Black hole jets and the membrane paradigm

Bob Penna (MIT)

A puzzle

- The **Blandford-Znajek** model is the Standard Model of spin-powered black hole jets.

- Unfortunately, the jet power depends on an arbitrary function:

$$\Omega_F(\theta) = \text{field line velocity}$$

- In practice, this ambiguity is fixed by solving force-free electrodynamics (or GRMHD) for black hole magnetospheres. This is hard!

- Amazingly, one always finds

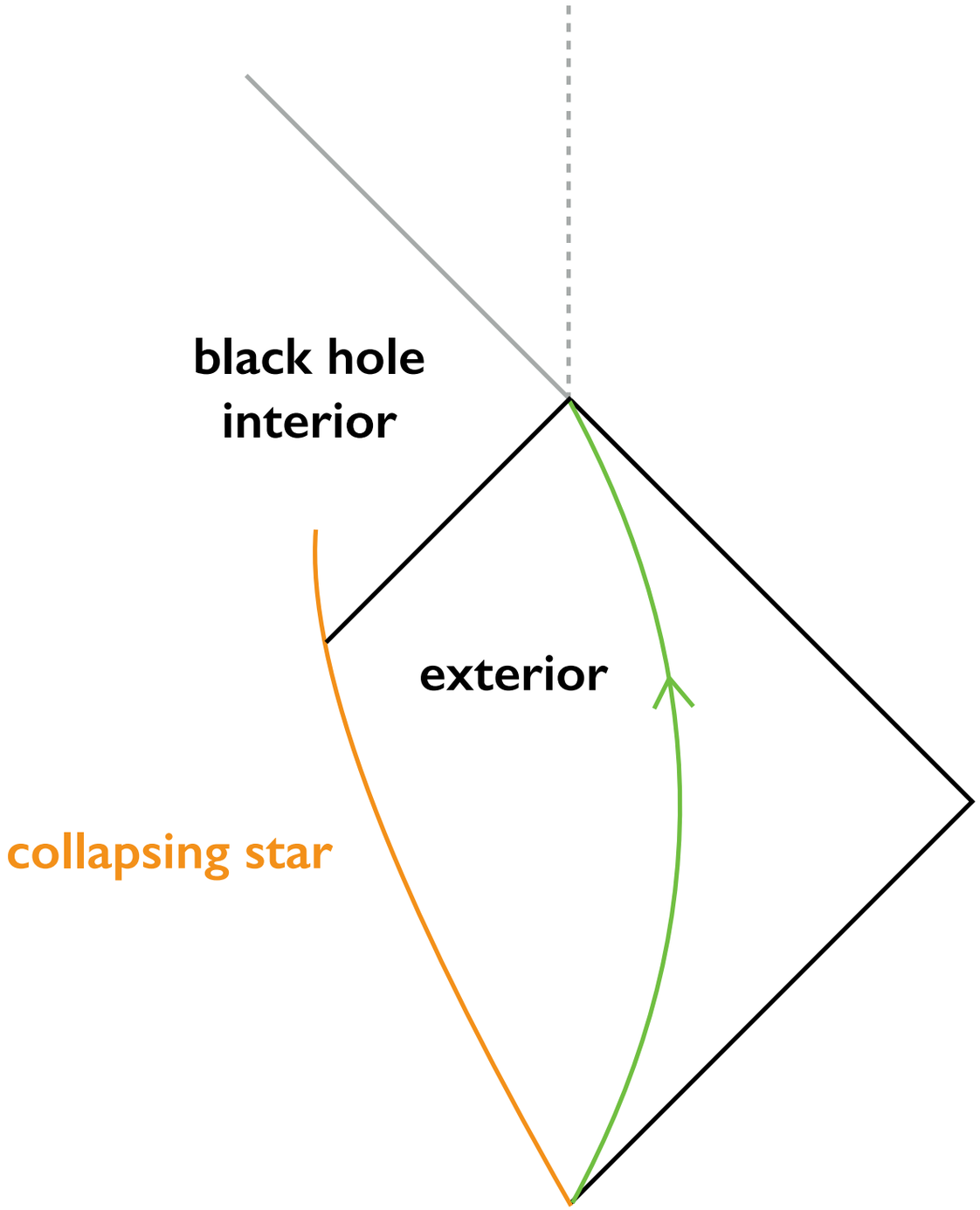
$$\Omega_F(\theta)/\Omega_H \sim 0.4 - 0.5$$

- I will give a new explanation of this fact (based on RFP (2015) arXiv:1504.00360).

Outline

- Black hole membrane paradigm
- The membrane at infinity
- Force-free black hole jets as circuits
- $\Omega_F/\Omega_H \sim 0.4$ - 0.5 from impedance matching

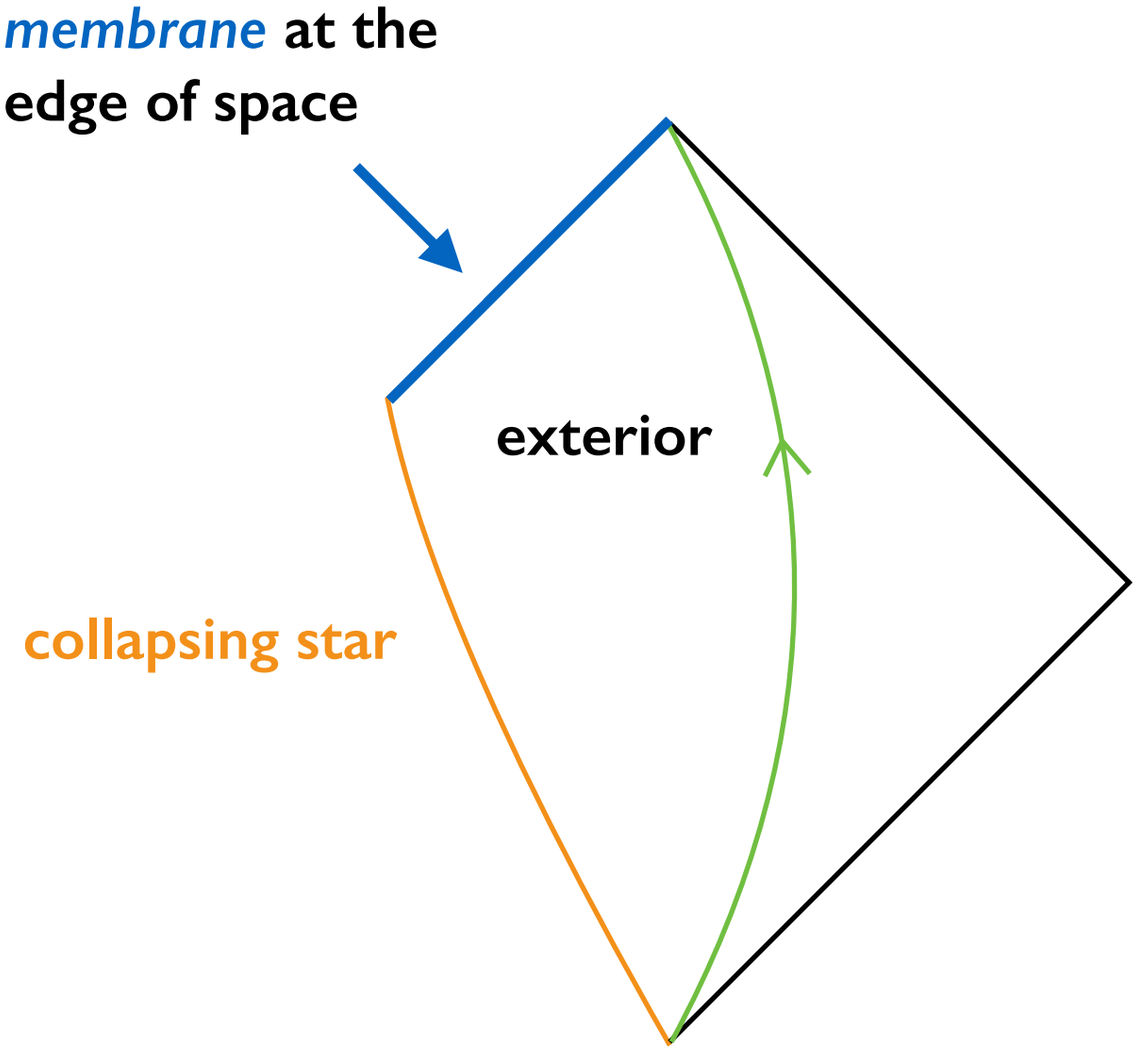
Why membranes?



