The non-idealness on Ideal MHD

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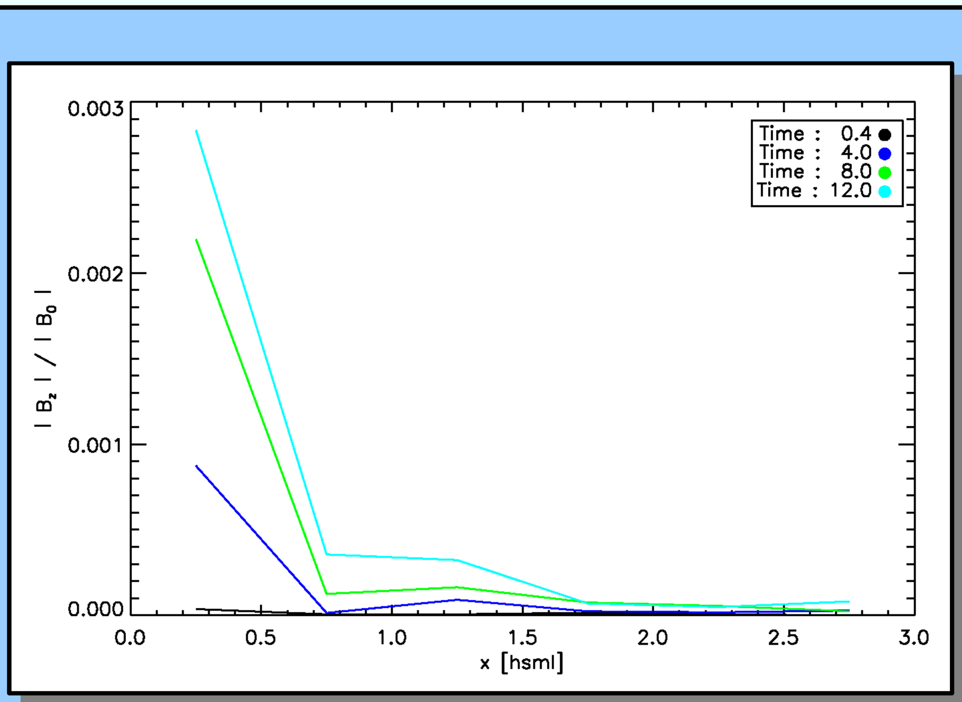
Astrophysical numerical Ideal MHD simulations have proven to successfully model a variety of astrophysical objects. However, the Ideal MHD approximation used by any numerical simulation is not valid bellow the resolution limit of the numerical representation. This also holds by any other hydrodynamical equation that we evolve in our numerical schemes, with the exception of the use of an additional *sub-grid* model to follow properties below the resolution. Particularly, in the case of Ideal MHD, that numerical artifacts that happen below the resolution limit can act as dissipative terms in the induction equation. We propose to calculate those dissipation numbers. In particular in the **SPMHD** implementation in **Gadget-3**, we are able to use different regularization/*div*(B) cleaning schemes. Allowing us to determine, with a full understanding the numerical limits, their effects and possible mimic as turbulent dissipation.



Reconnection Layer

One of the easiest tests that one can imagine to measure diffusive effects is to think situations in which we can push the system to naturally reconnect. Therefore we build a shock tube in which the left and right states, simply differ in the directions of the magnetic field and with a contracting velocity. Explicitly **B** = [0,0,+/- Bo] and **V** = [+/- Vo,0,0].
Assuming that the effect below the resolution will follow a resistive functional form, we can calculate the final induction equation in the contact discontinuity region as

$$\frac{\Delta B_z}{\Delta T} = -\eta_m \frac{4 B_0}{h^2}$$

Note that the terms corresponding to the cross product of the velocities and magnetic field are cancelled. The shock tubes were setted up using a glass particle distribution in a volume of 70x1x1, having a mean *Hsml* ~ 0.5.

In the upper figure we show the difference of the Bz component of the magnetic field renormalized to the corresponding value is is just taken an adiabatic compression of the field, as function of the distance from the contact discontinuity, for different times. One can observe how the difference increases with time, which corresponds to the acting numerical resistivity.

In the lower table we show the values found of the magnetic Reynolds number and the numerical resistivity. Note that the *Rm* varies two orders of magnitude between the regularisation schemes to *Dedner* and *Standard* implementations however the resistivity only varies one order.

