Fast Reconnection in Weakly Stochastic Magnetic Fields

Numerical Testing

3D modelling

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2D modelling

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Magnetic Fields on Scales from kilometers to kiloparsecs: properties and origin, Kraków, May 20, 2010

Turbulence as a Potential Answer for Fast Reconnection

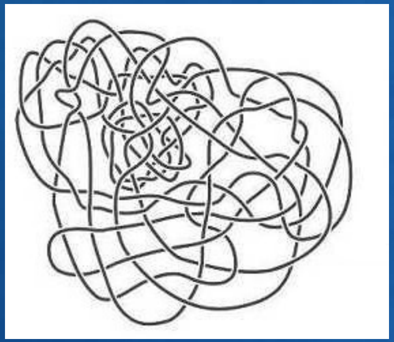
- Motivation
- Sweet-Parker model and its extension to LV99
- Building a numerical setup
- New measure of the reconnection rate
- Testing the LV99 predictions
- 3D vs. 2D models

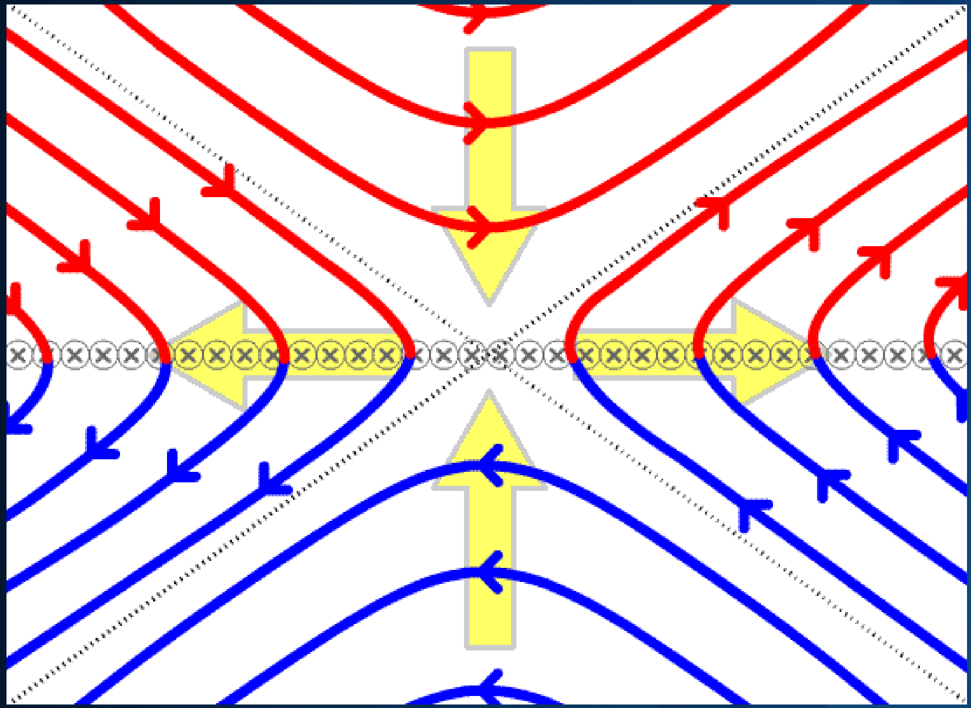
What is Magnetic Reconnection?

Induction equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} - \underline{\eta \nabla \times \vec{B}} \right), \nabla \cdot \vec{B} = 0$$

Magnetic field frozen in the plasma. It moves with plasma, thus in the presence of turbulence or other kind of motions after some time it can have very complex topology.

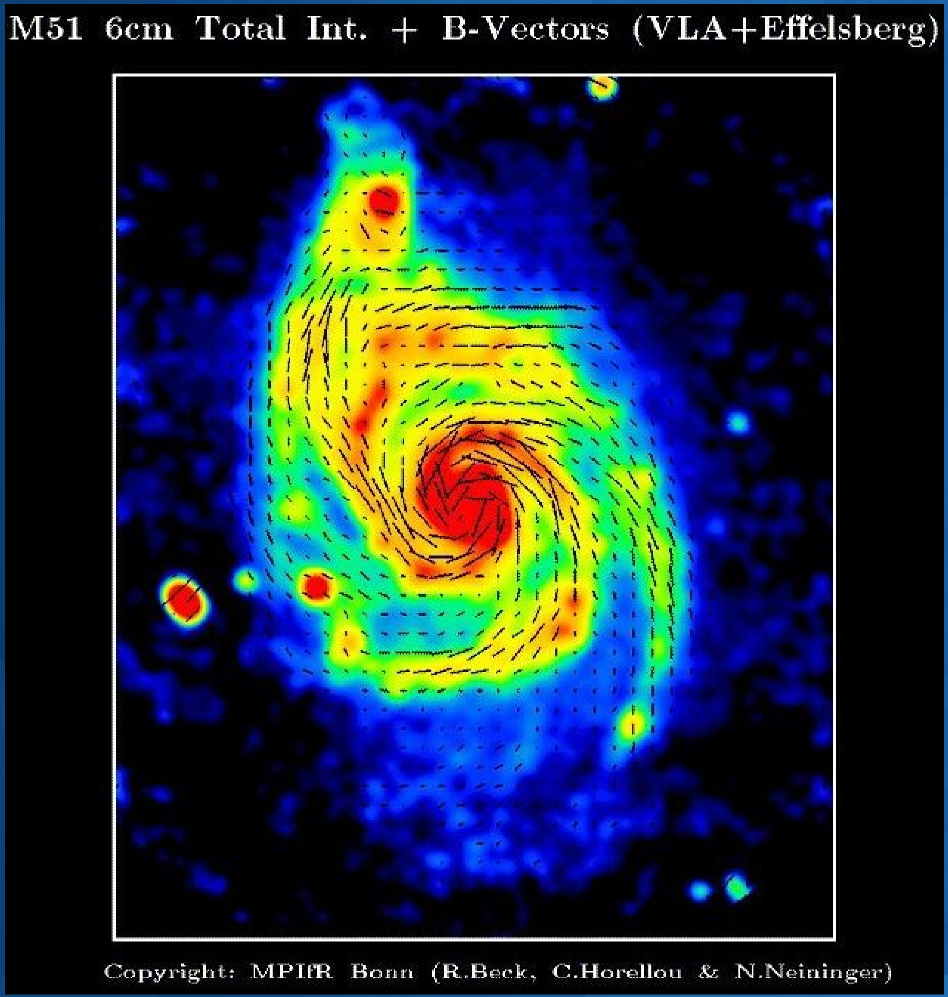


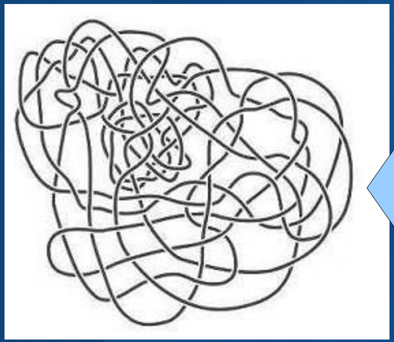


Magnetic reconnection is a process which violates conservation of the magnetic flux, changing it topology through reconnection of opositely directed magnetic field lines. It releases the energy of magnetic field into the kinetic motions and heat.

Where Fast Reconnection is Important?

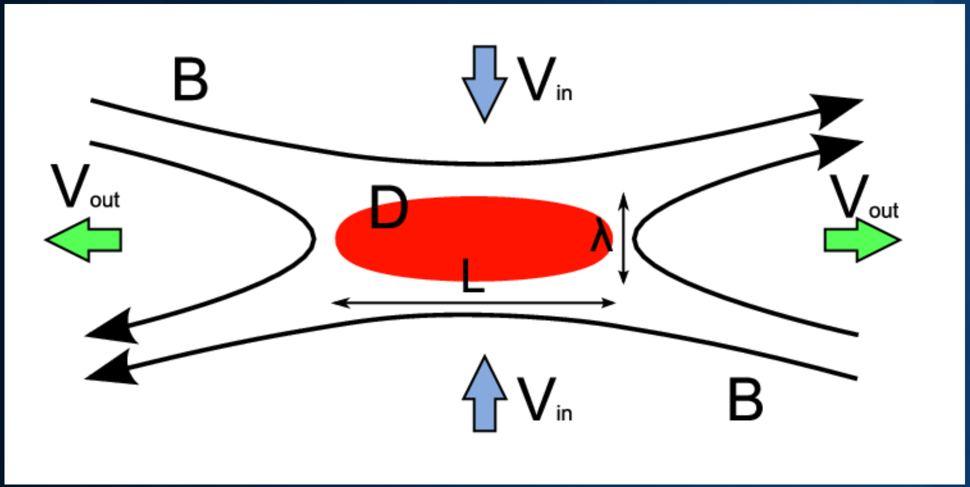
M. Aschwanden et al. (LMSAL), TRACE, NASA





$\langle B \rangle = 0$

Sweet-Parker model (1957)



$$V_{inflow} = \eta / \lambda$$

Ohmic diffusion

$$V_{inflow} L = V_{outflow} \lambda$$

Mass conservation

$$V_{outflow} = V_A$$

Free outflow

Reconnection Rate:

$$V_{inflow} = V_A \left(L V_A / \eta \right)^{-1/2} = V_A R_m^{-1/2}$$

Magnetic Reynolds Number

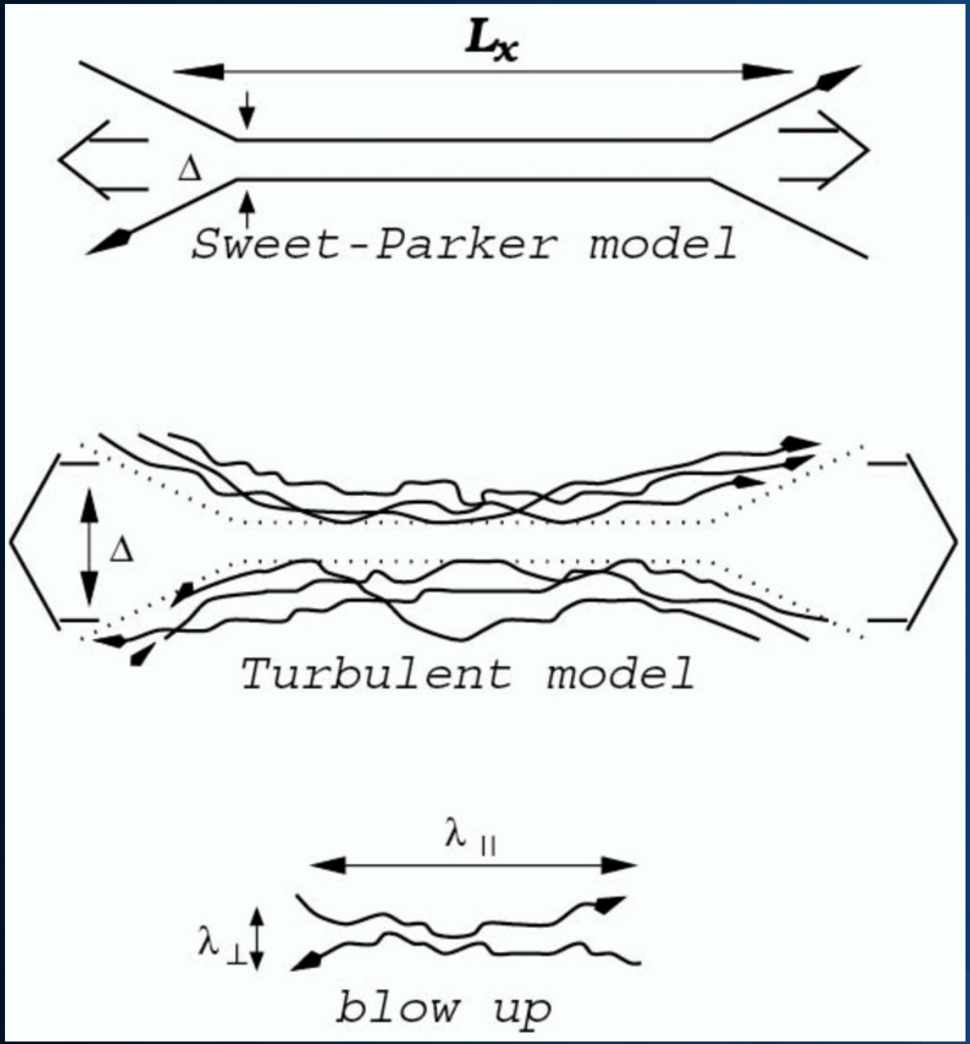
PROBLEM:

R_m very large for most of interstellar structures, $R_m \sim 10^{15}$-10^{20}!

