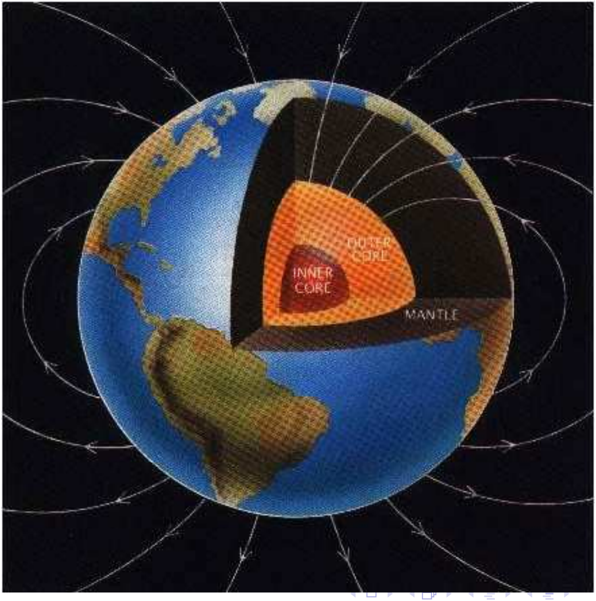
Compressibility and helicity in geodynamo

M.Yu.Reshetnyak

Institute of the Physics of the Earth RAS
Moscow, Russia

Structure of the Earth and geodynamo



What we think we know about geomagnetic field

- it should exist at least $3 \cdot 10^9$y (age of the Earth is $4.5 \cdot 10^9$y)
- it is non-stationary
- dipole strcuture
- reversals, excursons
- MAC waves

Boussinesq geodynamo model . . .

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \mathrm{q}^{-1}\,\Delta \mathbf{B}$$

$$\mathrm{E}\,\mathrm{Pr}^{-1}\left[\frac{\partial \mathbf{V}}{\partial t} - \mathbf{V} \times (\nabla \times \mathbf{V})\right] = -\nabla P - \mathbf{1_z} \times \mathbf{V} + \mathrm{Ra}\,T\,\mathbf{1_r} + \mathrm{E}\,\Delta \mathbf{V}$$

$$\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla)\,(T + T_0) = \Delta T \qquad (1)$$

$\mathrm{Pr} = \dfrac{\nu}{\kappa} \sim 10^{-1} \div 10$ – Prandtl number, $\mathrm{E} = \dfrac{\nu}{2\Omega \mathrm{L}^2} \sim 10^{-15}$ – Ekman number

$\mathrm{Ra} = \dfrac{\alpha g_0 \delta T \mathrm{L}}{2\Omega\kappa} \sim 10^9$ – modified Rayleigh number, $q = \dfrac{\kappa}{\eta} \sim 10^{-5}$ – Roberts number

Some results on Boussinesq-like geodynamo models

- self-consistent thermal and compositional dynamo
- Earth-like spectrum
- reversals and excursions
- inner core rotation
- scaling laws
- inverse cascades

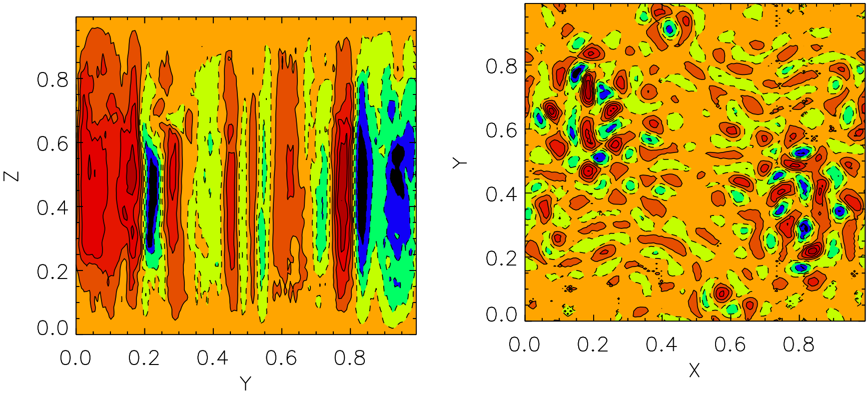


Figure: **Distribution of the V_z-component of the velocity field with ranges** $(-675, 701)$, $(-153, 157)$

Reshetnyak, Hejda, Nonlin. Proc. Geophys. 2008
Hejda, Reshetnyak, Phys. Earth Planet. Int. 2009.

