

# The regime of two off-phase waves of the background magnetic field in a solar cycle and predictions of Parker dynamo.

*Popova H. (1), Zharkov S.I. (2) and Zharkova V.V. (3)*

*1 – Faculty of Physics, Moscow State University, Russia*

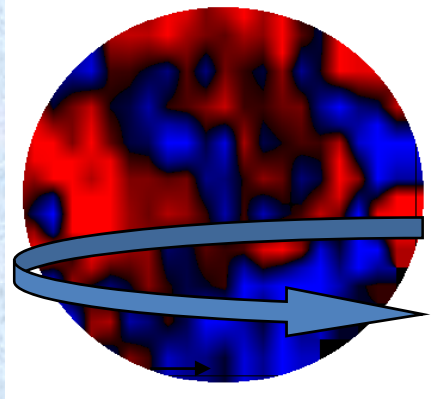
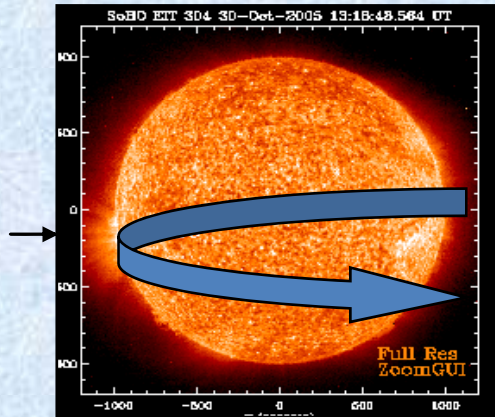
*2 – Mullard Space Science Laboratory, University College London, UK*

*3- Department of Mathematics, University of Bradford, UK*

## **ABSTRACT**

The variations in latitude and time of the solar background and sunspot magnetic fields in the cycle 21-23 are analyzed with the Principle Component Analysis technique. We identified the two main latitude periodical components of the opposite polarities reflecting two primary waves of the background magnetic field in each hemisphere travelling slightly off-phase. We built the latitudinal distribution for these waves and study the phase relations between the weak background solar magnetic (poloidal) field and strong magnetic field associated with sunspots (toroidal) field. We compare the results obtained from several modifications of Parker dynamo theory with the characteristics of derived waves and discuss their implications on the solar activity.

