The nonlinear low-mode model of Parker dynamo with the meridional circulation

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ABSTRACT

We investigated the Parker model with the meridional circulation by the low-mode approach. We obtained the dynamic system for Parker model and built the time distribution for the poloidal and the toroidal components of the solar (planetary) magnetic field for different values of the dynamo-number and the meridional circulation. We obtained the different regimes (oscillations, vacillations, dynamo-bursts) depending on the value of the dynamo-number and the meridional circulation.

PARKER DYNAMO

$$B_P \xrightarrow{\Omega} B_T$$

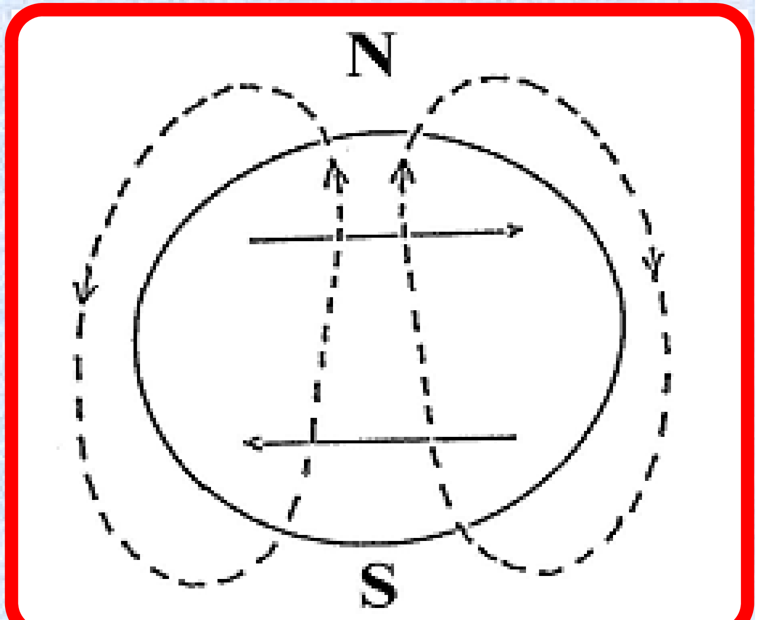
Differential rotation

$$B_T \xrightarrow{\alpha} B_P$$

Helicity

Parker presented a functional scheme for such dynamo as follows.

A toroidal magnetic fields is produced from the poloidal field by the action of differential rotation. The inverse process of transforming toroidal magnetic field into poloidal field is realized by the action of alpha-effect.



DYNAMO EQUATIONS

B is the toroidal magnetic field
A is proportional to the toroidal component of the vector potential, which determines the poloidal magnetic field
θ is the latitude measured from the equator
D is dynamo number

|D|>>1, V – meridional circulation

$$\frac{\partial A}{\partial t} + V(\theta)\frac{\partial A}{\partial \theta} = \alpha B + \frac{\partial^2 A}{\partial \theta^2},$$

$$\frac{\partial B}{\partial t} + \frac{\partial (V(\theta) B)}{\partial \theta} = D \cos\theta \frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2}.$$

The mathematical description of the dynamo leads to a cumbersome system of differential equations in partial derivatives, whose solution in the complete production is possible, but does not exhaust the problem. One of the ways to obtain simplified models is as follows. We assume that the excited magnetic field is simply arranged and it can be written relatively small number of parameters. That is, for its qualitative description the dynamo equation can be replaced by suitably chosen dynamical system which is not very high order.

This papers describe low-mode method and its applying:

1. D. Sokoloff, S. Nefedov, Numerical Methods and Programming, Vol. 8, pp. 195-204 (2007).
2. H. Xu, Yu Gao, E. P. Popova, S. N. Nefedov, H. Zhang, and D. D. Sokoloff, Astronomy Reports, Vol. 53, No. 2, pp. 160-165 (2009).
3. D.D. Sokoloff, S.N. Nefedov, A.A. Ermash, S.A. Lamzin, Pis’ma v Astronomicheskii Zhurnal, Vol. 34, No. 11, pp. 842–853 (2008).
4. S. Nefedov, D. Sokoloff, Astronomy Reports, Vol. 87, No. 3, pp. 1-8 (2010).
5. Sobko G.S., Zadkov V.N., Sokoloff D.D., Trukhin V.I. Inversion of the geomagnetic field in a simple model of the geodynamo (in print).

