# Magnetic Turbulence from Synchrotron Intensity Fluctuations

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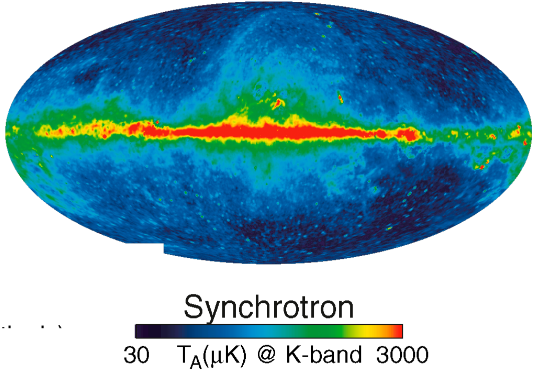
# Synchrotron on the sky

## In cosmology

- synchrotron is a nuisance foreground to CMB
- It is removed based on frequency information
- It is highly polarized, so ever more important for CMB polarization studies.

WMAP7 synchrotron map

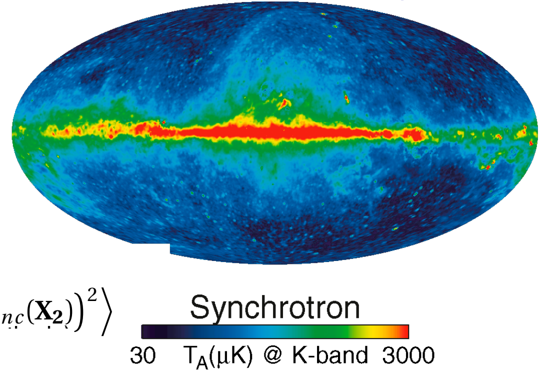


# Synchrotron on the sky

## But to study turbulence

- Synchrotron reflects the magnetic field $H$
- E.q. spatial structure reflects turbulent distribution of $H$
- We aim to learn about H from studying **correlation statistics** of $I_{sync}$

WMAP7 synchrotron map

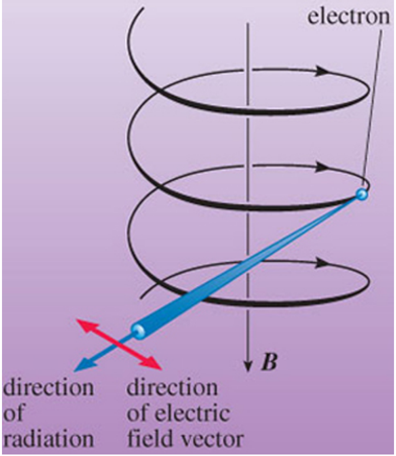



$$D_{sync}(\mathbf{R}) \equiv \left\langle \left( I_{sync}(\mathbf{X_1}) - I_{sync}(\mathbf{X_2}) \right)^2 \right\rangle$$

- Scaling $D_{sync} \propto R^m$
- Anisotropy $D_{sync}(R, \phi)$.

# Synchrotron is a complex signal

- Relativistic beaming of radiation
- Dependence of energy distribution of electrons
- $I_{sync}(\mathbf{X}) \propto \int dz H_{\perp}^{\gamma}(\mathbf{x})$
- $\gamma$ is a fractional power, $1.5 - 4$.
- only $H_{\perp}$ to line of sight contributes
- Accumulates along the line-of-sight



# Our goals

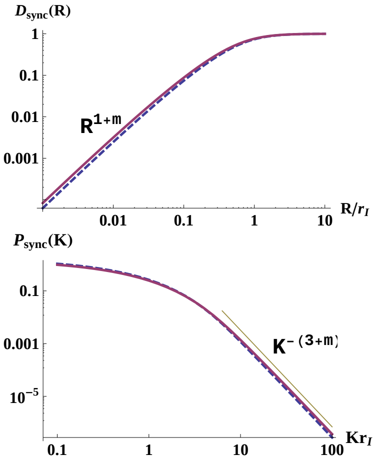
- Design $\gamma$ insensitive statistics
- Rigorously link observable on-sky properties of synchrotron intensity correlations to the properties of the statistically anisotropic magnetic field
- Consider correlation tensors induced by basic MHD modes
- Discuss how MHD modes are mapped into synchrotron properties and their distinctive signatures