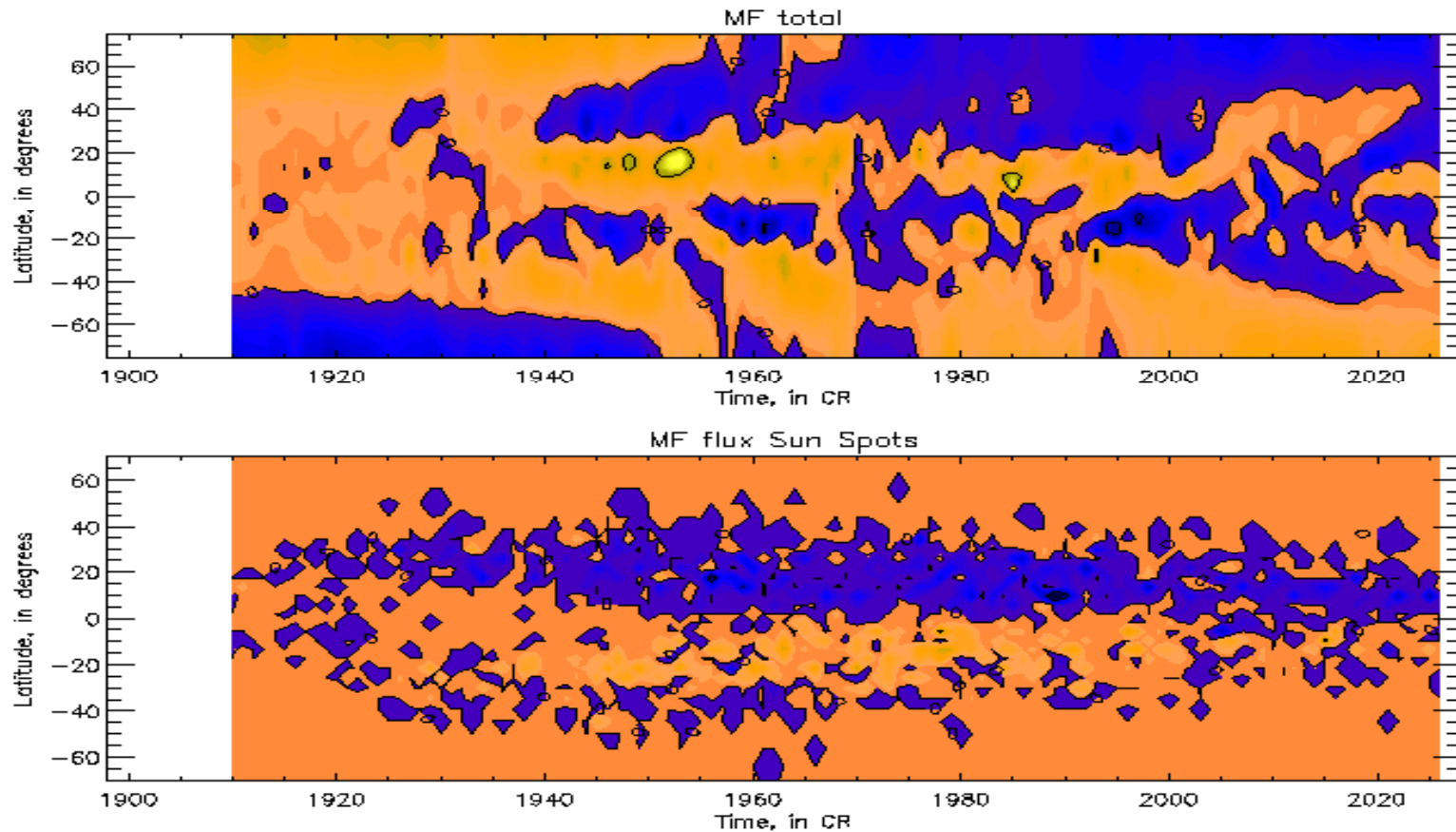
# Data analysis



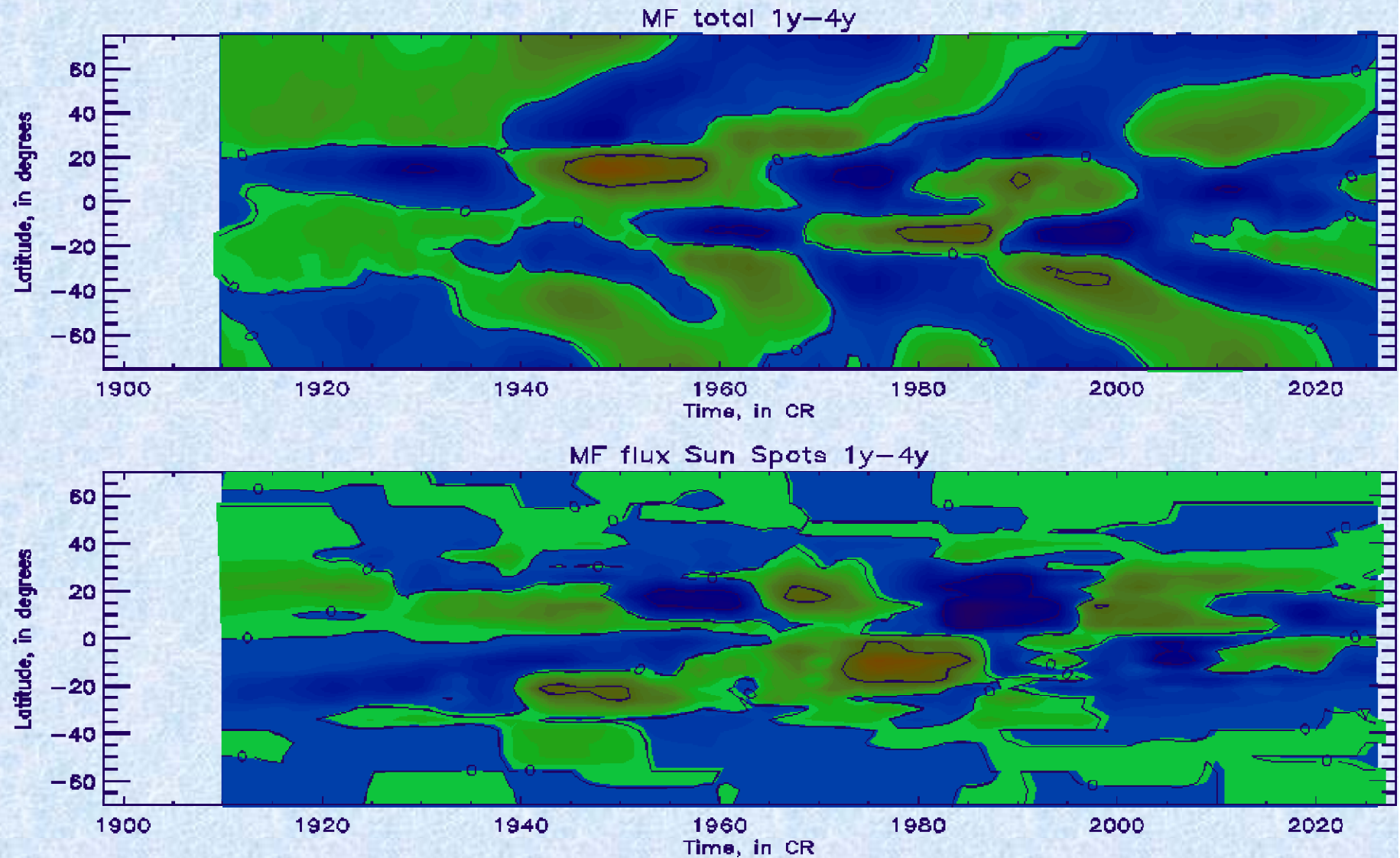
Solar Geophysical Data

- High resolution MF (SOHO/MDI)
  - sunspot magnetic field – SMF
  - sunspot areas
- Automated detection, CM data from Solar Feature Catalogues