We need fast reconnection to explain large scale magnetic fields in the ISM, galactic dynamo, etc.

Alternative approaches: Petschek reconnection, anomalous collisionless effects - they have limited application due to special conditions

Introducing a Weak Stochastic Component



B dissipates on a small scale $\lambda_{\parallel}$ determined by turbulence statistics.

Lazarian & Vishniac (1999)

Reconnection of 3D turbulent magnetic field involves simultaneous reconnection events

Key element: $L/\lambda_{\parallel}$ events!

Turbulent reconnection:

- Thick current sheet determined by field wandering.
- Reconnection is fast with Ohmic resistivity only.

$$V_{inflow} = \frac{L}{\lambda_{\parallel}} V_{rec,local}$$

$$V_{rec} < V_A \min\left[\left(\frac{L}{\lambda_{\parallel}}\right)^{1/2}, \left(\frac{\lambda_{\parallel}}{L}\right)^{1/2}\right]\left(\frac{V_T}{V_A}\right)^2$$

Equations and Physics Involved

MHD equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

isothermal EOS

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \left[\rho \vec{v} \vec{v} + \left(c_s^2 \rho + \frac{B^2}{8\pi} \right) \vec{I} - \frac{1}{4\pi} \vec{B} \vec{B} \right] = \rho \vec{f}$$

driving

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} - \eta \nabla \times \vec{B} \right), \nabla \cdot \vec{B} = 0$$

$$\vec{v}^{n+1} = \vec{f}^n \delta t_{force} + \vec{v}^n$$

(Alvelius, 1999)

resistivity

$$\underbrace{\eta = \eta_u}_{\text{uniform}} + \underbrace{\eta_a \left(\frac{|j|}{j_{crit}} - 1 \right) H \left(\frac{|j|}{j_{crit}} \right)}_{\text{anomalous}}$$

uniform **anomalous**

current density: $\vec{j} = \nabla \times \vec{B}$

- random with adjustable injection scale k_f

$$F(k) = A \exp\left(-\frac{(k - k_f)^2}{c} \right)$$

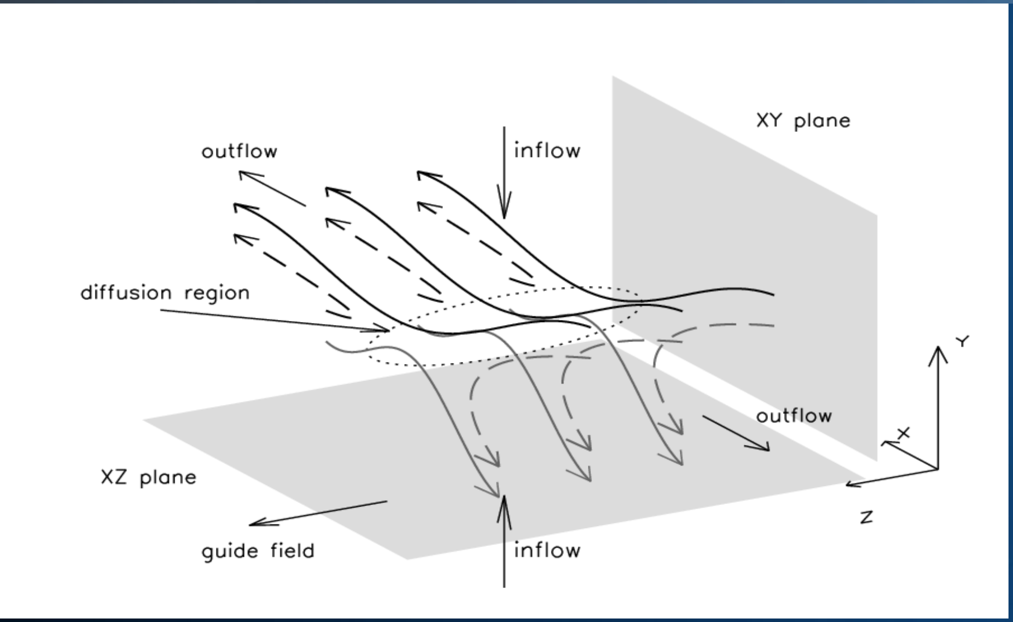
- divergence free (purely incompressible forcing)

$$\hat{f}_i(\vec{k}, t) = A_{ran}(\vec{k}, t) \vec{e}_1 + B_{ran}(\vec{k}, t) \vec{e}_2 \qquad \vec{e}_1 \perp \vec{e}_2 \perp \vec{k}$$

- zero correlation v-f (we control the power input)

$$\frac{K^{n+1} - K^n}{\delta t_{force}} = \frac{1}{2} \overline{f_k^n f_k^n} \delta t_{force} + \cancel{\overline{v_k^n f_k^n}}$$

Numerical Model of Turbulent Reconnection



- initial Harris current sheet setup

$$B_x(x,y)=B_0\tanh(y/h), B_y(x,y)=0$$

- initial density profile calculated from numerical the equilibrium of magnetic and gas pressures

$$p_{thermal}+p_{mag}=const$$

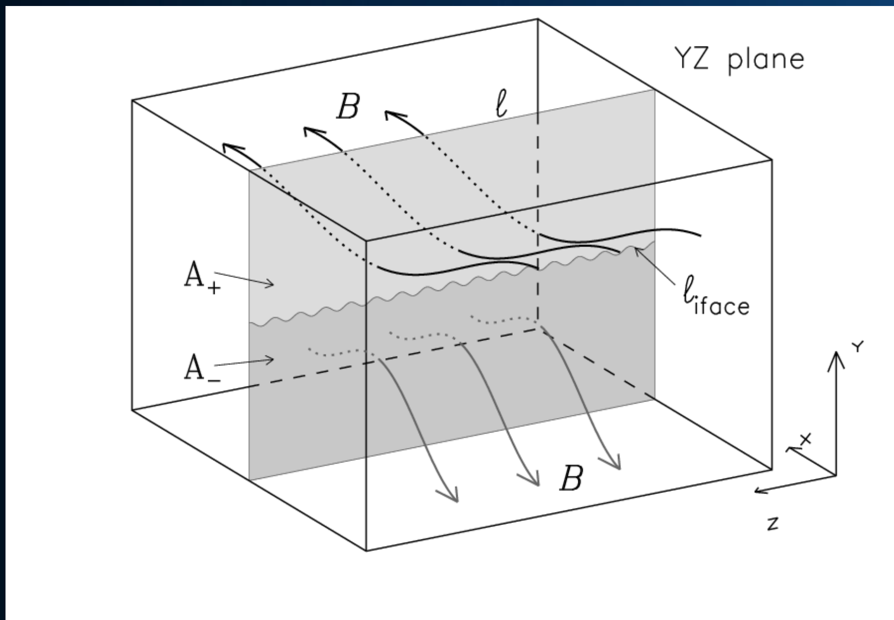
- reconnection develops as a result of initial vector potential perturbation

$$\delta A_z=-A_0\cos(2\pi x)\cos(\pi y)$$

RECONNECTION IS NOT DRIVEN

- no external forcing of reconnection!, V=0 at the boundaries initially, compressible flow (compressibility reduced by setting large sound speed)
- B_x, B_z components and density set to constant values at the inflow boundaries describing the ambient medium in which our box is embedded (this roughly guarantees conservation of mass and energy)
- open boundary conditions allow waves to leave the box without reflection

New Measure of Reconnection Rate



We estimate contributions to the integral over a plane YZ from different processes.

$$\partial_t \Phi = \oint \vec{E} \cdot d\vec{l} = \oint (\vec{V} \times \vec{B} - \eta \vec{J}) \cdot d\vec{l}$$

$$\partial_t \Phi_+ - \partial_t \Phi_- = \partial_t \int |B_x| dS$$

$$\partial_t \int |B_x| dS = \oint \vec{E} \cdot d\vec{l}_+ - \oint \vec{E} \cdot d\vec{l}_- = \oint sign(B_x) \vec{E} \cdot d\vec{l} + \int 2 \vec{E} \cdot d\vec{l}_{interface}$$

We define the interface term as

$$\boxed{\int 2 \vec{E} \cdot d\vec{l}_{interface} \equiv -2 V_{rec} |B_{x,\infty}| L_z}$$

Asymptotic absolute value of Bx

$$\boxed{V_{rec} = -\frac{1}{2|B_{x,\infty}| L_z} \left[\partial_t \int |B_x| dA - \oint sign(B_x) \vec{E} \cdot d\vec{l} \right]}$$

We Inject Weak Turbulence!