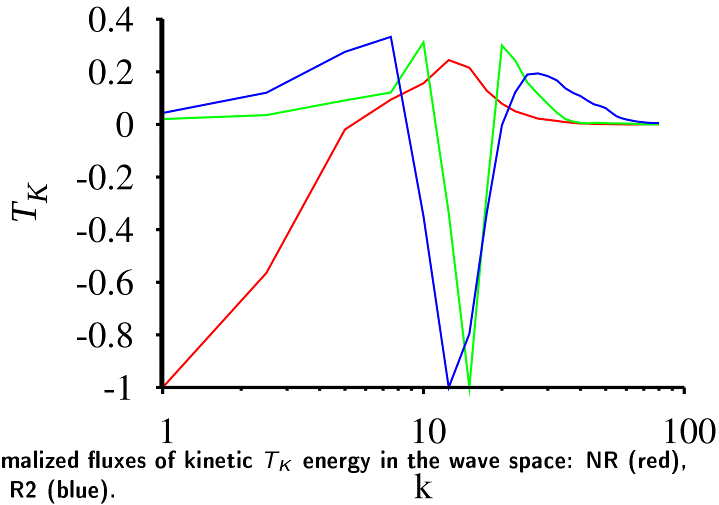
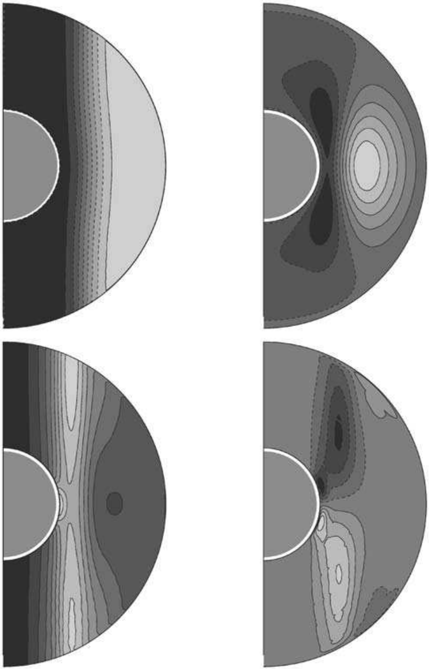
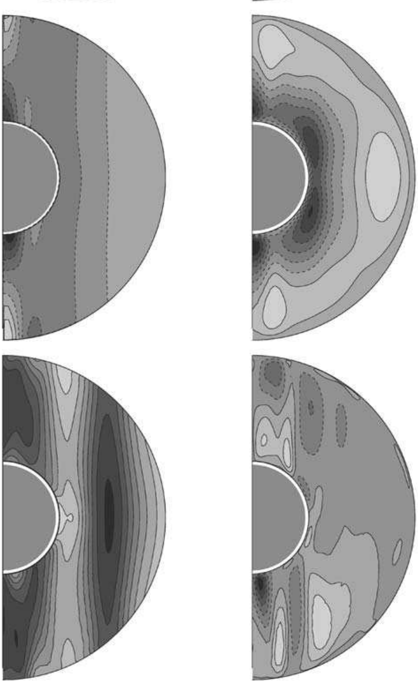


Figure: **Normalized fluxes of kinetic T_K energy in the wave space: NR (red), R1 (green), R2 (blue).**

