Basic MHD Turbulence

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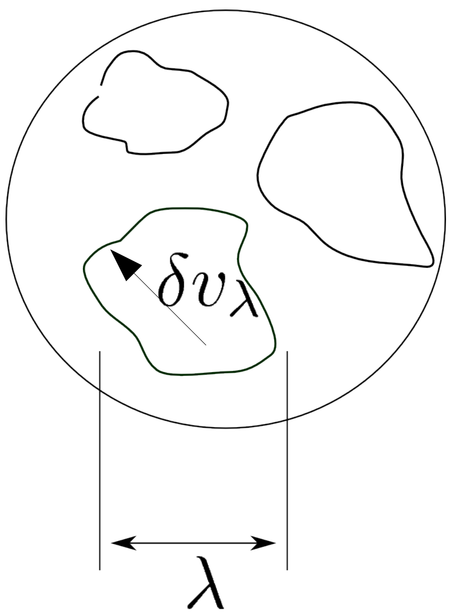
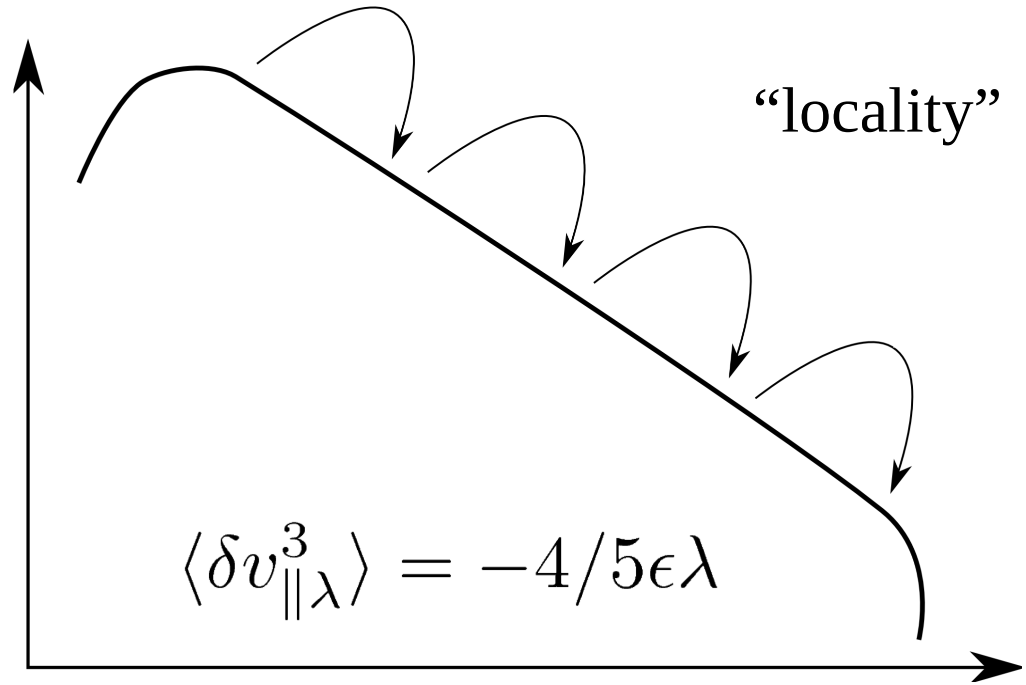
MFU III, 2011

The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers†

By A. N. Kolmogorov

$$\partial_t \mathbf{v} + \hat{S}(\mathbf{v} \cdot \nabla)\mathbf{v} = 0$$





“universality”

$$v \to vA, \quad \lambda \to \lambda B, \quad t \to tB/A$$

$$\tau_\lambda \sim \lambda / v$$

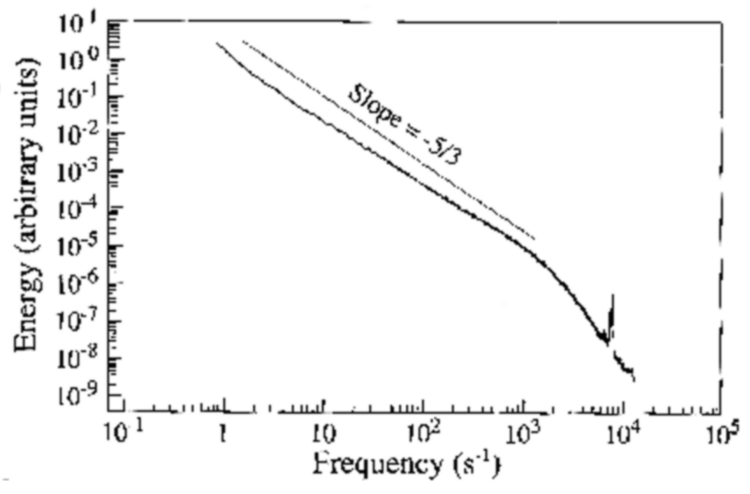
Universal hydrodynamic Turbulence

$$\epsilon = \frac{\rho v_\lambda^2}{2\tau_\lambda} \sim v_\lambda^3/\lambda \sim \text{const}$$

$$v_\lambda \sim \lambda^{1/3}$$

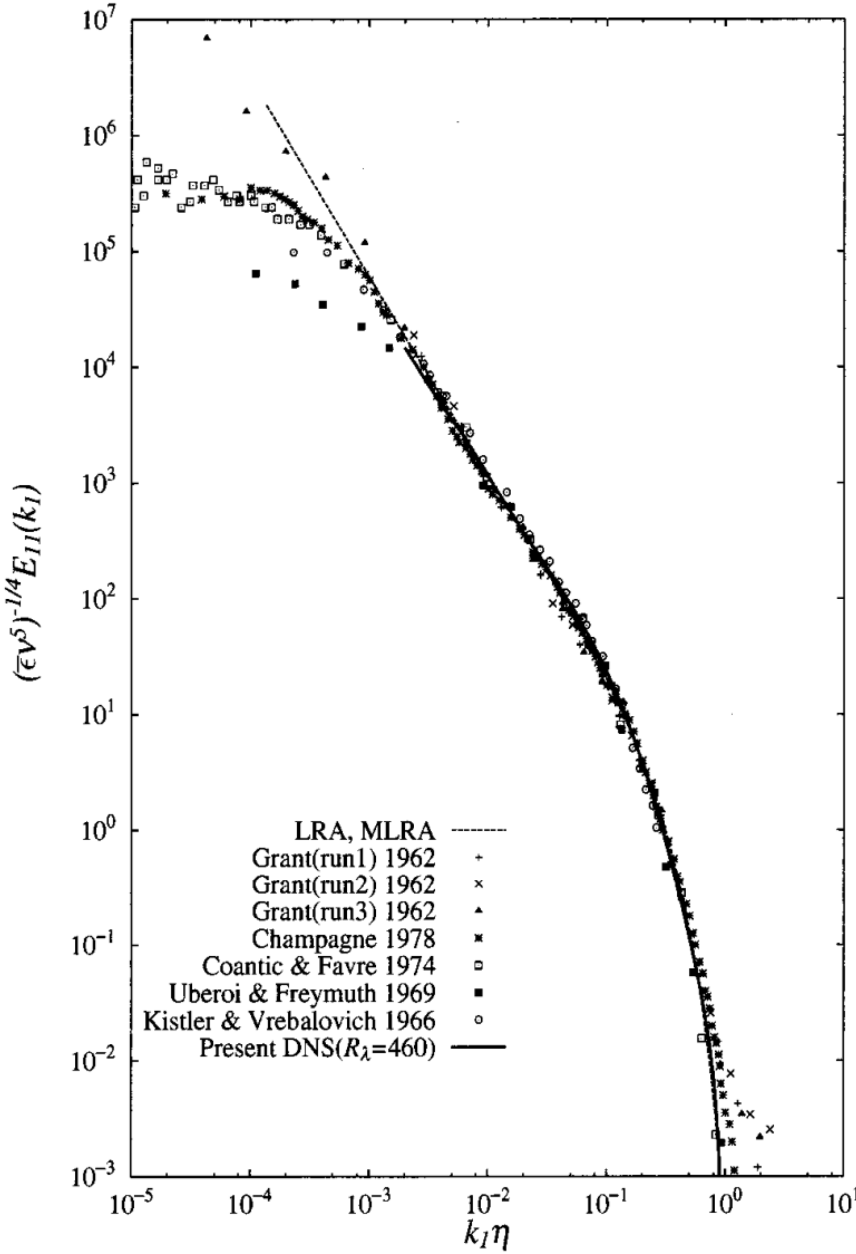
$$E(k) = C_K k^{-5/3} \epsilon^{2/3} (kL)^\alpha$$

$$\alpha \approx -0.035$$



$$C_K \approx 1.64$$

ONERA wind tunnel turbulence



Gotoh et al 2002, see also Kaneda et al 2003, Sreenivasan, 1995

MHD Turbulence

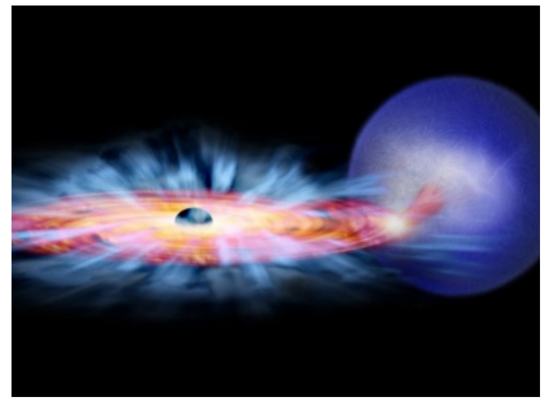
Plasma, cosmic rays and magnetic fields

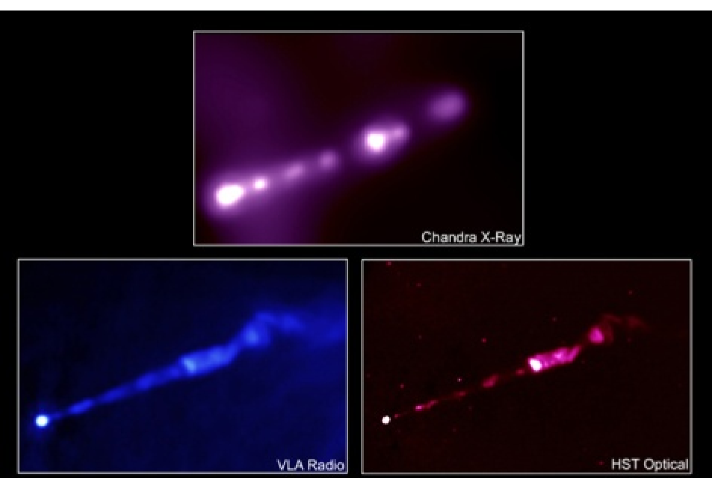
Energy density in our Galaxy

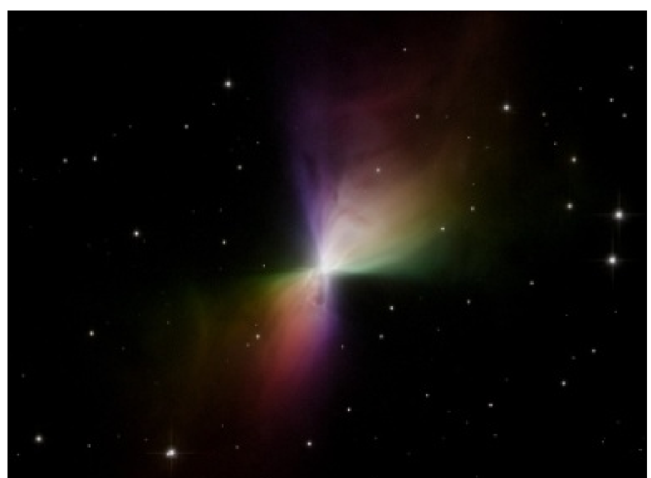
$$w_B \sim w_{CR} \sim w_{\text{kin.}} \sim 1\text{eV}/\text{cm}^3$$

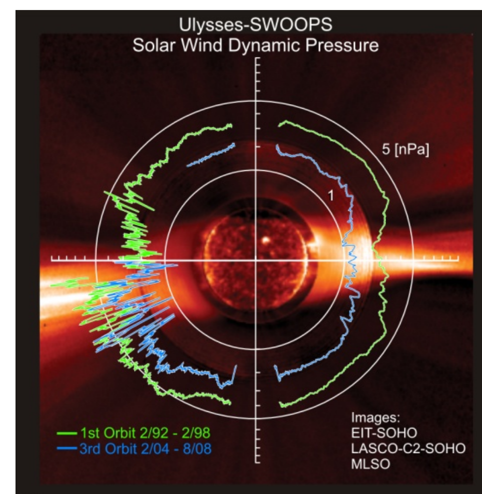
MHD turbulence is important for:

1. making accretion possible,
2. making fast reconnection possible,
3. heating in the solar wind, MC, ISM,
4. scattering of cosmic rays,
5. star formation.



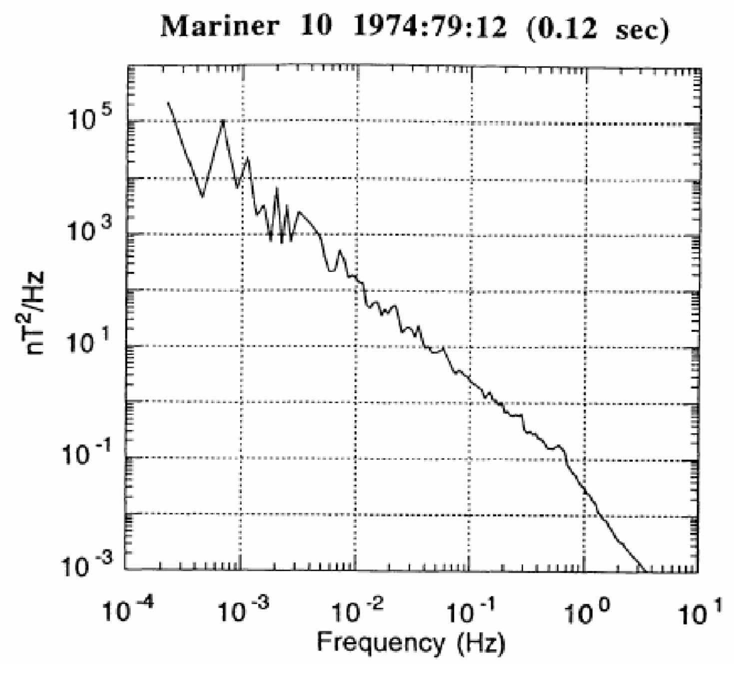






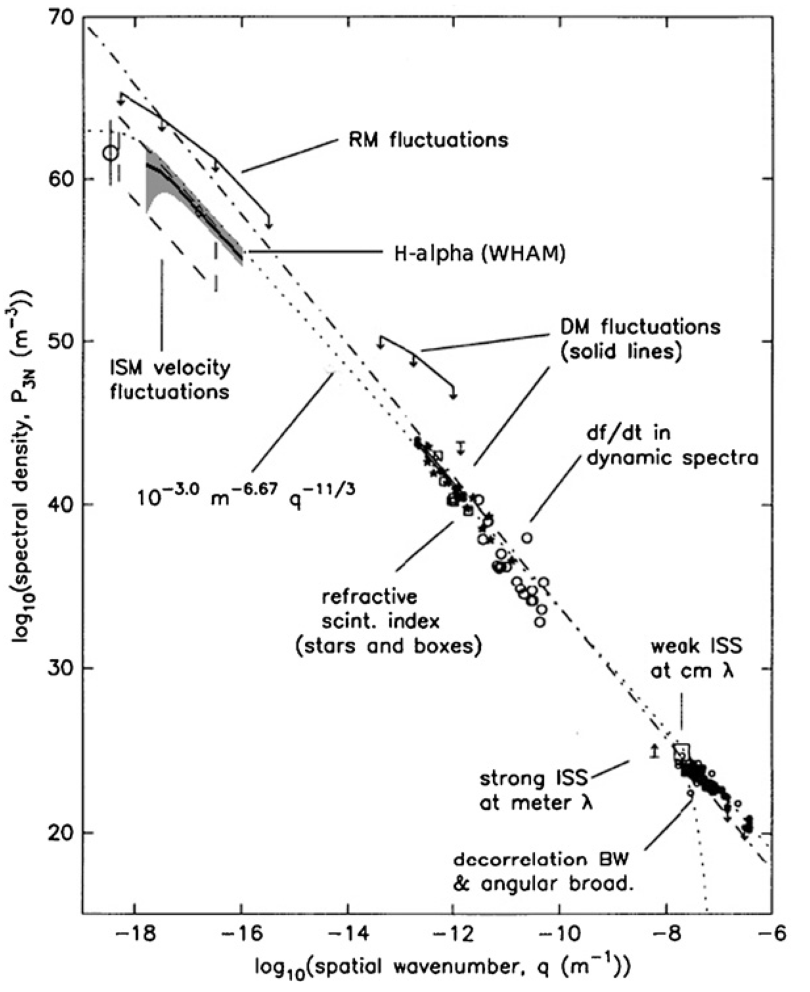
MHD Turbulence in space

Solar wind



Goldstein et al, 1995

Interstellar medium



Armstrong et al 1995
Chepurnov, Lazarian 2010

Basic properties of MHD turbulence

$$\partial_t \mathbf{w}^{\pm} + \hat{S}(\mathbf{w}^{\mp} \cdot \nabla)\mathbf{w}^{\pm} = 0$$

Elsasser variables: $\mathbf{w}^{\pm} = \mathbf{v} \pm \mathbf{B}/\sqrt{4\pi\rho}$ *Solenoidal projection:* $\hat{S}$

Energy flux described in terms of local quantities

$$\langle \delta w^{\pm}_{\|\lambda} (\delta w^{\mp}_{\lambda})^2 \rangle = -4/3 \epsilon^{\mp} \lambda$$

Basic properties of MHD turbulence

$$\partial_t \mathbf{w}^{\pm} + \hat{S}(\mathbf{w}^{\mp} \cdot \nabla)\mathbf{w}^{\pm} = 0$$

Elsasser variables: $\mathbf{w}^{\pm} = \mathbf{v} \pm \mathbf{B}/\sqrt{4\pi\rho}$ *Solenoidal projection:* $\hat{S}$

Dynamics is different from hydro, because there is a mean field.