# Designing $\gamma$ insensitive statistics

## Normalized statistics

$$\tilde{D}_{H_\perp^\gamma} = \frac{\left\langle \left( H_\perp^\gamma(\mathbf{x}_1) - H_\perp^\gamma(\mathbf{x}_2) \right)^2 \right\rangle}{\left\langle H_\perp^\gamma(\mathbf{x})^2 \right\rangle - \left\langle H_\perp^\gamma(\mathbf{x}) \right\rangle^2}$$

depends on $\gamma$ very weakly !




## Very good approximation

that allows to advance theory is

$$D_{H_\perp^\gamma}(\mathbf{r}) \approx \mathscr{A}(\gamma)\mathscr{P}(\gamma) D_{H_\perp^2}(\mathbf{r})$$

at scales smaller than energy injection scale.

# Anisotropic properties of the magnetic field

- Turbulent cascade proceeds anisotropically in the presence of the magnetic field $\mathbf{H}$ (e.g. Goldreich & Shridhar, 1995)
- Directions parallel to and perpendicular to the local $\mathbf{H}$, but we can think as having axial symmetry (statistically)
- Statistics of turbulent magnetic field is not isotropic. At best it is axisymmetric
- Symmetry axis is associated with the (local) direction of the mean field $\mathbf{H}$
- Axial symmetry is not exact over global scales due to “wandering” of the mean field

# H as axisymmetric random vector field

- Theory of axisymmetric statistics for Gaussian vector field has be developed by Batchelor, 1946, Chandrasekhar, 1950, and later by Mattheus and Smith, 1981 and Oughton, Radler, Mattheus, 1997.
- Symmetric part of the correlation tensor of solinoidal field is described in sufficient generality by two spectral functions

$$\left\langle H_i H_j \right\rangle = \frac{1}{(2\pi)^3} \int d^3\mathbf{k}\, e^{i\mathbf{k}\cdot\mathbf{r}} \Big[ E(\mathbf{k}) \left(\delta_{ij} - \hat{k}_i \hat{k}_j\right) + \\ + F(\mathbf{k}) \frac{(\hat{\mathbf{k}}\cdot\hat{\lambda})^2 \hat{k}_i \hat{k}_j + \hat{\lambda}_i \hat{\lambda}_j - (\hat{\mathbf{k}}\cdot\hat{\lambda})(\hat{k}_i \hat{\lambda}_j + \hat{k}_j \hat{\lambda}_i)}{1 - (\hat{\mathbf{k}}\cdot\hat{\lambda})^2} \Big]$$

- spectral functions $E(\mathbf{k})$ and $F(\mathbf{k})$ depend not only on the magnitude $k$, but on the angle of wave vector and the symmetry axis as well. In scaling regime

$$E(k,\mu) = A_E k^{-3-m_E} \widehat{E}(\mu) \ , \quad F(k,\mu) = A_F k^{-3-m_F} \widehat{F}(\mu)$$

# From magnetic field to synchrotron

- Anisotropy of the magnetic fluctuations lead to angular dependence of the synchrotron spectral measures. After some algebra :) one finds multipole expansion for small-scale $\tilde{D}_{sync}(R,\phi)$
- E tensor gives rise to multipole expansion of the synchrotron structure function

$$\tilde{D}_n(R) \approx A_E C_n(m) \left( \widehat{E}_n - \frac{1}{2}\epsilon \left( \widehat{E}_{n+2} + \widehat{E}_{n-2} \right) \right) R^{1+m}$$

$\epsilon(\theta) = \left( \langle H_x^2 \rangle - \langle H_y^2 \rangle \right) / \left( \langle H_x^2 \rangle + \langle H_y^2 \rangle \right)$

- and F-tensor ($G_p^F(\theta)$ are known coefficients)