- An **observer who does not jump in the black hole** has no access to the **interior**.

- Do not discuss things one cannot measure.

The membrane paradigm

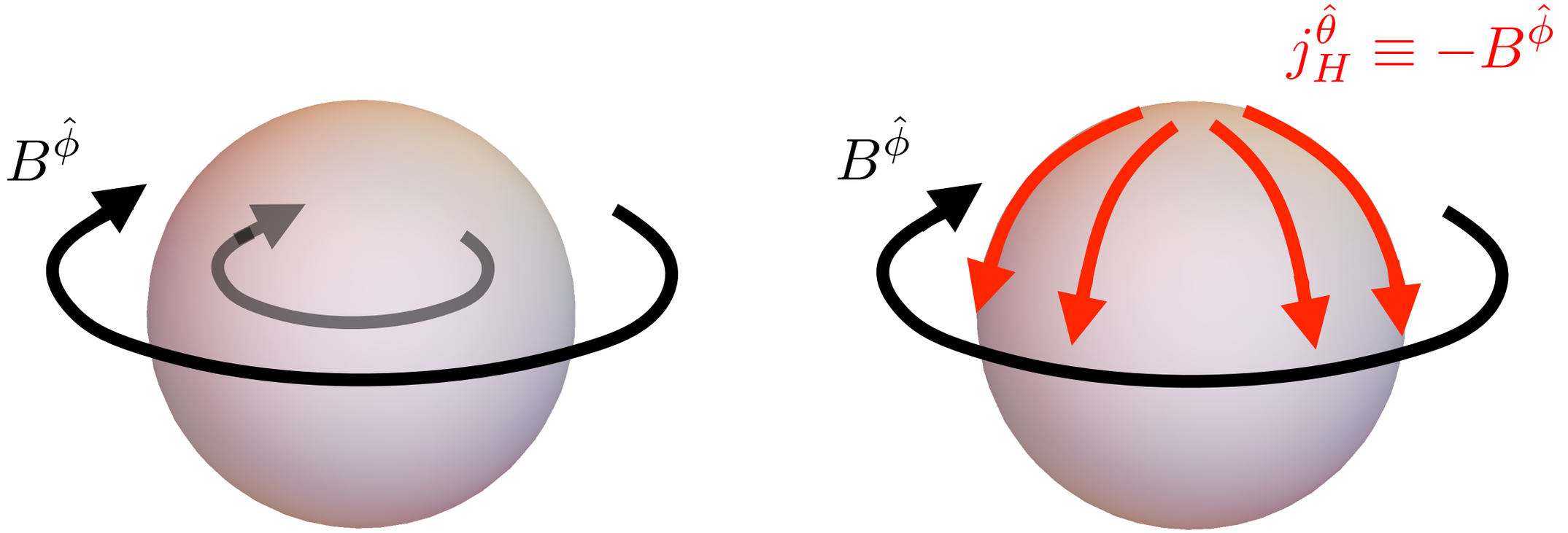


- **Idea**: throw away the black hole interior from all calculations (since the exterior observer cannot measure it).

- Add a **membrane** at the edge of space.

- Define the membrane so that exterior physics is reproduced correctly.