Test	Hsml	v_a	η_m	Re_m
Standard	0.50	$2.8e-1$	$1.8e-6$	$7.8e+4$
Dedner	0.50	$2.8e-1$	$4.8e-6$	$2.9e+4$
B-Smooth #15	0.46	$1.2e-2$	$1.7e-5$	$3.2e+2$
B-Smooth #30	0.47	$1.3e-2$	$1.5e-5$	$4.1e+2$
Art. Diss. $\alpha_B = 0.1$	0.49	$3.2e-2$	$6.3e-5$	$2.5e+2$
Art. Diss. $\alpha_B = 0.5$	0.51	$1.3e-2$	$2.0e-5$	$3.3e+2$

Numerical Schemes

We make use of 4 different numerical schemes implemented in **Gadget-3** [see, Dolag2009]. They have been proved to have distinct properties and handle different problems successfully.

•Standard: this is the implementation with the basic improvements of MHD equations, as defined in Dolag2009.

•*div*(B) cleaning: follows the div(B) errors and subtracting them from the induction equation. Is locally driven (without any smearing of features) and generally improves stability.

$$\frac{\Delta B}{\Delta T} = -\nabla \phi$$

•Artificial Dissipation: this implementation is referred as a regularization schemes that is meant to stabilize the code artificially using a similar formulation as the physical dissipation.

$$\frac{\Delta B}{\Delta T} = \eta_M \nabla^2 B$$

•B-Smoothing: this regularization technique smoothes the field each N global timesteps. This regularization of the field smears out features, and as been not physically motivated can lead to spurious over-smoothing of the field.

$$\hat{B} = \frac{\Sigma B w}{\Sigma w}$$

Resolution Considerations

Additionally we study the problems in several resolutions. Surprisingly the Magnetic Reynolds number keeps constant for Standard implementation and the *div*(B) cleaning scheme, whereas for the regularization schemes it decreases.
This can be understood in the terms that the diffusive approach of the regularization schemes keeps constant as independent of the resolution. In the Art. Dissipation case because we use a constant value, whereas in the Magnetic field smoothing because we keep constant the smoothing frequency.
This will imply that the regularization schemes will converge rapidly with increasing resolution, because the numerical resistivity acts as a physical one. In the other cases, one expects to resolve better the turbulent cascade allowing to the magnetic field to grow faster [see Alexander Beck poster].

Introduction

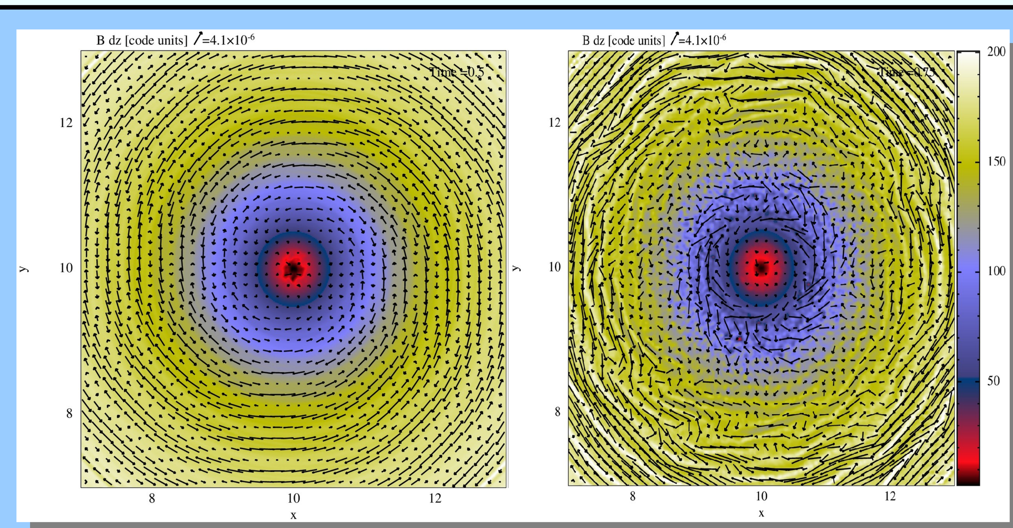
Simulations have been able to reproduce quite well several astrophysical MHD situations (see, Kotarba2010, Dolag2009 & Sur2010). Those simulations rely only in Ideal MHD regime (no explicit physical diffusion considered). An effective dynamo action requires the non-ideal MHD terms to be considered, however is not usually the case. Therefore in the numerical schemes some reconnection is being done. Actually numerical codes are meant to be accurate in scales larger that in intrinsic numerical resolution that each implementation is able to resolve. Inside those scales (i.e. below the resolution), the codes can break, in our case, the Ideal MHD constraint and allow the code to reconnect the field lines in some undefined manner.

Depending in the numerical scheme used, this effect can be larger or smaller, however generally is neglected just not considering those scales. Gracefully, we can explain physically this numerical effect.
Is commonly accepted (Ruzmaikin1988) that the magnetic field is coupled with the turbulence of the system. In the same way as the Kolmogorov turbulent theory, the coherence length of the magnetic turbulence energy scales from larger to smaller ones.