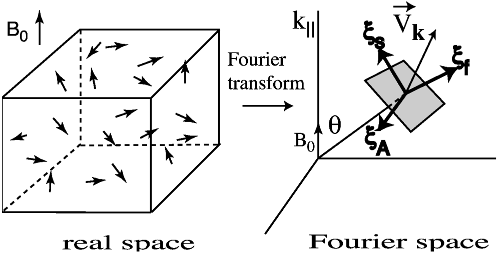
$$\tilde{D}_n(R) \sim A_F C_n(m) \sin^2\theta \sum_{p=-\infty}^{\infty} \left[ \widehat{F}_p - \frac{1}{2}\epsilon \left( \widehat{F}_{p-2} + \widehat{F}_{p+2} \right) \right] G_{n-p}^F(\theta) R^{1+m}$$

- Synchrotron signal depends on the angle $\theta$ between the mean magnetic field and the line-of-sight

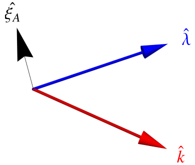
# Anisotropic statistics of MHD modes

## Alfven, Fast and Slow modes (Cho & Lazarian, 2002)

- Are defined relative to the local $\mathbf{H}$ ($\hat{\lambda}$)
- Modes are shown to be statistically independent



# Basic mode structure

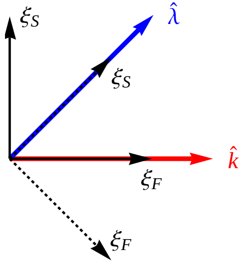



## Alfven waves

- Displacement $\xi_A$ is perpendicular to the plane spanned by $\hat{\lambda}$ and $\hat{\mathbf{k}}$

## Fast modes

- are compressible modes
- Displacements $\xi_F$ lie in $\hat{\lambda} - \hat{\mathbf{k}}$ plane
- In high $\beta \gg 1$ plasma $\xi_F$ are potential, i.e $\xi_F \,||\, \mathbf{k}$.
- In low $\beta \ll 1$ plasma $\xi_F \perp \hat{\lambda}$.




## Slow modes

- are orthogonal to both Alfven and Fast ones
- In high $\beta \gg 1$ plasma $\xi_S$ are transverse, $\xi_S \perp \mathbf{k}$. Second to Alfven “polarization” of transverse waves
- In low $\beta \ll 1$ plasma $\xi_S \,||\, \hat{\lambda}$ and subdominant

# Anisotropic statistics of MHD modes

Anisotropic correlation tensor for magnetic field can be readily obtained from the frozen-in condition (Yan & Lazarian, 2002,2004 ...)

$$\delta\mathbf{H} \propto \mathbf{k}\times(\mathbf{v}\times\hat{\lambda})/\omega(\mathbf{k})$$

Alfven modes are the mix $F(\mathbf{k}) = -E(\mathbf{k})$ with $m \approx 2/3$ Kolmogorov-like scaling

$$\left\langle H_i(\mathbf{k})H_j^*(\mathbf{k})\right\rangle_A \propto E(\mathbf{k})\left[\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)-\frac{(\hat{\mathbf{k}}\cdot\hat{\lambda})^2\hat{k}_i\hat{k}_j+\hat{\lambda}_i\hat{\lambda}_j-(\hat{\mathbf{k}}\cdot\hat{\lambda})(\hat{k}_i\hat{\lambda}_j+\hat{k}_j\hat{\lambda}_i)}{1-(\hat{\mathbf{k}}\cdot\hat{\lambda})^2}\right]$$

Fast modes are of $F$-type with isotropic $F(k)$ and $m \approx 1/2$

$$\left\langle H_i(\mathbf{k})H_j^*(\mathbf{k})\right\rangle_F \propto F(k)\,\frac{(\hat{\mathbf{k}}\cdot\hat{\lambda})^2\hat{k}_i\hat{k}_j+\hat{\lambda}_i\hat{\lambda}_j-(\hat{\mathbf{k}}\cdot\hat{\lambda})(\hat{k}_i\hat{\lambda}_j+\hat{k}_j\hat{\lambda}_i)}{1-(\hat{\mathbf{k}}\cdot\hat{\lambda})^2}\times\begin{cases}1 & \beta\ll 1\\ \frac{2}{\beta}\left(1-(\hat{\mathbf{k}}\cdot\hat{\lambda})^2\right) & \beta\gg 1\end{cases}$$

