Variation of the Schumann resonance frequency

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1. Observations

Typical spectrum of Schumann resonances (SR)

Resonance frequency experiences variations [Balser and Wagner, 1962].
1.1. Effect of the solar activity

*Sao et al.* [1973] compared the first mode frequency at Tottory observatory (35º latitude) and solar X-ray intensity during the December 1971 – February 1972 period. A good similarity between day-to-day variations of both parameters was found.

*Schlegel and Füllekrug* [1999] examined changes of the daily averaged first SR frequency at Arrival Heights, Antarctica, for nine strong SPEs and found that the frequency increased by about 0.04-0.14 Hz for the SPE days. The authors related this increase with proton precipitation and not with X-ray bursts.

*Kulak et al.* [2003a,b] examined the changes of the characteristic frequencies of the first resonance during the minimum and maximum of the solar cycle, based of the measurements held in the Carpatian mountains, and found that the frequency grew up clearly during the active phase of solar cycle. It is possible that the increase is caused by the enhanced X-ray activity.


*Roldugin et al.* [2004a] studied variations of the Schumann resonance frequency in the Kola and Kamchatka Peninsulas during seven days of March – April, 2001, when the intensive solar X-ray bursts occurred. All X-ray bursts were accompanied by ~0.2 Hz increase in the first mode frequency, at least in one of the magnetic components. For the second mode the increase (in average by ~0.3 Hz) was registered in most events, when the ELF noise level was not very high.
Solid lines: the 1st Schumann resonance frequency at the Kola peninsula. Dotted lines: the solar X-ray intensity onboard GOES 10 satellite. Dashed lines: the frequency diurnal variations for undisturbed days.
The 1st Schumann resonance frequency in Kamchatka peninsula in $H$, $D$ and $Z$ components.

The vertical lines mark the peaks of the solar X-ray bursts.

November 6, 1997.
5 minute data

Thick solid line: the first-order Schumann resonance frequency in Lovozero.
Thin solid line: solar proton flux with $E > 100$ MeV.
Thin dotted line: 0.5-4 Å X ray intensity on board GOES 8 satellite.
Thick dashed line: the smoothed diurnal variation for adjacent days.

The first mode SR frequency appeared to decrease by about 0.2 – 0.4 Hz during the main phase of the energetic proton precipitation.
Superimposed events

Top: the first order Schumann resonance frequency for each of the four events centered on the SPE onsets. Bottom: the averaged frequency.
Thin line: the 1\textsuperscript{st} Schumann resonance frequency in Lovozero. 
Thick line: energetic proton flux. 
Dotted line: solar X-ray intensity on board of GOES 10 satellite. 
Thick dashed line: the smoothed diurnal variation of SR frequency for adjacent quiet days.

Thus, we think that X-rays increase the resonance frequency, whereas solar protons decrease the resonance frequency.
1.2. Diurnal variation

Very intriguing is the diurnal variation of the frequency [Mitchell, 1976; Sentman, 1987, 1989; Füllekrug, 1995; Schlegel and Füllekrug, 2000]. They are not equal in different magnetic components.

Roldugin et al. [2004b] studied diurnal variations of the Schumann resonance frequency at three points:
Spitsbergen (obs. Barentsburg, $\lambda = 78.9^\circ$ N, $\varphi = 11.9^\circ$ E),
the Kola Peninsula (obs. Lovozero, $\lambda = 68.0^\circ$ N, $\varphi = 35.1^\circ$ E),
the Kamchatka Peninsula (obs. Karimshino, $\lambda = 52.9^\circ$ N, $\varphi = 158.3^\circ$ E).
The diurnal variation of the first SR mode frequency in the NS component in Barentsburg

Top: the variation for each of five days near the autumnal equinox. The frequency is indicated correctly for the first day only, the second and the following curves are shifted downward by 0.5 Hz relative to the preceding one.