- Low resolution MF (WSO) – synoptic maps of BMF

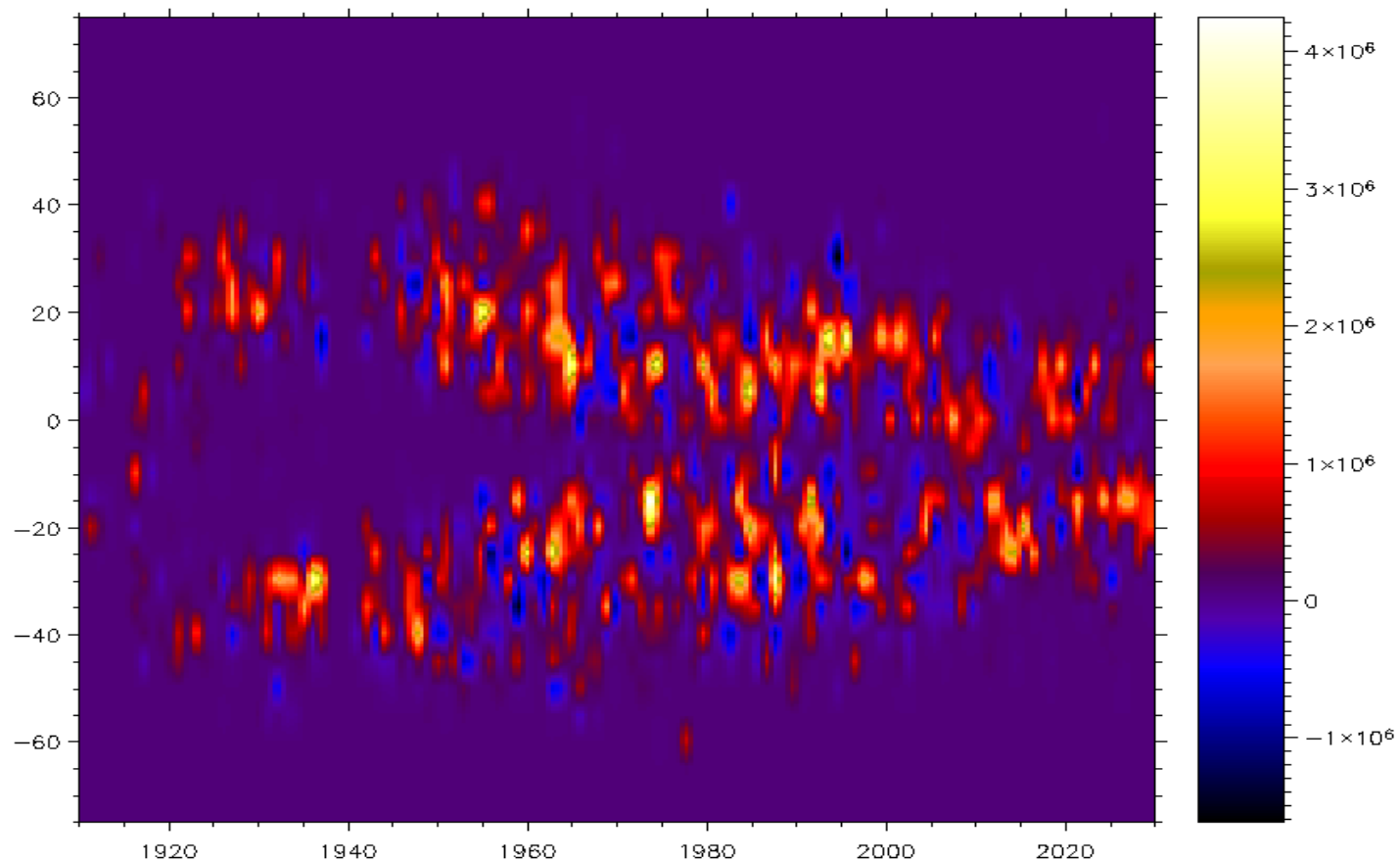
**BMF (top) and excess SMF (bottom) by 1 year – phase  
between PF and TF is  $\pi/2 \sim 11$  (Stix, 1986)  
(blue - negative, orange – positive polarities)**