At small scales (in our case the resolution of our simulation), the ideal MHD constraint is not hold any more because the MHD equations are resolved between interpolants that, by definition, lay outside that length. Usually, when is needed to follow small scale physics, a sub-grid model is added to the schemes using additional physical recipes to evolve processes that should occur inside the resolution length.

We propose to estimate how much is the numerical dissipation in numbers, therefore being be able to estimate in which regimes our equations are valid.

Particularly we study this within the SPMHD implementation in Gadget-3, which offer us different regularization schemes and div(B) cleaning schemes (See Numerical Schemes in this poster).

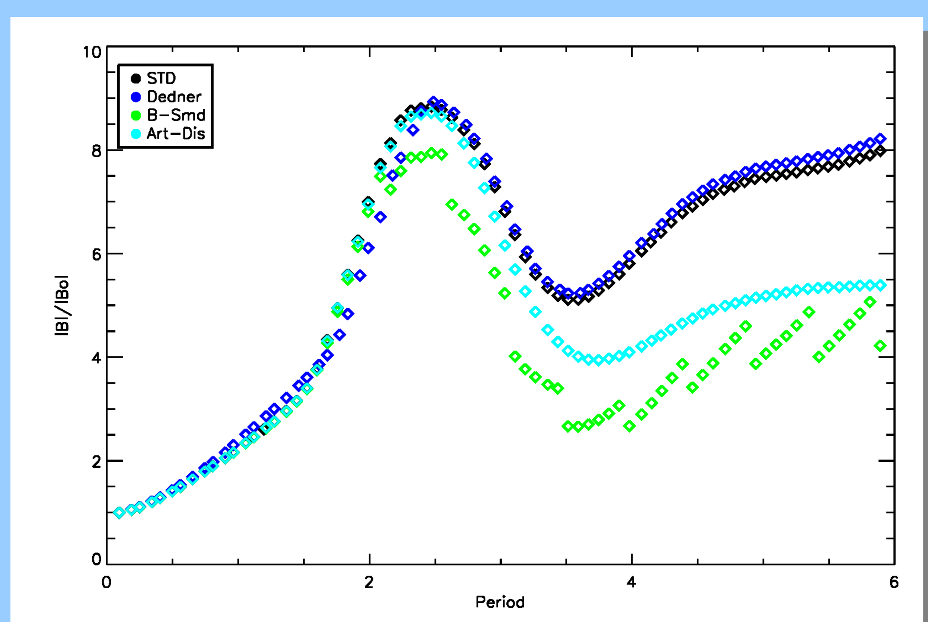


Winding-up Disk

When magnetic field lines parallel to the plane of rotating disk evolve, they wind-up until reconnection happens and the central region is isolated from the external magnetic field [Parker1979, Weiss1966]. This reconnection will be function of the number of windings and the magnetic resistivity of the system. After the reconnection and isolation of the center, there is a decay in the total magnetic field inside the disk.

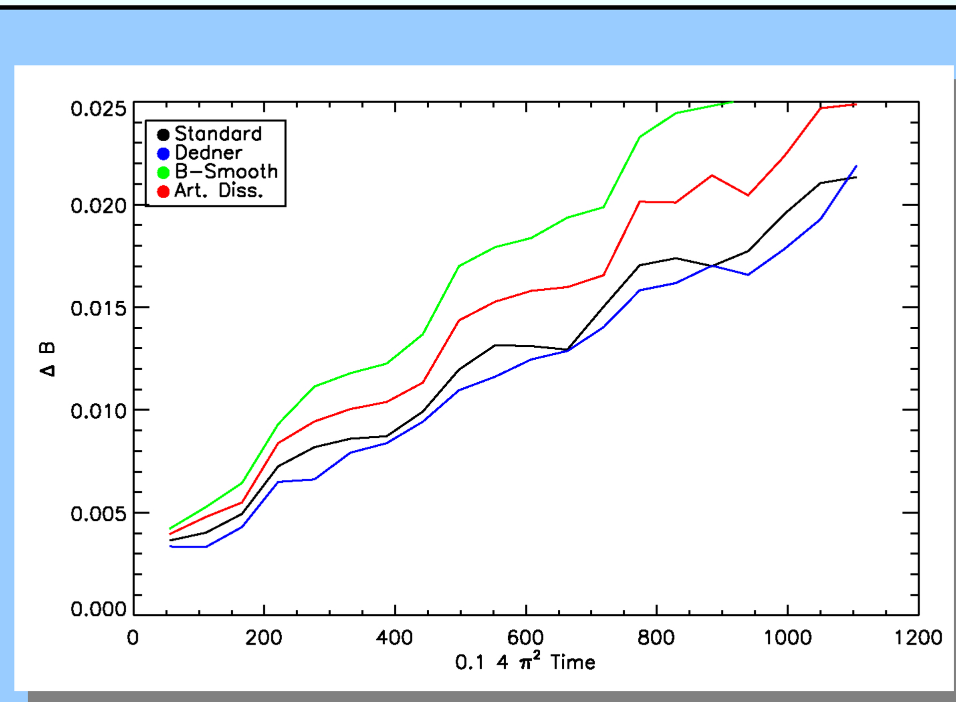
This is shown in the upper figures. We show in color the velocity strength and the magnetic field vector for two different times, before (right) and after (left) the reconnection occurs. In particular the maximum magnetic field reached before the reconnection occurs is given by