Bottom: the variation averaged over the five days. The thin line is observation, the thick line is approximation.
The diurnal variations of the first SR mode frequency in the NS and EW components in Lovozero (thin lines) and Barentsburg (thick lines) near the spring equinox.
The diurnal variations of the first SR mode frequency in the NS and EW components in (top) Lovozero and (bottom) Karimshino outlined in the (left) universal and (right) local time.
The frequency variation is 0.2-0.3 Hz at all the observatories. The semi-diurnal harmonics dominates in the frequency variation. The variation seems to be controlled by LT rather than by UT. The frequency of the NS component has maxima approximately at 07 and 19 LT. The diurnal variation of the frequency of the EW component reveals the anti-phase behavior, so that the maximum in the EW component frequency occurs at about 01 and 13 LT.
2. Theory of frequency variation

2.1. The simplest model

The cavity between the Earth and ionosphere is a resonator for electromagnetic waves. The simplest vacuum model confined with two concentric perfectly conducting spheres yields the frequency [Schumann, 1952]

\[ \omega_n = \frac{c}{R_E} \sqrt{n(n+1)}, \]

where \( n = 1, 2, 3, \ldots \) is the mode number, \( c \) is the light velocity, \( R_E \) is the Earth radius.

Three independent polarization modes corresponding to the first Schumann resonance mode \((n = 1)\). The axes \( x \), \( y \), and \( z \) are orthogonal. The Earth is shown by thin circle. The thick lines are the oscillating magnetic fields. If the medium is horizontally homogeneous, the fields of all three modes oscillate with equal frequencies.
The simplest model predicts the first order frequency $f_1 = 10.6$ Hz that is invariable.

Observations yield in average $f_1 = 7.8$ Hz [Balser and Wagner, 1962], which is variable. The frequency became smaller because the ionosphere is not a perfect conductor. The ionosphere has the dielectric permeability

\[
\varepsilon = 1 - \frac{\omega_o^2}{\omega(\omega - i\nu)}
\]

where $\omega_o = (4\pi e^2 N/m)^{1/2}$ is the plasma frequency, $\nu$ is the electron collision frequency. Variations in the electron number density $N$ can cause variations in the Schumann resonance frequency.

### 2.2. Models with height-dependent atmospheric conductivity

Bliokh et al. [1980], using a model with

- $\varepsilon = 1$ for $0 < z < a$,
- $\varepsilon = \varepsilon_i = \text{const}$ for $z > a$,

obtained the following formula

\[
\omega_n = \frac{c}{R_E} \left( \sqrt{n(n+1) - \frac{Z^2 R_E^2}{4 a^2}} + iZ \frac{R_E}{2 a} \right)
\]

where

\[
Z = \frac{1}{\sqrt{\varepsilon_i}}
\]

is the impedance of the upper edge ($r = R_E + a$).

This model can roughly describe the frequency variations. However it predicts rather unrealistic undisturbed frequencies.
*Greifinger and Greifinger* [1978] as well as *Sentman* [1983, 1990] considered models with $\varepsilon$ exponentially growing with altitude. The models yield a good agreement with the observed undisturbed frequencies.

*Roldugin et al.* [2003] considered a slightly simpler model:

$$
\varepsilon = 1 \quad \text{for } 0 < z < a,
$$

$$
\varepsilon = \varepsilon_2 e^{\frac{z-a}{h}} \quad \text{for } z > a,
$$

The following formula was obtained:

$$
\omega = \frac{c \sqrt{n(n+1)}}{R_E} \left[ \frac{1}{1 + \frac{2h}{a} \gamma - \ln \left( \frac{\omega_{o2}}{\omega} \right) \left( \frac{h}{c} \right)^{i\nu_{o2}}} \right], \quad (6)
$$

For $n = 1$:

Re $f_1 = 7.8 + 0.32 \log_{10} \left( \frac{N_2}{100} \right) - 0.38 (h - 4) + 0.049 (a - 61)$,

Im $f_1 = -0.22 - 0.028 \log_{10} \left( \frac{N_2}{100} \right) - 0.024 (h - 4) - 0.00055 (a - 61)$.