Example: the membrane current

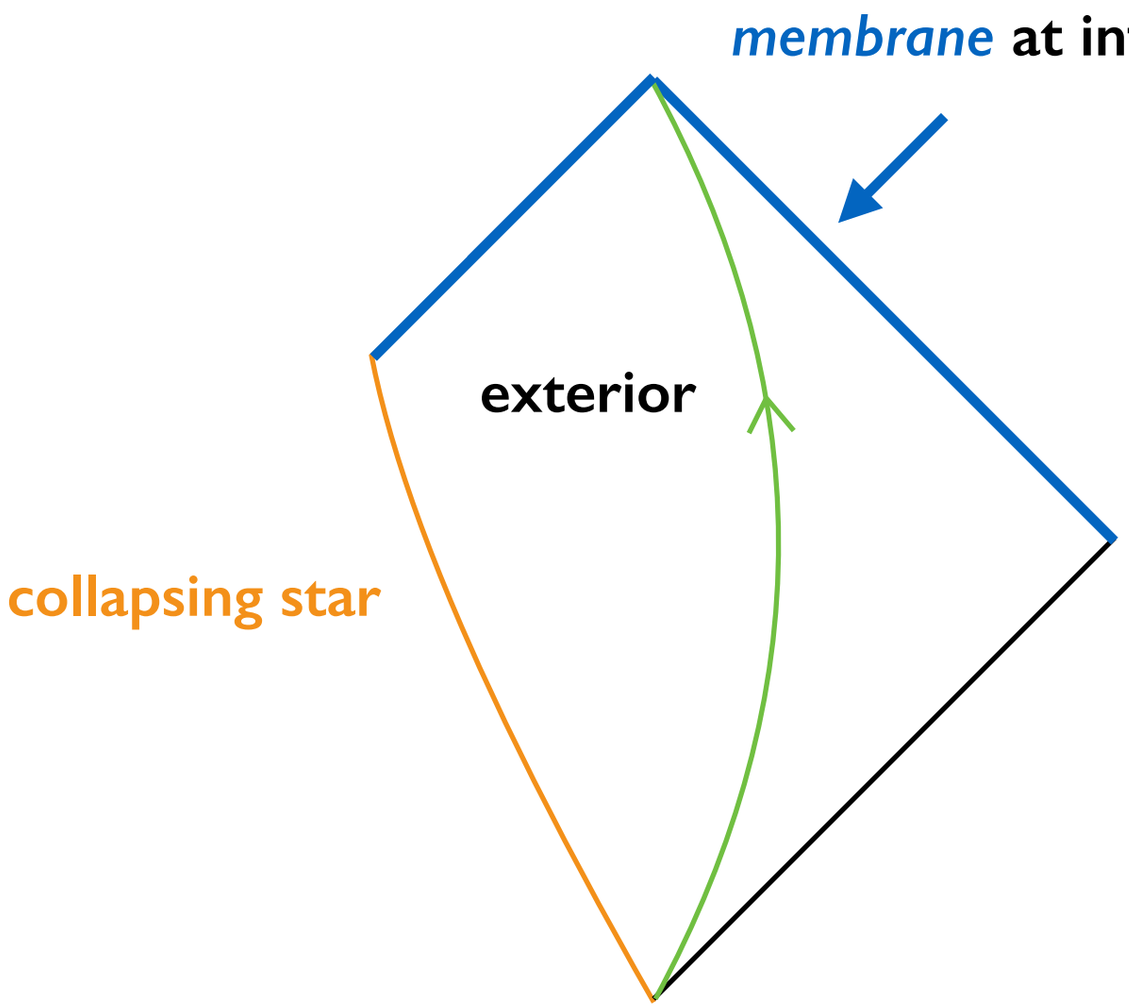


To satisfy Ampere's law, the membrane must carry a **current density.**

Properties of the membrane

- The membrane is a fluid obeying the Navier-Stokes equations.
- It has a viscosity and a resistivity.
- The energy and angular momentum of the black hole are stored in the membrane's stress-energy tensor.
- For a modern, mathematically rigorous derivation based on an action principle see Parikh and Wilczek (1998).

The membrane at infinity

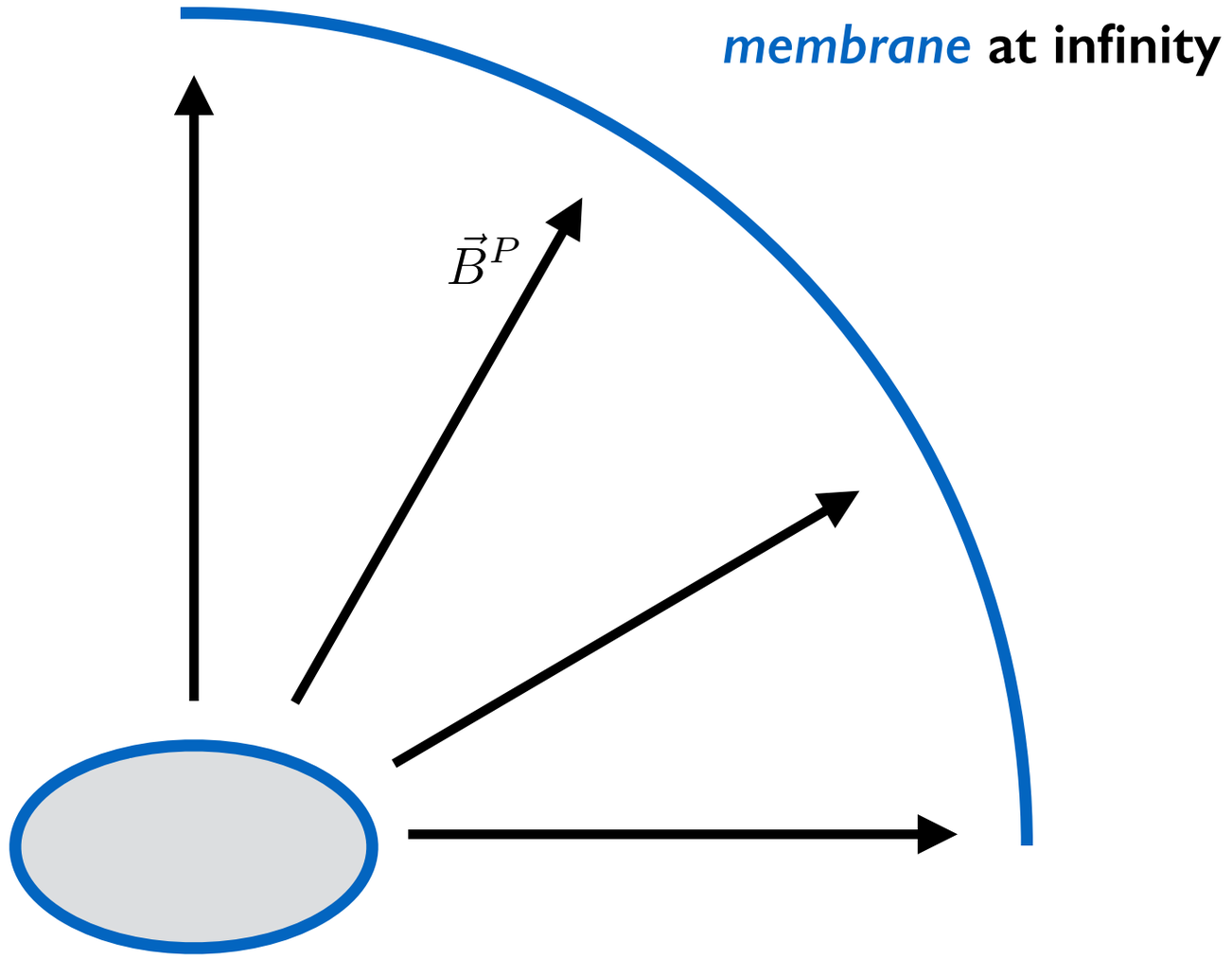


- Recently, I've shown that **future null infinity** also has a dual membrane description (RFP (2015), Phys. Rev. D, in press).

- The membranes at the horizon and infinity have the same **surface resistivity:**

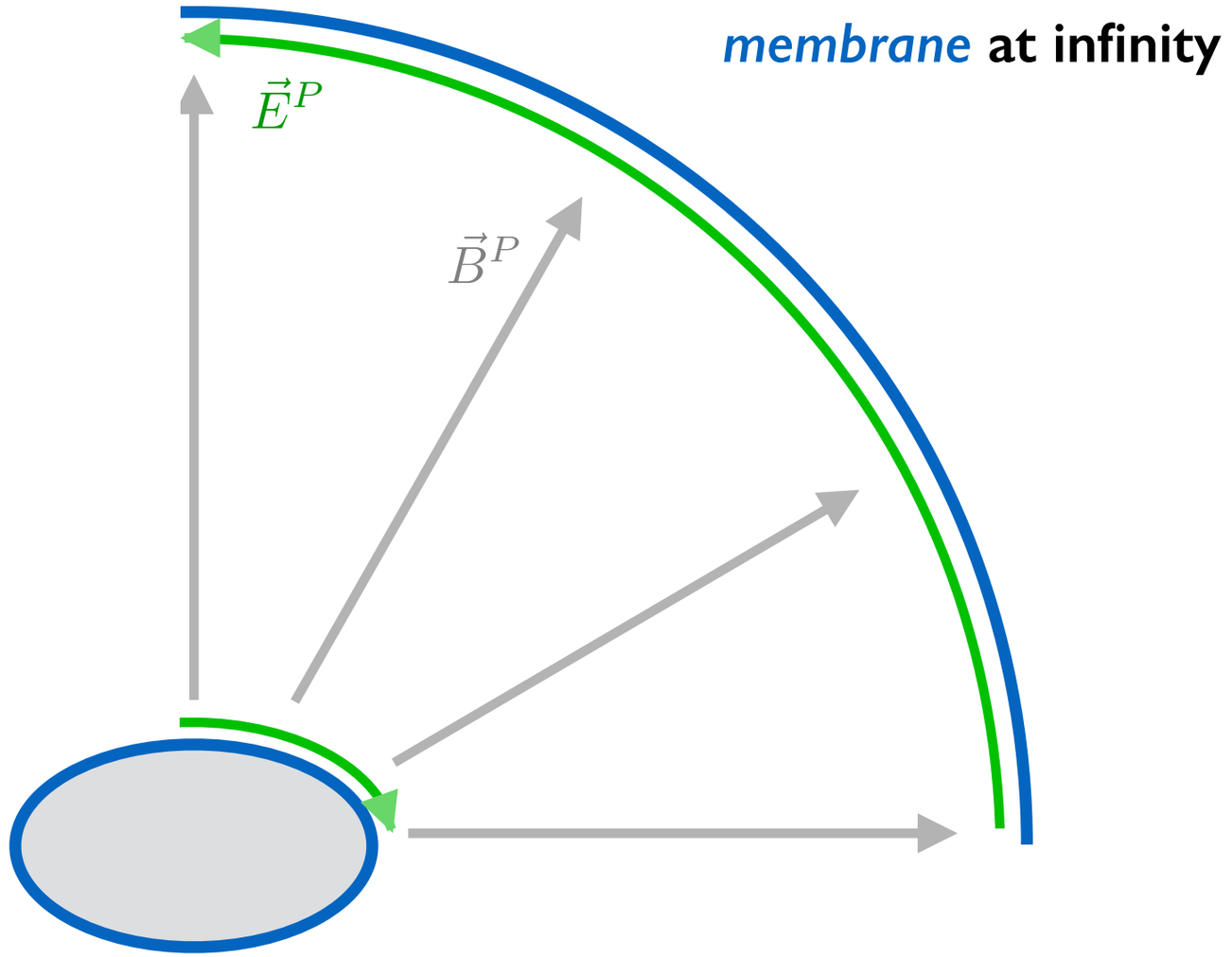
$$R_H = R_\infty = 377\,\mathrm{Ohms}.$$