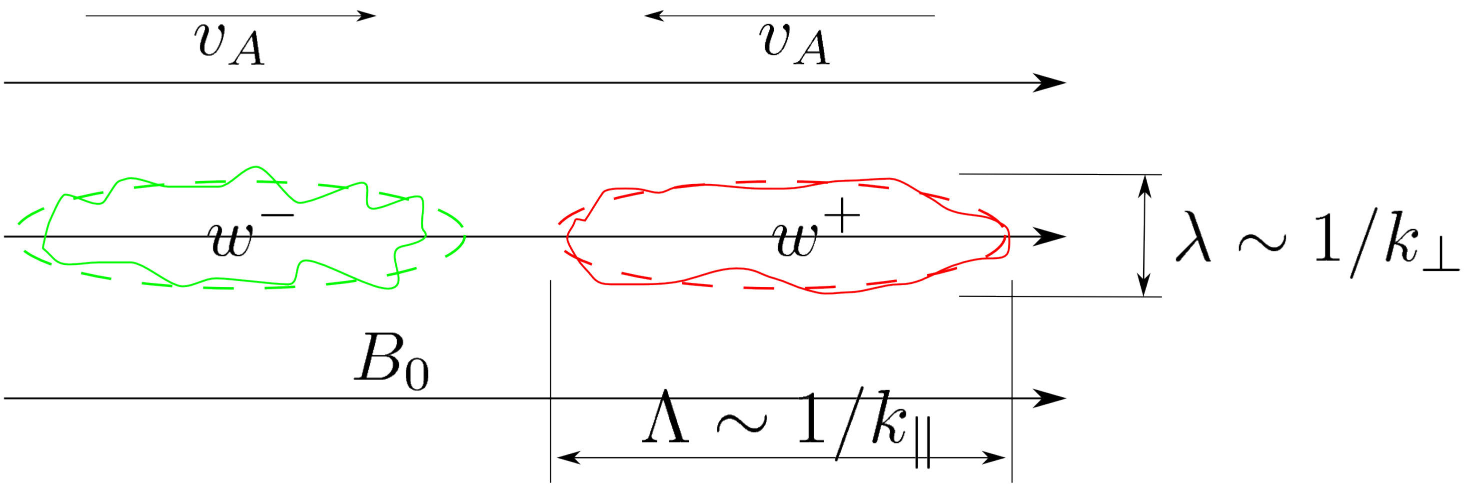
$$\partial_t \delta\mathbf{w}^{\pm} \mp (\mathbf{v_A} \cdot \nabla)\delta\mathbf{w}^{\pm} + \hat{S}(\delta\mathbf{w}^{\mp} \cdot \nabla)\delta\mathbf{w}^{\pm} = 0$$

↑

Mean field (Kraichnan 1965, Iroshnikov 1963)

If universality exists, it is different from hydro.

Basic properties of MHD turbulence

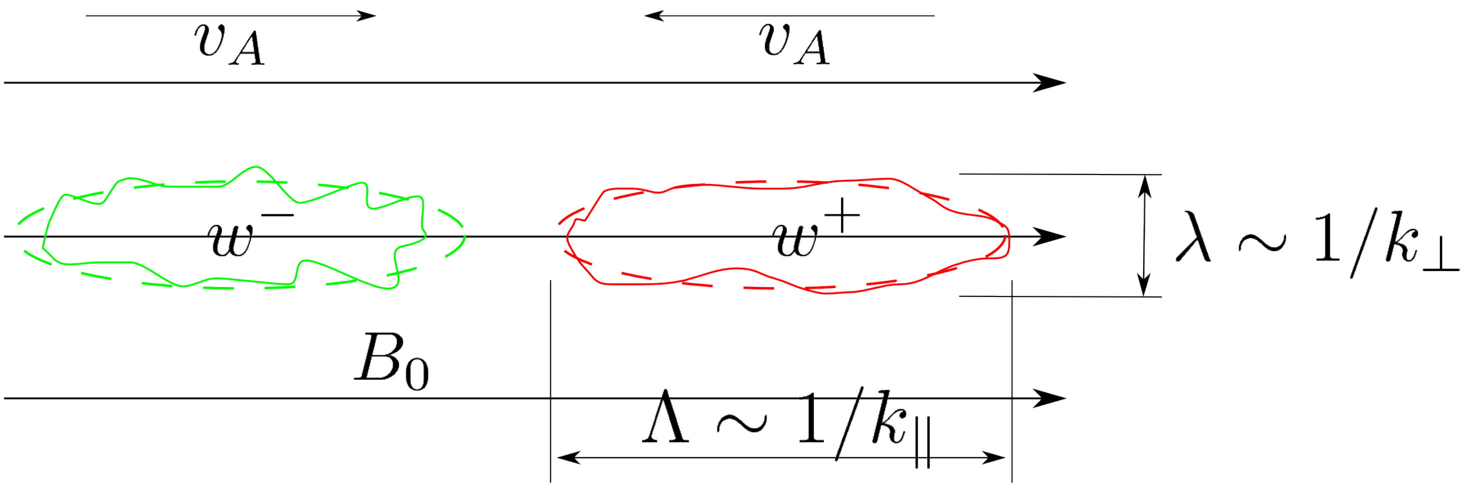


Consider weak interaction and the dominant interaction is 3-wave:

$$\mathbf{k_1} + \mathbf{k_2} = \mathbf{k_3}$$

$$\omega_1 + \omega_2 = \omega_3$$

Basic properties of MHD turbulence

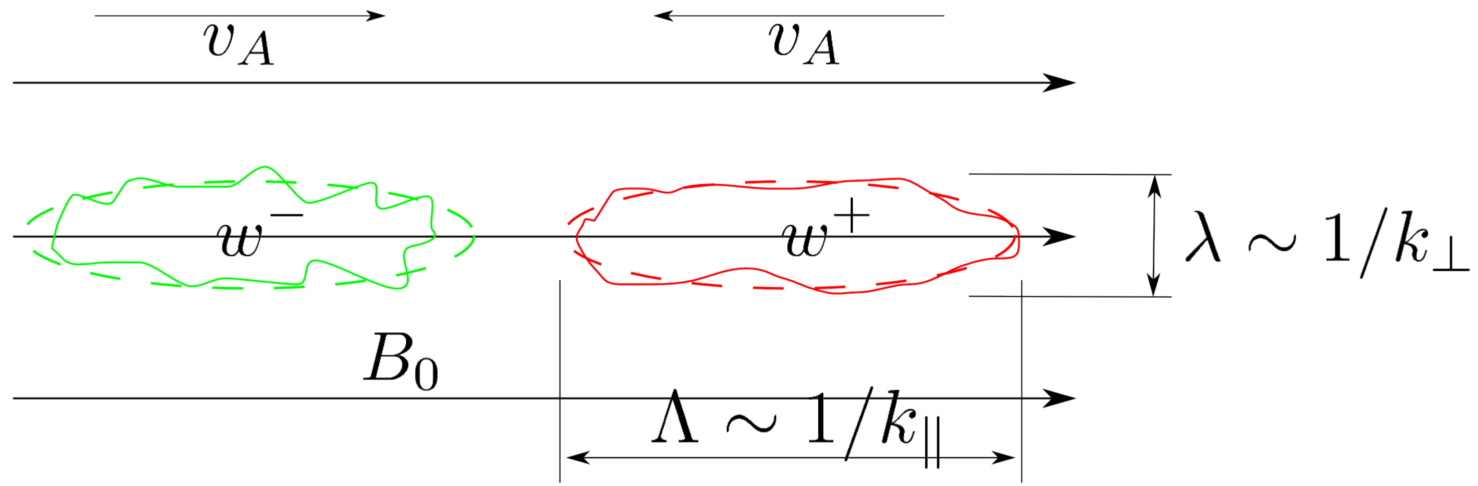


$$\mathbf{k}_{\perp\mathbf{1}} + \mathbf{k}_{\perp\mathbf{2}} = \mathbf{k}_{\perp\mathbf{3}}$$

$$k_{\|1} + k_{\|2} = k_{\|3}$$

$$\omega_1 + \omega_2 = \omega_3$$

Basic properties of MHD turbulence

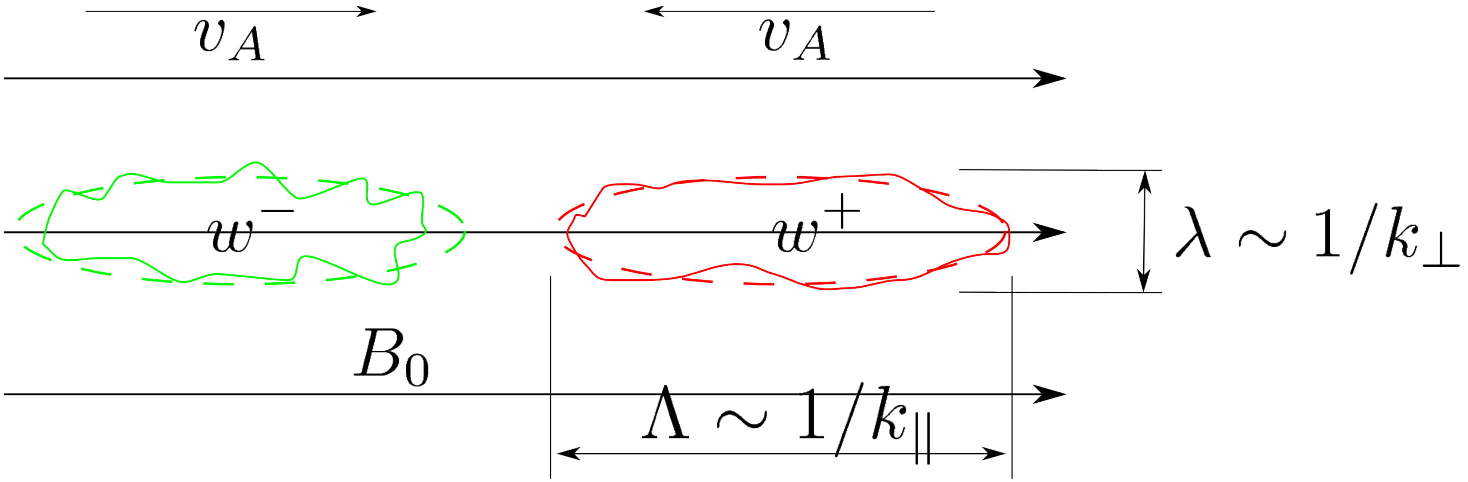


$$\mathbf{k}_{\perp\mathbf{1}} + \mathbf{k}_{\perp\mathbf{2}} = \mathbf{k}_{\perp\mathbf{3}}$$

$$\pm\omega_1 \pm \omega_2 = \pm\omega_3$$

$$\omega_1 + \omega_2 = \omega_3$$

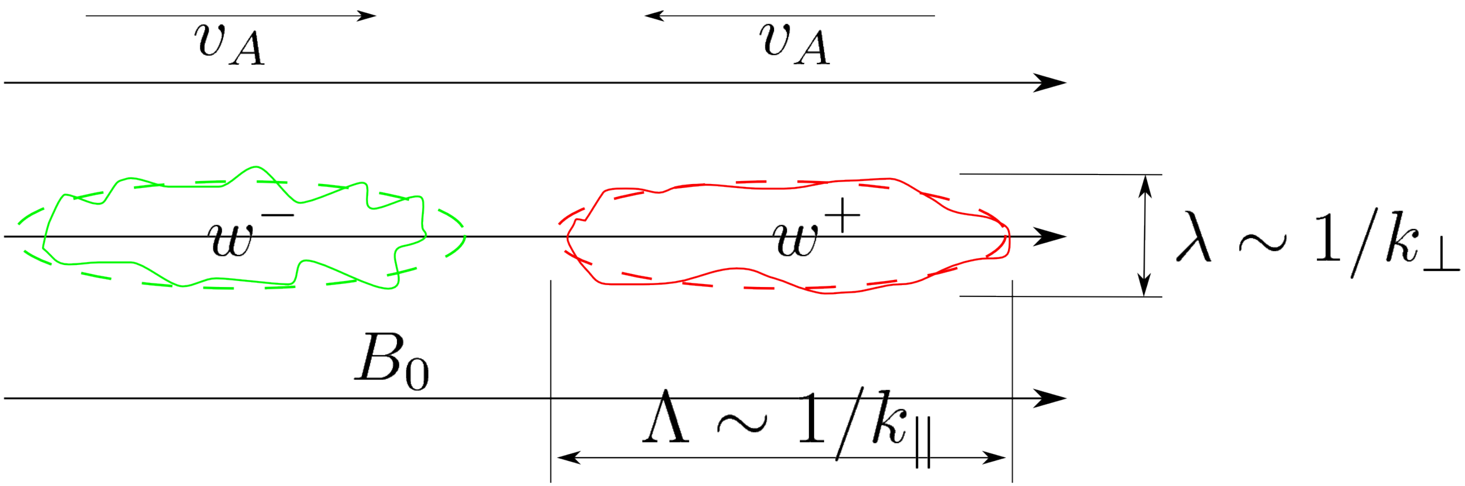
Basic properties of MHD turbulence



$$\mathbf{k_{\perp 1}} + \mathbf{k_{\perp 2}} = \mathbf{k_{\perp 3}}$$

$$\omega_1 = -\omega_2;\ \omega_3 = 0$$

Basic properties of MHD turbulence

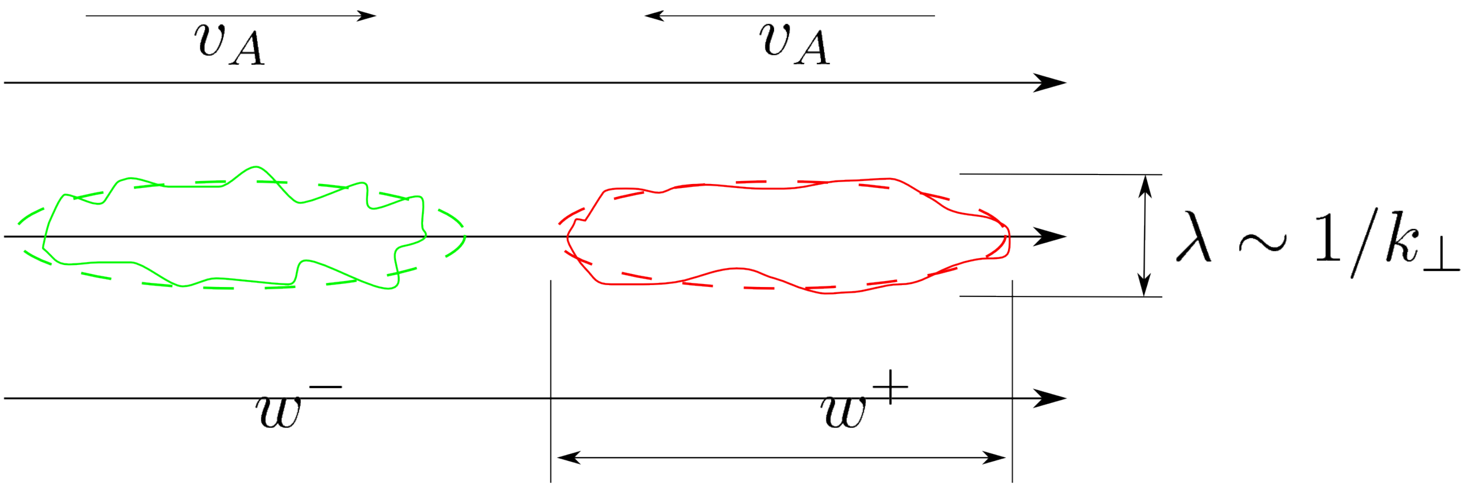


$$\mathbf{k}_{\perp \mathbf{1}} + \mathbf{k}_{\perp \mathbf{2}} = \mathbf{k}_{\perp \mathbf{3}}$$

$$\omega_1 = -\omega_2; \; \omega_3 = 0$$

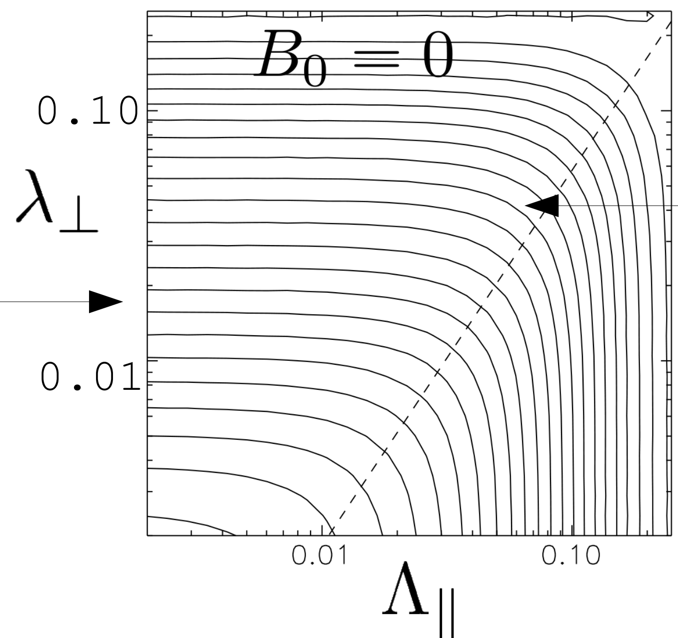
$k_{||}$ is conserved, $k_\perp$ is increasing

