Dissipation? (Non-ideal flow)

$$\Rightarrow \sigma(R_F) \sim \sigma_L^{2/3} >> 1 \rightarrow \sigma(R_{\text{shock}}) << 1$$

Equatorial outflow

Return current

“sheet” (current closes in Nebula - ion doped shock model has ~ full current to Nebulae in equator)

Oblique rotator’s sheet = frozen in wave: dissipative?

Bogovalov’s (1999) monopole solution

Arons & Spitkovsky PIC simulation (unpublished)

“Reconnection” dissipates sheets and all B field between sheets in equator - “resistivity” = ? observational signature? How complete can the dissipation be? (Kirk & Skaeraasen 2003; next talk)

II. Current Sheet = Force-Free structure (Arons & Spitkovsky, in progress) - radiation losses kill pressure of sheet pinch

sheet unstable to shear generation of strong EM waves, waves damp (surf-riding acceleration), magnetic pressure outside sheet regenerates sheet, kills overall magnetic field
Sheet thickness $> R_L \left(\frac{n_{\text{Goldreich-Julian}}}{n_{\text{pairs}}}\right)$
from $J/(2ne\gamma) < 1$ - could be much thicker, depends on latitude distribution of return current
- observations of torus injection suggest final low sigma outflow has thickness $\sim 0.3 R_{\text{shock}} \gg R_L$
No dissipation model says much (new) about observations
Conclusions

A. Sigma and gamma problems are still with us - dissipation in the wind may be most likely solution - observations of small scale structure close to PSRs might help - global wind theory needs to define the return current structure better (connect the dipole to the wind)

B. Modeling the wind termination and the nebular response has much improved
Ion Doped Equatorial Shock (Kinetic Ions/1DMHD pairs)

X-Ray Variability
Ion doped shock model
(see also Amato’s poster)
Synthetic brightness map

Parameters of ion doped shock model
\[ \sigma_1 \sim 5 \times 10^{-3}, \gamma_1 \sim 3 \times 10^6, m_1 c^2 \gamma_1 = 0.15 e \Phi \text{ magnetosphere} \]

\[ \dot{N}_i \approx \dot{N}_{GJ} = 2 \times 10^{34} \text{ s}^{-1}, \dot{N}_\pm \approx 3 \times 10^{38} \text{ s}^{-1} \sim 10^4 \dot{N}_i \Rightarrow \text{MHD} \]

Equatorial flow = Return Current + pairs

Pair injection rate comparable to polar cap and outer gap model predictions - does not address higher latitudes/jets

UHE ions yield UHE neutrinos, perhaps observable in Crab by Ice Cube (Amato, Guetta & Blasi; Bednarek)
Inject a wind in a broad cone, Poynting flux \( \sim (\lambda \sin \lambda)^2 \), \( \gamma >> 1 \).

1st rational model for torus asymmetry

Model requires \( \sigma > 0.01 \), other parameters =?

Ion doped shock structure model and K-L global model perhaps consistent
Torus Assymmetry explained?

Approaching side brighter than receding side
1D models have $v \sim c/12$, too small
Energy in equatorial outflow, jet is interesting but not of major dynamical significance

$v \sim 0.5c$: good for Doppler boost but
How come no Doppler asymmetry for inner ring?
Where did all the polar Poynting flux in waves go? (should still be in KE flux)
The injection rate problem

Radio observations suggest average pair injection rates >> polar cap, outer gap models (~$10^{40}$/sec for Crab); issue appears to be common to many PWN

Reconnection estimates of current sheet dissipation (Kirk and Skaeraasen) suggest high rates (~$10^{40}$/sec for Crab)

If injection all in equator, shock(s) must convert flow energy to broken power law particle spectrum, $\gamma_{\text{wind}} \sim 10^3$ (Gallant et al) - no VHE neutrinos
Or, higher latitude wind is slower, denser than equatorial wind (Begelman, JA), radio from high(er) latitude injection

Either way, pair creation in the magnetosphere needs some new thinking (and has needed such for a long time)