Force-free black hole magnetosphere



Force-free black hole magnetosphere

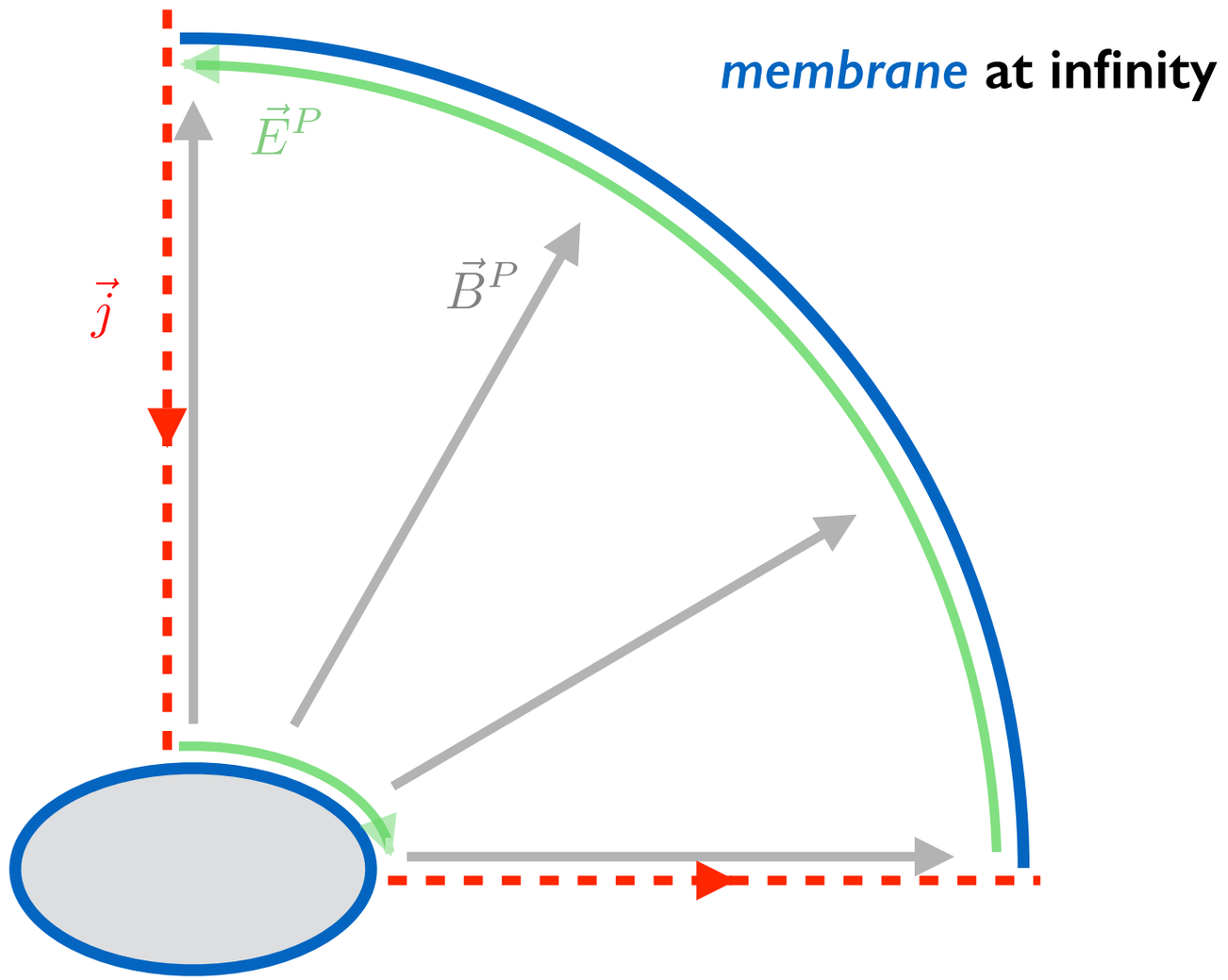
The poloidal electric field is orthogonal to the magnetic field.



Force-free black hole magnetosphere

The poloidal electric field is orthogonal to the magnetic field.

The current in the magnetosphere follows magnetic field lines.