Name	B_{0z}	η_u $[10^{-3}]$	η_a $[10^{-3}]$	P_{inj}	k_{inj}	Δk_{inj}	N_f	$\tilde{v}_f$	v_l (t=12)	$\langle\lvert\boldsymbol{v}\rvert\rangle$ (t=12)	$\tilde{V}_A$	$\beta \equiv p/p_{mag}$
PD	0.1	1.0	0.0	0.1	8	0.5	96	0.0015	0.031	0.085	1.05	31.7
	0.1	1.0	0.0	0.2	8	0.5	96	0.0022	0.041	0.107	1.05	31.7
	0.1	1.0	0.0	0.5	8	0.5	96	0.0035	0.051	0.158	1.05	31.7
	0.1	1.0	0.0	1.0	8	0.5	96	0.0049	0.065	0.197	1.05	31.7
	0.1	1.0	0.0	2.0	8	0.5	96	0.0069	0.084	0.262	1.05	31.7
	1.0	1.0	0.0	0.1	8	0.5	96	0.0015	0.042	0.099	1.41	16.0
	1.0	1.0	0.0	0.2	8	0.5	96	0.0022	0.043	0.121	1.41	16.0
	1.0	1.0	0.0	0.5	8	0.5	96	0.0035	0.056	0.171	1.41	16.0
	1.0	1.0	0.0	1.0	8	0.5	96	0.0049	0.068	0.212	1.41	16.0
	1.0	1.0	0.0	2.0	8	0.5	96	0.0069	0.083	0.293	1.41	16.0

V_l V_A

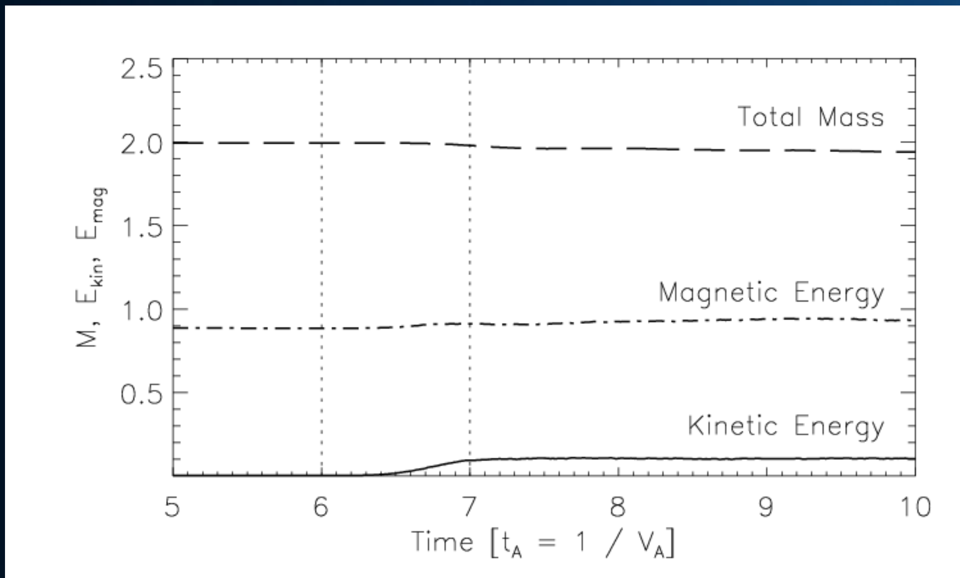
Peak amplitude of the driving in the Fourier space.

Velocity at the injection scale taken from spectrum after the full turbulence development.

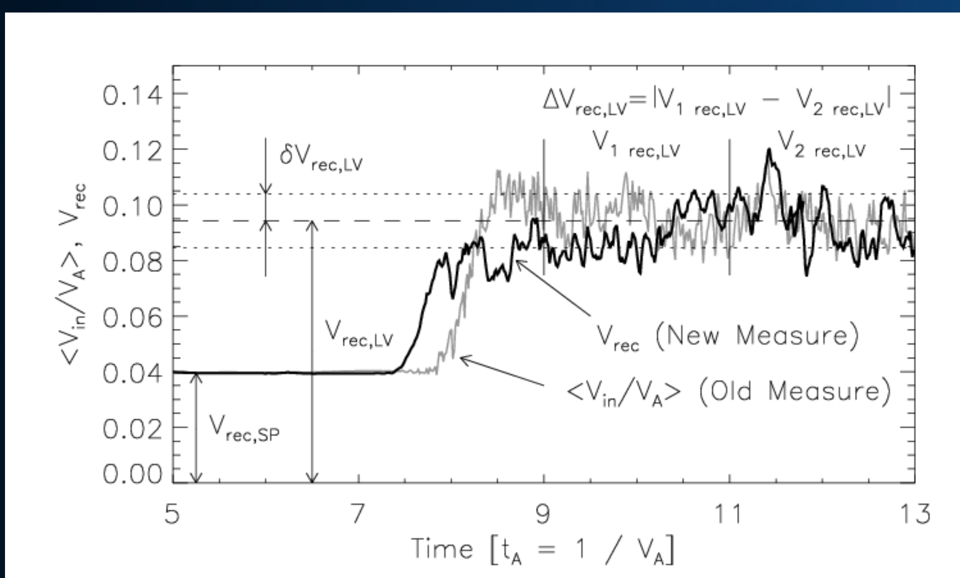
$$\tilde{v}_f < v_l \ll V_A$$

$$\tilde{v}_f \ll v_l \approx V_{rec,SP}$$

What Happens When Turbulence Appears?

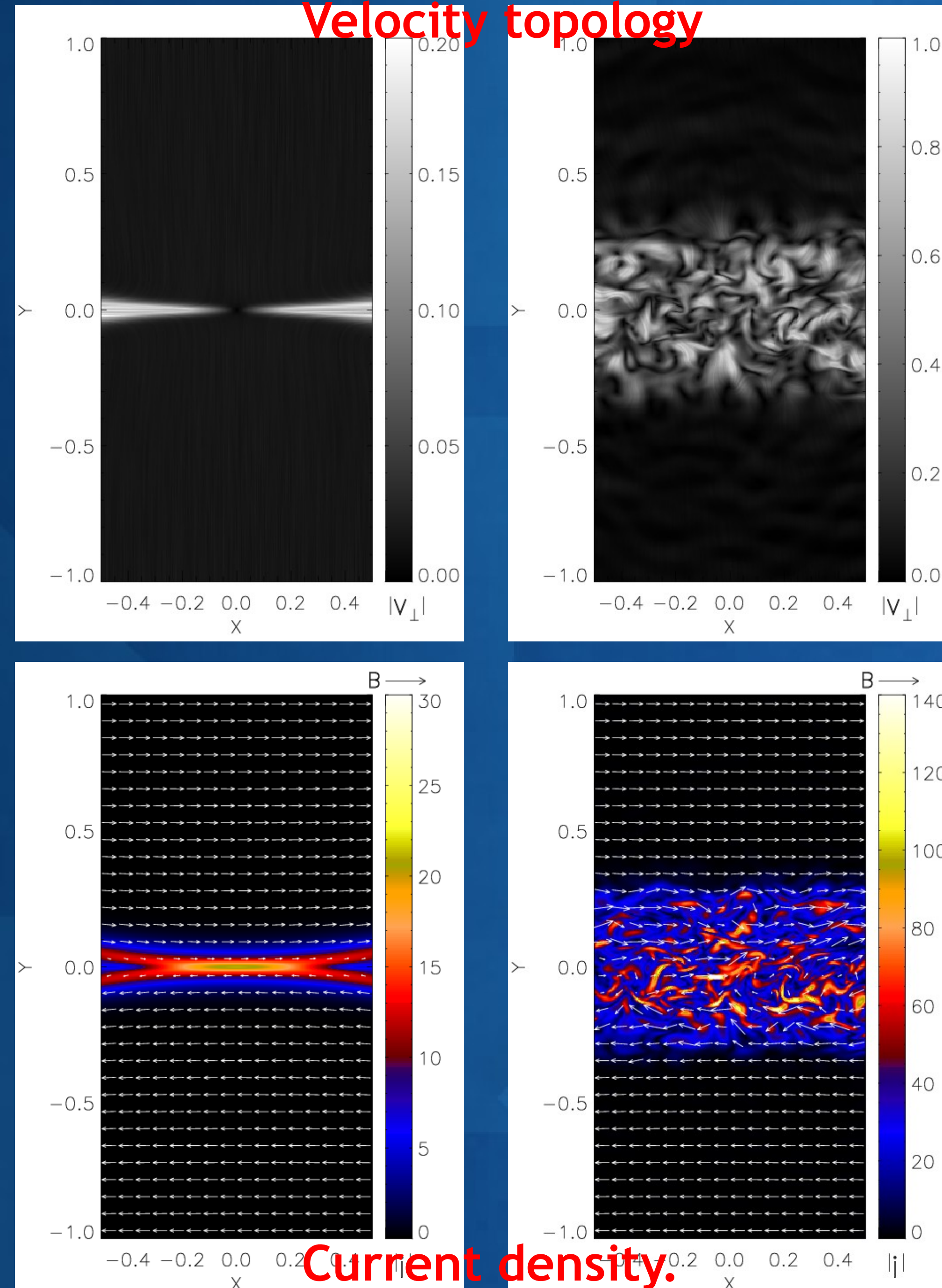


Mass conservation and energy evolution



Reconnection rate change.

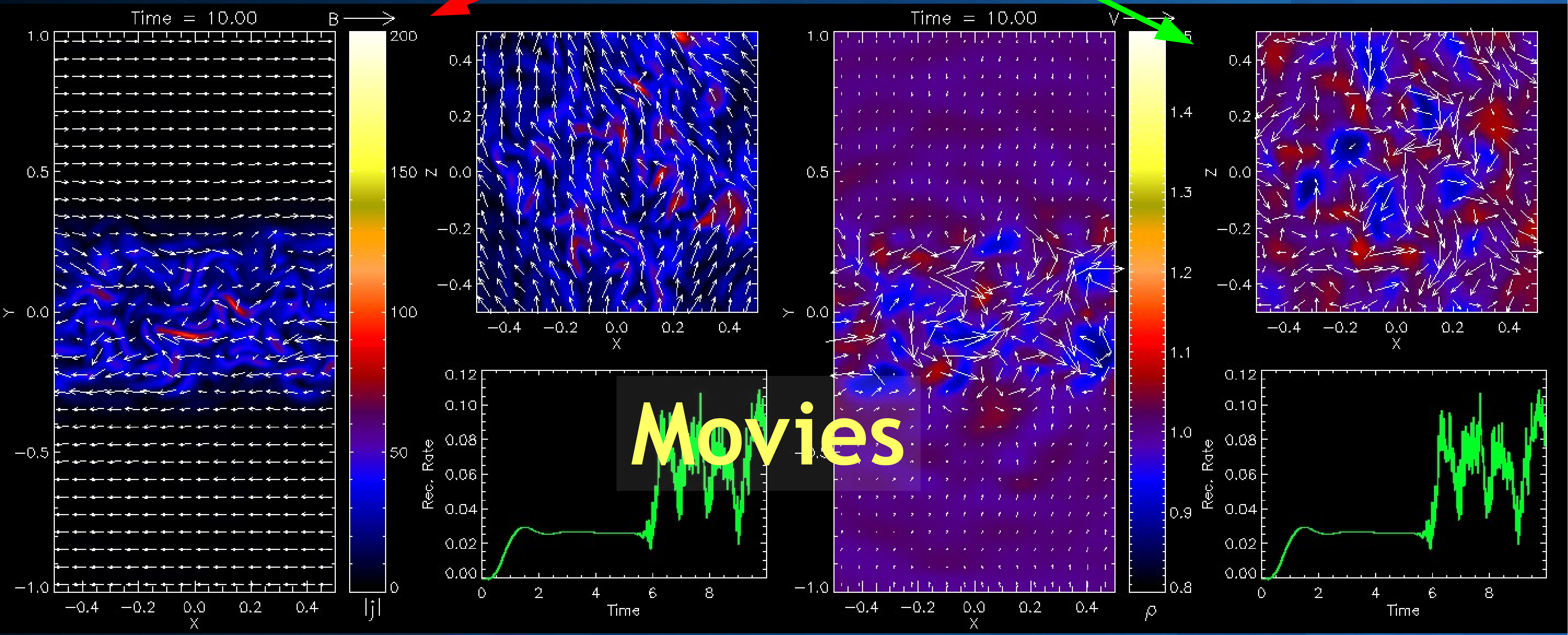
Velocity topology



Current density.