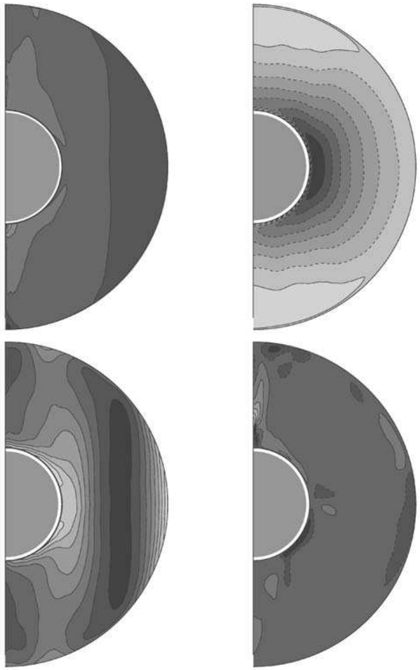
ω, T, E_K, χ for R1



ω, T, E_K, χ for R2



ω, T, E_K, χ for R3



$\omega(r)$, $Nu(r)$ for R1–R3

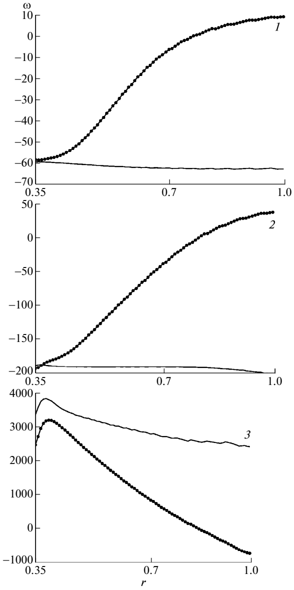


Fig. 2. The profiles of the rotation angular velocity (ω) along the radius for the *R*1 (*1*), *R*2 (*2*), and *R*3 (*3*) regimes within (a solid line) and outside (circles) TC.

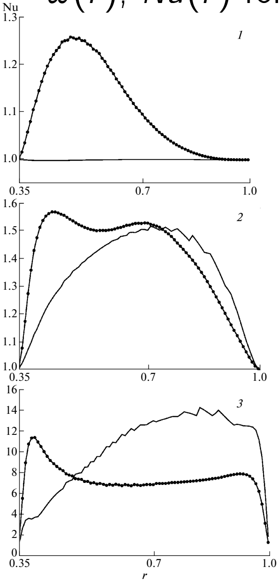


Fig. 3. The profiles of the Nusselt number (Nu) along the radius for the *R*1 (*1*), *R*2 (*2*), and *R*3 (*3*) regimes within (a solid line) and outside (circles) TC.

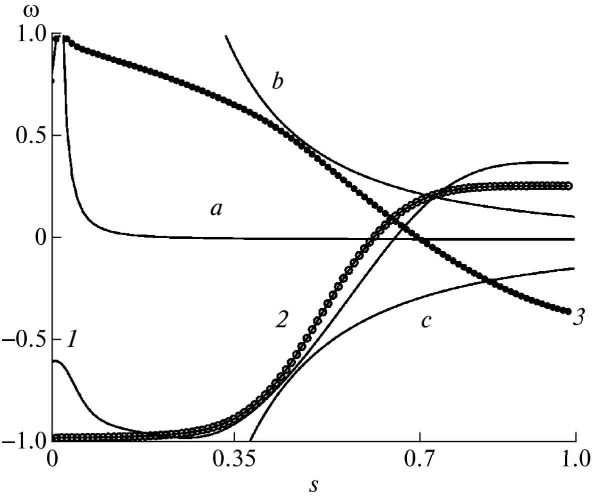


Fig. 7. The profiles of the rotation angular velocity $\omega(s)$ for the $R1$ (*1*), $R2$ (*2*), and $R3$ (*3*) regimes. Parabolas (*a*) and (*b*) are specified by the $f = A_i/s^2$ function, where $A_a = \omega_{R1}(0.01)$, $A_b = \omega_{R3}(0.46)$, and $A_c = -\omega_{R1}(0.46) \approx \omega_{R2}(0.46)$.

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$\frac{\delta\rho}{\rho} \sim 20\%$ – we need anelastic model!
Boussinesq or anelastic, $\nabla \cdot \mathbf{V} \neq 0$, $\left(\frac{\partial \rho}{\partial t} = 0\right)$?

- 15 years ago: "Can 3D thermal convection generate magnetic field at all?"
- Even for Boussinesq we have quite enough parameters: kinematic viscoity, thermodiffusion, magnetic diffusion, intensity of thermal sources (including various b.c.), daily angular rotaion (which is too rapid for simulations)
- $\mathrm{Re} \sim 10^8 - 10^9$, $q = \kappa/\eta = 10^{-5}$, $\mathrm{R_m} \sim 10^3$.
- Anisotropy: $l_{||}/l_{\perp} \sim \mathrm{E}^{-1/3} \sim 10^5$ (at least at the onset of convection)

It is only some of the reasons why Boussinesq approximation lived so long in geodynamo!