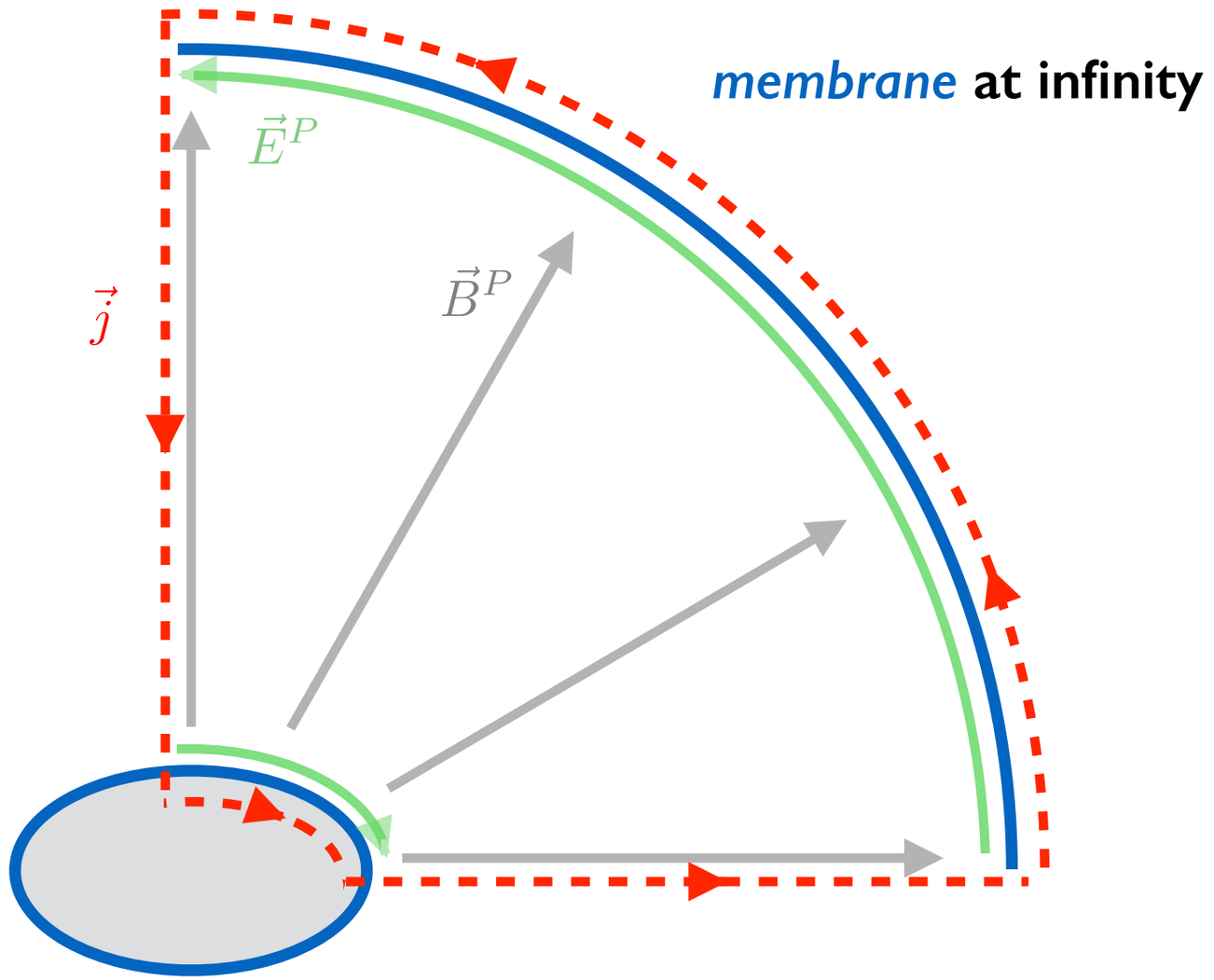
Force-free black hole magnetosphere

The poloidal electric field is orthogonal to the magnetic field.

The current in the magnetosphere follows magnetic field lines.

The currents on the membranes follow electric field lines (membrane Ohm's law).

The total current forms a closed circuit!

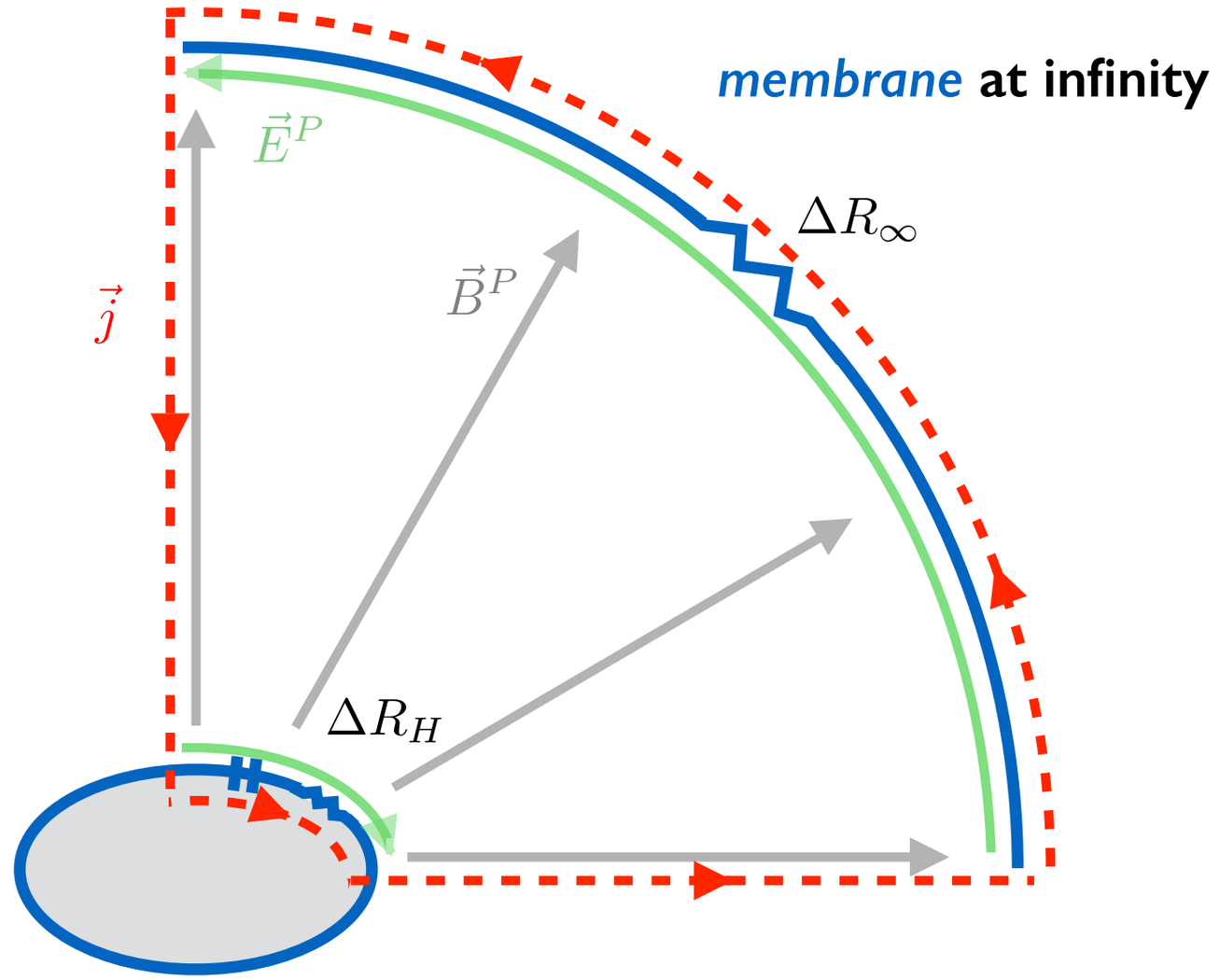


RFP (2015) arXiv:1504.00360

Force-free black hole magnetosphere

The black hole acts as a battery driving the circuit.

The BZ jet power is the power dissipated on the membrane at infinity.