What Happens When Turbulence Appears?

- Evolution of magnetic field and current density, velocity and density

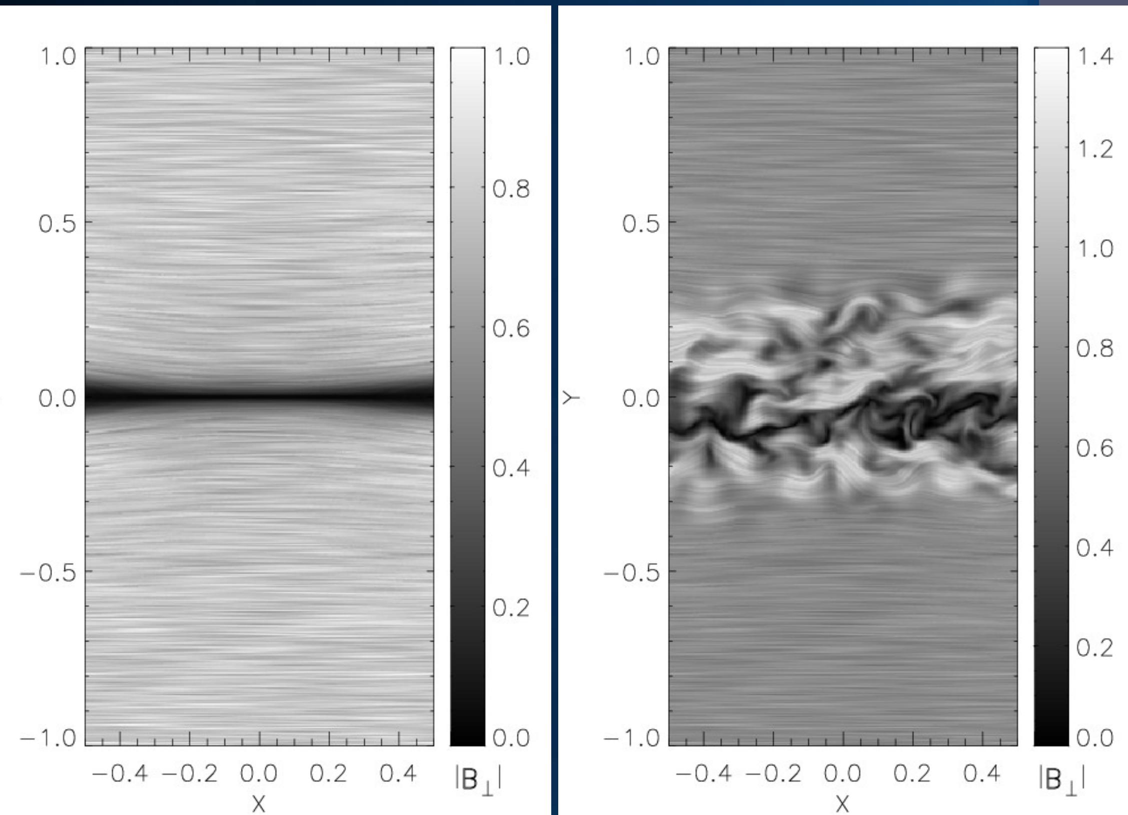


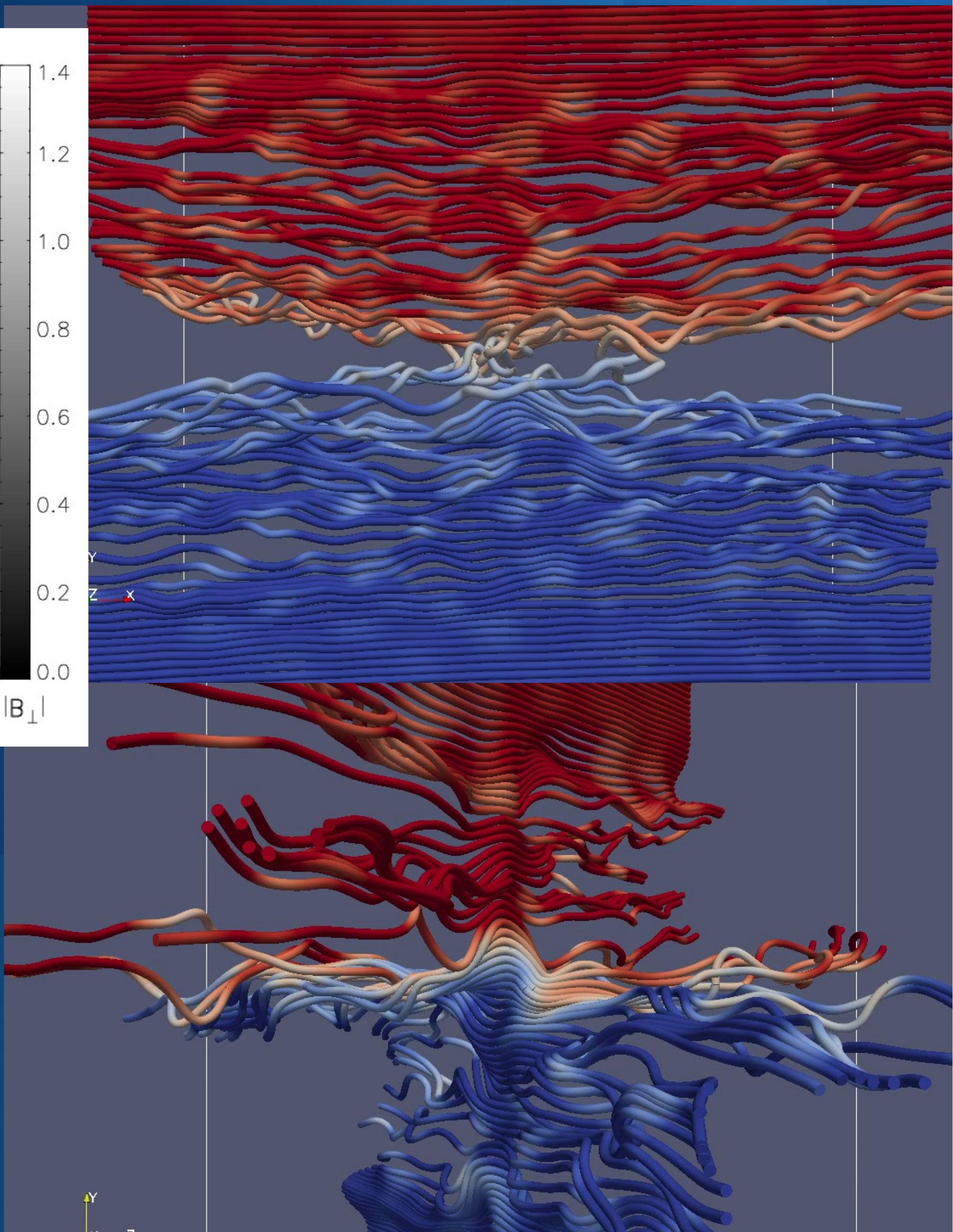
Parameters: B_x=1.0, B_z=1.0, P_f=1.0, k_f=5, η_u=5·10^{-3}

Reconnection rate:

$$\langle |V_y / V_A| \rangle_{inflow\ boundary}$$

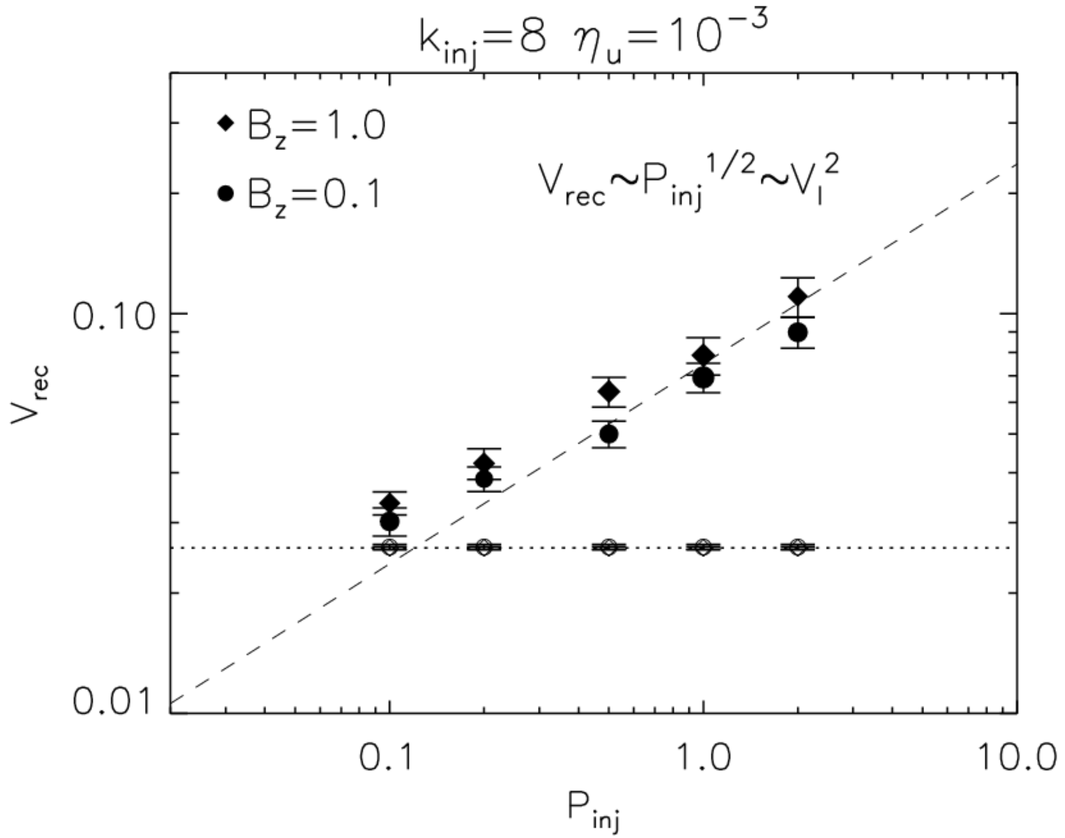
Change of Magnetic Topology in 3D





without   with
turbulence

Turbulent Power and Injection Dependence



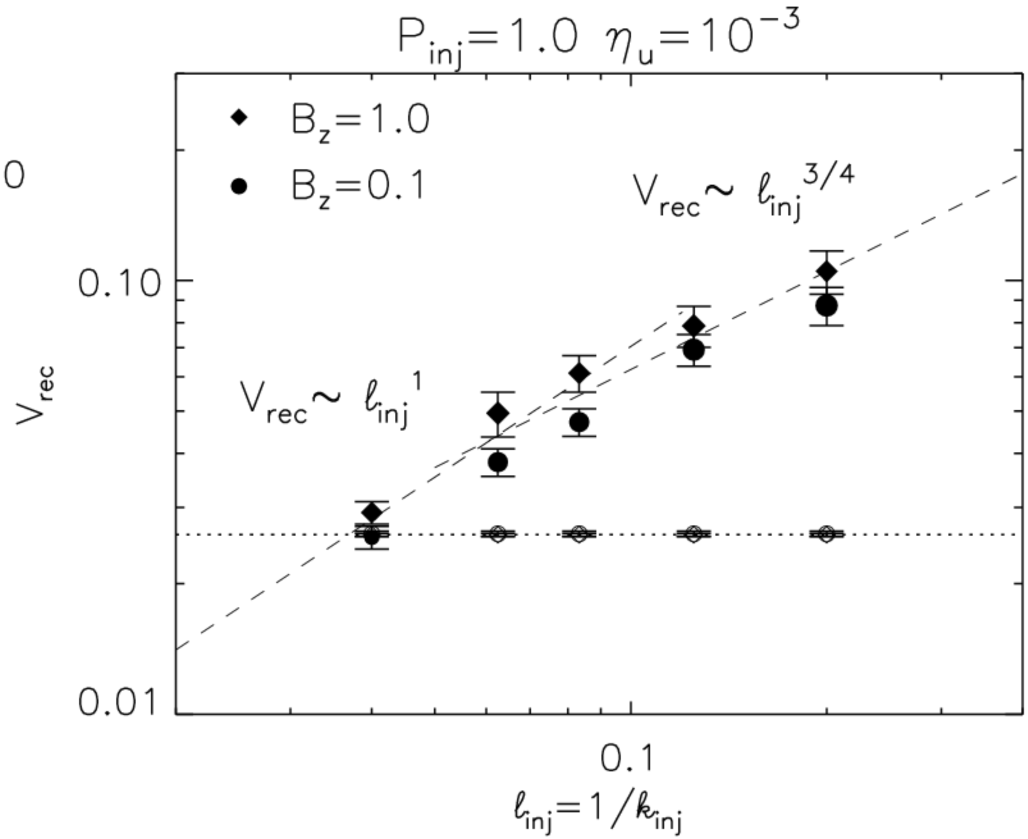
Upper limit imposed by the large-scale field line diffusion

$$V_{\rm rec} < V_{\rm A}\ \min\left[\left(\frac{L_x}{l}\right)^{1/2}, \left(\frac{l}{L_x}\right)^{1/2}\right]\left(\frac{v_l}{V_{\rm A}}\right)^2 .$$

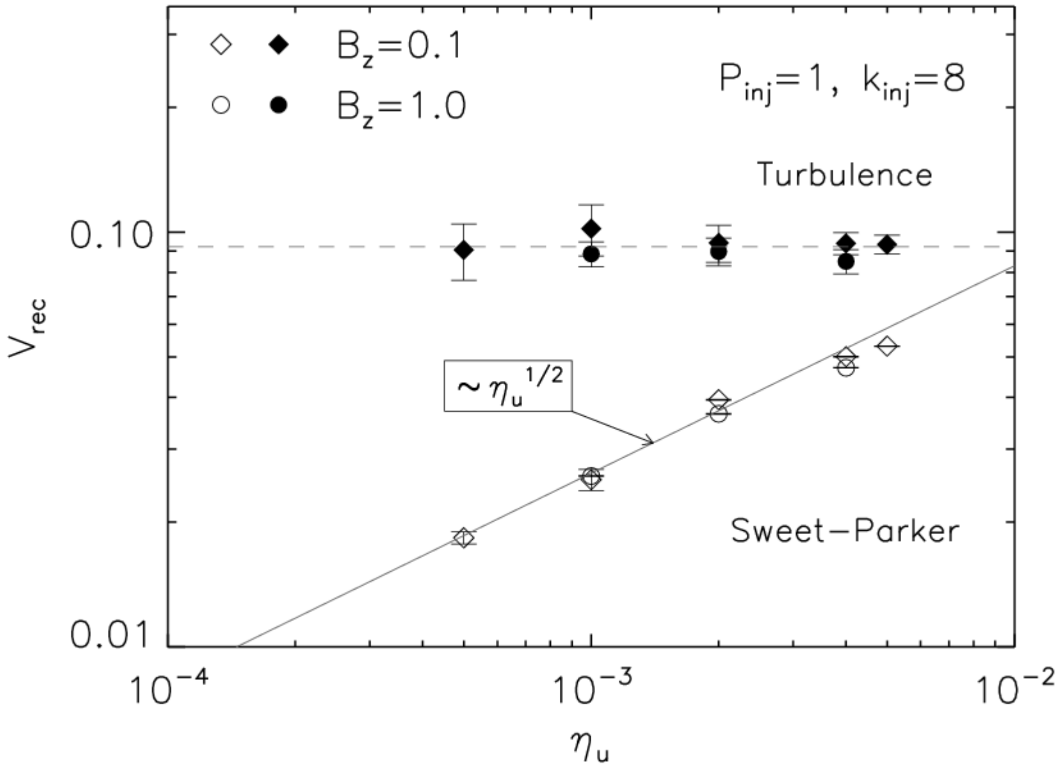
Lazarian & Vishniac (1999)

Power-velocity conversion

$$P \sim v_{turb}^2 (V_A/l) \text{ or } v_l^4/(l V_A)$$

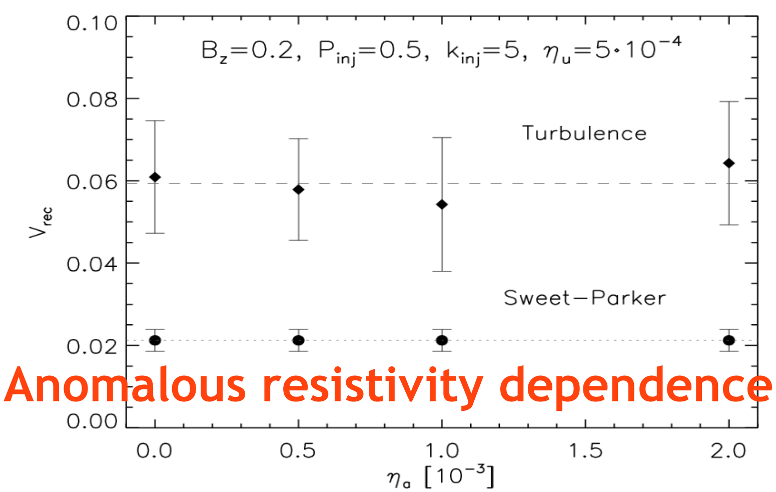


Uniform Resistivity Dependence

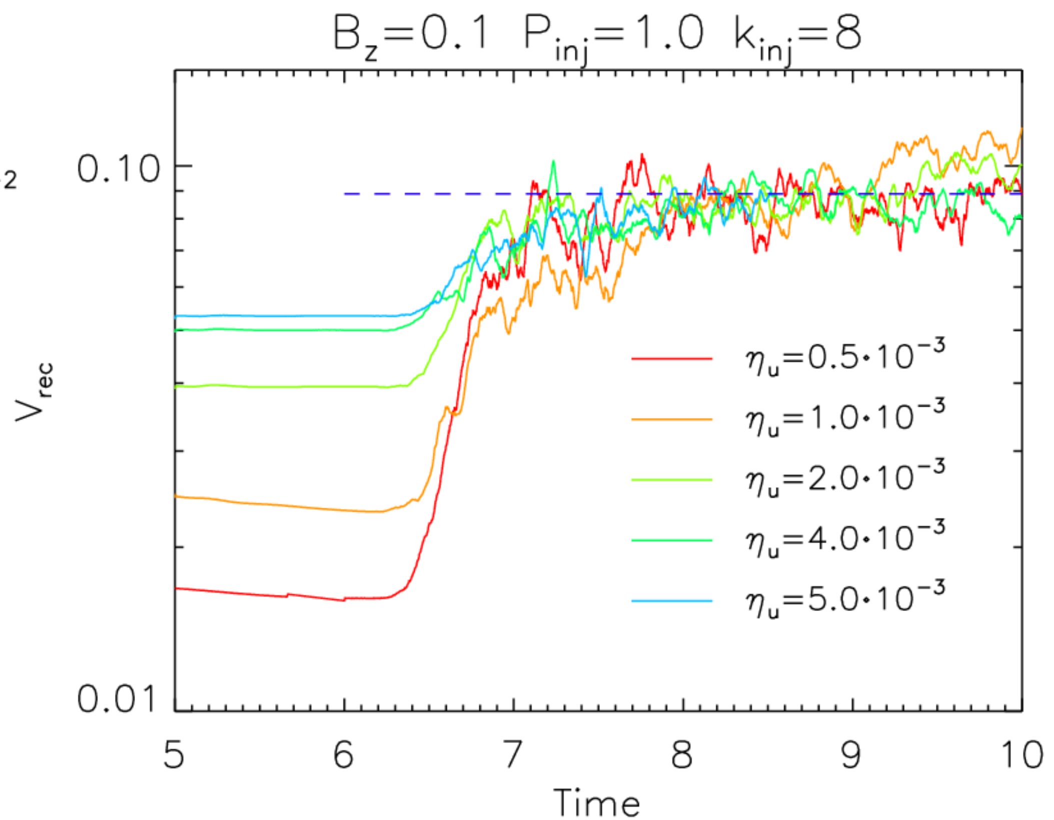


The reconnection rate does not depend on the Ohmic resistivity, thus the reconnection is FAST!

No eta-dependence!