Basic properties of MHD turbulence



$k_\parallel$ is conserved, $k_\perp$ is increasing $k_\parallel \ll k_\perp$

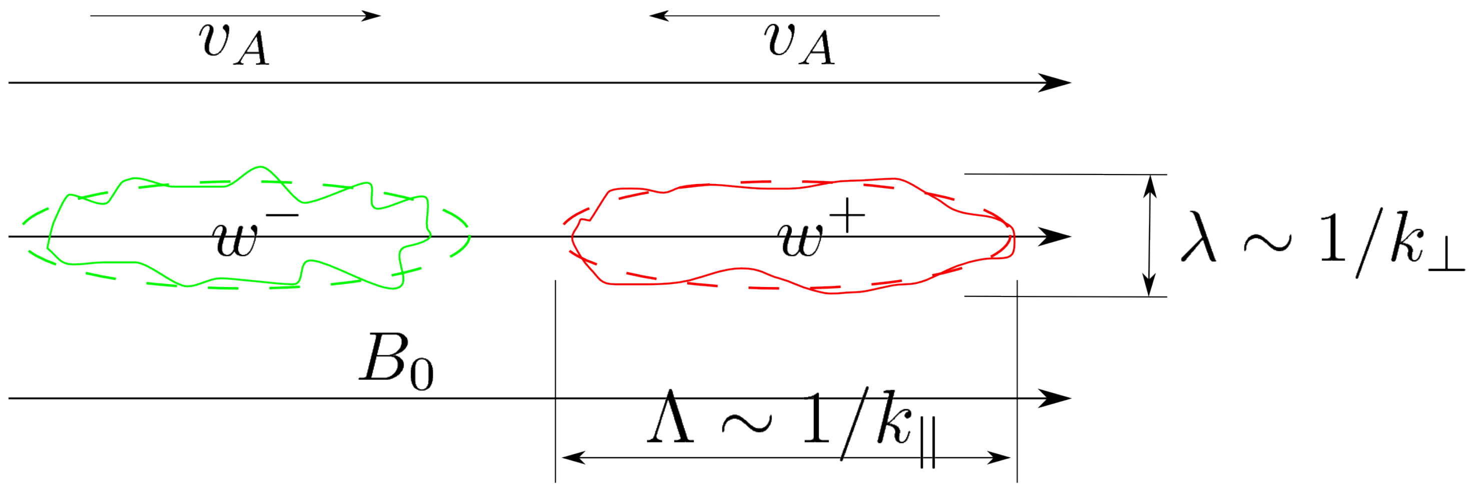
Isotropically driven MHD turbulence



Contours of 2nd order SF, parallel & perpendicular directions are wrt to **local** mean field

From Beresnyak & Lazarian 2009
See also Cho & Vishniac 2000

Basic properties of MHD turbulence



$k_{||}$ is conserved, $k_\perp$ is increasing

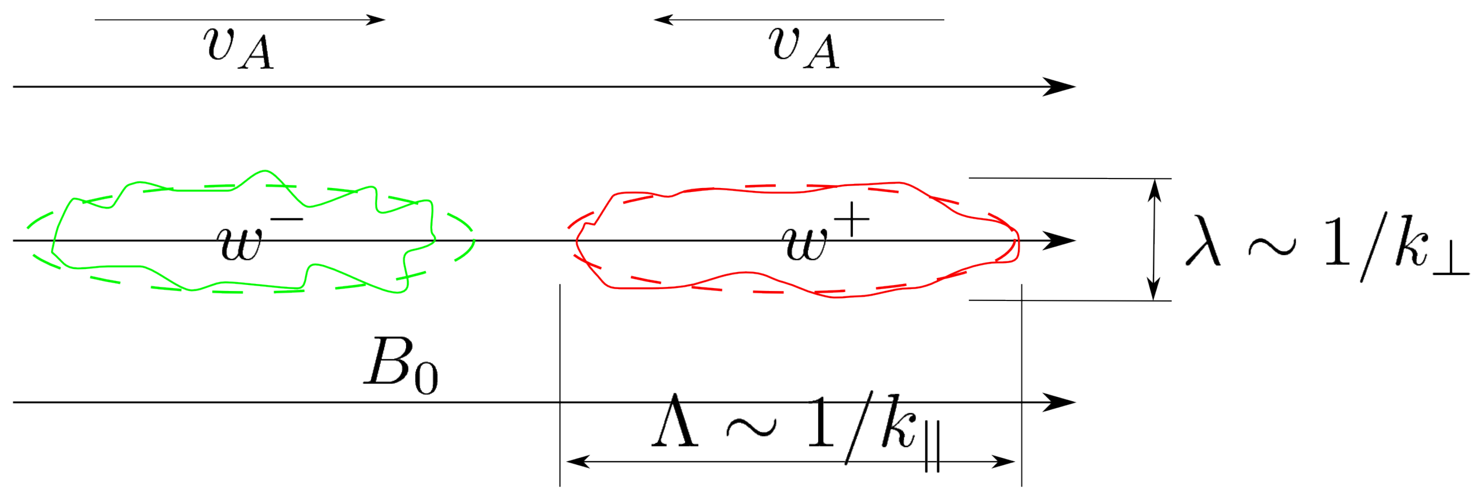
$$k_{||} \ll k_\perp$$

$$\partial_t \delta \mathbf{w}^\pm \mp (\mathbf{v_A} \cdot \nabla) \delta \mathbf{w}^\pm + \hat{S}(\delta \mathbf{w}^\mp \cdot \nabla) \delta \mathbf{w}^\pm = 0$$

$\not{k}_{||}, k_\perp$

could be split in two equations

Basic properties of MHD turbulence



$$k_\parallel \ll k_\perp$$

Alfvenic dynamics (a.k.a. “reduced MHD”) has essential nonlinearity:

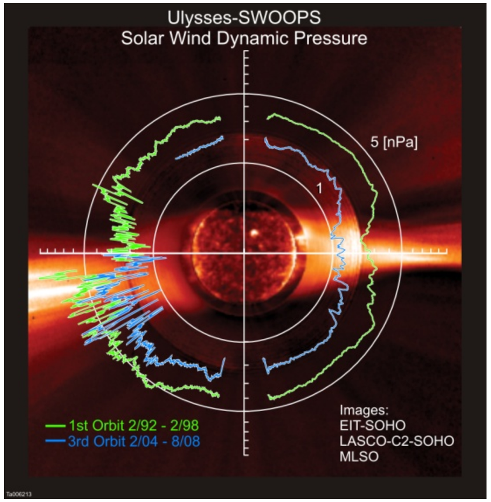
$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel)\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

Slow mode is passively mixed:

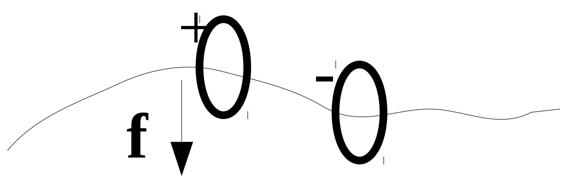
$$\partial_t w_\parallel^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel)w_\parallel^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)w_\parallel^\pm = 0$$

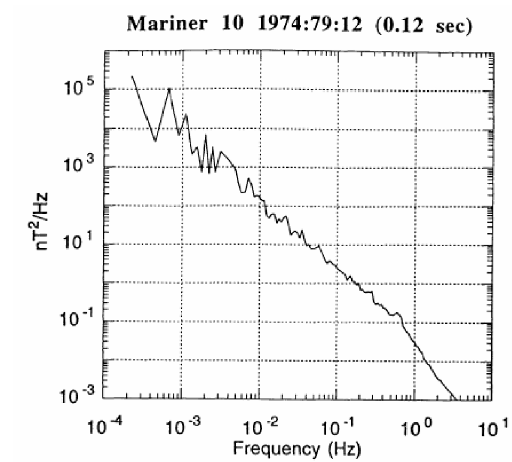
Basic properties of MHD turbulence

Reduced (Alfvenic) MHD could be derived for weakly collisional plasmas as Alfven mode does not require pressure support.



Density fluctuations in the solar wind are much smaller than you would expect from transonic flow-- it is mostly an Alfvenic flow.



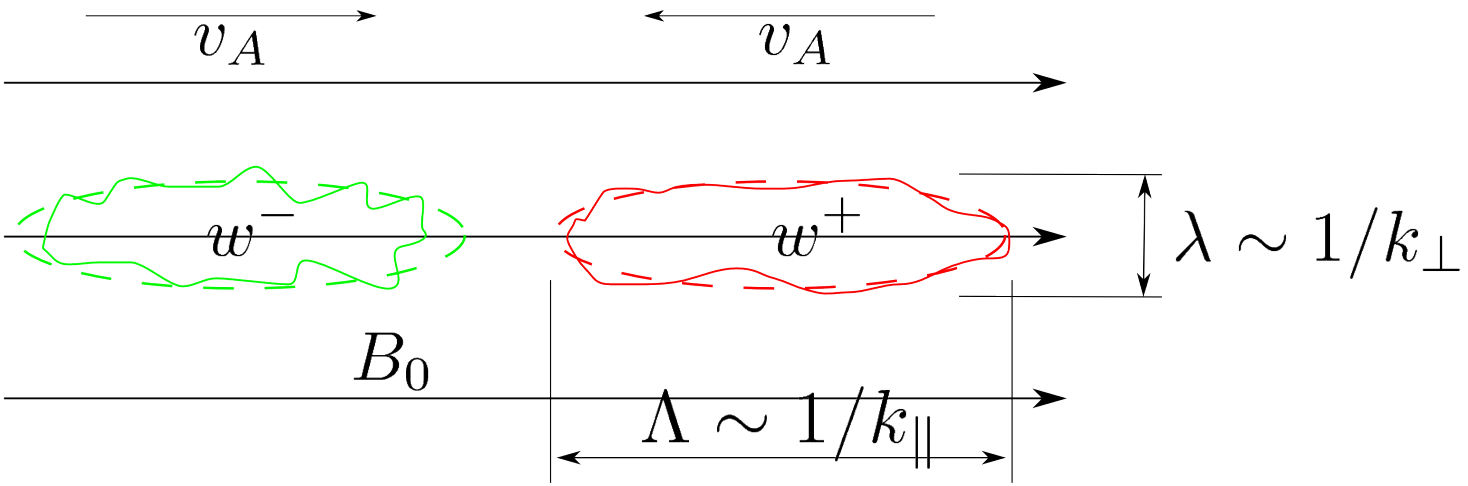


$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_{||})\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

A new universality is possible:

$$w \to wA, \quad \lambda \to \lambda B, \quad t \to tB/A, \quad \Lambda \to \Lambda B/A$$

Basic properties of MHD turbulence



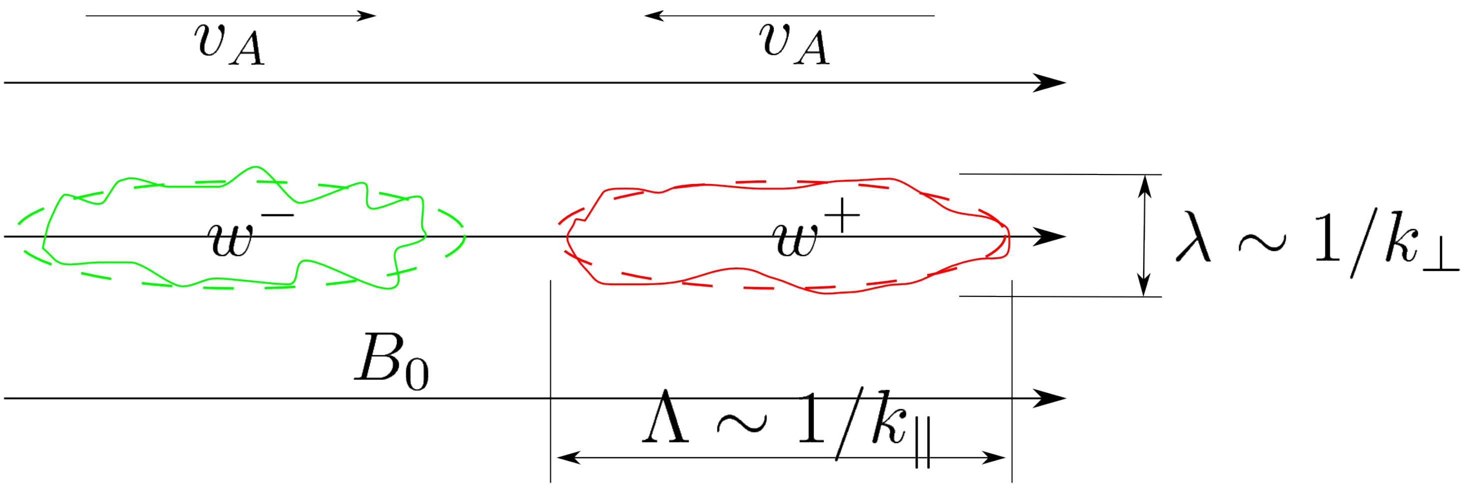
$$k_{||} \ll k_\perp$$

$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_{||})\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

Contribution of nonlinear term has a tendency to increase, thus leading to "strong turbulence", despite a strong mean field, i.e. v_A>>w.

$$v_A k_{||}/\delta w k_\perp \sim 1$$

Basic properties of MHD turbulence



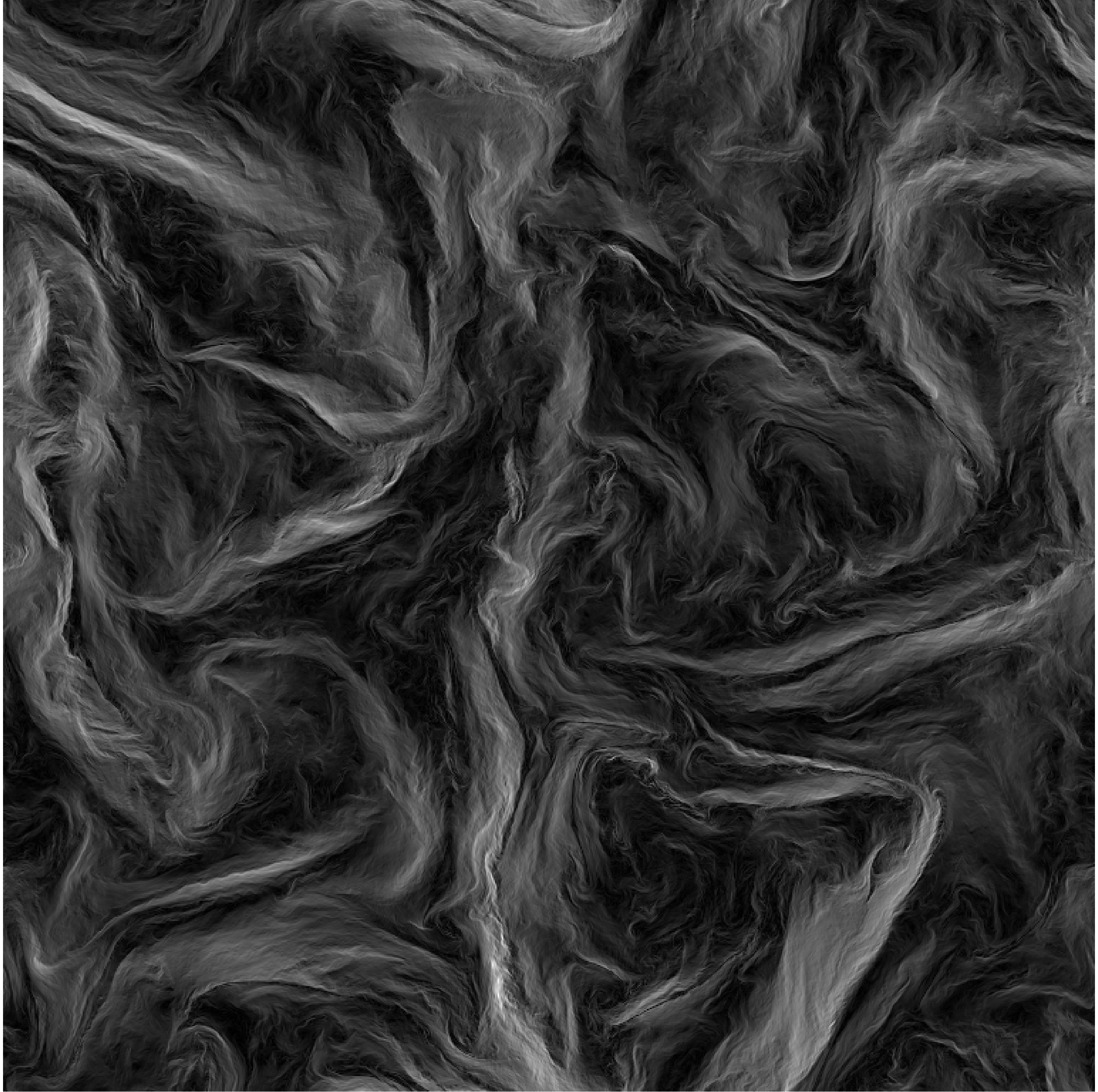
Goldreich-Sridhar (1995) model:
critical balance, an uncertainty
relation $\omega\tau_{\rm cas} \sim 1$

$$v_A k_{||}/\delta w k_\perp \sim 1$$

Confirmed by Cho & Vishniac 2000

Strong cascading, -5/3 spectra:

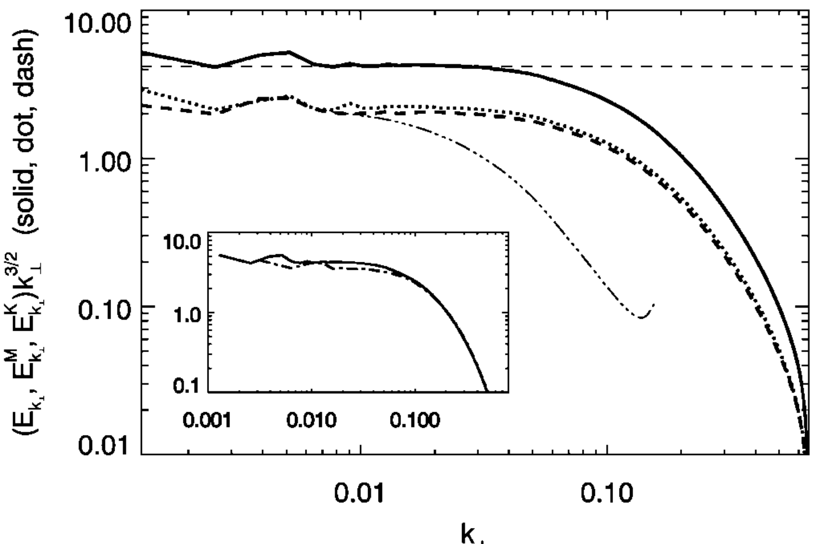
$$\epsilon^+ = \frac{(w_\lambda^+)^2 w_\lambda^-}{\lambda}; \qquad \epsilon^- = \frac{(w_\lambda^-)^2 w_\lambda^+}{\lambda}.$$



Energy spectral slopes: -5/3 or -3/2?

Goldreich-Sridhar model predicts -5/3 but
shallower slopes are often observed in simulations.

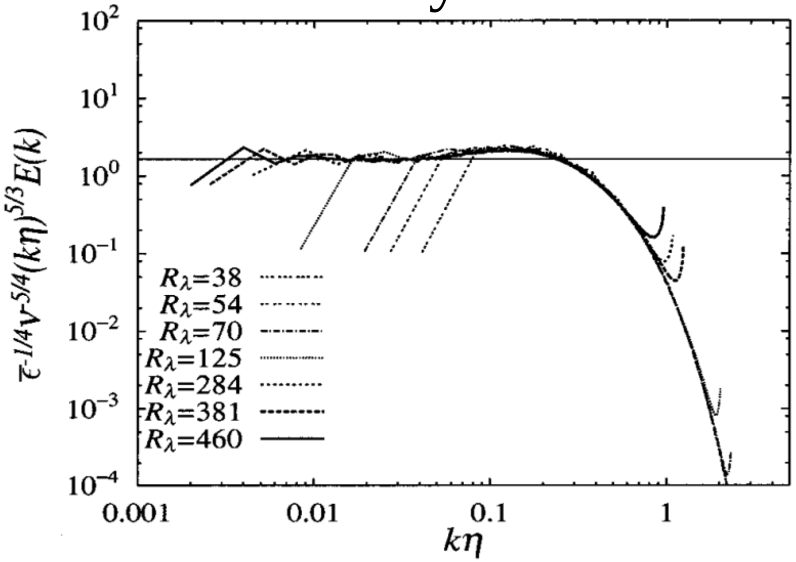
MHD, strong mean field



Muller & Grappin 2005

(same paper claims -5/3 without bottleneck for B0=0 case)

Hydro

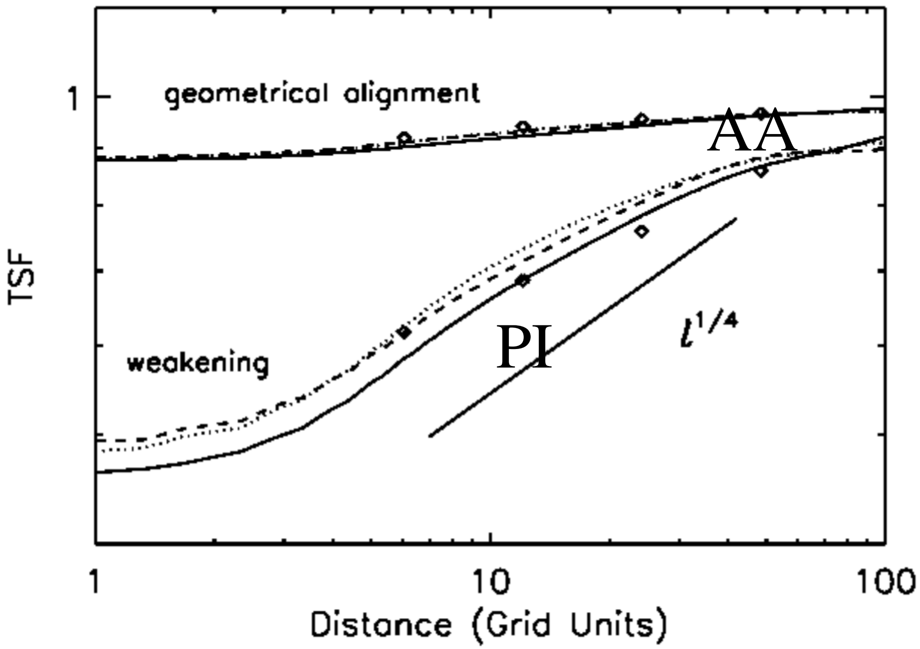


Gotoh et al 2002

"Dynamic alignment"

Boldyrev (2005) proposed "dynamic alignment" which will weaken the interaction and produce -3/2 slope (could be -13/9~-1.44 though).

Beresnyak & Lazarian (2005):



$$\mathrm{AA} = \langle |\sin\theta| \rangle$$

$$\mathrm{PI} = \langle |\delta\mathbf{w}^+ \times \delta\mathbf{w}^-| \rangle / \langle |\delta w^+ \delta w^-| \rangle$$

$$\mathrm{DA} = \langle |\delta\mathbf{v} \times \delta\mathbf{b}| \rangle / \langle |\delta v \delta b| \rangle$$

note that $\delta\mathbf{w}^+ \times \delta\mathbf{w}^- = -2\delta\mathbf{v} \times \delta\mathbf{b}$

Mason, Cattaneo & Boldyrev (2006) measured DA, similar to PI, and claimed precise correspondence with Boldyrev model and a new universal -3/2 spectral slope for MHD.

What is the physics behind alignment?

Boldyrev (2006) proposed that alignment is dynamically created on each scale and is limited by the field wandering. This gives alignment proportional to the amplitude.

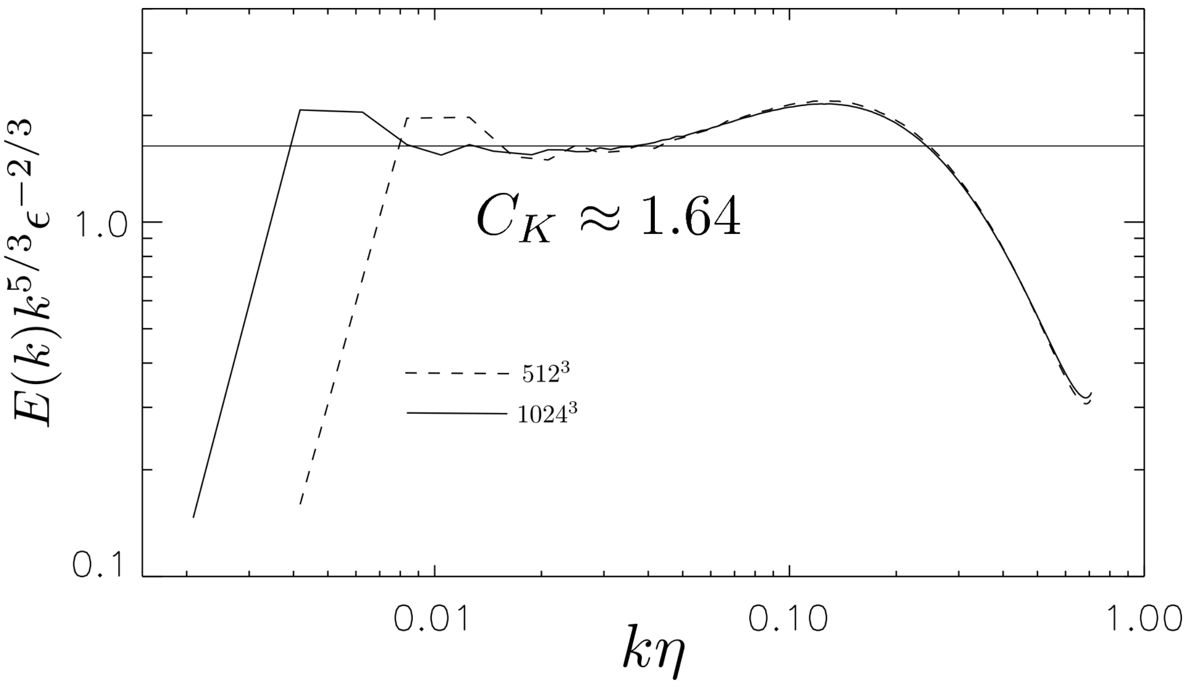
But this directly contradicts the above-mentioned precise symmetry

$$w \to wA, \quad \lambda \to \lambda B, \quad t \to tB/A, \quad \Lambda \to \Lambda B/A$$

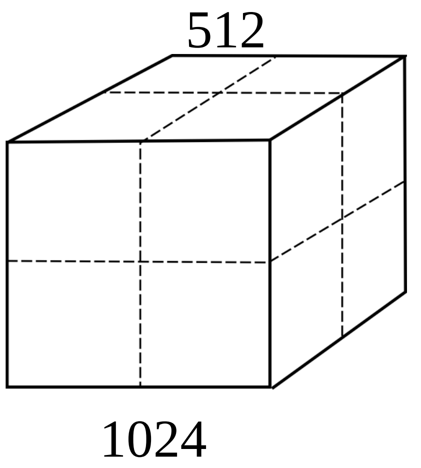
But why alignment is scale-dependent?

Resolution study

Hydro:

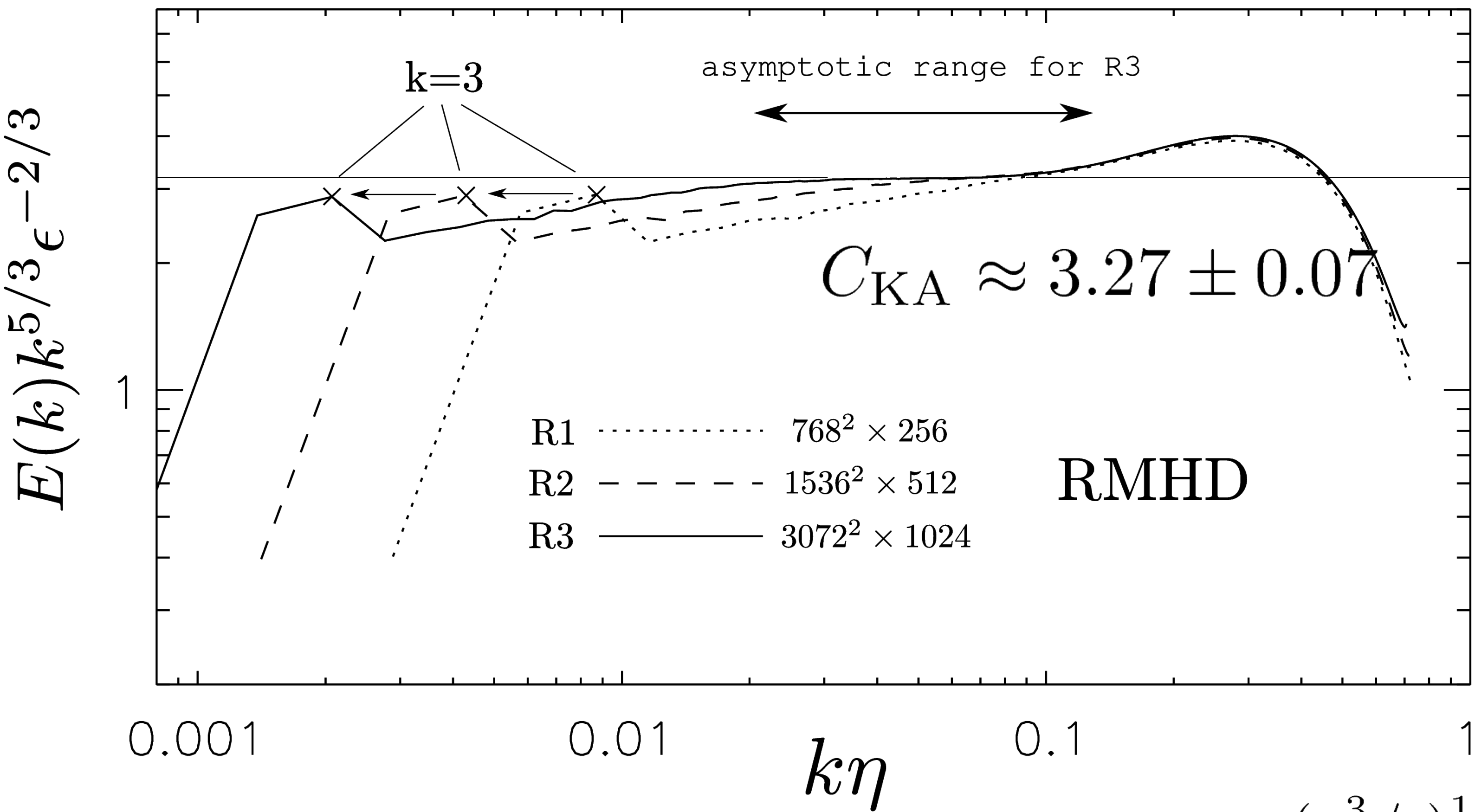


Larger resolution could mean larger scales



Resolution study is a **rigorous** way to claim a correspondence or a lack of it with a particular universal scaling