RFP (2015) arXiv:1504.00360

Impedance matching

- The arbitrary function in the BZ model is the **circuit efficiency**:

$$\Omega_F/\Omega_H = \frac{\Delta R_\infty/\Delta R_H}{1+\Delta R_\infty/\Delta R_H}.$$

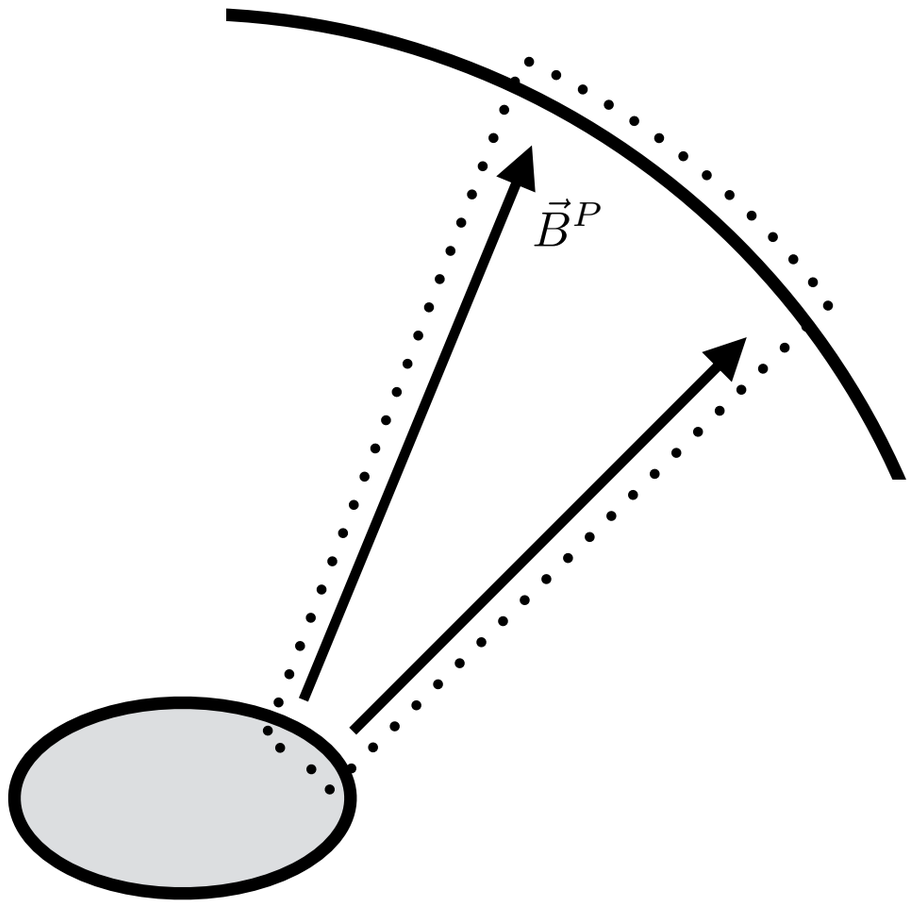
- Just as for ordinary circuits, **50% efficiency** ($\Omega_F/\Omega_H = 0.5$) gives maximum power.

- Maximum output power corresponds to **perfect impedance matching** between horizon and infinity ($\Delta R_\infty/\Delta R_H = 1$).

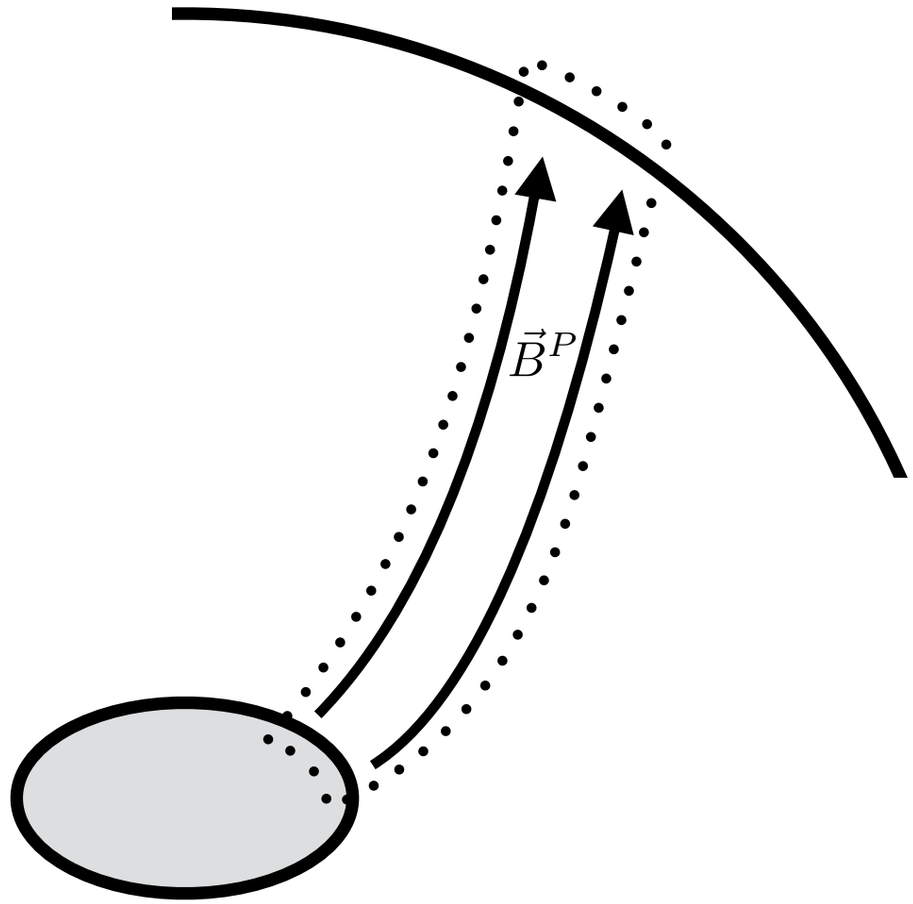
- Both membranes have the same **surface resistivity**: 377 Ohms. So **Ω_F/Ω_H** only depends on the field line distributions at the horizon and at infinity.

RFP (2015) arXiv:1504.00360

Impedance matching



$$\Delta R_\infty = \Delta R_H$$



$$\Delta R_\infty < \Delta R_H$$

RFP (2015) arXiv:1504.00360

Impedance matching

Explicitly,

$$\Omega_F/\Omega_H = \frac{\Delta R_\infty/\Delta R_H}{1+\Delta R_\infty/\Delta R_H},$$

where,

$$\Delta R_\infty/\Delta R_H = \frac{\left[\frac{\sqrt{g_{\theta\theta}}}{\varpi}\frac{d\theta}{dA_\phi}\right]_\infty}{\left[\frac{\sqrt{g_{\theta\theta}}}{\varpi}\frac{d\theta}{dA_\phi}\right]_H}.$$

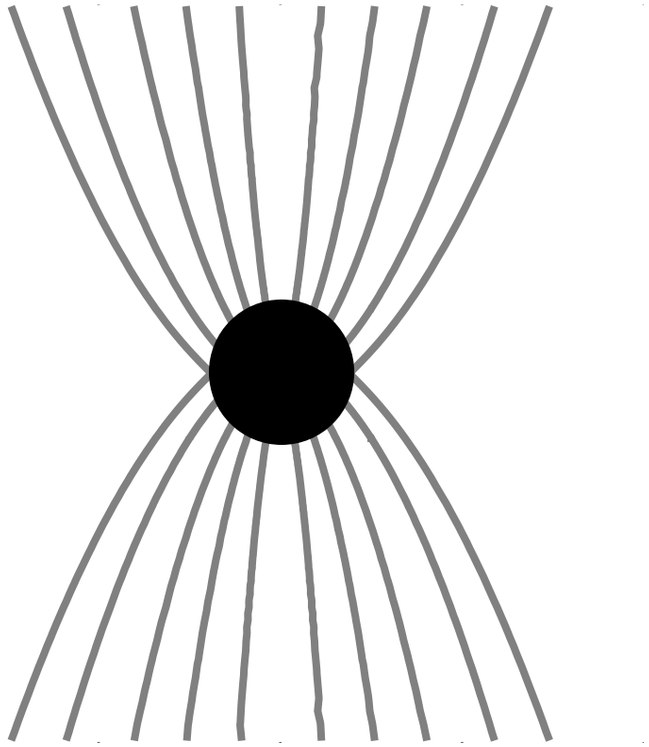
ϖ is a cylindrical radius. It depends only on the metric.