**1y-4y residuals for BMF (top) and excess SMF (bottom)  
blue –negative excess above mean, green –positive**

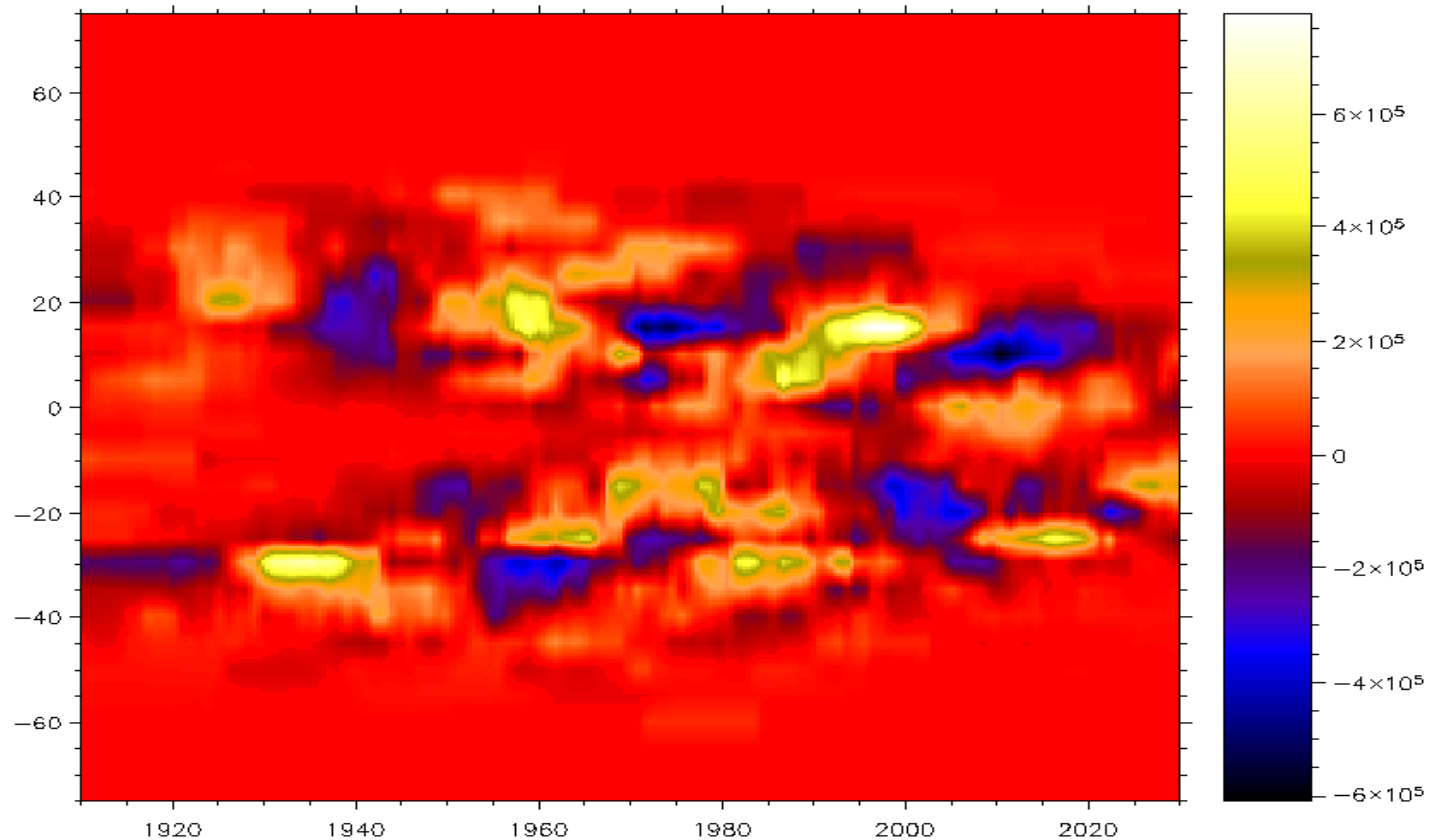


# Excess magnetic flux in sunspots (ESMF) vs latitude



# Excess magnetic field of sunspots (SMF) vs latitude averaged over 1 year (blue – negative excess, yellow – positive)

shows fine periodical patterns with  $\Delta t \sim 2.5\text{-}3$  years

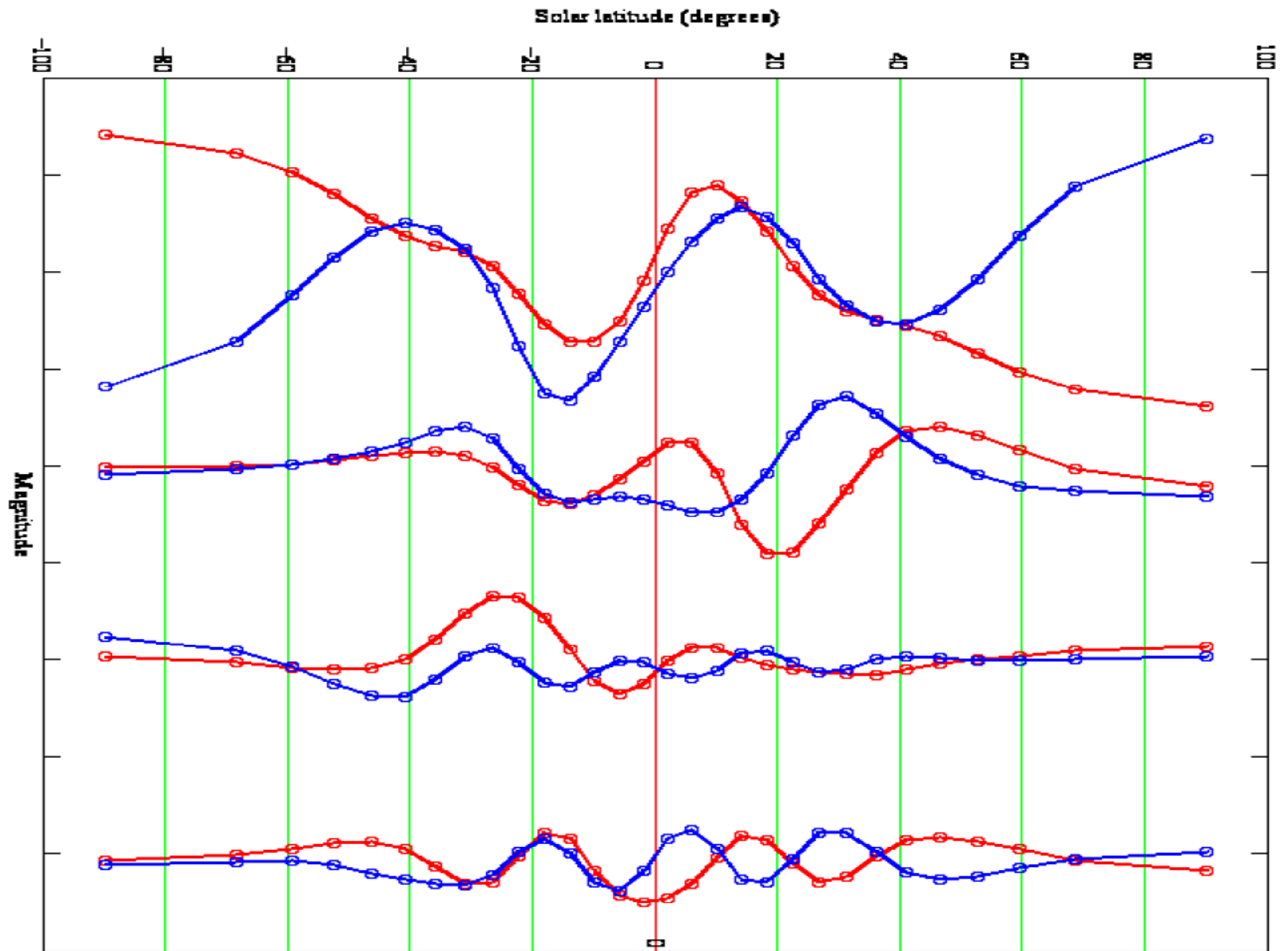


# Principal Component Analysis

In order to derive the main periods present in the observational data of solar activity: sunspot areas and sunspot magnetic fields at given latitudes affected by various processes (dynamo waves, meridional flows, torsional oscillations etc.), let us apply the Principal Component Analysis (PCA) and derive the strongest eigenfunctions from the available data.

PCA is the orthogonal linear transformation allowing a vector space of  $N$  observations  $\vec{X}_N$  each containing  $M$  points to be transformed to a new coordinate system reducing the multi-dimensional data  $X(M \times N)$  to lower dimensions for analysis, so that the greatest variance by any projection of the data lies on the first coordinate called the principal component.<sup>18</sup> The eigenvalues and eigenvectors  $\vec{V}$  are paired; the eigenvectors  $\vec{V}$  with the highest eigenvalues are the principal components of the original matrix  $X$ .

The observations of BMF (background magnetic field) were carried out from the beginning of the Carrington Rotation 1642, up to the year 2005.16, or CR 2027. The data covers the solar activity cycles 21, 22, and 23. In the current study, we consider the averaged magnetic field in the cycles 21-23 only within the  $30 \mu$ -hemispheres in the heliographic latitude sines (from  $75.2^\circ$  North to  $75.2^\circ$  South) averaged in heliographic longitude within the latitudinal strip of  $5^\circ$ .





**The 2 main eigenvalues of the latitudinal BMF distribution in the total set of cycles 21-23 (upper plot), in the separate sets for the cycle 21, cycle 22 and cycle 23 (from the top to the bottom, respectively).**

**We derived the two main latitude components in sunspot magnetic flux which are rather different in the Northern and Southern hemispheres that points out to a presence of the two waves travelling in each hemisphere (possibly, in the opposite directions) or having the opposite polarities. These waves are modulated by the two primary waves of the background magnetic field where the maximums in sunspot components correspond to the minimums in the BMF components.**

## PARKER DYNAMO

$$\mathbf{B}_P \xrightarrow{\Omega} \mathbf{B}_T$$

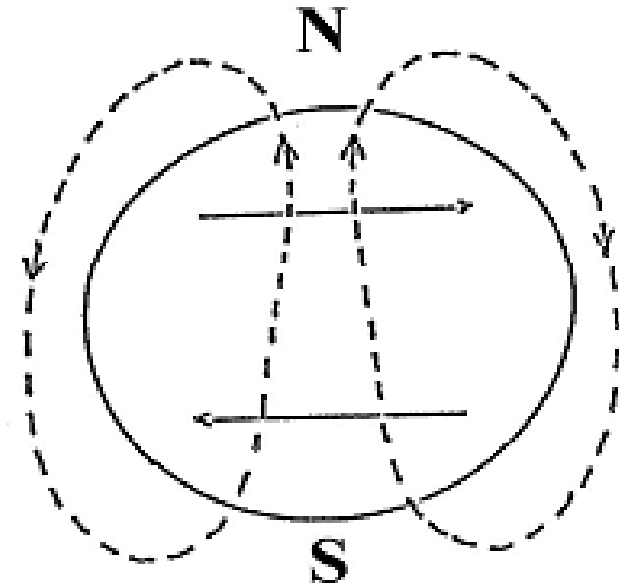
Differential rotation

$$\mathbf{B}_T \xrightarrow{\alpha} \mathbf{B}_P$$

Helicity

Parker presented a functional scheme for such dynamo as follows.

A toroidal magnetic field is produced from the poloidal field by the action of differential rotation. The inverse process of transforming toroidal magnetic field into poloidal field is realized by the action of alpha-effect.



# DYNAMO EQUATIONS

$B$  is the toroidal magnetic field

$A$  is proportional to the toroidal component of the vector potential, which determines the poloidal magnetic field