Latitudinal distribution of the magnetic field components

$$A = a_1 \sin\theta + a_2 \sin 3\theta$$

$$B = b_1 \sin 2\theta + b_2 \sin 4\theta$$

$D = R_\alpha R_\omega$ $\alpha = \alpha_0(\theta)/(1+\xi^2 B^2) \approx \alpha_0(1-\xi^2 B^2)$

α_0 - value of the helicity in the nonmagnetized medium

$B_0 = \xi^{-1}$ - significant suppression of alpha-effect is at this value of magnetic field

Here $\alpha_0(\theta) = \cos\theta$ $\xi = 1$

$$A(0) = B(0) = A(\pi) = B(\pi) = 0$$

Dynamical system

$$\frac{\partial a_1}{\partial t} = \frac{R_\alpha b_1}{2} - a_1 - \xi^2 \frac{3R_\alpha b_1}{8}(b_1^2 + 2b_2^2) - \frac{V^2 a_1}{4},$$

$$\frac{\partial a_2}{\partial t} = \frac{R_\alpha}{2}(b_1 + b_2) - 9a_2 - \xi^2 \frac{3R_\alpha(b_1 + b_2)}{8}(b_1^2 + b_1 b_2 + b_2^2) - \frac{V^2 a_2}{4},$$

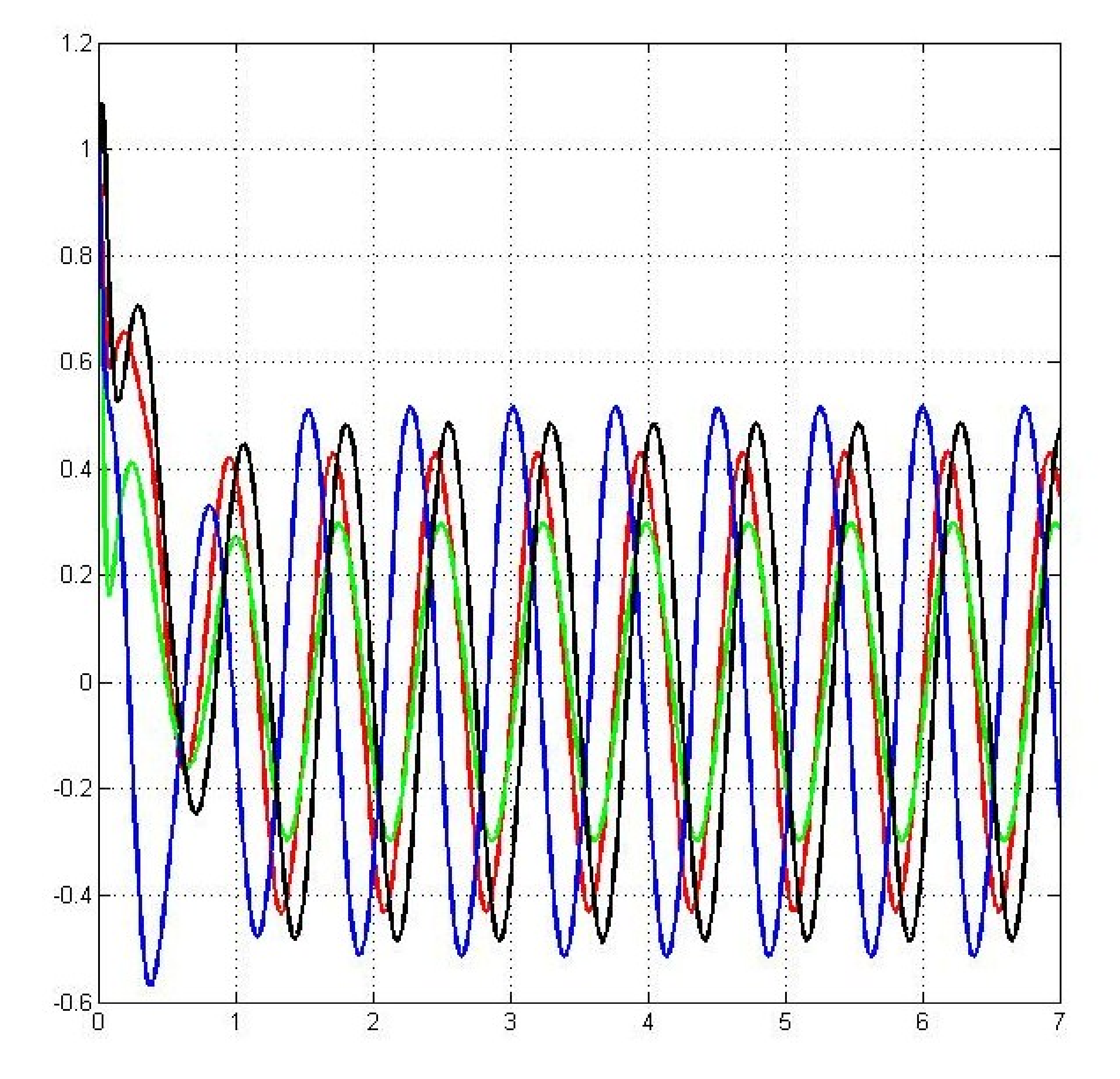
$$\frac{\partial b_1}{\partial t} = \frac{R_\omega}{2}(a_1 - 3a_2) - 4b_1 - \frac{V^2 b_1}{4},$$

$$\frac{\partial b_2}{\partial t} = \frac{3R_\omega a_2}{2} - 16b_2 - \frac{V^2 b_2}{4}.$$

The toroidal field is much stronger than the poloidal field in our approximation. So we deleted non-linear terms for poloidal modes from our system. We obtained this dynamical system.