Resolution study for Alfvenic turbulence

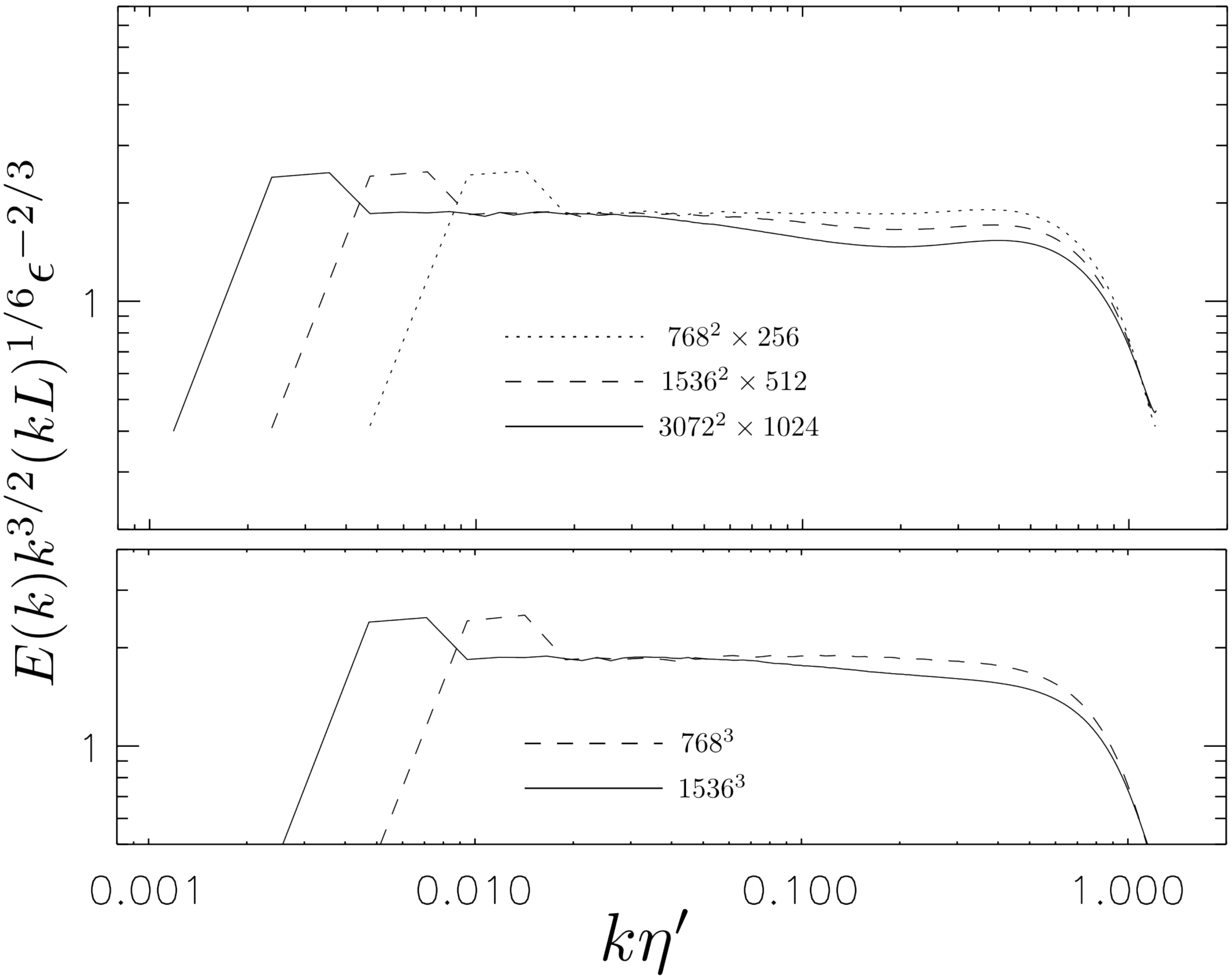


$\eta = (\nu_n^3/\epsilon)^{1/(3n-2)}$

Due to the presence of the slow mode (1-1.3 energy of the Alfvenic mode), the full Kolmogorov constant will be:

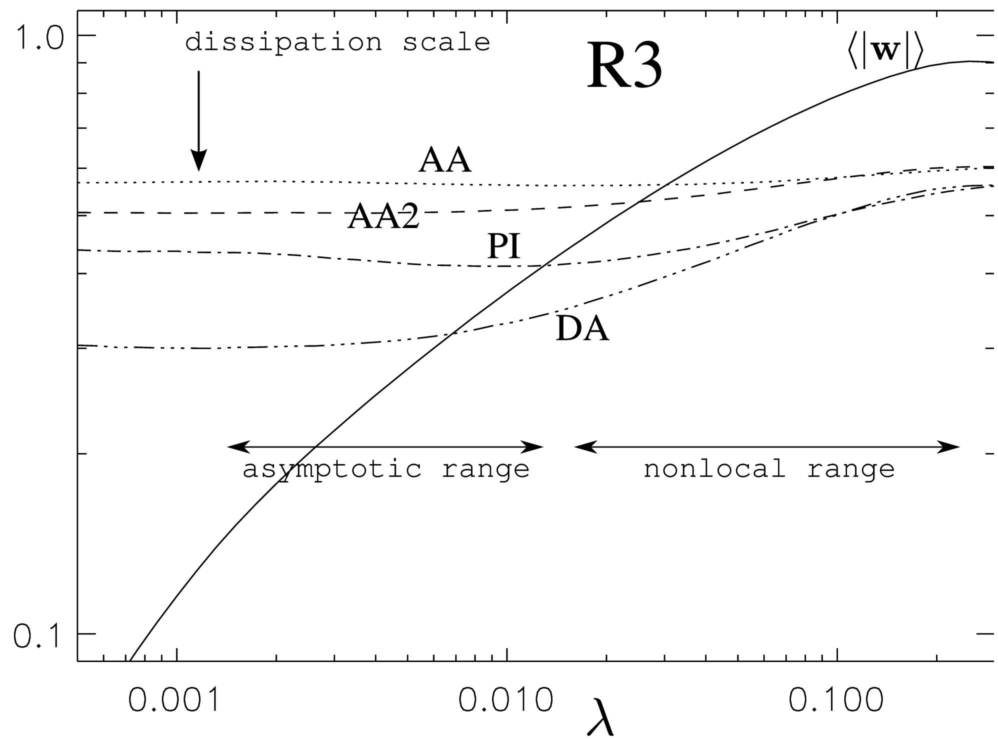
$$C_{\rm K}(\mathrm{MHD}) \approx 4.2 \pm 0.2$$

Resolution study for 3/2 model



Alignment effects are limited to nonlocal range, and do not modify the -5/3 slope of MHD turbulence

$3072^2 \times 1024$

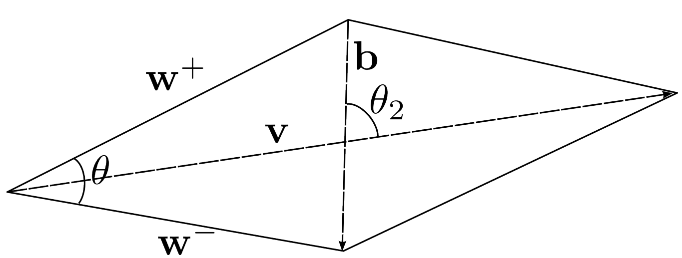


$$\mathrm{AA} = \langle |\sin\theta| \rangle$$

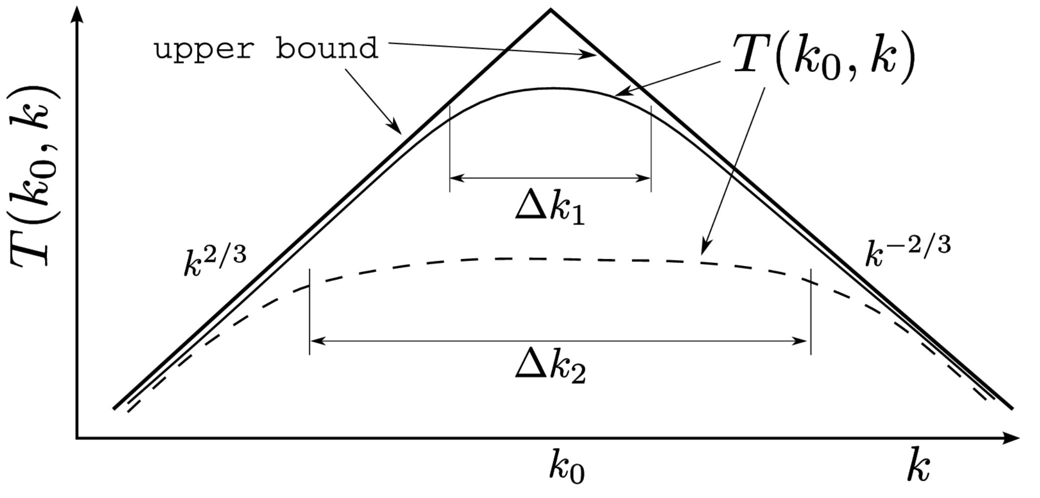
$$\mathrm{AA2} = \langle |\sin\theta_2| \rangle$$

$$\mathrm{PI} = \langle |\delta w^+ \delta w^- \sin\theta| \rangle / \langle |\delta w^+ \delta w^-| \rangle$$

$$\mathrm{DA} = \langle |\delta v \delta b \sin\theta_2| \rangle / \langle |\delta v \delta b| \rangle$$



Locality of energy transfer

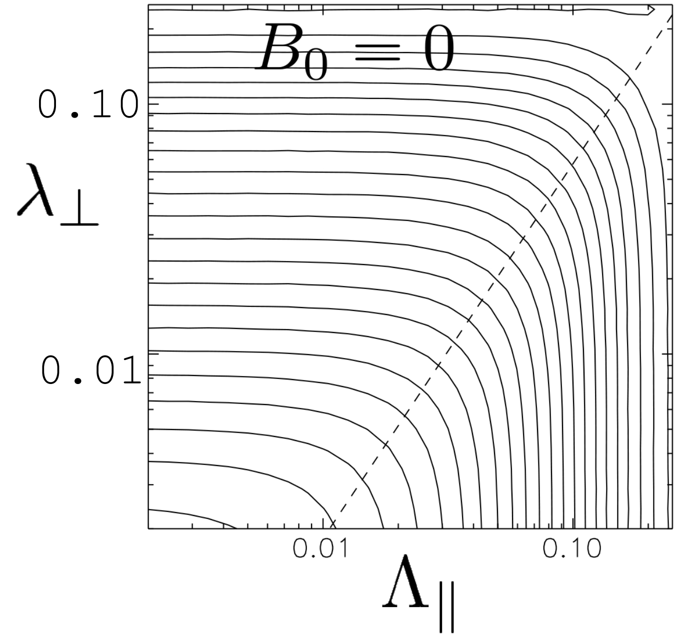


Diffuse locality of MHD turbulence is consistent with high value of the Kolmogorov constant. This explains wide transition towards asymptotic regime.

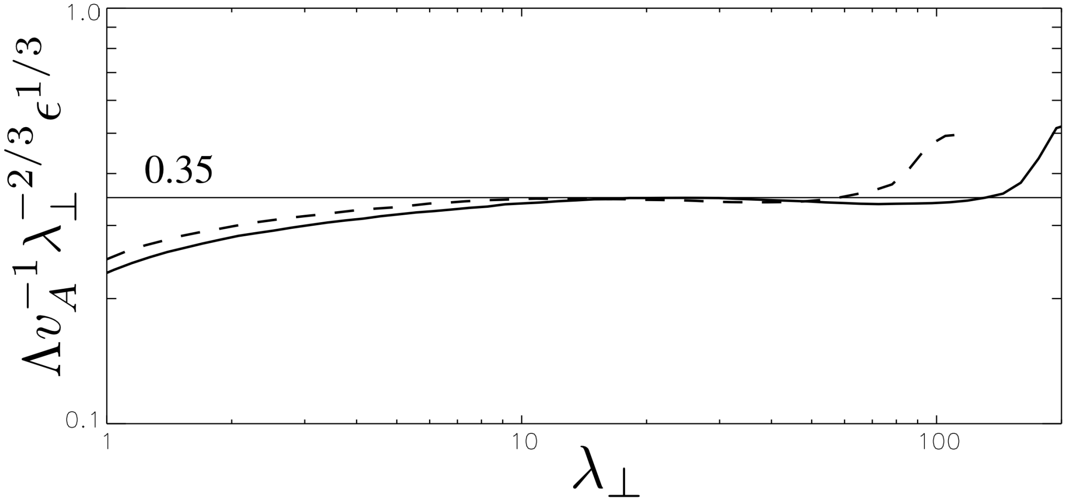
1) statistics of the asymptotic regime are very different from random,
2) it takes one order of magnitude in scale for turbulence to adjust
3) wider locality(x4.7 wider) explains lack of bottleneck in earlier numerics

Universal anisotropy

$$\Lambda_{\|} = C_A v_A \lambda_{\perp}^{2/3} \epsilon^{-1/3}$$



Anisotropy constant:

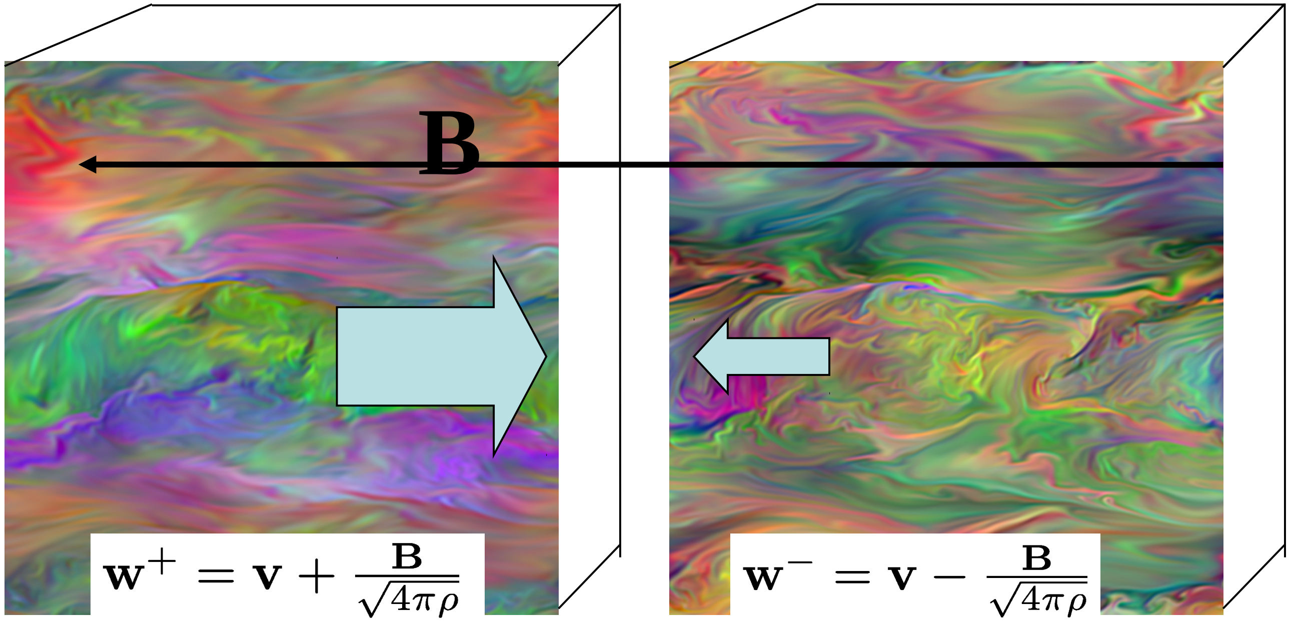


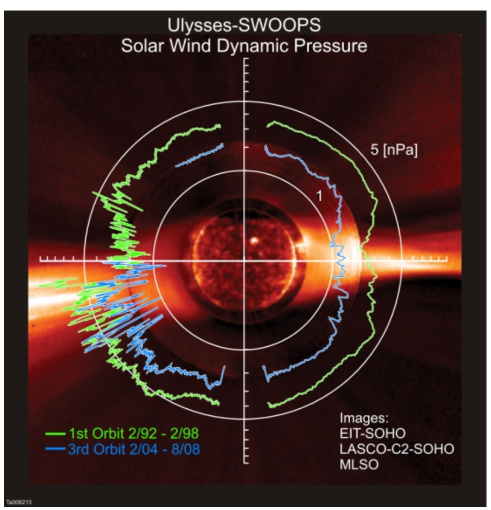
(preliminary)

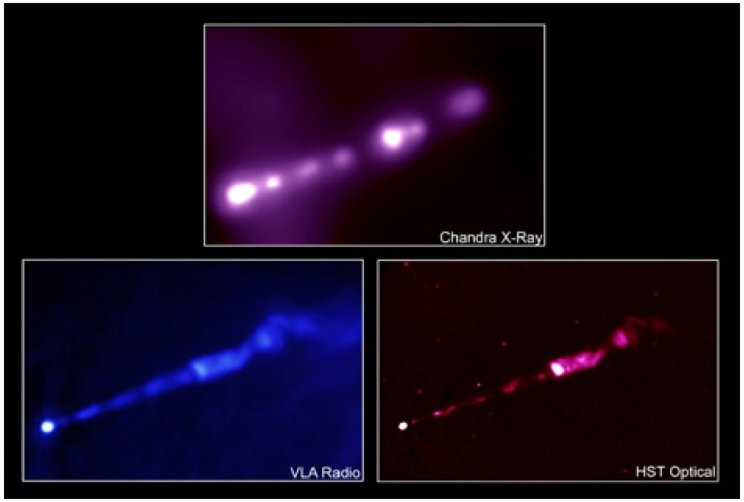
Summary for balanced turbulence

- MHD turbulence has a -5/3 spectrum and a Kolmogorov constant which is much higher than hydrodynamic constant, i.e., in MHD turbulence the energy transfer is much less efficient.
- This has implications for turbulence decay times and turbulence heating rates. E.g., the turbulent heating rate calculated using the measured energy spectrum and hydrodynamic value of the constant will be off by a factor of (4.1/1.6)^1.5 ~ 4.
- More details in Phys. Rev. Lett. 106, 075001

Imbalanced turbulence

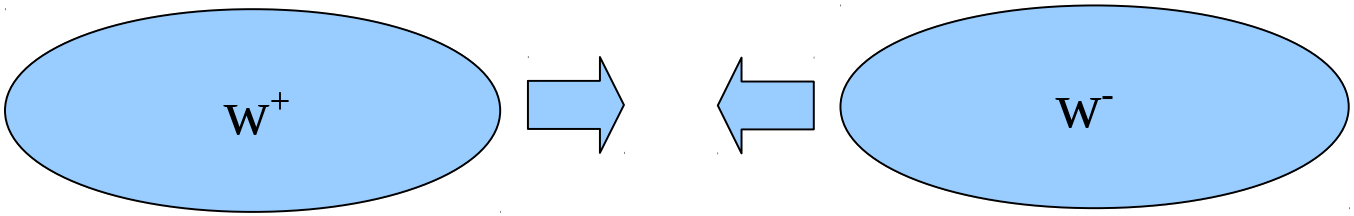






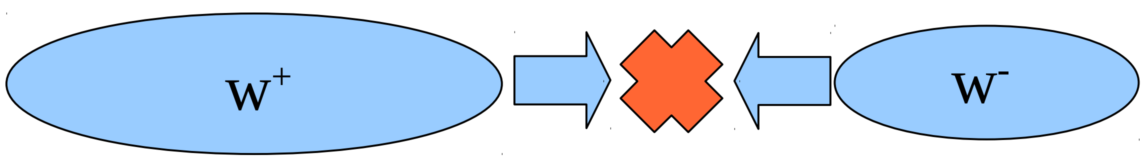
Serious conseptual problem in the imbalanced case

GS95: uncertainty relation $\tau_{cas}\omega\sim 1$, i.e. $\Lambda\sim\lambda v_A/\delta w$.