$\theta$  is the latitude measured from the equator

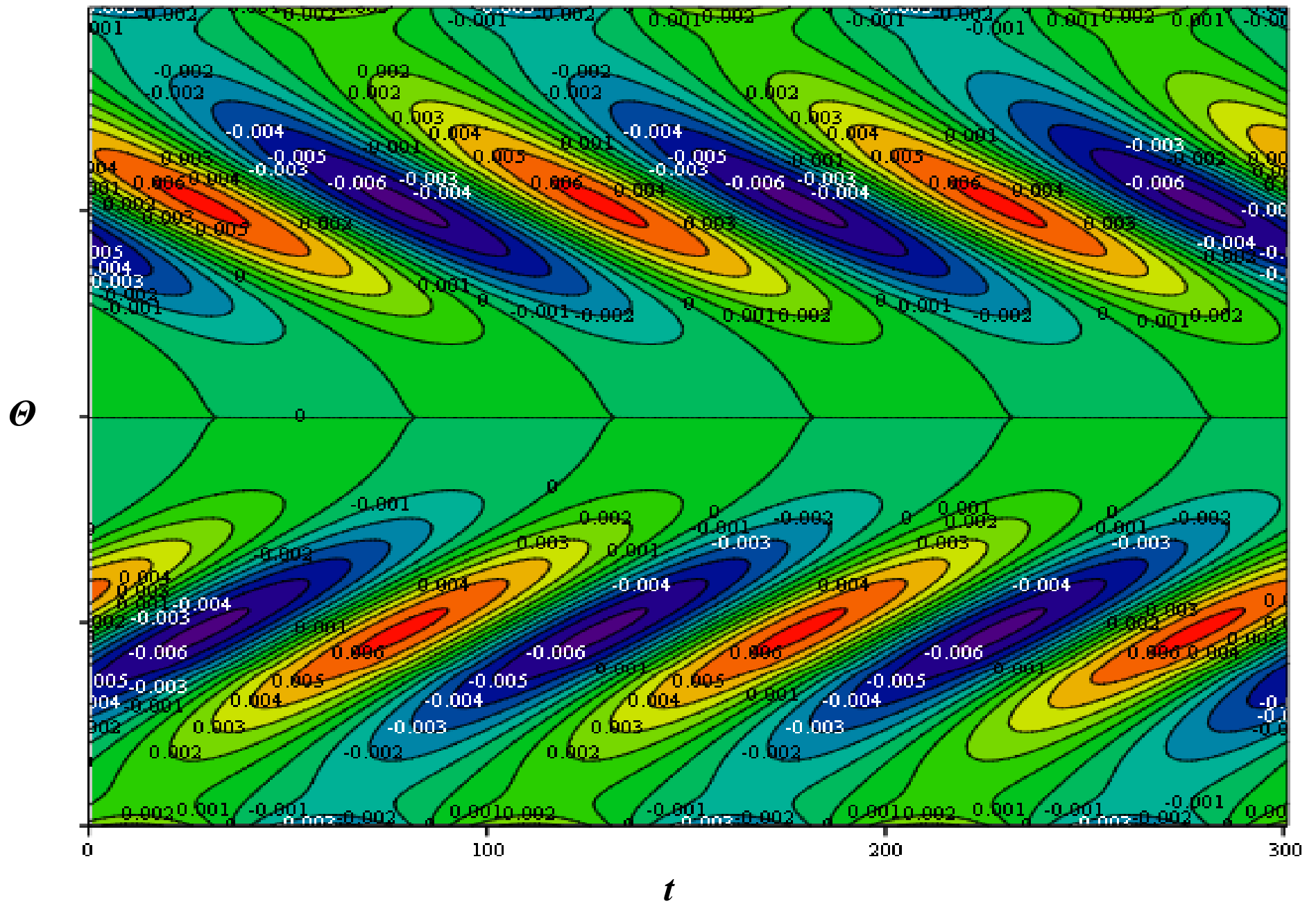
$D$  is dynamo number

$|D| \gg 1$ ,  $V$  – meridional circulation

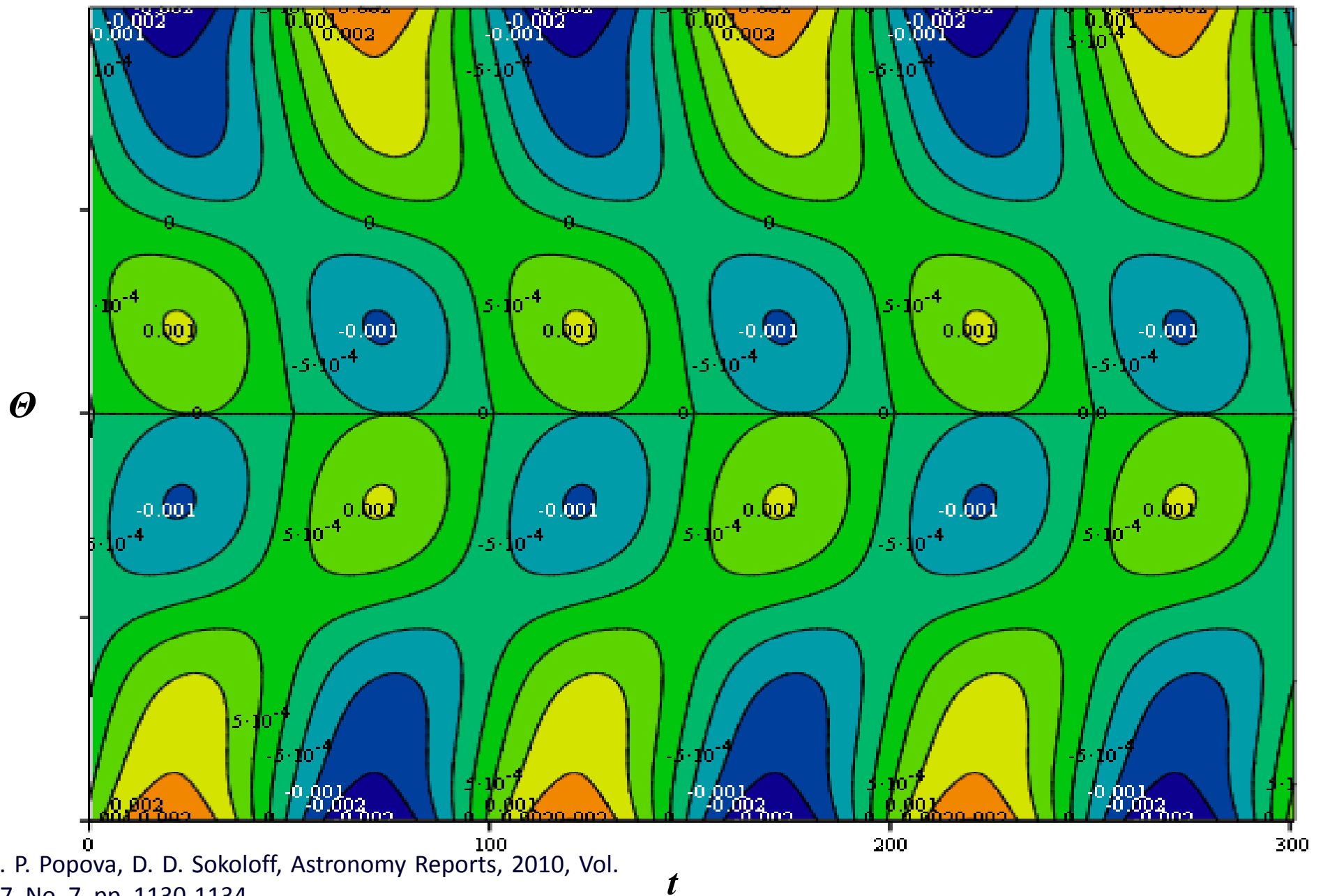
$$\frac{\partial A}{\partial t} + V(\theta) \frac{\partial A}{\partial \theta} = \alpha B + \frac{\partial^2 A}{\partial \theta^2}, \quad (1.1)$$

$$\frac{\partial B}{\partial t} + \frac{\partial(V(\theta)B)}{\partial \theta} = D \cos \theta \frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2}.$$