Strong turbulence: “unpolarized” mix of Alfven and Slow waves in high $\beta$ plasma

$$\left\langle H_i(\mathbf{k})H_j^*(\mathbf{k})\right\rangle_{A+S} \propto E(\mathbf{k})\left(\delta_{ij}-\hat{k}_i\hat{k}_j\right)$$

# Distinctive angular structure of synchrotron correlations

Alfven modes with $m \approx 2/3$ Kolmogorov-like scaling

$$\tilde{D}_n(R) \approx A_E C_n(2/3) \sum_{p=-\infty}^{\infty} \left[ \widehat{E}_p - \frac{1}{2}\epsilon \left( \widehat{E}_{p-2} + \widehat{E}_{p+2} \right) \right] G^A_{n-p} R^{5/3}$$

Fast modes isotropic $F(k)$ and $m \approx 1/2$

$$\tilde{D}_n(R) \sim A_F C_n(1/2) \sin^2\theta \left( G^F_n(0) - \frac{1}{2}\epsilon \left[ G^F_{n-2}(0) + G^F_{n+2}(0) \right] \right) \widehat{F}_0 \, R^{3/2} \qquad (high\ \beta)$$

$$\tilde{D}_n(R) \sim A_F C_n(1/2) \sin^2\theta \left( G^F_n(\theta) - \frac{1}{2}\epsilon \left[ G^F_{n-2}(\theta) + G^F_{n+2}(\theta) \right] \right) \widehat{F}_0 \, R^{3/2} \qquad (low\ \beta)$$

Strong turbulence: “unpolarized” mix of Alfven and Slow waves in high $\beta$ plasma

$$\tilde{D}_n(R) \approx A_{AS} C_n(2/3) \left( \widehat{E}_n - \frac{1}{2}\epsilon \left( \widehat{E}_{n-2} + \widehat{E}_{n+2} \right) \right) R^{5/3}$$

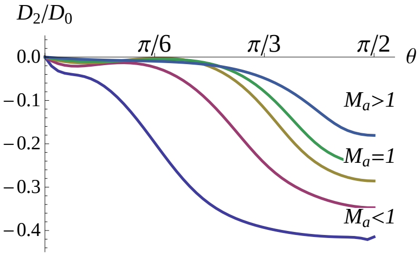
provides new discriminative info about different MHD models

# Axisymmetry and wandering of the magnetic field

- How to take into account that the direction of the magnetic field is not uniform ? How useful is the extension of our formalism to global scales ?
- Actually, if $H$ field wanders around some globally defined direction $\hat{\lambda}_0$, the axisymmetric formalism still holds ! Global direction plays the symmetry axis role, but local correlation tensors need to be smoothed, considering distribution of $\hat{\lambda}$. The outcome is tractable isotropization of the statistics

## Quadrupole to monopole ratio of the synchrotron structure function in the model of Alfvenic turbulence with different Alfvenic Mach numbers, $M_a$

Expected wandering of the local magnetic field, larger for larger $M_a$ is accounted for, which leads to isotropization especially when $H$ has small angle with the line of sight.



# Summary

- We developed a framework to related the anisotropic properties of observable synchrotron correlations to the properties of the underlying magnetic turbulence.
- Useful rule of thumb is that the scaling and angular dependence of synchrotron statistics can be bootstrapped from the tractable $\gamma = 2$ case.
- Anisotropy of the turbulence lead to largely quadrupole anisotropy of the synchrotron, with extended correlations along the projection of the magnetic field.
- Details of synchrotron anisotropy contain infromation about mode content of MHD turbulence and, in particular, presence of compressible modes.
- Potentially spatial properties of synchrotron give additional input to be used in CMB cleaning mechanisms.
- Polarization studies are expected to be even more informative.