Oscillations

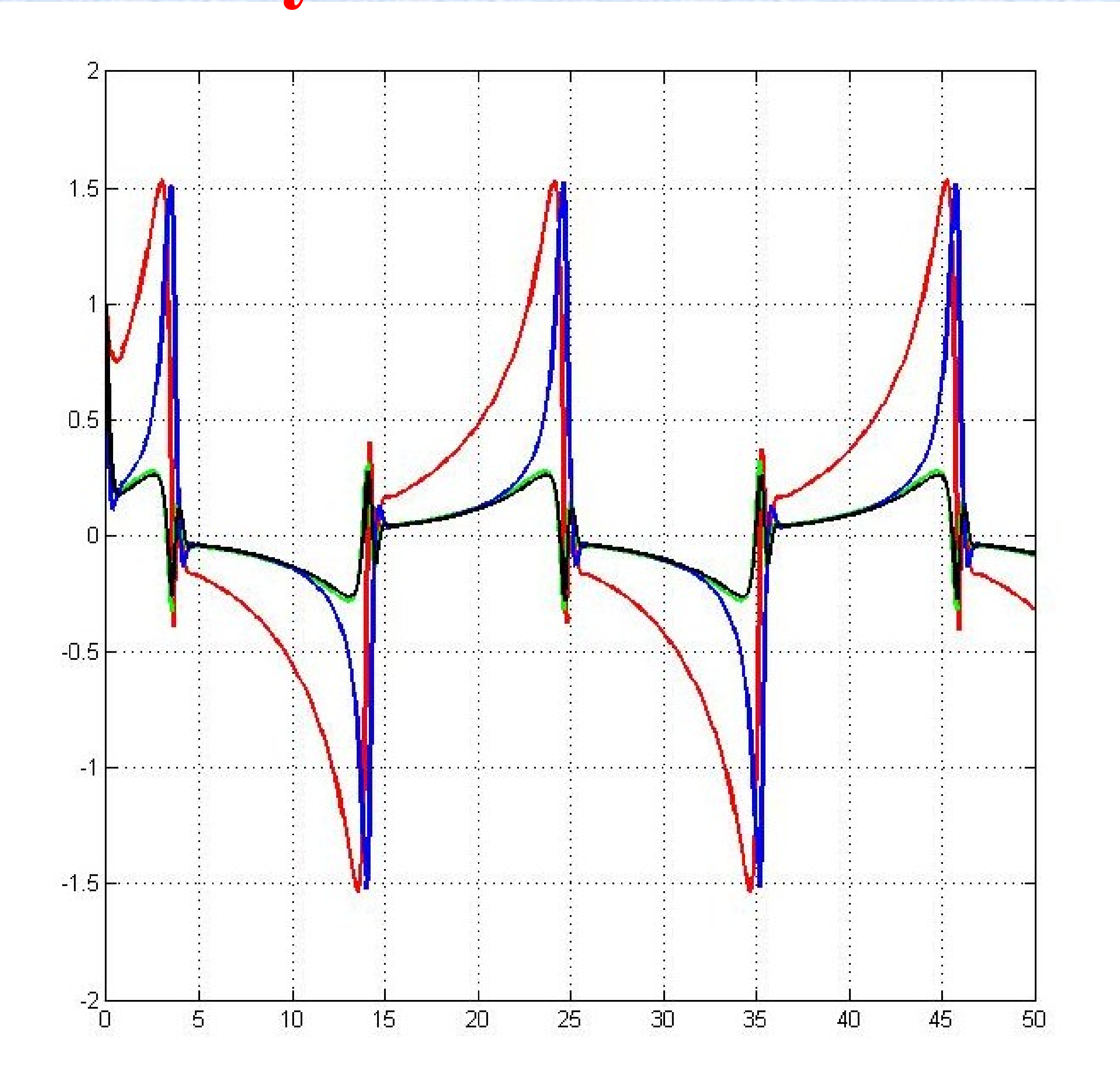
a_1 -red
a_2 - green
b_1 - blue
b_2 - black