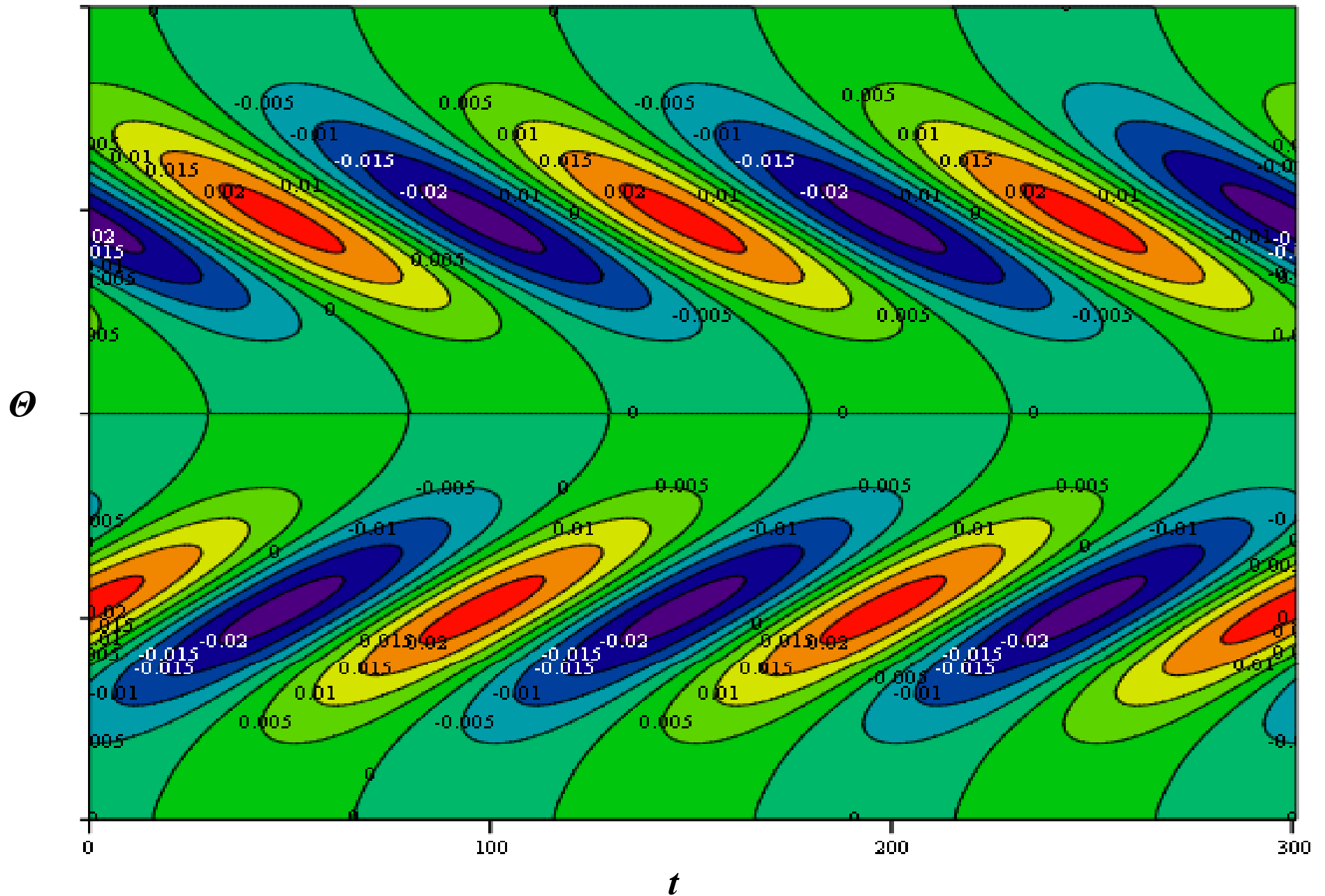
# Poloidal magnetic field, $D = -10000$ , $V = 0.5$ , simulations



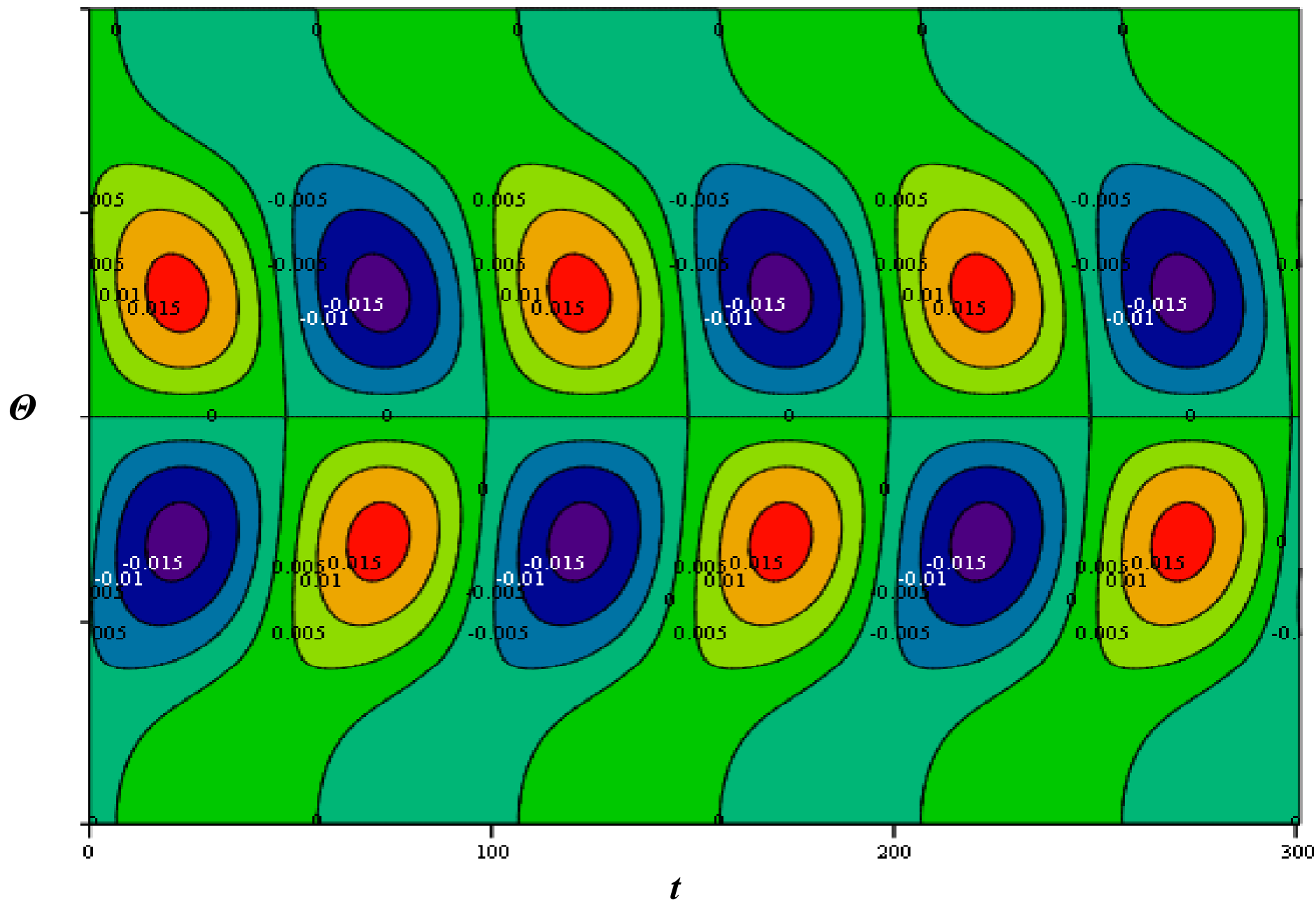
## Poloidal magnetic field, $D = -100$ , $V = 0.5$ , simulations