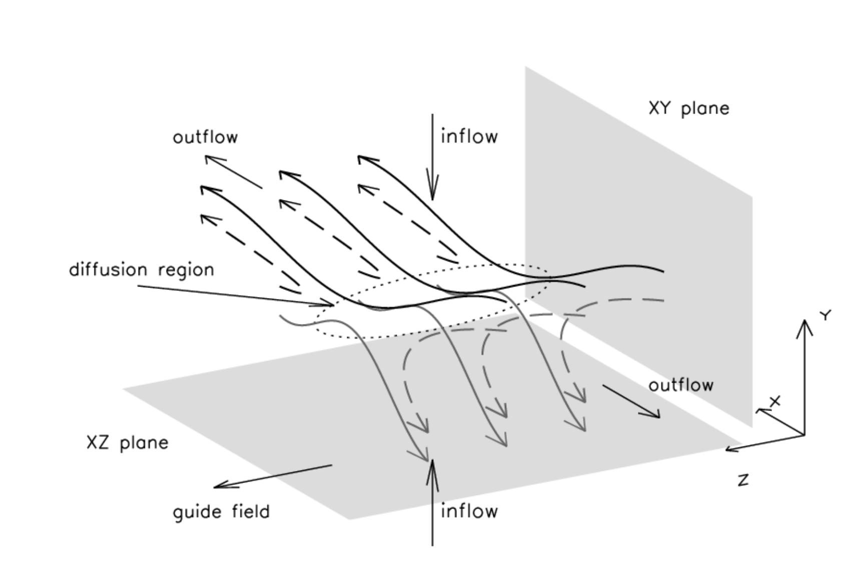


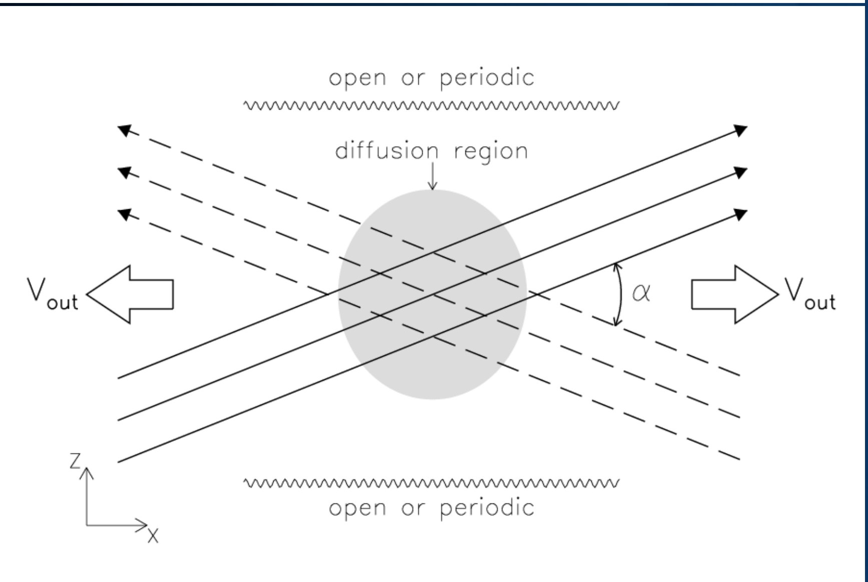
Anomalous resistivity dependence

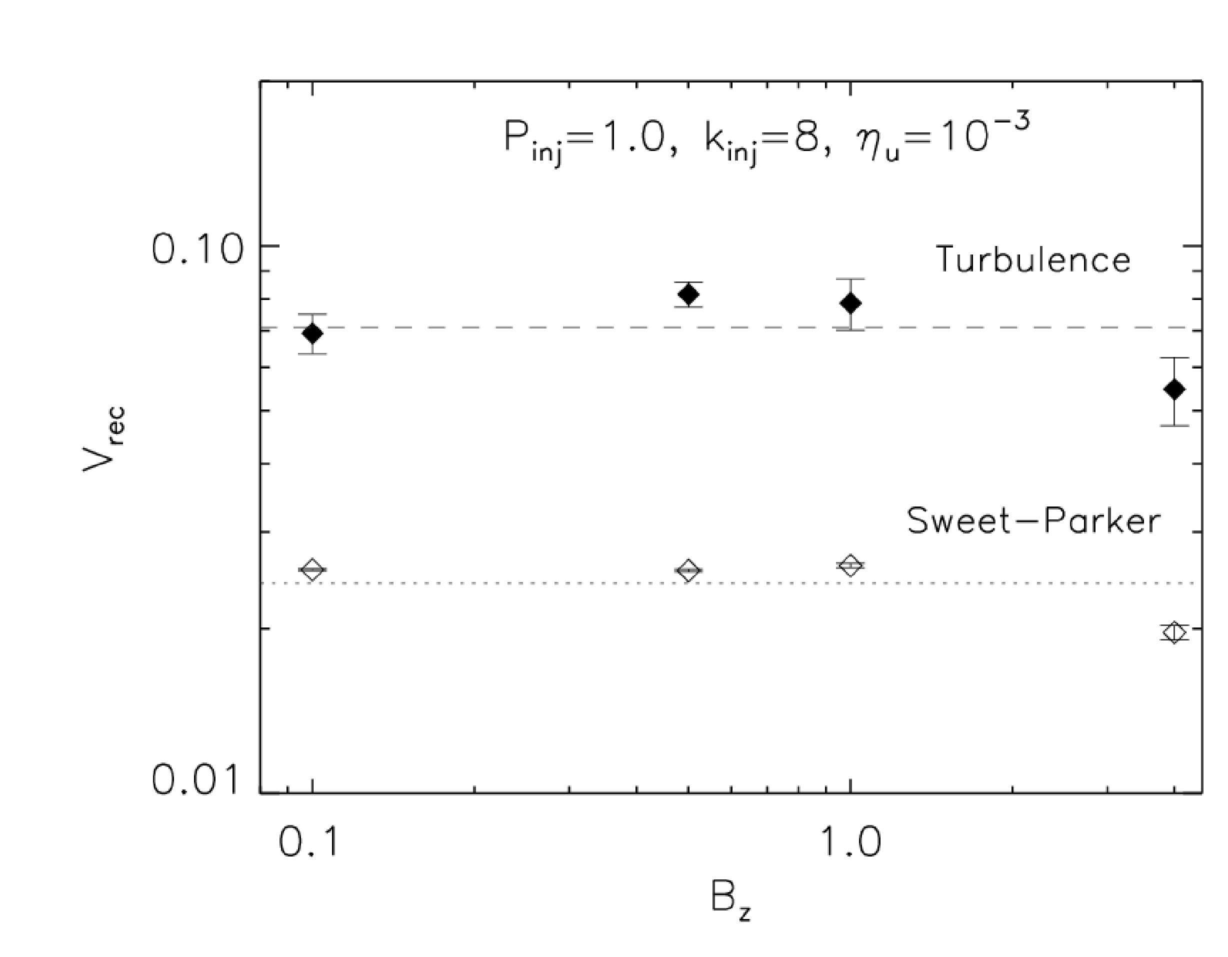


Universality of the Model

B_z - dependence

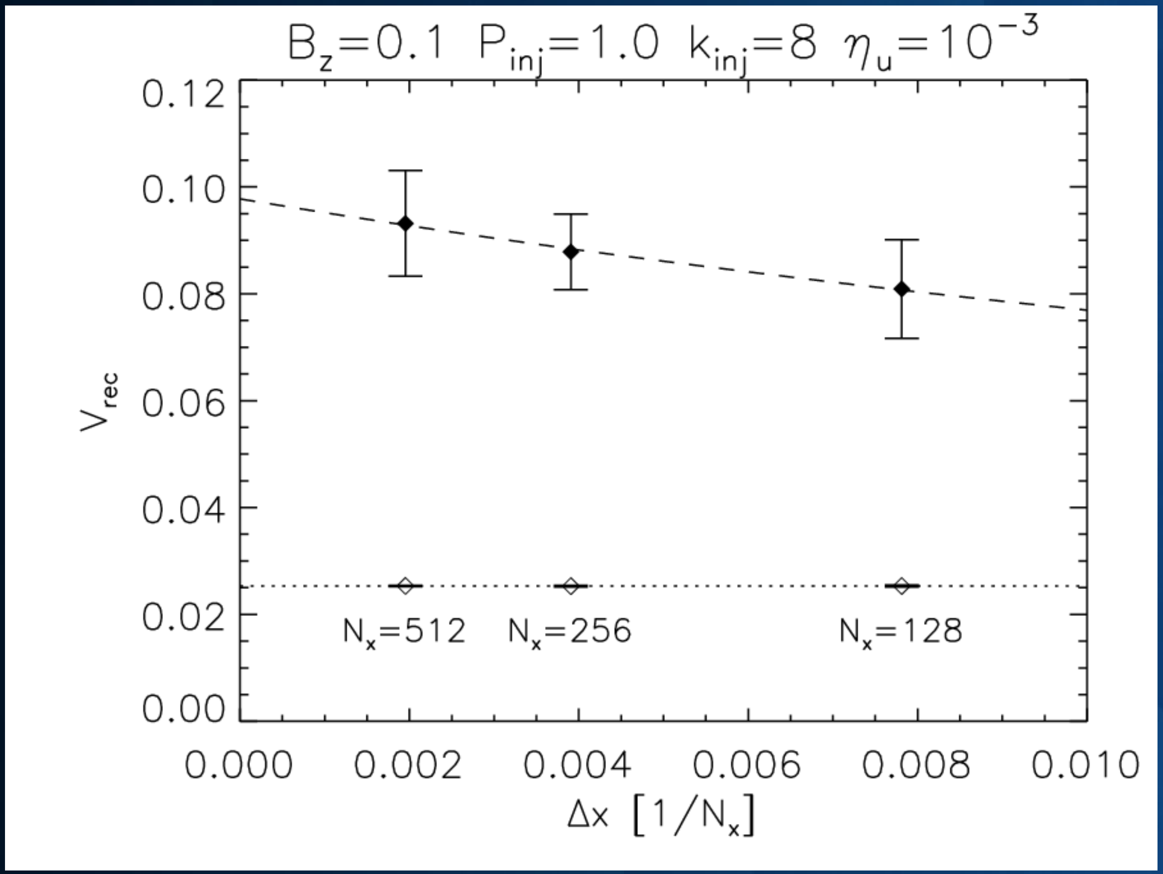






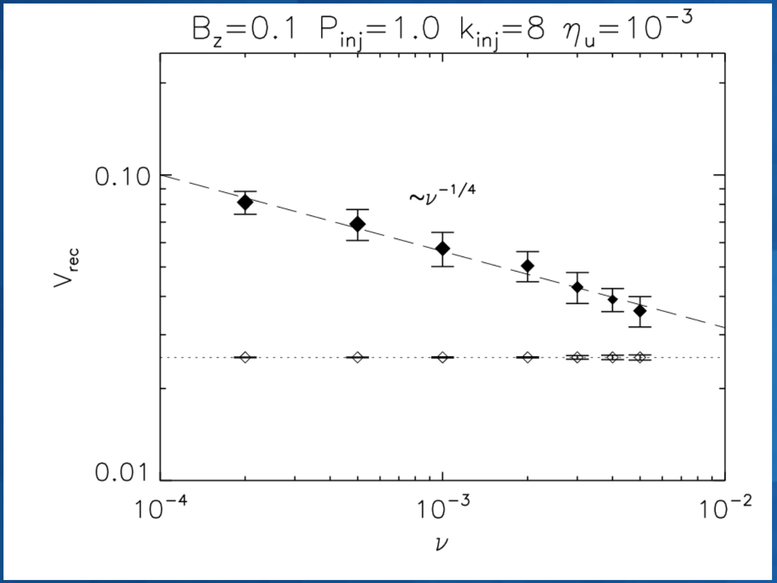
$0° < \alpha < 152°$

Resolution Dependence - Convergence



Reconnection rate shows good convergence for three different resolutions.