For weak interaction $\Lambda\sim$const.

If for w^+ $\Lambda\sim$const, but for w^- Λ is decreasing, cascade stops?



Without resolving this paradox the theory of imbalanced turbulence is <u>impossible</u>!

Notation

$$w^{\pm} = z^{\pm} = v \pm b \qquad \text{Elsasser variables}$$

w's – used in Goldreich's papers
z's – used in Biskamp book

$(w^{\pm})^2$ – Elsasser energy

$\tau^{\pm}$ – nonlinear timescale

$\epsilon^{\pm}$ – dissipation rate

} Energy cascade

$\lambda^{\pm}$ – perpendicular scale

$\Lambda^{\pm}$ – parallel scale

} Geometry

Basic Measurements

$$(w^{\pm})^2 = E(1 \pm \sigma_C)/2 \text{– Elsasser energy}$$

$$\tau^{\pm} \text{ – nonlinear timescale}$$

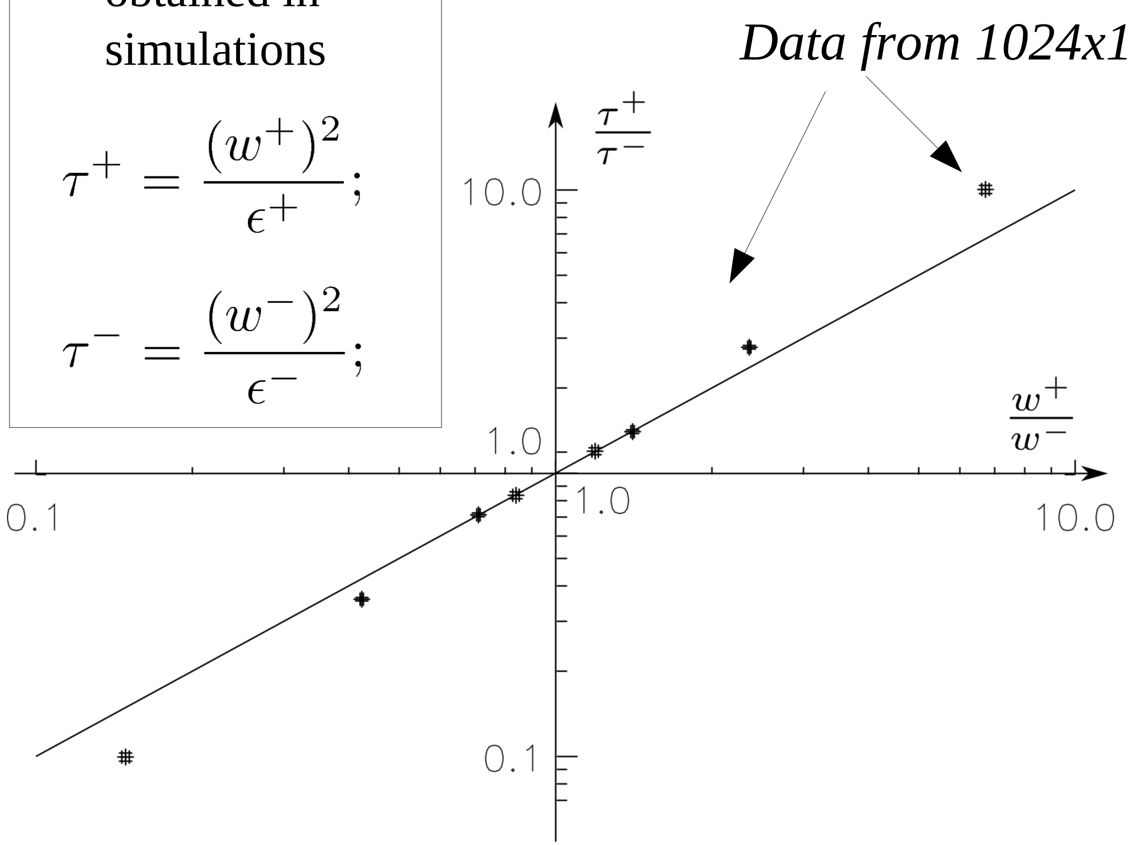
$$\epsilon^{\pm} \text{ – dissipation rate}$$

} Energy cascade

obtained in simulations

$$\tau^+ = \frac{(w^+)^2}{\epsilon^+};$$

$$\tau^- = \frac{(w^-)^2}{\epsilon^-};$$

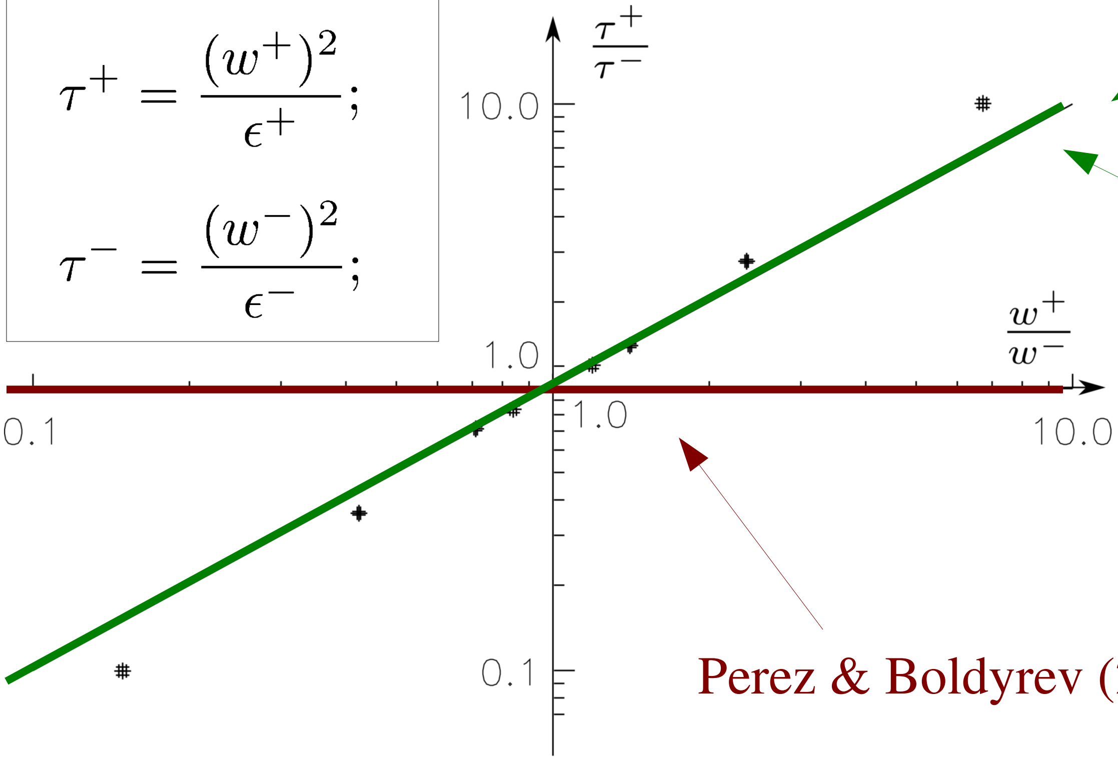


Energy cascade

obtained in simulations

$$\tau^+ = \frac{(w^+)^2}{\epsilon^+};$$

$$\tau^- = \frac{(w^-)^2}{\epsilon^-};$$



GS95,
Lithwick et al (2007)

Strong cascading

$$\tau^+ = \lambda / w_\lambda^-;$$
$$\tau^- = \lambda / w_\lambda^+;$$

Perez & Boldyrev (2009)

Powerful message from numerics:

$$\frac{(w_\lambda^+)^2}{(w_\lambda^-)^2} \geq \left(\frac{\epsilon^+}{\epsilon^-}\right)^2,$$

which is also makes sense from theory.

Cascade in Imbalanced Turbulence?

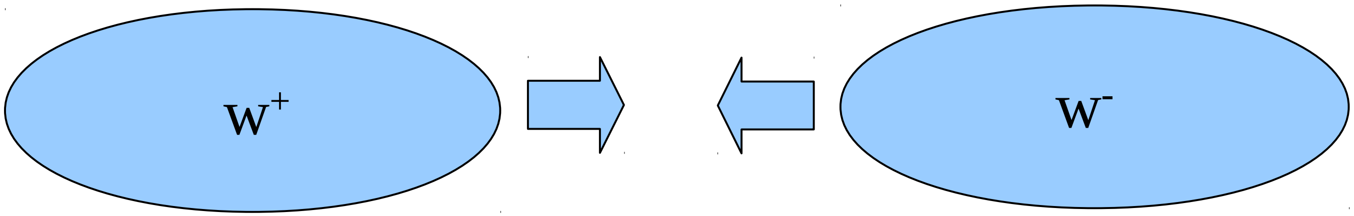
$$\epsilon^- = \frac{(w_\lambda^-)^2 w_\lambda^+}{\lambda}; \qquad \epsilon^+ = \frac{(w_\lambda^+)^2 w_\lambda^-}{\lambda} \cdot o(1)$$

Suppose, one of the waves is cascaded somewhat weaker than strong. If "-" wave have insufficient amplitude to provide strong cascading, then:

$$\frac{(w_\lambda^+)^2}{(w_\lambda^-)^2} \geq \left(\frac{\epsilon^+}{\epsilon^-}\right)^2$$

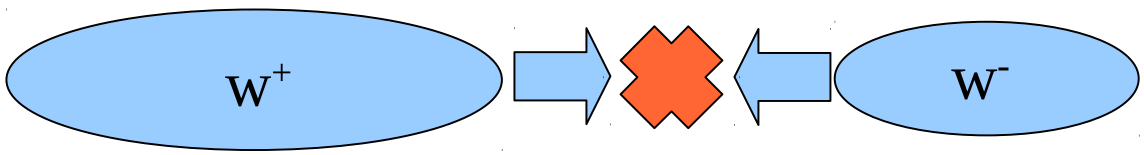
Serious conseptual problem in the imbalanced case

GS95: uncertainty relation $\tau_{cas}\omega\sim1$, i.e. $\Lambda\sim\lambda v_A/\delta w$.



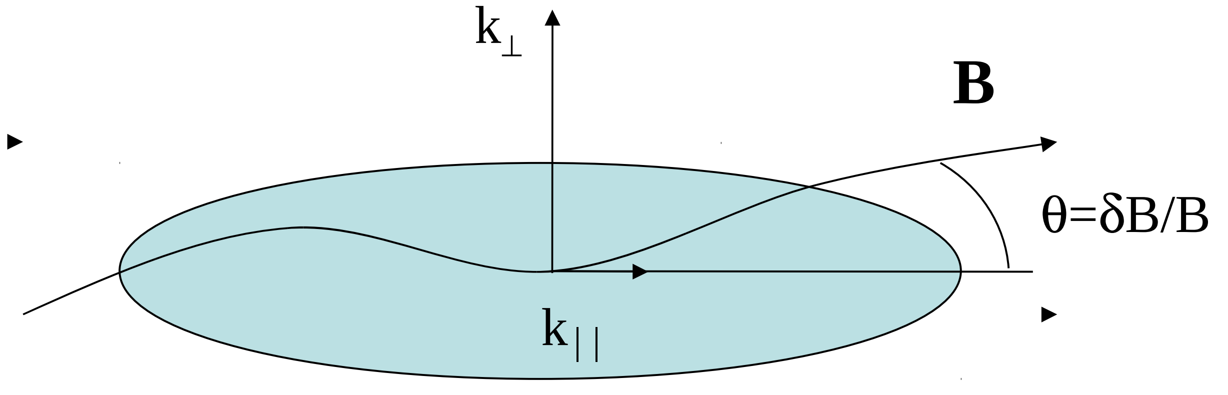
For weak interaction $\Lambda\sim$const.

If for w^+ $\Lambda\sim$const, but for w^- Λ is decreasing, cascade stops?



Without resolving this paradox the theory of imbalanced turbulence is <u>impossible</u>!

Field wandering argument



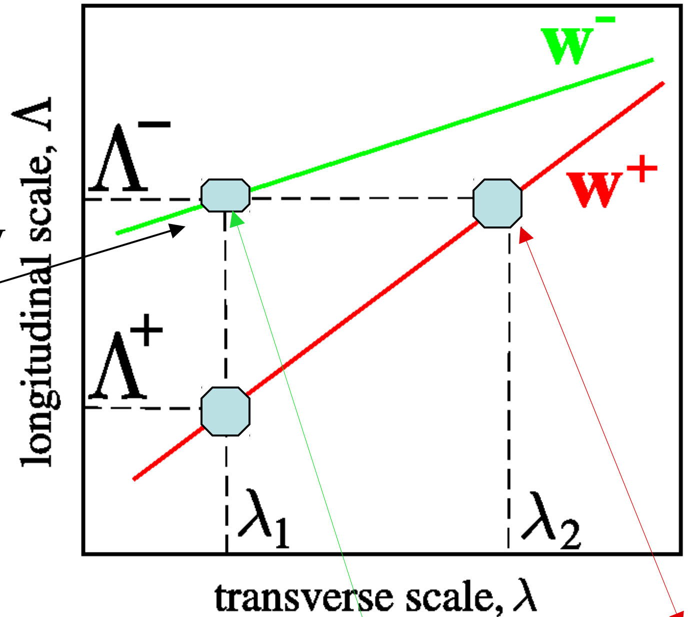
$k_{||} \sim 1/\Lambda$; $\qquad k_\perp \sim 1/\lambda$;

$\delta k_{||} \sim k_\perp \theta$; $\qquad \delta\Lambda \sim \lambda B/\delta B \sim \lambda v_A/\delta w$.

From the point of interacting eddies,
mean field is not well-defined.
This is the unique feature of strong turbulence.