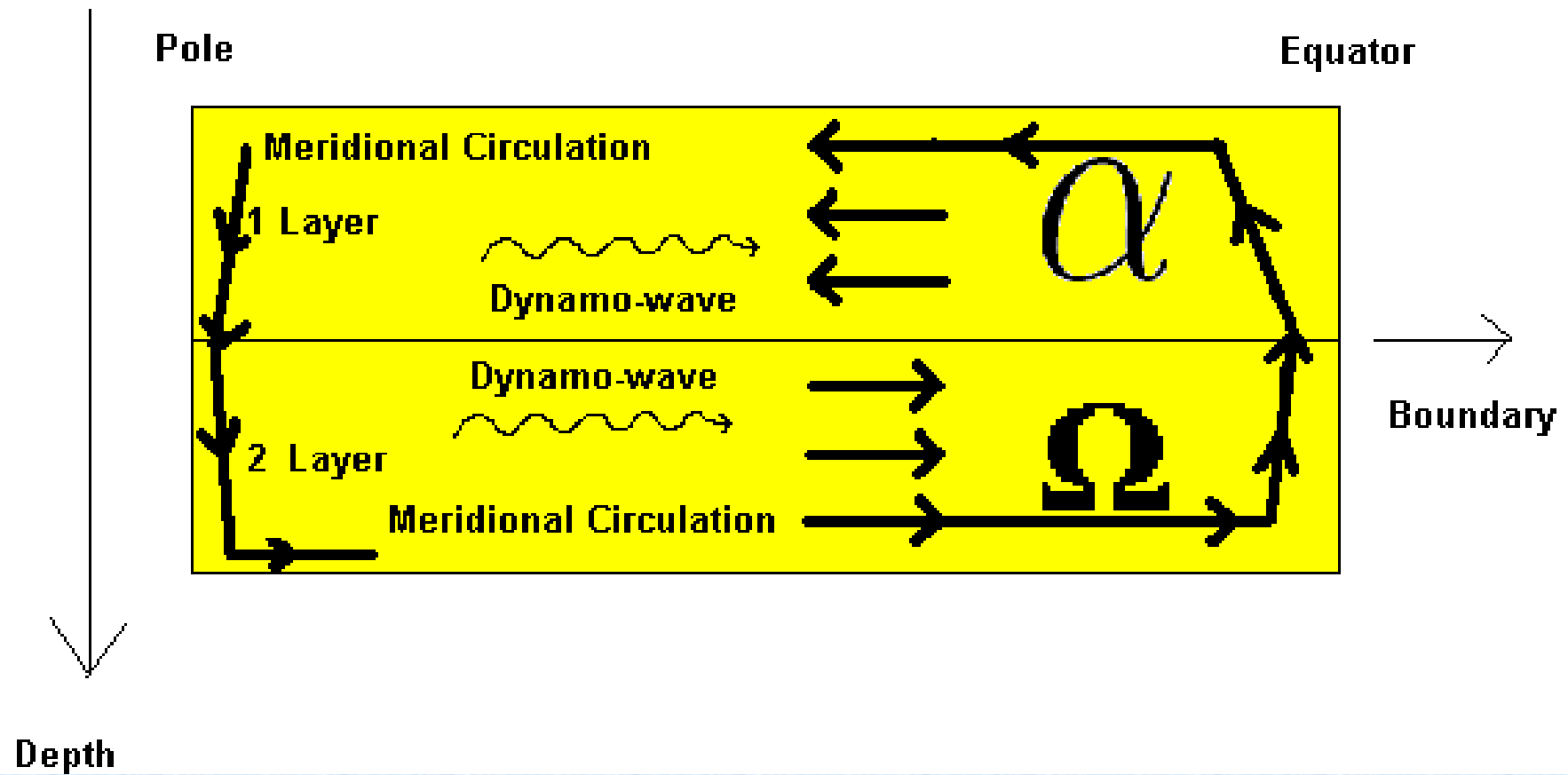
# Toroidal magnetic field, $D = -10000$ , $V = 0.5$ , simulations



# Toroidal magnetic field, $D = -100$ , $V = 0.5$ , simulations



# Dynamo in Two-Layer Medium





Parker (1993) proposed the following governing equations

$$\frac{\partial B}{\partial t} = \beta \Delta B, \quad \frac{\partial A}{\partial t} = \alpha B + \beta \Delta A, \quad (2.1)$$

$$\frac{\partial b}{\partial t} = D \cos \theta \frac{\partial a}{\partial \theta} + \Delta b, \quad \frac{\partial a}{\partial t} = \Delta a \quad (2.2)$$

for a dynamo with the  $\alpha$ -effect present in one radial layer and shear of differential rotation present in the other. Here  $\beta$  is the ratio of the diffusion coefficients in first and second layers,  $B(r, \theta, t), b(r, \theta, t)$  are the corresponding toroidal magnetic fields in these layers, and  $A(r, \theta, t), a(r, \theta, t)$  are proportional to the corresponding toroidal components of the vector potential (which determines the poloidal magnetic field). Parker assumed the differential rotation to dominate in one layer and to vanish in the other, and, conversely, the  $\alpha$ -effect to prevail in the second layer and to vanish in the first.

We included the meridional flows in each layer:

$$\frac{\partial B}{\partial t} + \frac{\partial(VB)}{\partial \theta} = \beta \Delta B, \quad \frac{\partial A}{\partial t} + V \frac{\partial A}{\partial \theta} = \alpha B + \beta \Delta A, \quad (2.3)$$

$$\frac{\partial b}{\partial t} + \frac{\partial(vb)}{\partial \theta} = D \cos \theta \frac{\partial a}{\partial \theta} + \Delta b, \quad \frac{\partial a}{\partial t} + v \frac{\partial a}{\partial \theta} = \Delta a, \quad (2.4)$$

here  $V(\theta)$ ,  $v(\theta)$  are the meridional flows in the respective layers.

Following Parker we prescribe  $r = 0$  for the radial boundary between two layers and use boundary conditions:

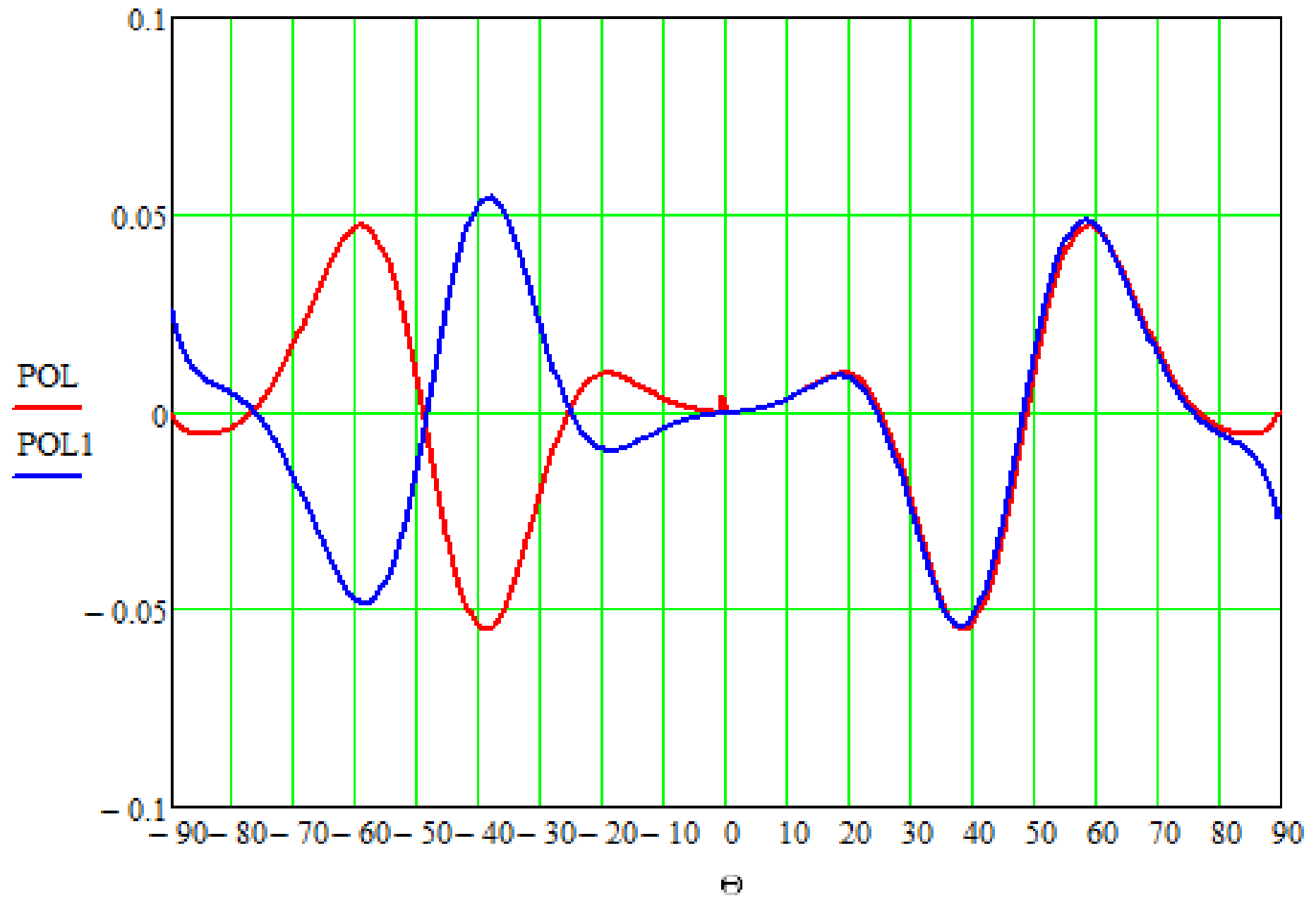
$$b = B, \quad a = A, \quad \frac{\partial b}{\partial r} = \beta \frac{\partial B}{\partial r}, \quad \frac{\partial a}{\partial r} = \frac{\partial A}{\partial r}. \quad (2.5)$$