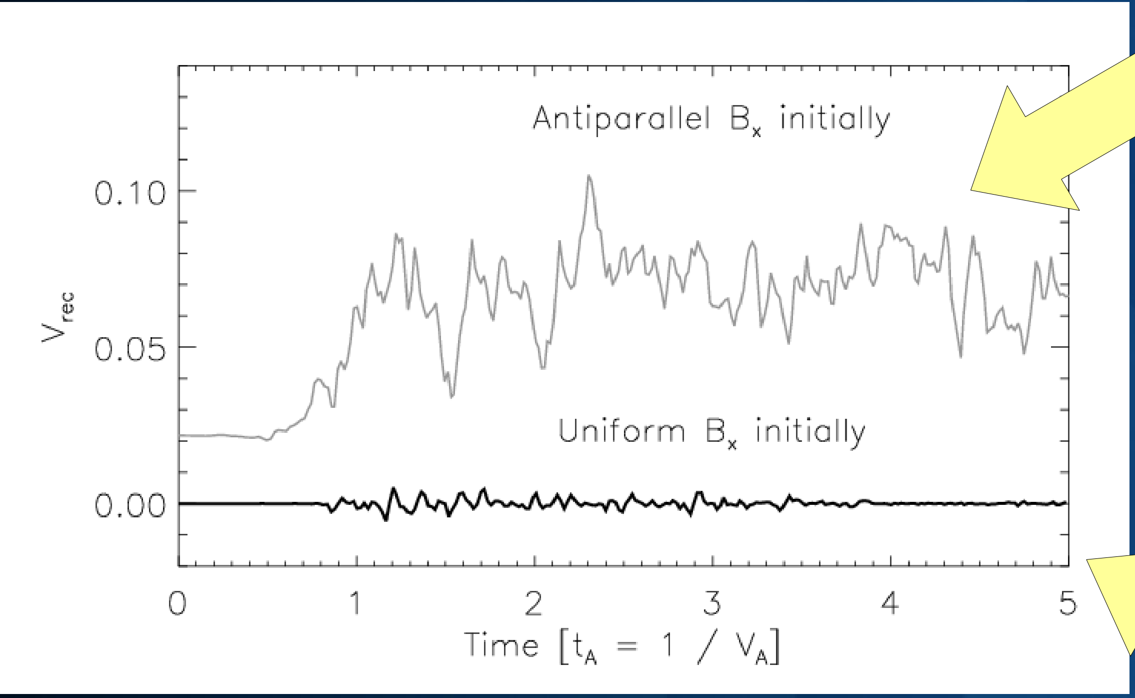
Small increase in the reconnection rate can be attributed to the numerical viscosity decreasing with the resolution (weaker dissipation at the current sheet scale).



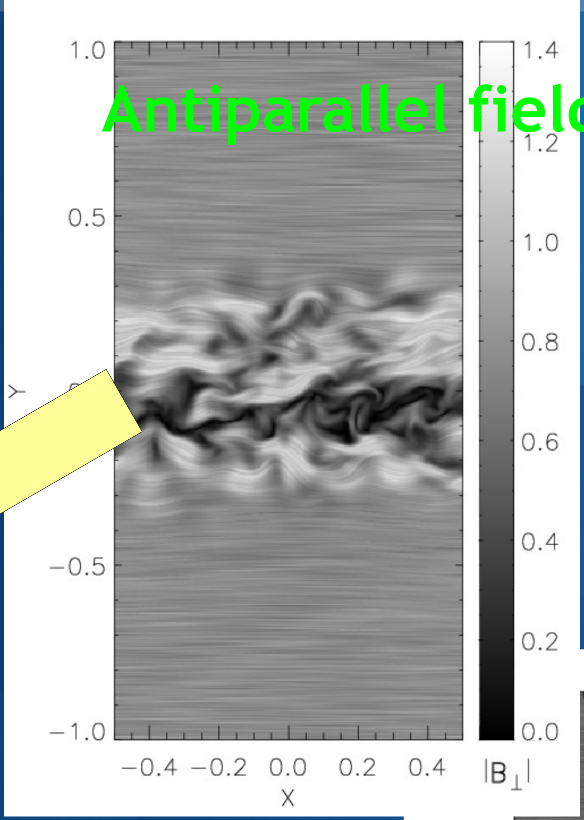
Viscosity dependence is weak: $V_{rec} \sim \nu^{-1/4}$

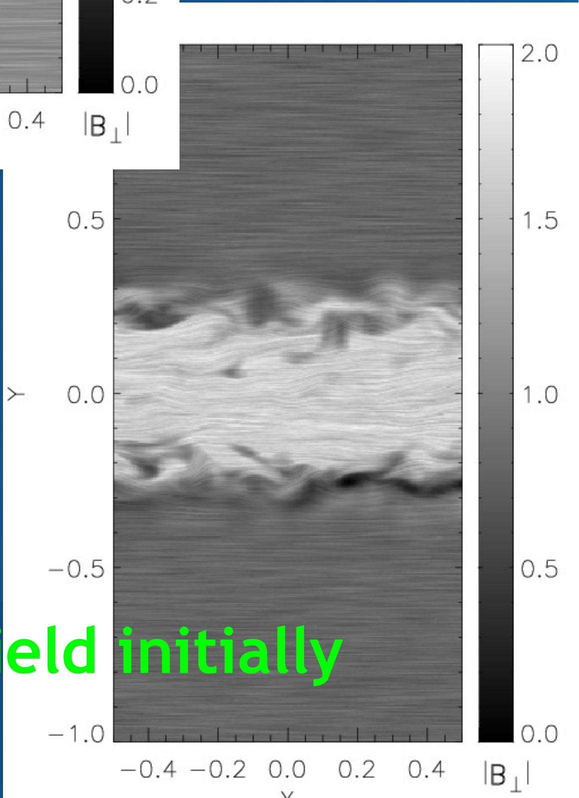
No Field Reversal Case

The rate of reconnection



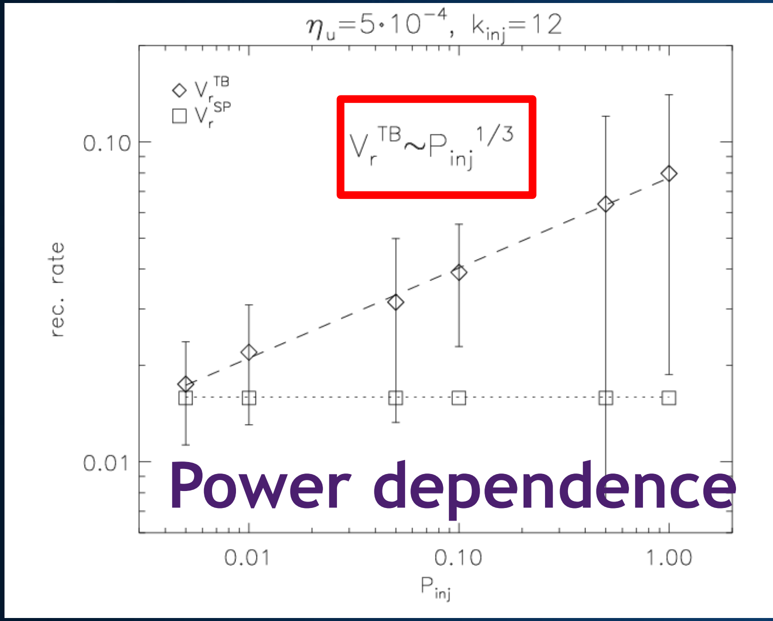
Antiparallel field initially



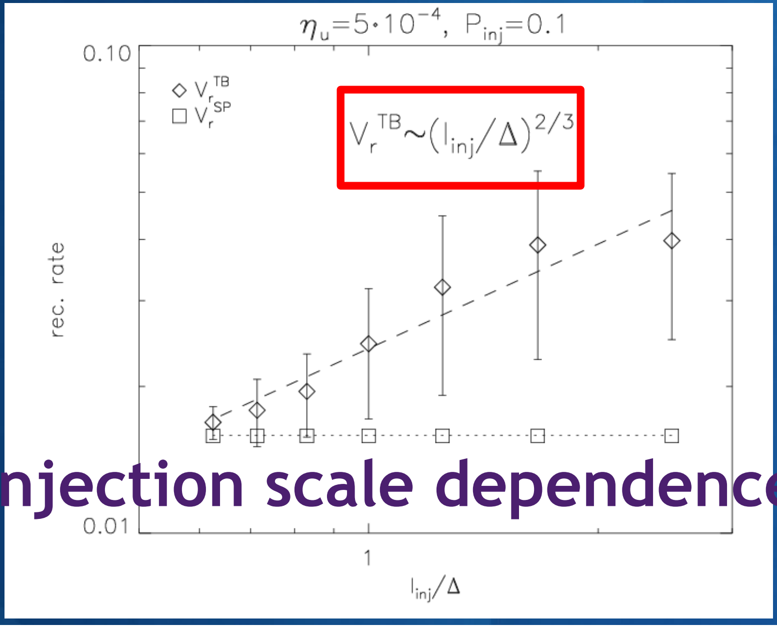


Uniform field initially

2D Models



Power dependence

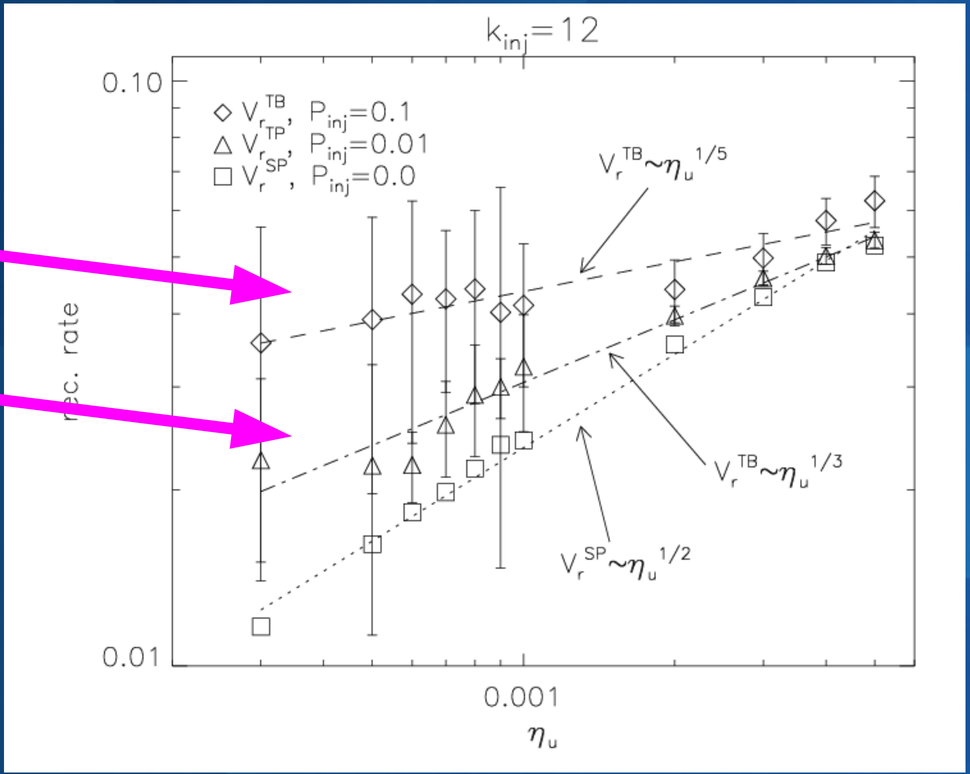


Injection scale dependence

Resistivity dependence!!!