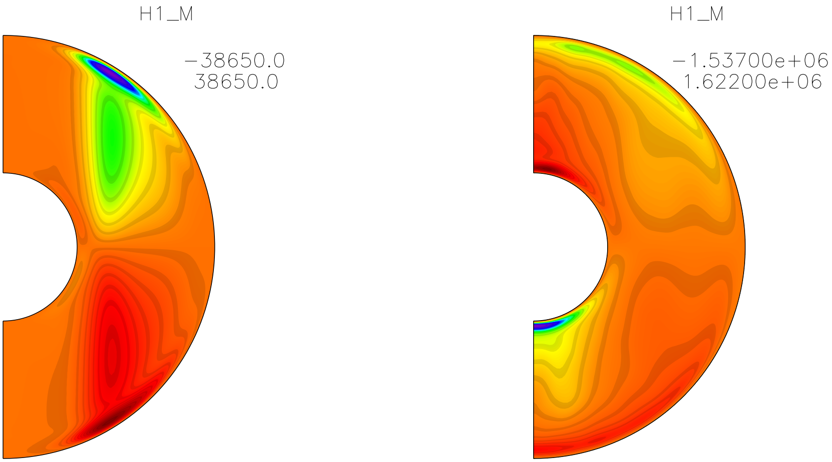
Kinetic helicity generation

Kinetic helicity $\chi =< \mathbf{v} \cdot \mathrm{rot}\, \mathbf{v} >$, closely related to the α-effect – the reason why we have a large-scale magnetic field in the body for $\mathrm{R_m} \gg 1$.

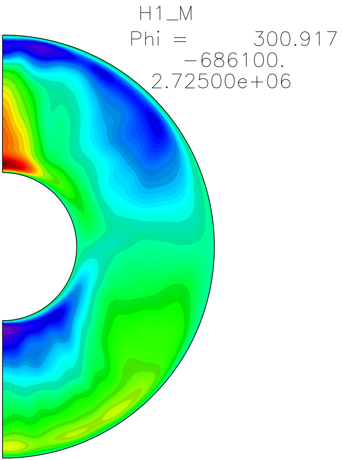
Sources:

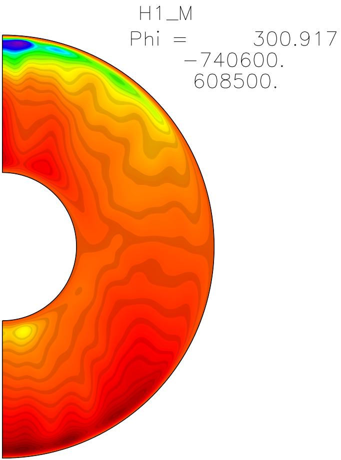
- viscous stresses, e.g., no-slip b.c, generation in the Ekman layer of thikness $\delta_E \sim E^{1/2}$?
- rotation+boundaries: $\dfrac{dE_K}{dz} \neq 0$ (violation of geostrophic balance)
- rotation+density gradient:
$$\chi \sim \frac{(\mathbf{\Omega} \cdot \nabla \rho)}{\rho}\, l\, v,\ \tau \sim l/v,\ \alpha = -\frac{\tau}{3}\chi \sim -\frac{(\mathbf{\Omega} \cdot \nabla \rho)}{\rho}\, l^2$$

Meridional section of kinetic helicity χ for $\mathrm{E} = 2 \cdot 10^{-4}$, $\mathrm{Pr} = 1$ for $\mathrm{Ra} = 1.5 \cdot 10^2$ and $\mathrm{Ra} = 8 \cdot 10^2$, $\nabla \cdot \mathbf{V} = 0$.



Meridional section of kinetic helicity χ, $\mathrm{E} = 2\,10^{-4}$, $\mathrm{Pr} = 1$, $\mathrm{Ra} = 8 \cdot 10^2$, $\dfrac{\delta\rho}{\rho} = 0.2$ and $\dfrac{\delta\rho}{\rho} = 1$.





Estimation of vorticity. Observations. $\nabla \cdot \mathbf{V} = 0$ limit.