In view of the symmetry conditions  $\alpha(-\theta) = -\alpha(\theta)$ ,  $V(-\theta) = -V(\theta)$  the above system of equations can be considered in only one (e.g., the northern) hemisphere using anti-symmetry (dipolar symmetry) or symmetry (quadrupolar symmetry) conditions at the equator.

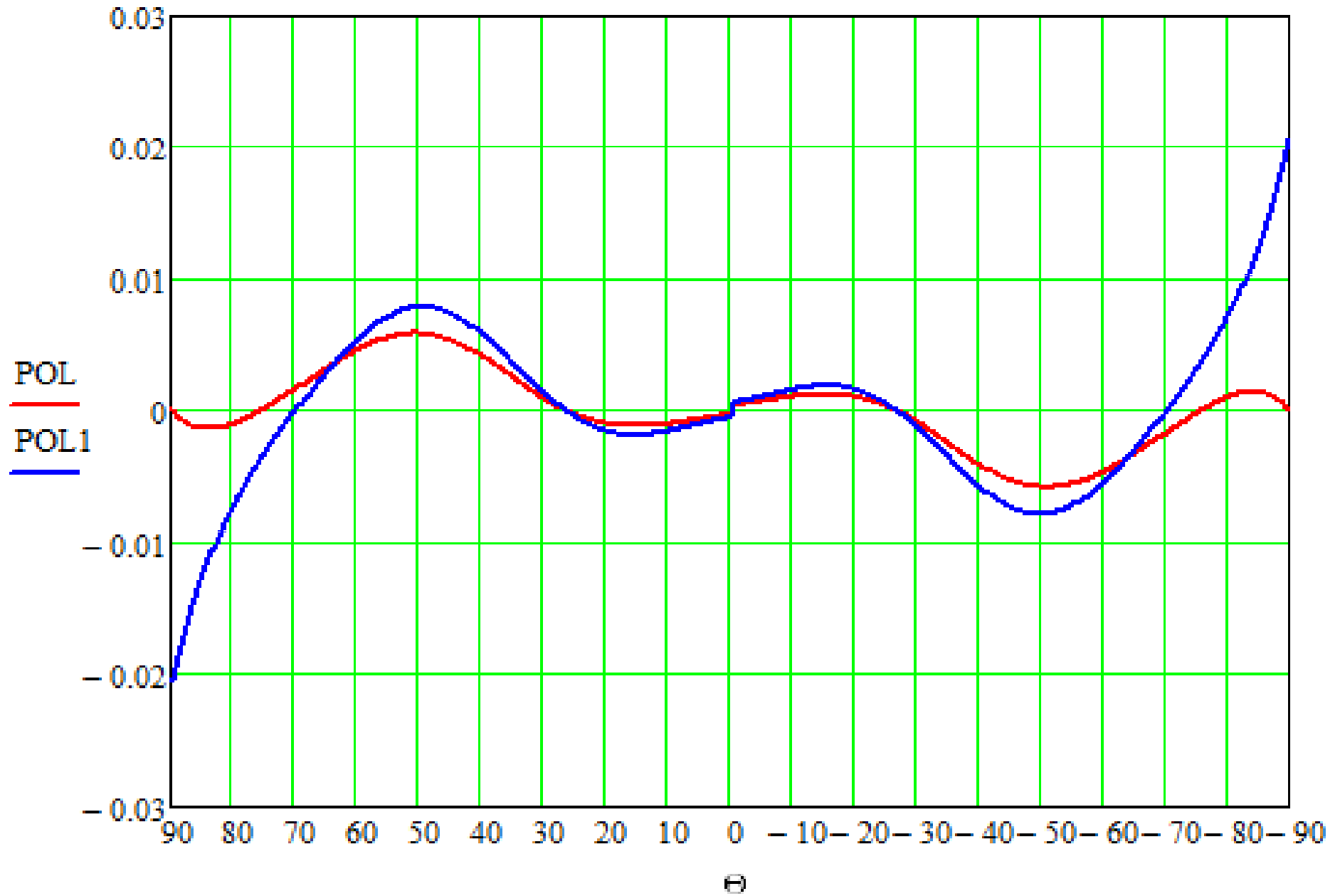
We obtained Hamilton-Jacobi equation for eqs. (2.3) and (2.4) by a method similar to the method described in Popova et al. (2010).

H. Popova, M. E. Arthuskova, D. Sokoloff, *Geophys. Astrophys. Fluid Dyn., Geophysical & Astrophysical Fluid Dynamics*, 2010, Vol. 104, No. 5, pp. 631 — 641.

# Dipole and quadrupole components of poloidal magnetic field (BMF), simulations



# Two dipole components with different boundary conditions of BMF, simulations



# Conclusions

- ◆ A latitudinal distribution is derived for two primary waves of the background magnetic field (BMF) and two periods: 11 and 2.5 years
- ◆ Observations show that maximums in sunspot components (SMF) correspond to minimums in the BMF components
- ◆ According to the dynamo theory BMF waves are result of a composition of two dipoles or one dipole and one quadrupole modes of the poloidal magnetic field
- ◆ Simulations illustrated that the toroidal and the poloidal magnetic fields have a small phase shift if the intensity of the dynamo waves is equal for both magnetic components
- ◆ If the dynamo number is a threshold in the upper layers of the convective zone (where observations data for BMF are available), but this number is big in the inner layers (where is generation of the toroidal magnetic field according Parker two-layers model), then maximums in SMF correspond to the BMF minimums