***t* (dimensionless units)**

Dynamo-bursts

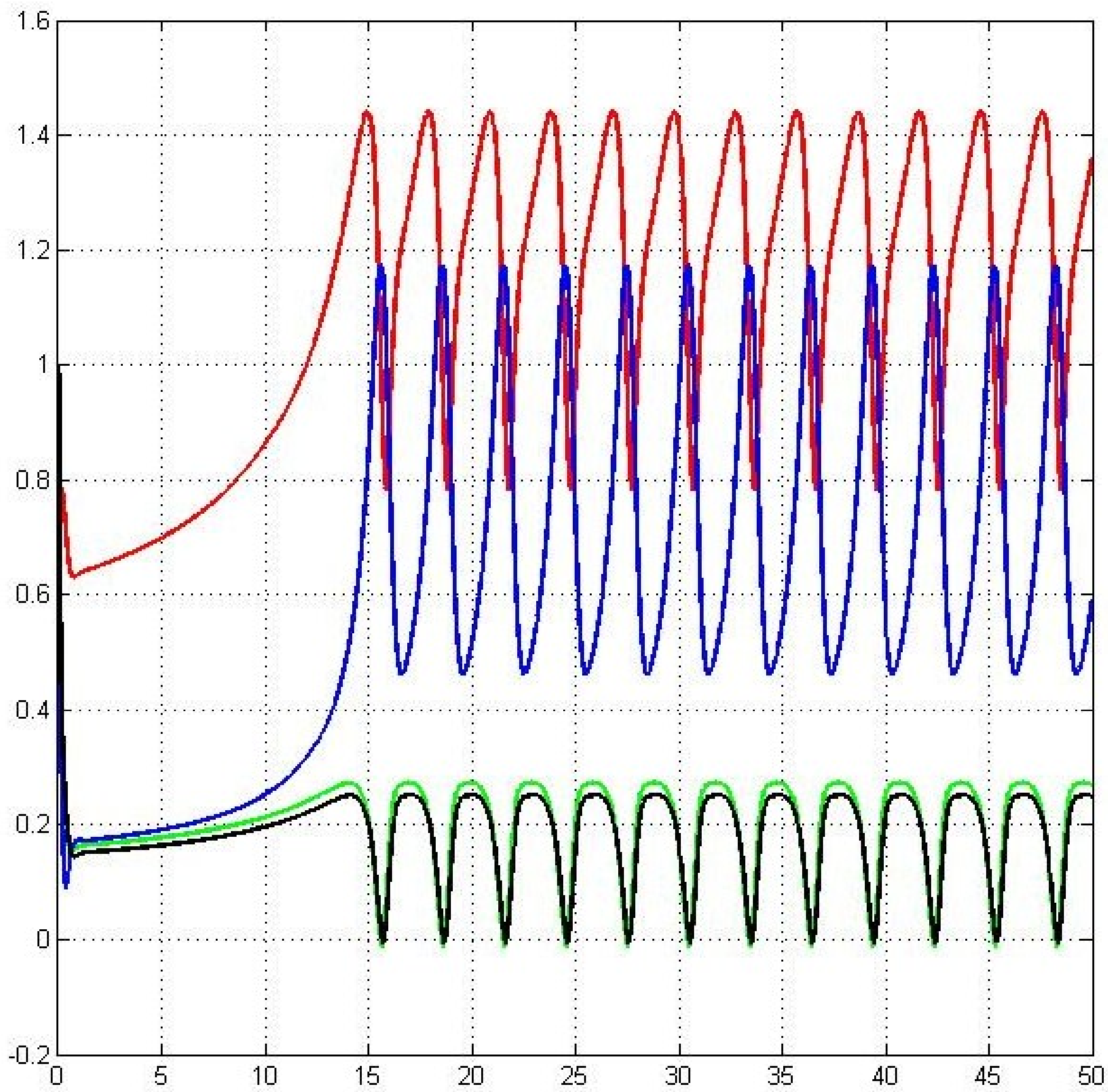
a_1 -red
a_2 - green
b_1 - blue
b_2 - black



t (dimensionless units)

Vacillations

a_1 -red
a_2 - green
b_1 - blue
b_2 - black



t (dimensionless units)

Conclusions

Three regimes (oscillations, vacillations, dynamo-bursts) exist at small values (<1, model units) of the meridional circulation. Two regimes (oscillations, vacillations) exist at middle values (1<V<3, model units). Only oscillations exist at big values (>3). We obtained diapason of the dynamo numbers for each regime.

Ratio for the values of cycle time for these regimes is “period of oscillations”/ “period of vacillations”/ “period of dynamo-bursts”=1/4/30. We obtained the diapason of the dynamo numbers for oscillation regime when superposition or interaction of harmonics exists.

We obtained a case when the meridional circulation can switch regimes. For D=100 vacillations exist at small values of the meridional circulation and dynamo-bursts exist at middle values.

According to this model if celestial body has intensive meridional flows only oscillations exist in its magnetic activity. If celestial body has slow meridional flows, three regimes can exist in its magnetic activity. If meridional flows are not so strong, oscillations and vacillations can exist.