Imbalanced cascade is more complex

$\tau_{cas}\omega \sim 1$
(uncertainty relation)

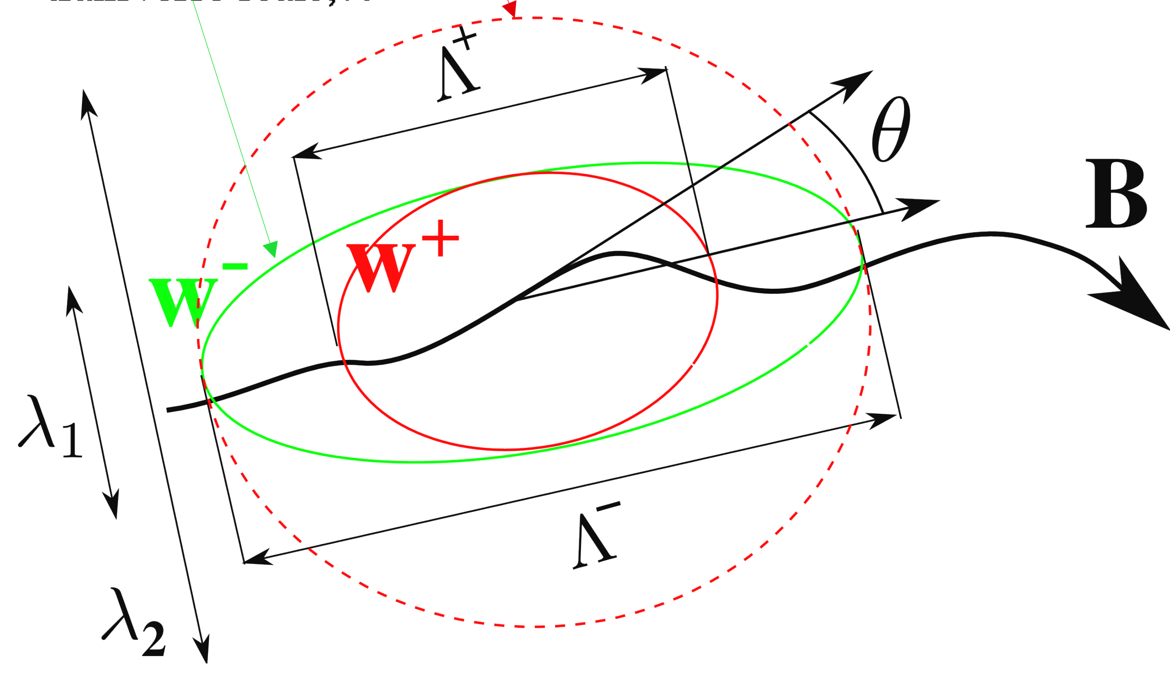


Difference in local field direction

$$\theta \sim \delta b^{+}(\lambda_2)/v_A$$

correspond to anisotropy

$$\lambda_1 = \theta \Lambda^{+}$$



A model of strong imbalanced turbulence

(*Beresnyak & Lazarian, ApJ, 2008*)

Old critical balance (causality) $\Lambda^- = v_A \left(\frac{w^+(\lambda_1)}{\lambda_1}\right)^{-1}$; $\left(\frac{\Lambda^+}{\lambda_1}\right)^{-1} = \frac{w^+(\lambda_2)}{v_A}$ New critical balance (field wandering)

energy — shear rate

$$\epsilon^- = \frac{(w^-(\lambda_1))^2 w^+(\lambda_1)}{\lambda_1}$$

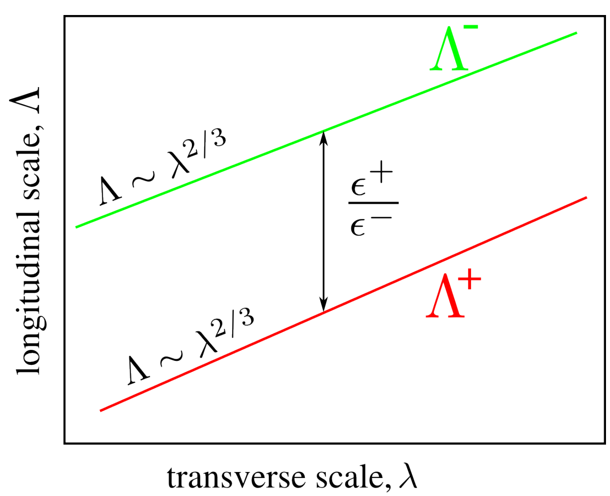
Strong cascading of weak wave

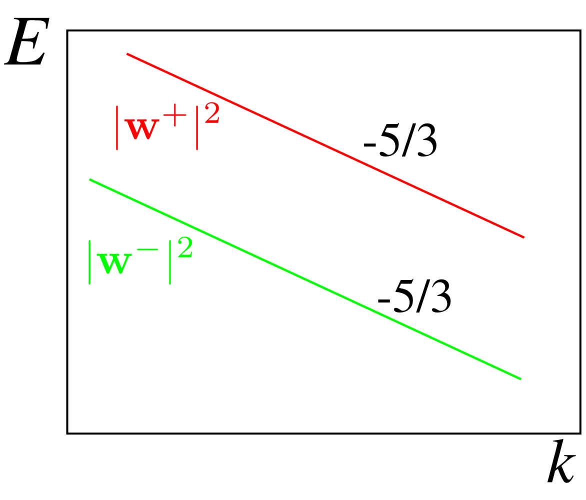
$$\epsilon^+ = \frac{(w^+(\lambda_2))^2 w^-(\lambda_1)}{\lambda_1} \cdot \frac{w^-(\lambda_1)\Lambda^-}{v_A \lambda_1} \cdot f(\lambda_1/\lambda_2)$$

weakening factor

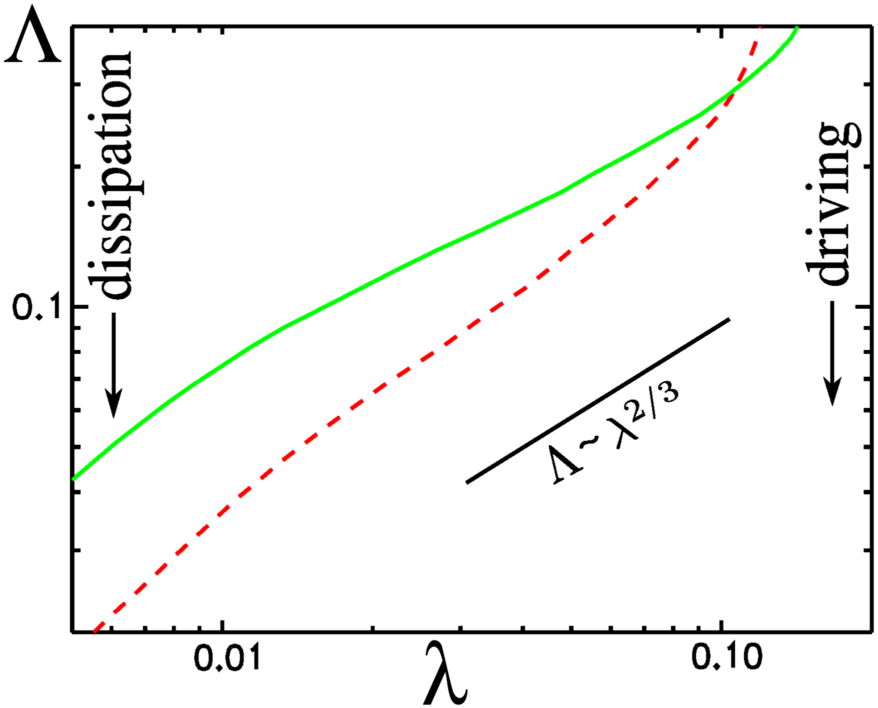
Weak cascading of strong wave

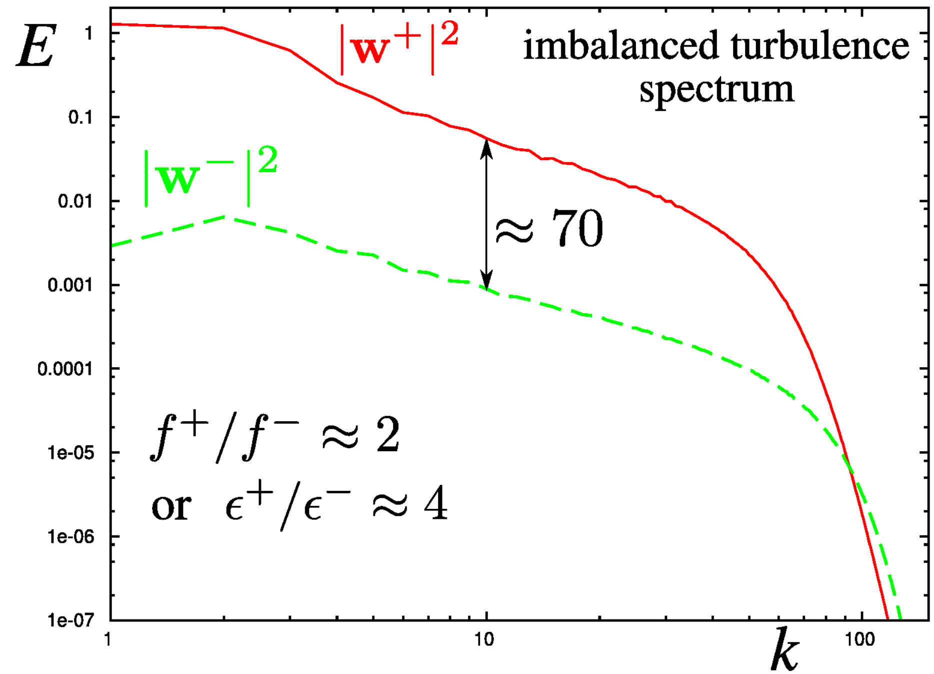
Asymptotic power-law solutions:





Numerical data support this model





Our model vs numerics:

a) the energy imbalance is higher than in the case when both waves are cascaded strongly, which suggest that dominant wave is cascaded weakly
b) Time evolution of spectra suggests that strong wave have a longer dissipation timescale
c) the anisotropies are different and the strong wave anisotropy is smaller
d) subdominant wave eddies are aligned with respect to the local field, while dominant wave eddies are aligned with respect to larger-scale field
e) the inertial range of the dominant wave is shorter
f) there is no "pinning" on dissipation scale, which suggest nonlocal cascading

Other models vs numerics:

numerics \ models	*Lithwick et al (2007)*	*Beresnyak & Lazarian (2008)*	*Chandran (2008)*	*Perez & Boldyrev (2009)*
cascading timescales	✔✘	✔	✘	✘
spectral slopes	-	-	✘	-
anisotropies	✘	✔	✘	✘?
time evolution	✔	✔	✘	✘
dissipation scale	✔	✔	✘	✘

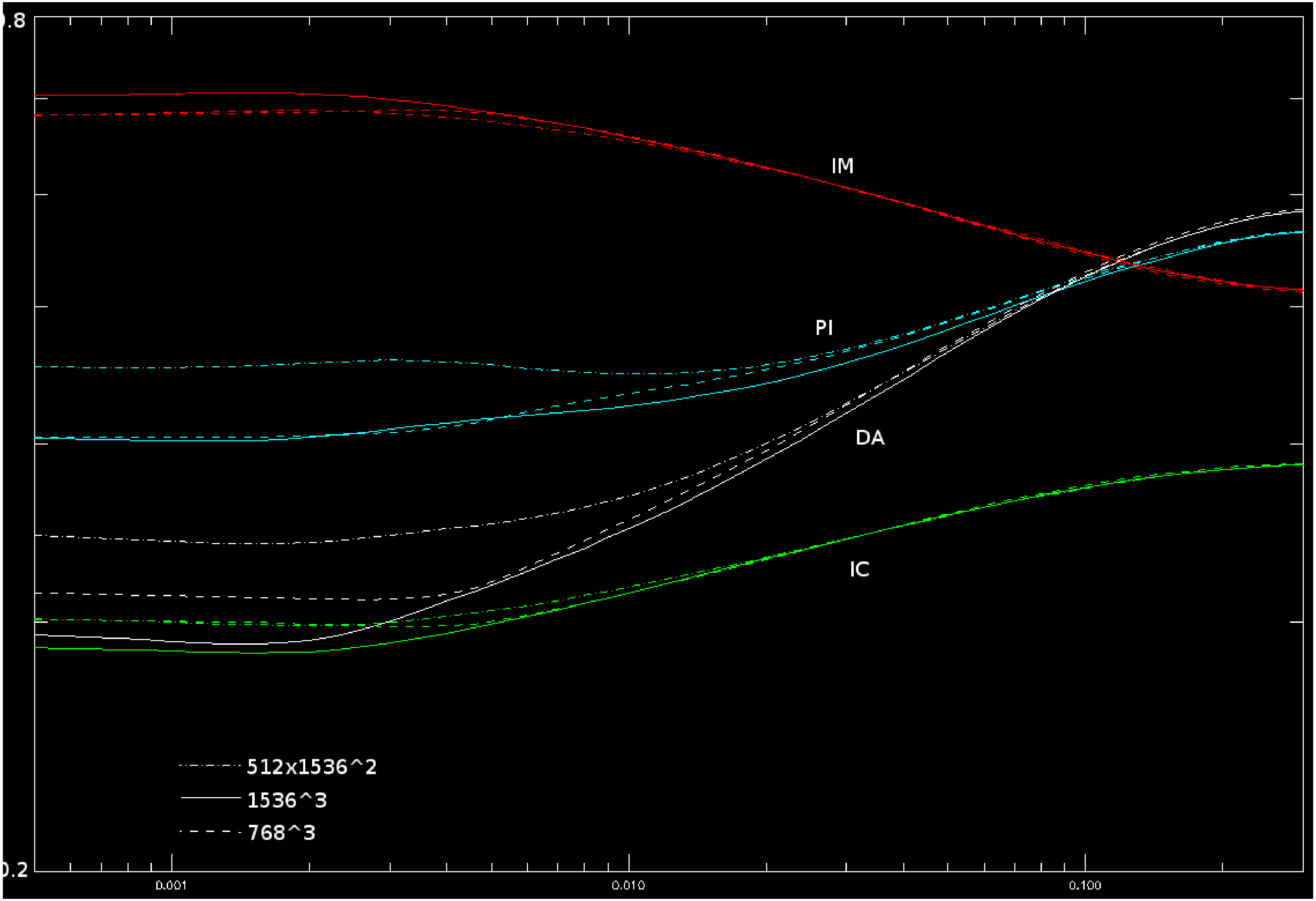
Summary

• MHD turbulence has a *universal cascade,* although different from hydrodynamic cascade.

• For the first time, we were able to measure the *Kolmogorov constant*, i.e. the efficiency of the energy transfer in MHD turbulence and explained the lack of bottleneck effect in earlier MHD simulations.

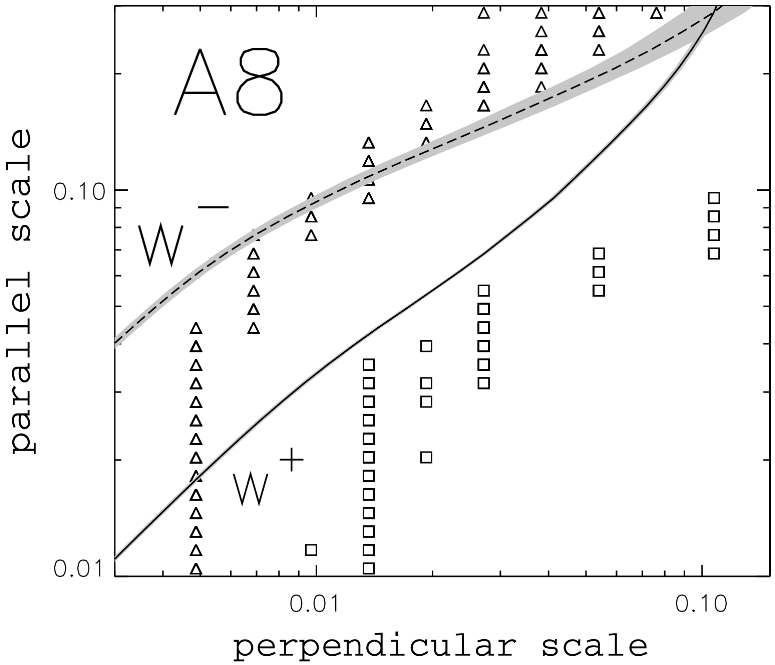
• We now have a good idea how cascading happens in the general case, i.e., in *imbalanced turbulence*. In nature, imbalanced turbulence is more common than the balanced one, as sources and sinks of energy exist in a large scale mean magnetic field.

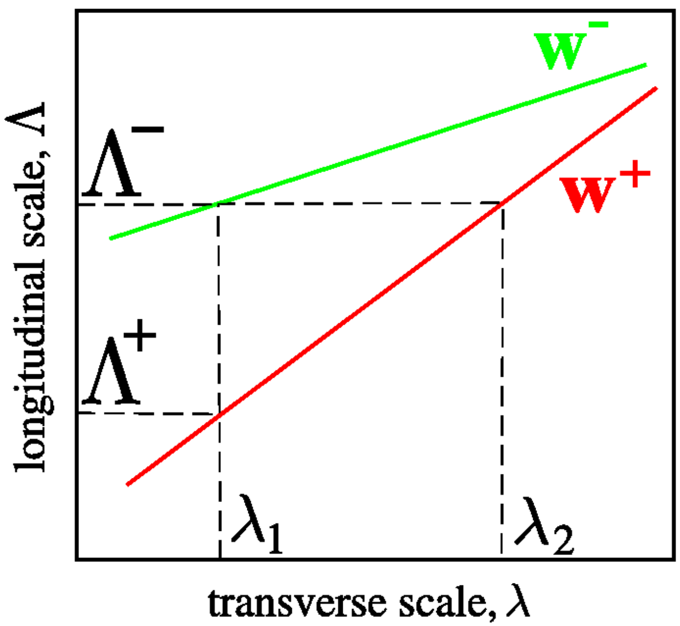
• Numerics is an efficient tool to discriminate between models, by both qualitative and quantitative means.