$\Omega_F(\theta)/\Omega_H$ is fixed by the vector potential, $A_\phi(\theta)$, at the horizon and at infinity.

All roughly uniform force-free fields have $\Omega_F/\Omega_H \sim 0.4$ - 0.5.

RFP (2015) arXiv:1504.00360

Example: paraboloidal field



The distribution of magnetic flux at infinity is

$$A_\phi = r(1-\cos\theta)/2,$$

and the distribution of flux at the horizon is

$$A_\phi = 2\log 2 - (1+\cos\theta)\log(1+\cos\theta).$$

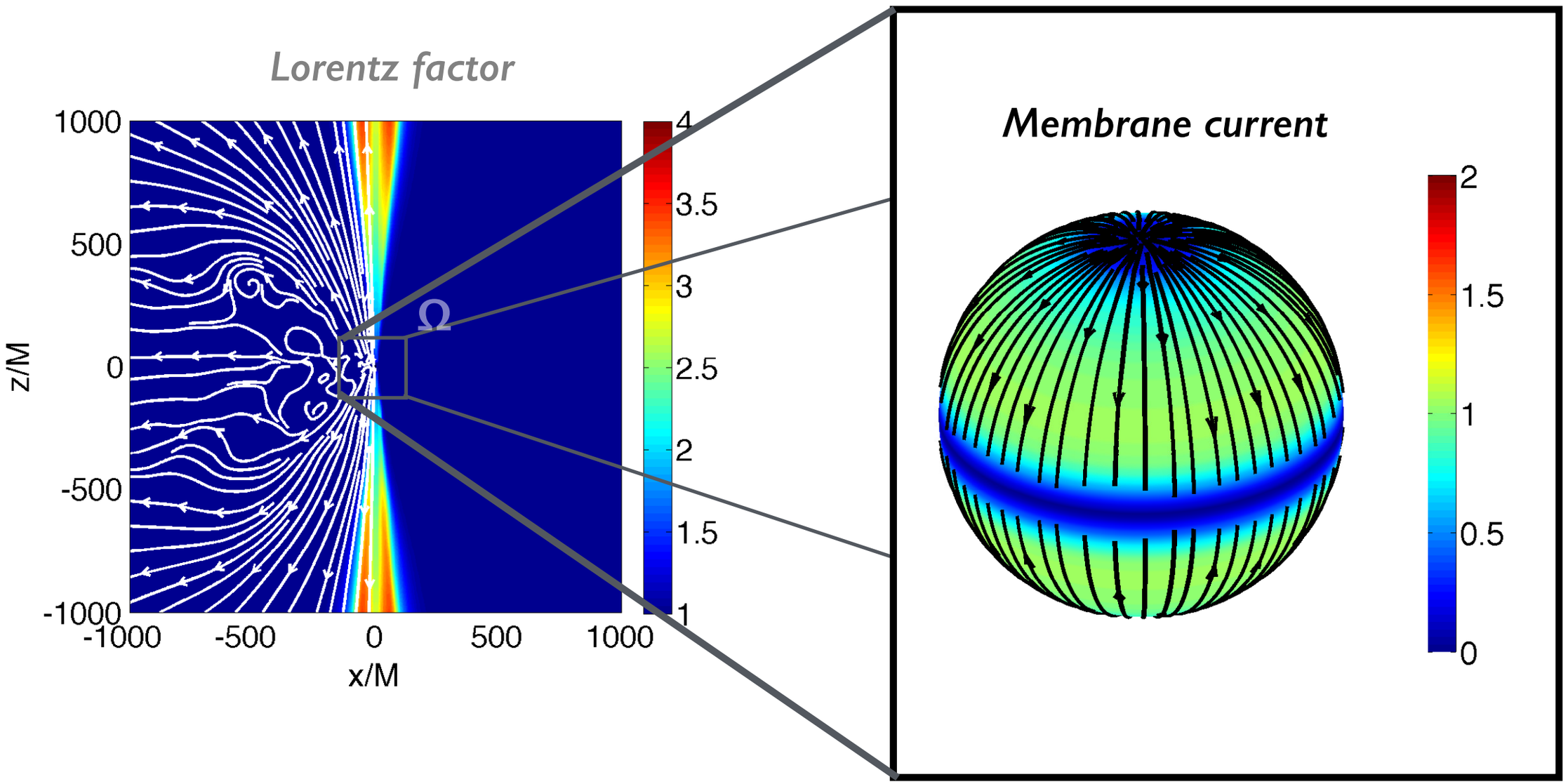
Plugging into the previous formula gives

$$\Omega_F/\Omega_H = \frac{\sin^2\theta(1+\log(1+\cos\theta))}{4\log 2 + \sin^2\theta + [\sin^2\theta - 2(1+\cos\theta)]\log(1+\cos\theta)}.$$

RFP (2015) arXiv:1504.00360

Example: GRMHD simulations

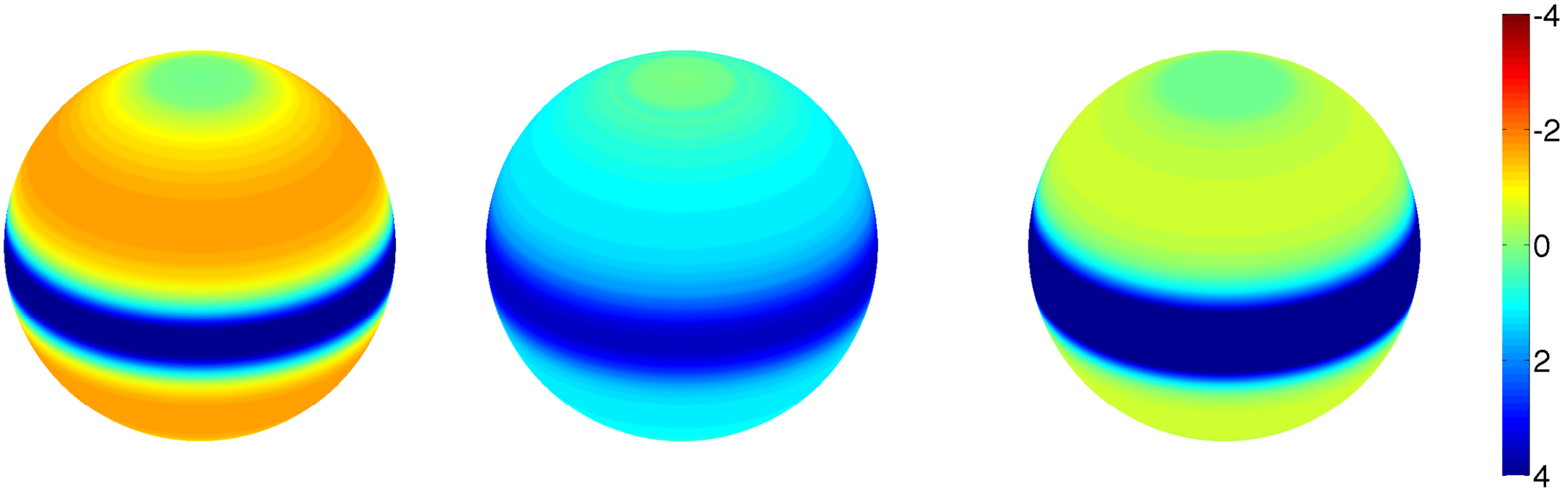
t- and ϕ-averaged GRMHD simulation data (Ω_H = 0.34):