For $n = 2$:

Re $f_2 = 13.7 + 0.56 \log_{10} \left( \frac{N_2}{100} \right) - 0.65 (h - 4) + 0.086 (a - 61)$,

Im $f_2 = -0.40 - 0.050 \log_{10} \left( \frac{N_2}{100} \right) - 0.045 (h - 4) - 0.00094 (a - 61)$.

**Since the model contains many parameters, it can explain both increase and decrease of the SR frequency.**
2.3. Model of the diurnal variation

2.3.1. Physics of the model

Maltsev and Roldugin [2004] take into account that there are three slightly differing frequencies (triplet) in the first Schumann resonance. Each frequency is related to a certain wave polarization fixed in the GSM coordinates. The effective frequency of the summary field is a combination of the triplet frequencies and depends on the latitude and local time. When an observatory rotates together with the Earth, it registers the effective frequency varying with the local time, the variation being different in the $H$ (north-south) and $D$ (east-west) components. The choice of the modes is related to pronounced ionospheric inhomogeneities. In particular, the $x$ mode corresponds to the day-night asymmetry, the $z$ mode is related to the auroral precipitation.

\begin{itemize}
  \item \textit{x}-mode, $x$ axes is sunward
  \item \textit{y}-mode, $y$ axes is duskward
  \item \textit{z}-mode, $z$ axes is northward
\end{itemize}
2.3.2. Basic equations

Effective frequencies for the $H$ and $D$ magnetic components:

$$\omega_H = \frac{H_x \omega_x + H_y \omega_y + H_z \omega_z}{H_x + H_y + H_z}$$  \hspace{1cm} (7) \\
$$\omega_D = \frac{D_x \omega_x + D_y \omega_y + D_z \omega_z}{D_x + D_y + D_z}$$  \hspace{1cm} (8)  \\

Fields of the first mode ($n = 1$) harmonics:

$$H_x = A_x \sin^2 \varphi$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} D_x = A_x \cos^2 \varphi \sin \Lambda$$  \\
$$H_y = A_y \cos^2 \varphi$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} D_y = A_y \sin^2 \varphi \sin \Lambda$$  \\
$$H_z = 0$$  \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} D_z = A_z \cos \Lambda$$  \\

where $A_x$, $A_y$, and $A_z$ are the amplitudes of the $x$, $y$, and $z$ modes (they do not experience diurnal variation);  \\
$\omega_x$, $\omega_y$, and $\omega_z$ are the frequencies of the $x$, $y$, and $z$ modes (they do not experience diurnal variation);  \\
$\Lambda$ is latitude,  \\
$\varphi = 2\pi \text{ LT}/24$ is longitude,  \\
LT is local time.

2.3.3. Calculation results

We fitted $A_x, A_y, A_z, f_x, f_y, f_z$.  \\
\[ f = \frac{\omega}{2\pi} \]

Fitting yielded:

$f_x = 8.10$ Hz, $f_y = 7.84$ Hz, $f_z = 7.75$ Hz.
Observed and modeled diurnal variation of the effective frequency of the Schumann resonance in Lovozero (the Kola peninsula).
Observed and modeled diurnal variation of the effective frequency of the Schumann resonance in Karimshino (the Kamchatka peninsula).
2.3.4. Prediction

At the equator ($\Lambda = 0$) we have $D_x = 0$, $D_y = 0$. We observe there $D_z = A_z$. Since $A_z$ has no diurnal variation, we will not observe the diurnal variation in the $D$ component at the equator.

3. Conclusions

Variations of the SR frequency are caused by variations of the ionospheric conductivity.
- X-rays cause the increase of the frequency.
- SPEs cause the decrease of the frequency.
- Diurnal variation of the SR frequency can be explained by the splitting of the SR mode.

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