$$Re_m = \left(\frac{B_{max}}{B_0}\right)^3$$



The figure above shows the mean value of the magnetic field inside the disk, and how does it evolve until is damped by the reconnection, for the different implementations cases. Therefore, finding the peak in the magnetic field evolution we can calculate the magnetic Reynolds number, independent on any assumption on the characteristic length or velocity of the system. The resulting values are shown in the table below.

Test	Re_m
Standard	691
Dedner	713
B-Smooth #15	502
Art. Diss. $\alpha_B = 0.1$	613



Circularly polarized Alfven Wave

Toth2000 described this test and it is used to verify a several MHD Eulerian schemes. The test involves a circularly polarized Alfven wave (CPAW) propagating into the domain which solution turns out to be exact solution of the MHD equations (as difference of the linearly polarized case). The solution after one period should exactly match the initial conditions, if there is no presence of any kind of dissipative process.

The particles setup are *glass*-like distribution, within a completely three dimensional setup. The initial conditions are Rho=1, P=0.1, Vx = 0, Bx = 1, Vy = By = 0.1 sin(2 Pi x) and Vz = Bz = 0.1 cos(2 Pi x). Solving analytically the induction equation for the initial condition in the non-ideal case, we conclude that at first approximation in each period an additional, to the ideal case, term survives in the Z-component and it can be written as

$$\frac{\Delta B_z}{\Delta T} = -0.1\, \eta_m 4 \pi^2$$

Therefore, we can measure the change in the amplitude of the wave in the Z-component in different times and estimate a value for the numerical resistivity.

In the upper figure we shown change of the amplitude for the Z- component of the magnetic field as function of time for the different numerical implementations. We scale the time evolution with the proper factor so the slope corresponds to the numerical dissipation constant. In the lower table we summarize the values obtained for this test.

Test	η_m	Re_m
Standard	$1.69e-5$	$7.1e+3$
Dedner	$1.67e-5$	$7.2e+3$
B-Smooth #15	$2.42e-5$	$4.9e+3$
Art. Diss. $\alpha_B = 0.1$	$2.00e-5$	$6.0e+3$

Conclusion

All numerical schemes rely on different kinds of discretizations of the physical system. How this affects the result is a complex task and should be carefully taken into account. We concentrate our efforts in the case of Ideal MHD and the effects that happen below the resolution. We identify them as resistivity terms from Non-Ideal MHD. Note that there is no strict reason to be so, because this effect should be completely numerical and does not necessary have to obey any kind of dissipation equation. However, the interpretation as resistivity allow us to understand previous successful astrophysical applications. Solving the induction equation for simple test cases we identify terms that exists in the Non-Ideal case, being able to measure the numerical resistivity and numbers for the magnetic Reynolds number.

We study this effect in tree different tests. In two of them, Reconnection Layer and CPAW, we need to decide which will be our characteristic speed and length. Doing so we infer magnetic Reynolds number that in general are higher than 5e+3. The Winding-up disk problem allow us to infer this value without any additional assumption, giving us values of the order 5e+2. When we translate this values into physical turbulent resistivity we obtain values between 1e+20 to 1e+22 [cm^2/s] wich are below to estimated for turbulent resistivity [see, Lesch2003, Bonafede2011]. We believe that this explains the reconnection happening by turbulent motions in previous works [Kotarba2010, Dolag2009]. Note that this effect is quite complex, because is determined locally by the morphology of the field and characteristic lengths compromised, as is shown in the disagreement between the Winding-up disk and the other problems suggested. However, the global system still is characterized by Ideal MHD.

Finally, we show that some schemes keep the Magnetic Reynolds number with different resolution, whereas others keep the magnetic diffusivity. The latter means that will act as a physical diffusivity term, allowing to a quicker convergence with increasing resolution.

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