RFP, Narayan, Sadowski, MNRAS (2013), 436

Example: GRMHD simulations

$$I\Delta V \quad - \quad I^2\Delta R_H \quad = \quad I^2\Delta R_\infty$$

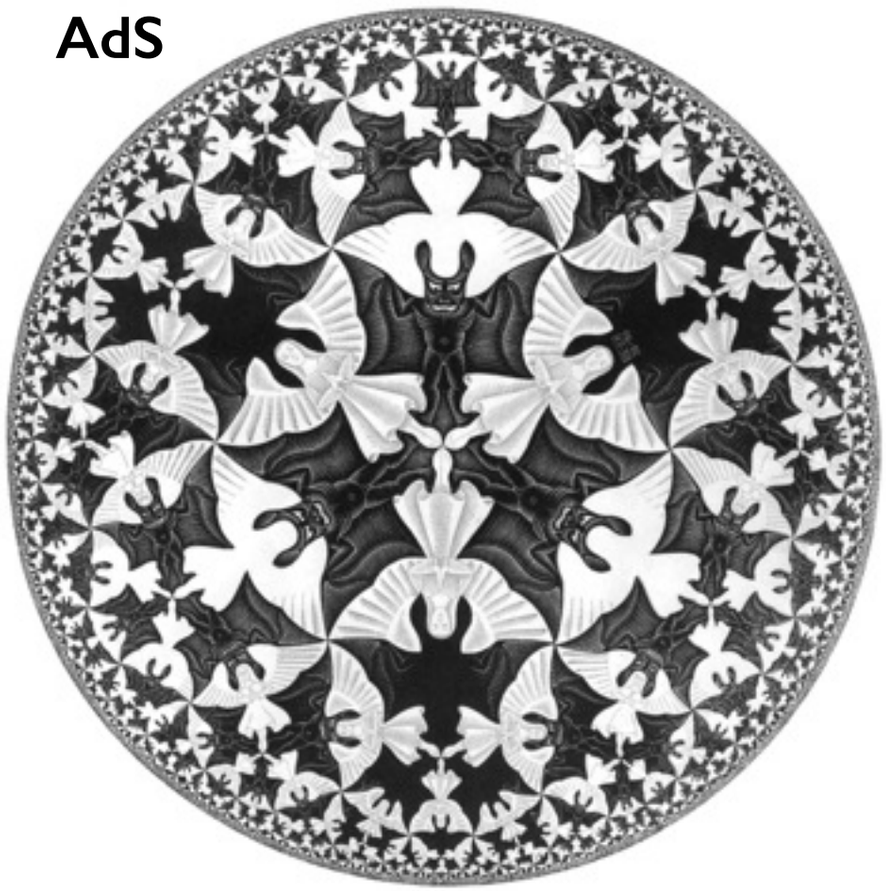


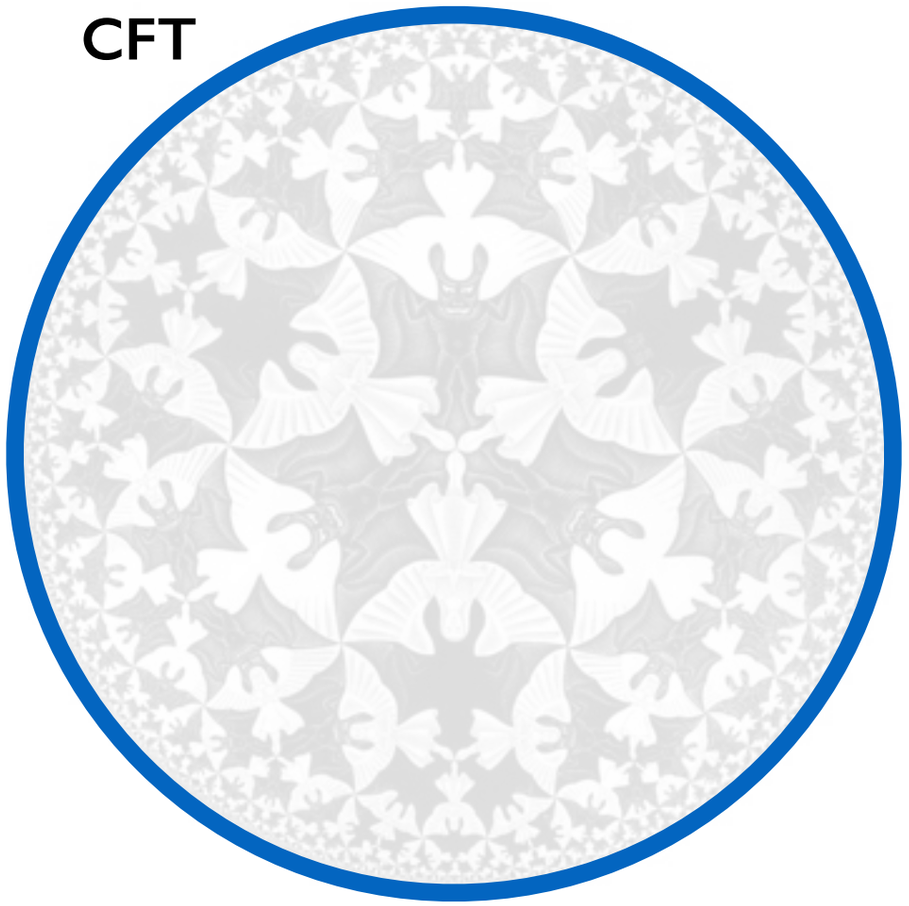
t- and ϕ-averaged GRMHD simulation data (Ω_H = 0.34).

RFP, Narayan, Sadowski, MNRAS (2013), 436

Membranes and fundamental physics

- AdS/CFT





Conclusions

- I have extended the membrane paradigm to include a “membrane at infinity.”
- This gives a dual description of force-free jets as closed circuits.
- Ω_F/Ω_H is the circuit efficiency. It is completely fixed by $A_\phi(\theta)$ at the horizon and $A_\phi(\theta)$ at infinity.
- All roughly uniform force-free fields have $\Omega_F/\Omega_H \sim 0.4$ - 0.5.
- This universality follows from the universality of $R_H = R_\infty = 377$ Ohms.
- This fixes the arbitrary function Ω_F in the BZ jet power prediction.