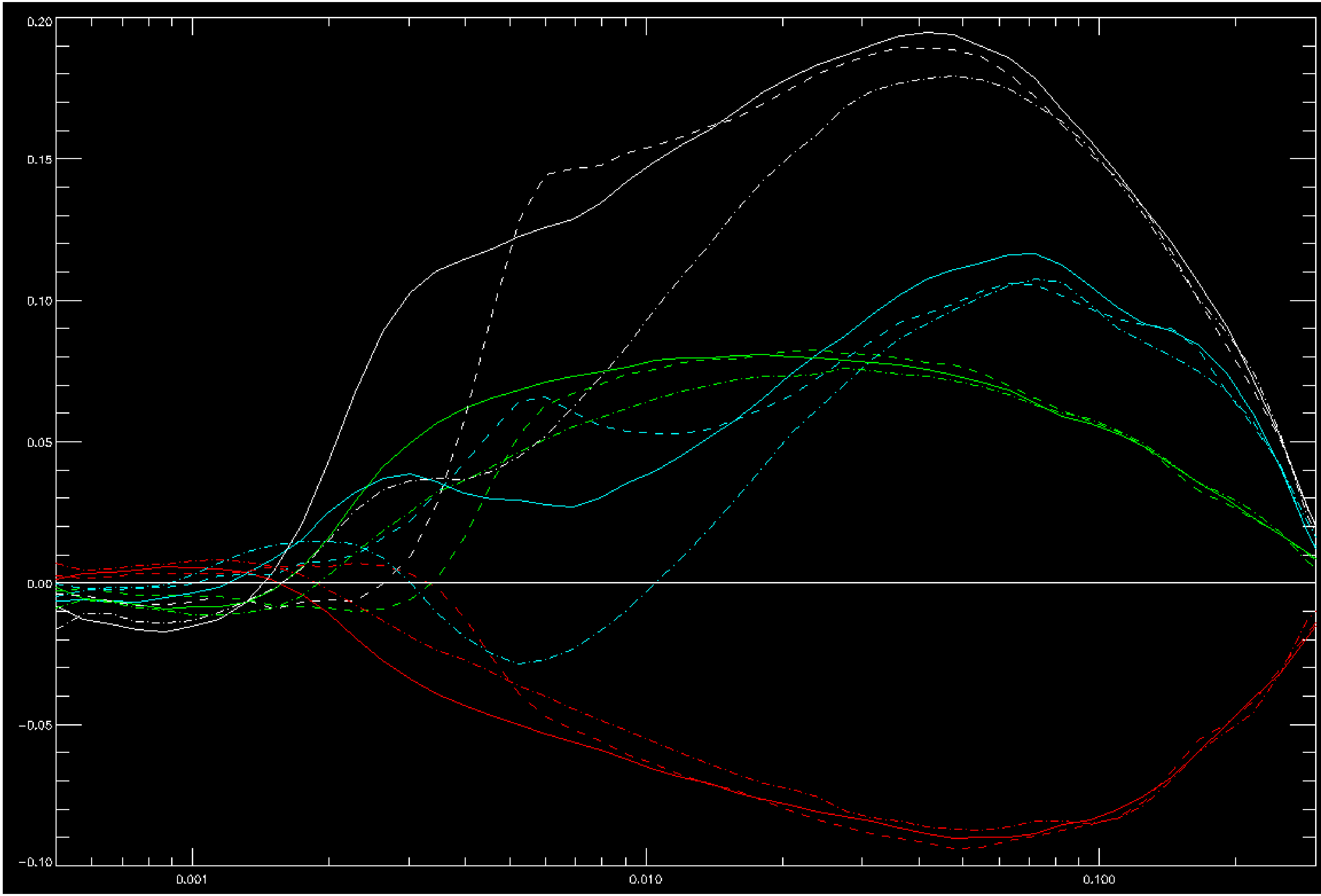


0.8
0.2
IM
PI
DA
IC
512x1536^2
1536^3
768^3
0.001
0.010
0.100

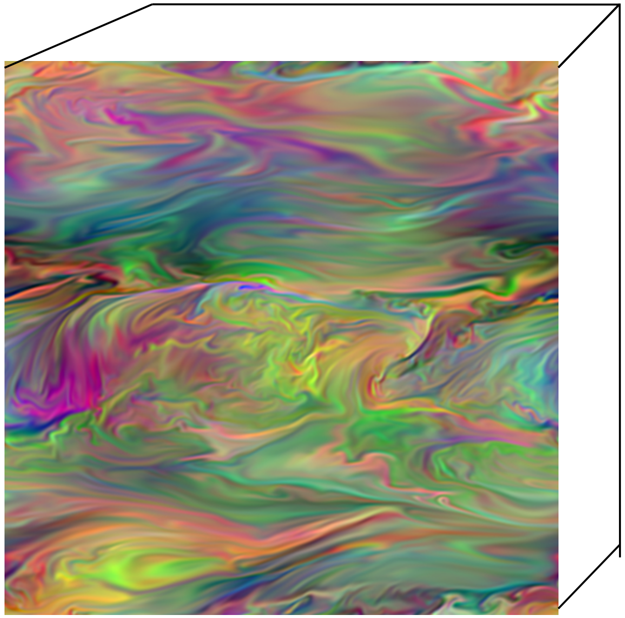
Strong eddies are aligned with respect to larger-scale field



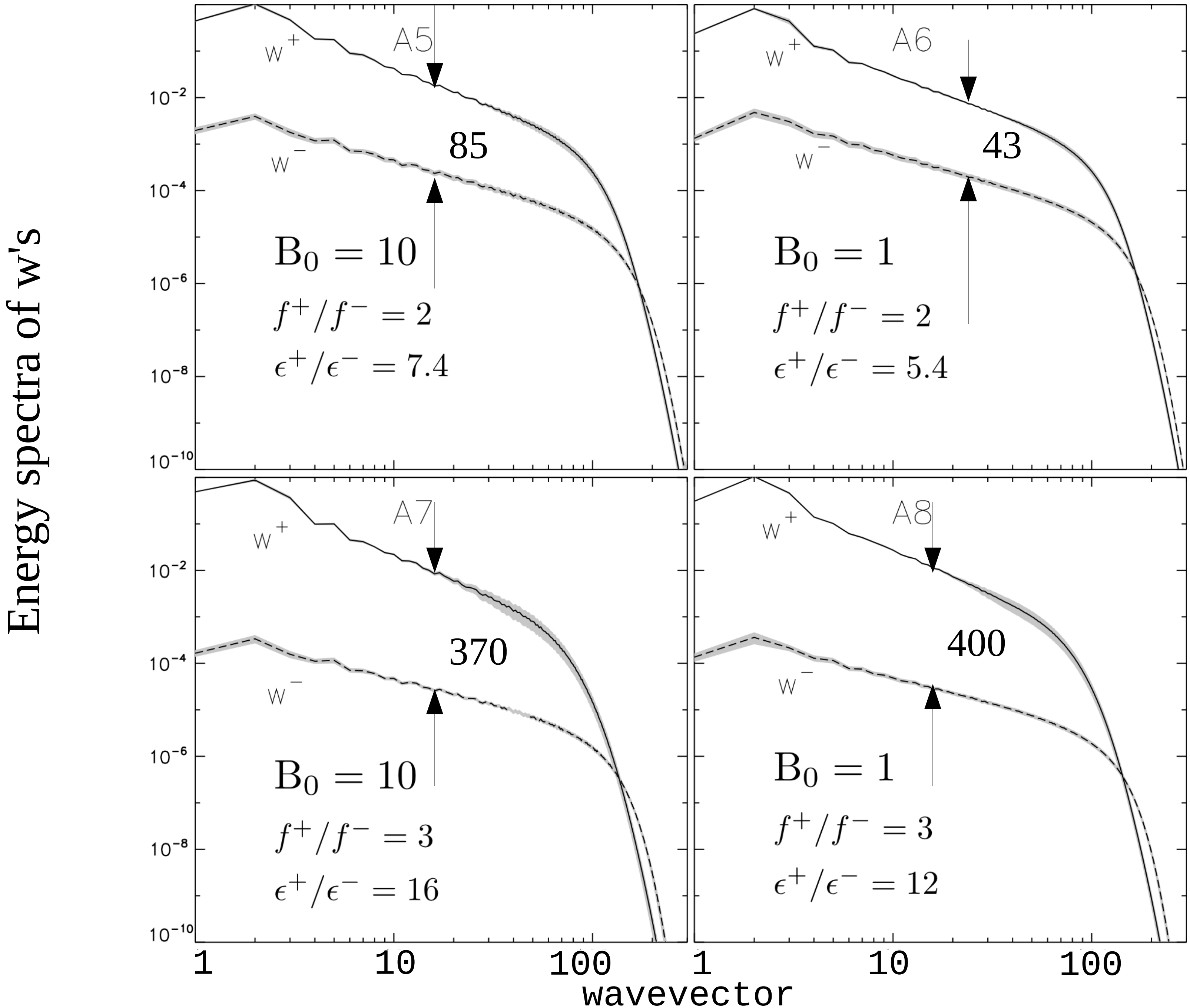



0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
0.001
0.010
0.100

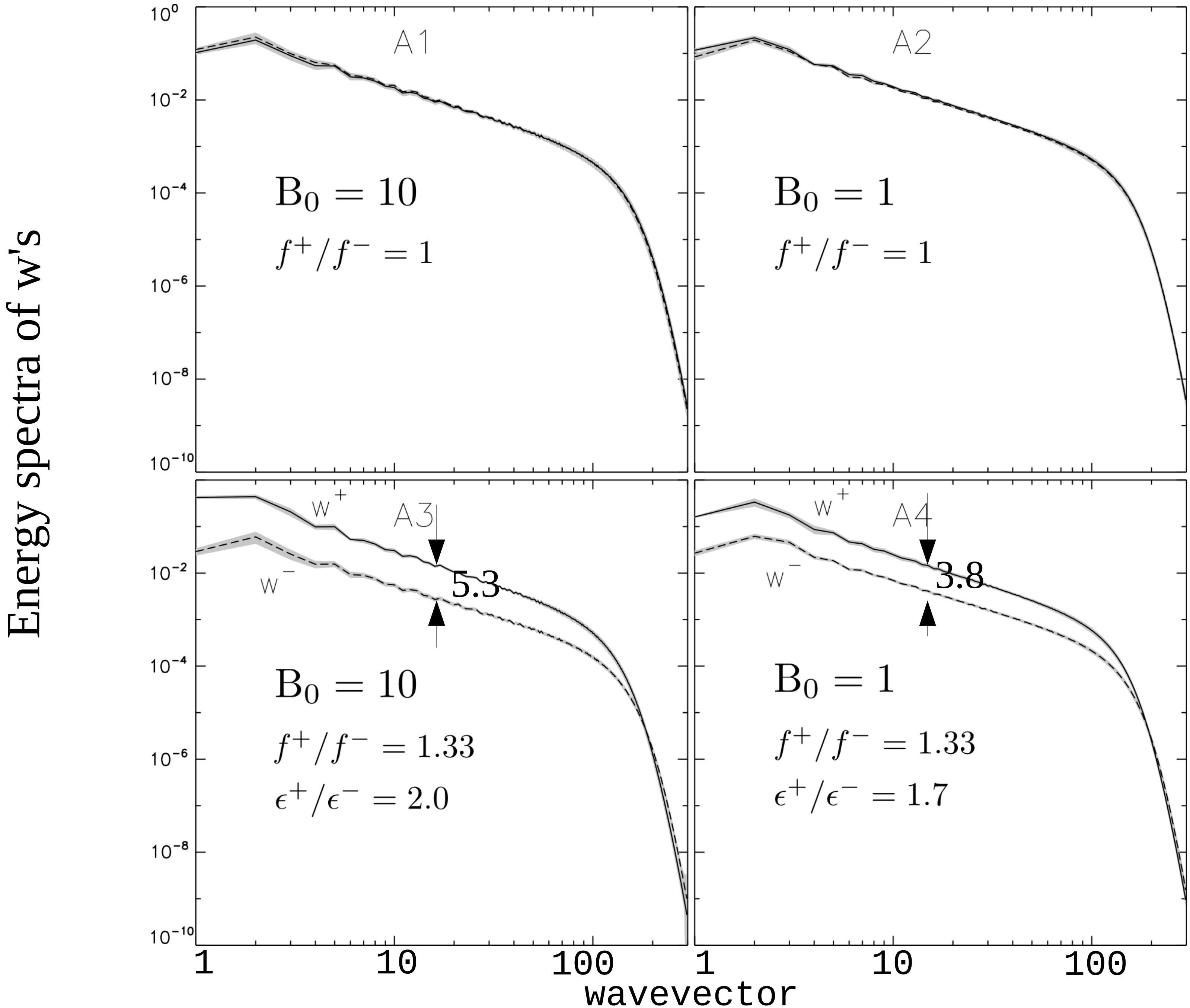
Numerical simulations



Pseudospectral code solves MHD/hydro/RMHD equations on a grid, three-dimensional, stationary driven turbulence, explicit dissipation, up to 3072^2x1024 (balanced), 1536^3(imbalanced).

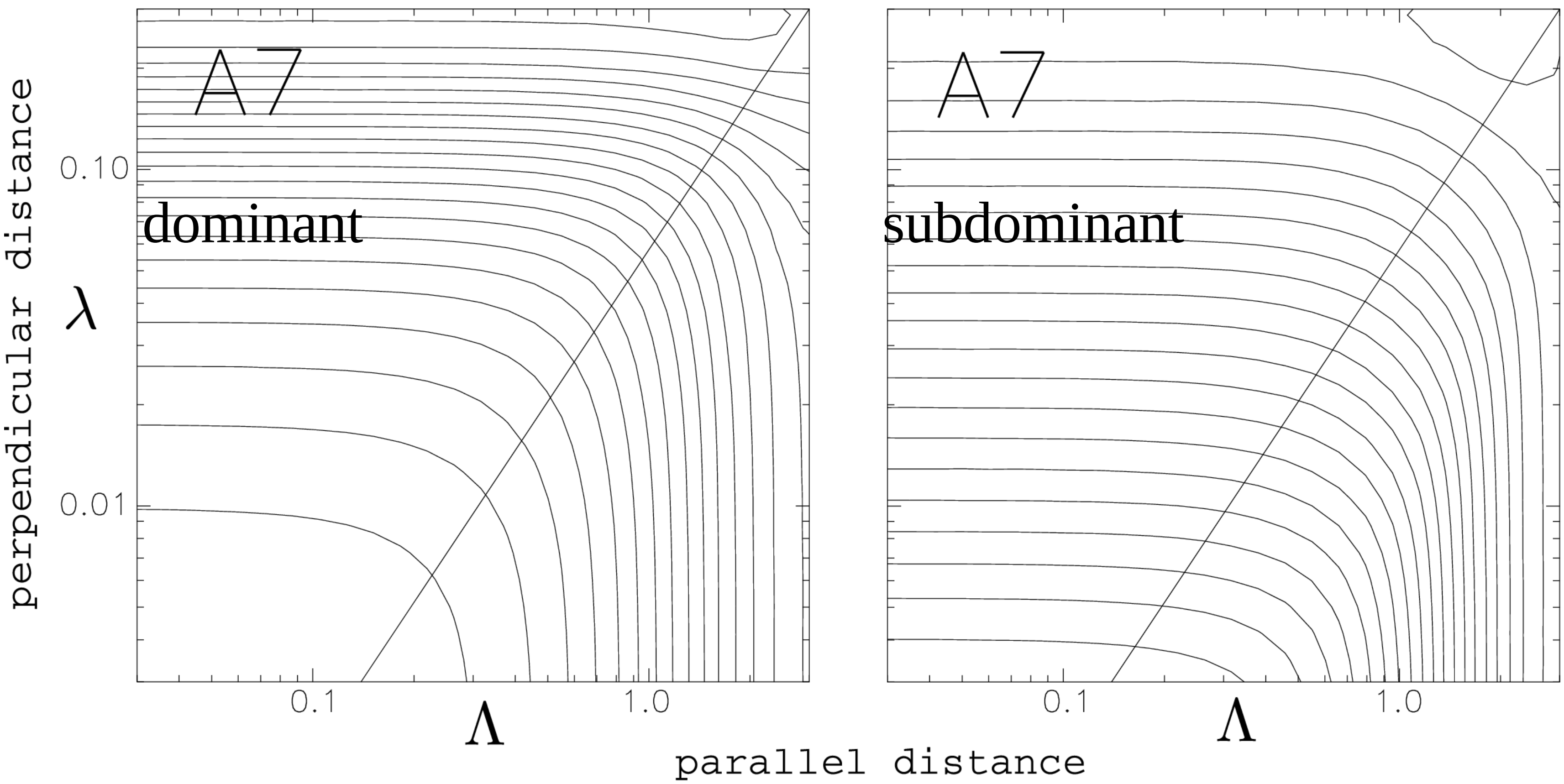


Energy spectra of w's
wavevector
A5
A6
A7
A8
w+
w−
85
43
370
400
$B_0 = 10$
$f^+/f^- = 2$
$\epsilon^+/\epsilon^- = 7.4$
$B_0 = 1$
$f^+/f^- = 2$
$\epsilon^+/\epsilon^- = 5.4$
$B_0 = 10$
$f^+/f^- = 3$
$\epsilon^+/\epsilon^- = 16$
$B_0 = 1$
$f^+/f^- = 3$
$\epsilon^+/\epsilon^- = 12$
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1
10
100



Energy spectra of w's
wavevector
A1
$B_0 = 10$
$f^+/f^- = 1$
A2
$B_0 = 1$
$f^+/f^- = 1$
A3
w^+
w^-
5.3
$B_0 = 10$
$f^+/f^- = 1.33$
$\epsilon^+/\epsilon^- = 2.0$
A4
w^+
w^-
3.8
$B_0 = 1$
$f^+/f^- = 1.33$
$\epsilon^+/\epsilon^- = 1.7$

imbalanced



A7
0.10
dominant
0.01
λ
perpendicular distance
0.1
1.0
Λ
A7
subdominant
0.1
1.0
Λ
parallel distance

Imbalanced turbulence