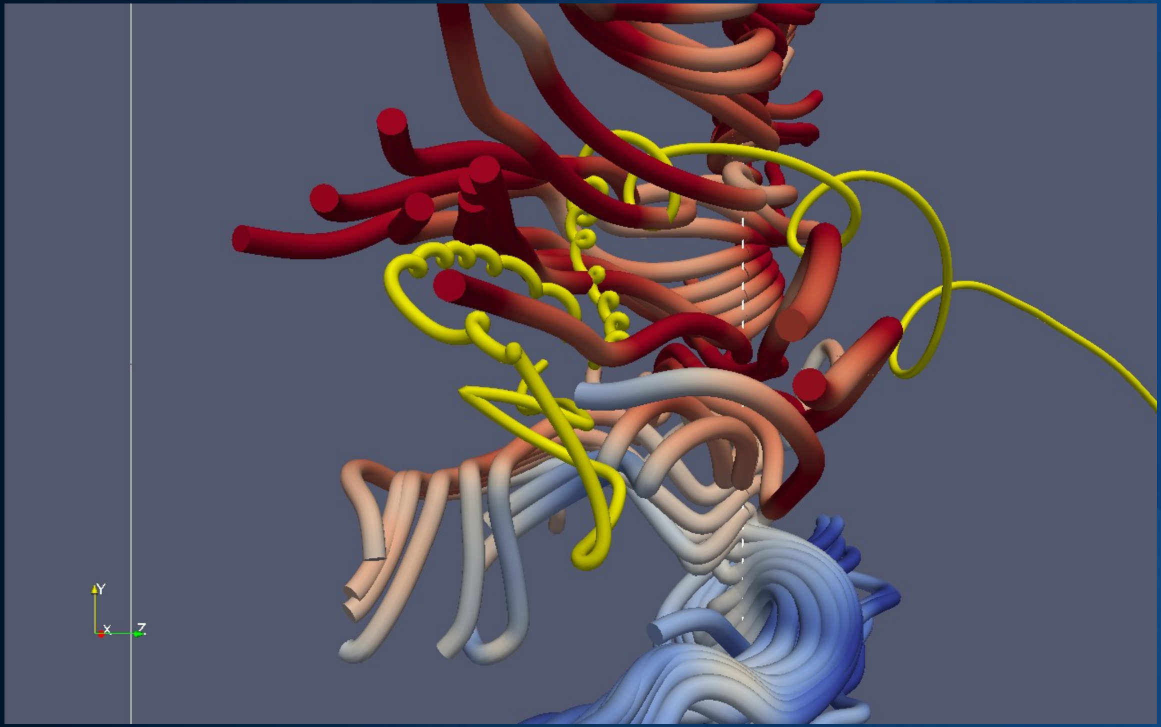
$$V_{rec}\sim\eta_u^{1/5}\ for\ P_{inj}=0.1$$

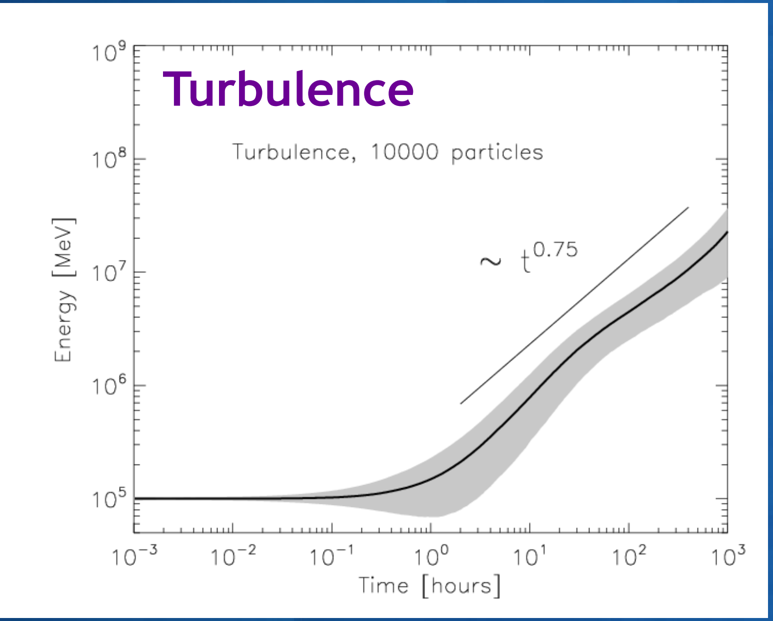
$$V_{rec}\sim\eta_u^{1/3}\ for\ P_{inj}=0.01$$

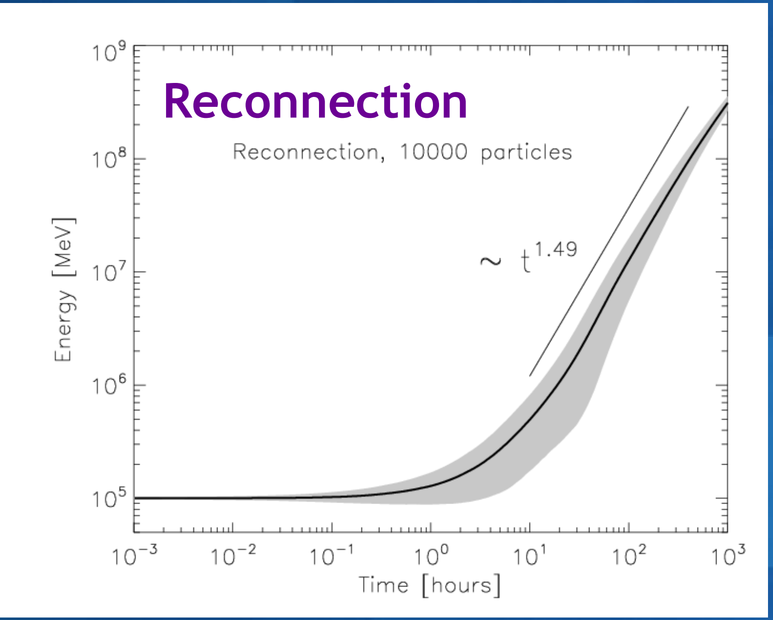


Ongoing Projects: Particle Acceleration

In collaboration with
Elisabete M. G. Dal Pino
Alex Lazarian





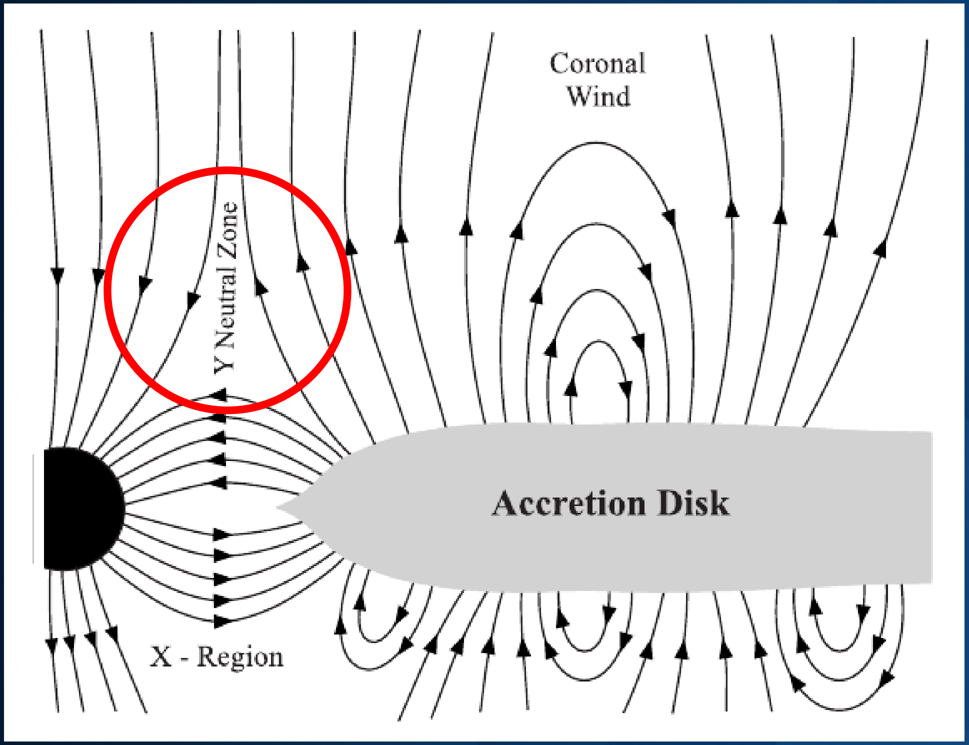


Fast magnetic reconnection is a possible candidate for the first order Fermi acceleration (Dal Pino & Lazarian, 2001, 2003, 2005)

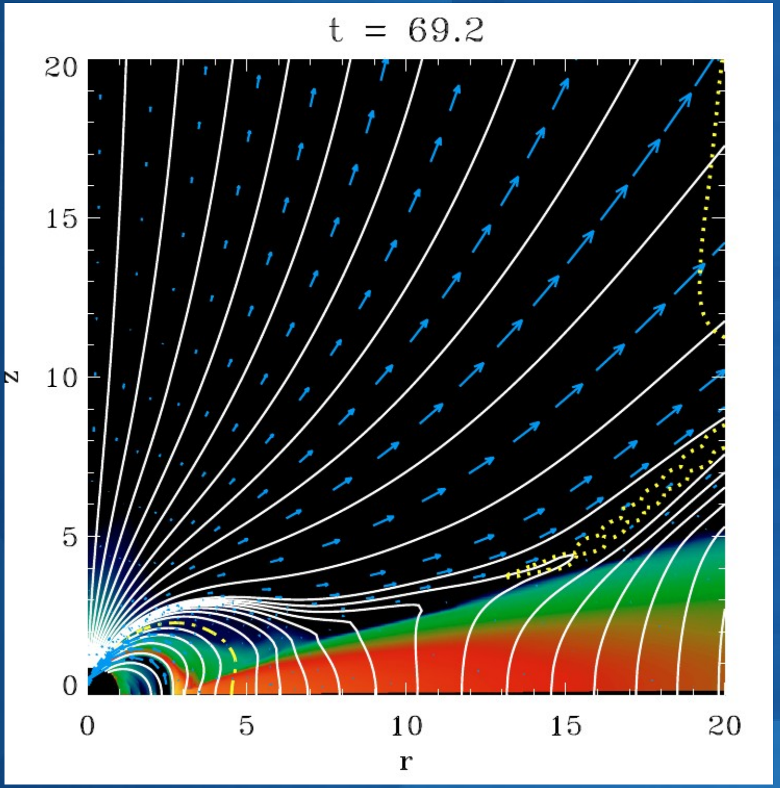
Ongoing Projects: Reconnection in Accretion Disks

In collaboration with
Luis H. S. Kadowaki
Elisabete M. G. Dal Pino

MHD simulations of the accretion onto a dipolar magnetosphere (Zanni & Ferreira, 2009)



Violent magnetic reconnection is possible in the Y Neutral Zone resulting in energy release ~10^{39} ergs/s (Dal Pino & Lazarian, 2005)



Density color map with magnetic field lines

Conclusions

- Numerical studies of stochastic reconnection finally possible, even though reconnection in numerical simulations is always fast
- Turbulence significantly affects the topology of B near the diffusion region
- Fragmentation of current sheet decreases the disparity in inflow/outflow ratios
- For large scale turbulence reconnection is the enhancement of reconnection rate V_{rec} grows as $\sim V_T^2$ and $\sim (l_{inj}/\Delta)^{3/4}$
- Reconnection speed is independent of the resistivity
- We define a new measure which precicely shows the amount of reconnected flux in the presence of turbulence