Let $l_\perp = \mathcal{C}_l\, L$ and $v_\omega^{observ} = \mathcal{C}_v\, V_{wd}$
For $l_\perp \sim \mathrm{E}^{1/3}\, L \sim 10^{-5}\, L = 10\, m$ $(\mathcal{C}_l = 10^{-5})$ and $\mathcal{C}_v = 1$ one has exactly

$$\mathrm{rot}\, \mathbf{v}_\omega^{observ} = \frac{\mathcal{C}_v}{\mathcal{C}_l} \frac{V_{wd}}{L} \sim 3 \cdot 10^{-5}\, \mathcal{C}_v\, s^{-1}$$

This scale is too small for geodynamo: $\mathrm{R_m} \sim 10^{-2}$.

Effect of $\nabla\rho$

$\Omega = 7.3\cdot 10^{-5}\,s^{-1}$, $\nu = 10^{-6} m^2/s$,
$V_{wd} = 3\cdot 10^{-4}\,m/s$, $L = 10^6\,m$, $\mathcal{C} = \delta\rho/\rho = 0.2$.

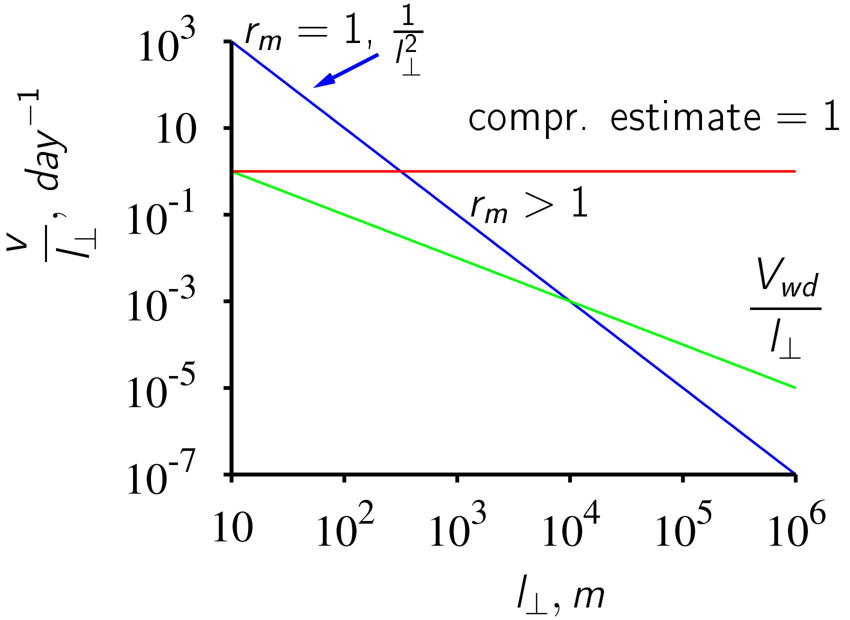
$v_\perp \sim \mathcal{C}\,\frac{l_\perp}{L}\,v_z$ — horizontal velocity of cell with v_z

$F_c \sim 2\mathcal{C}\Omega v_\perp \sim 2\mathcal{C}\Omega\,\frac{l_\perp}{L}\,v_z$ — Coriolise force

$v_\omega = \tau\,F_c \sim \frac{l_\perp}{v_\perp}\,2\mathcal{C}\Omega\,\frac{l_\perp}{L}\,v_z \sim 2\mathcal{C}\Omega l_\perp$

$$\boldsymbol{\omega} \sim \mathrm{rot}\,\mathbf{v}_\omega \sim 2\mathcal{C}\Omega \sim \overbrace{3\cdot 10^{-5}\,s^{-1}}^{\sim day^{-1}} - \text{ for } \nabla\cdot\mathbf{v} \neq 0 \tag{2}$$

Here we do not know $l_\perp$ because diffusion does not introduced!


$r_m = 1, \frac{1}{l_\perp^2}$
compr. estimate = 1
$r_m > 1$
$\frac{V_{wd}}{l_\perp}$
$\frac{v}{l_\perp}$, day^{-1}
10^3
10
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10
10^2
10^3
10^4
10^5
10^6
$l_\perp, m$