Semi-Analytical GRMHD Jet Model

-an complementary approach to GRMHD simulations-

Hung-Yi PU

Academia Sinica, Institute of Astronomy and Astrophysics (ASIAA)

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Important Ingredients of “BH powered” Relativistic Jet

- **Extraction of BH rotational energy by a hole-threading field line:**
    - MHD version of BZ process (Takahashi et al. 1990)
    - Poynting flux dominated GRMHD flow
    - the development of outflow is constraint by the inflow
- **Jet acceleration:**
    - efficient conversion from magnetic to kinetic energy
    - fast magnetosonic surface exists, such that magnetic nozzle effect can take place (e.g. Camenzind 1989; Li et al. 1992; Begelman & Li 1994; Takahashi & Shibata 1998)
- **Parabolic field line** (e.g. observation of M87; Asada & Nakamura 2012)

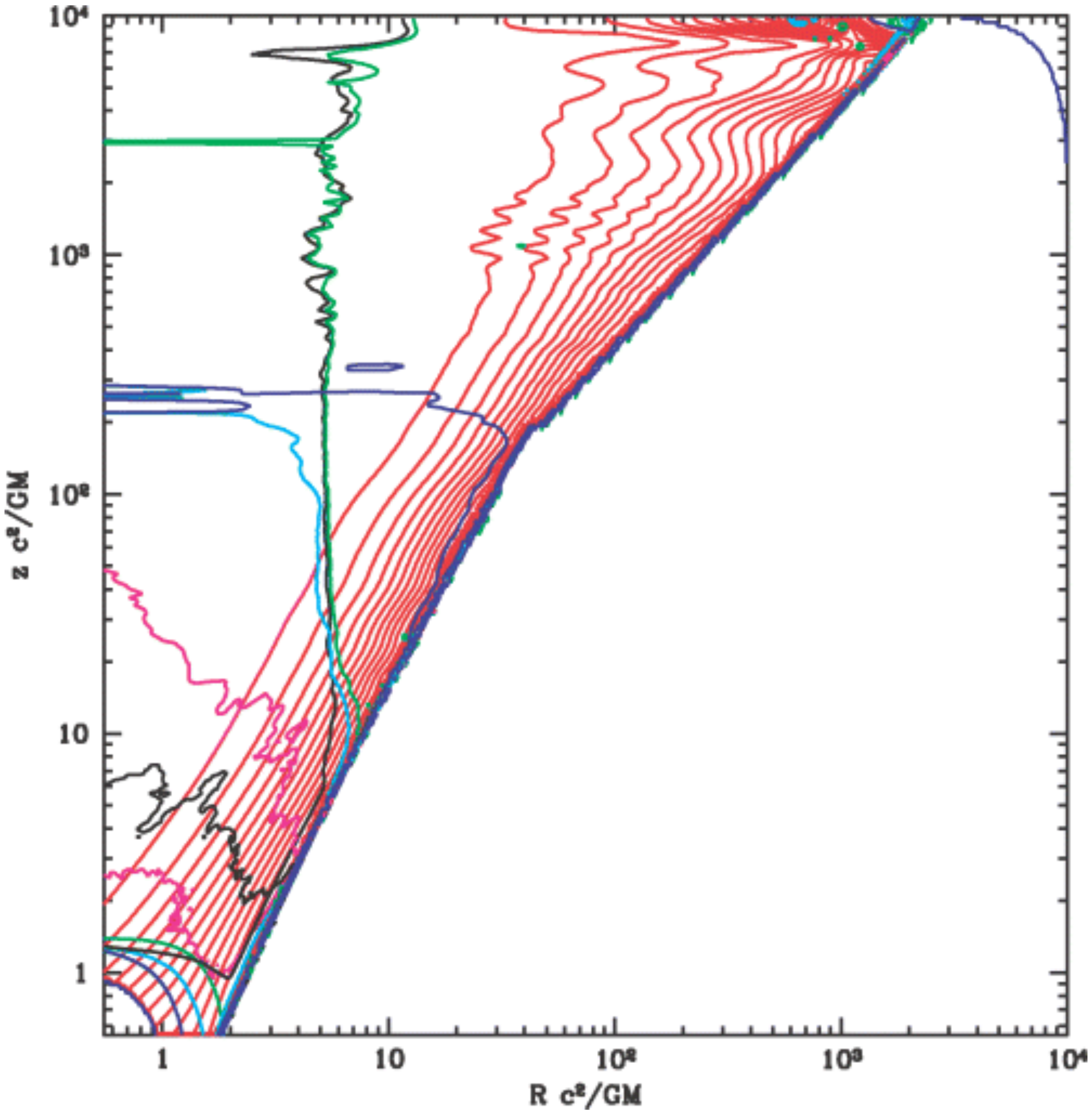
Outline

- GRMHD flow structure in the vicinity of BH
- Model setup and Results
  (based on ApJ 801:56 2015)
- Next Setup and Preliminary results
- Summary

GRMHD Flow Structure

GRMHD Simulation (a/M=0.9375)

B_p field lines and characteristic surfaces



time-averaged GRMHD simulation result, McKinney 2006

(see also, McKinney and Gammie 2004, Hawley and Krolik 2006)

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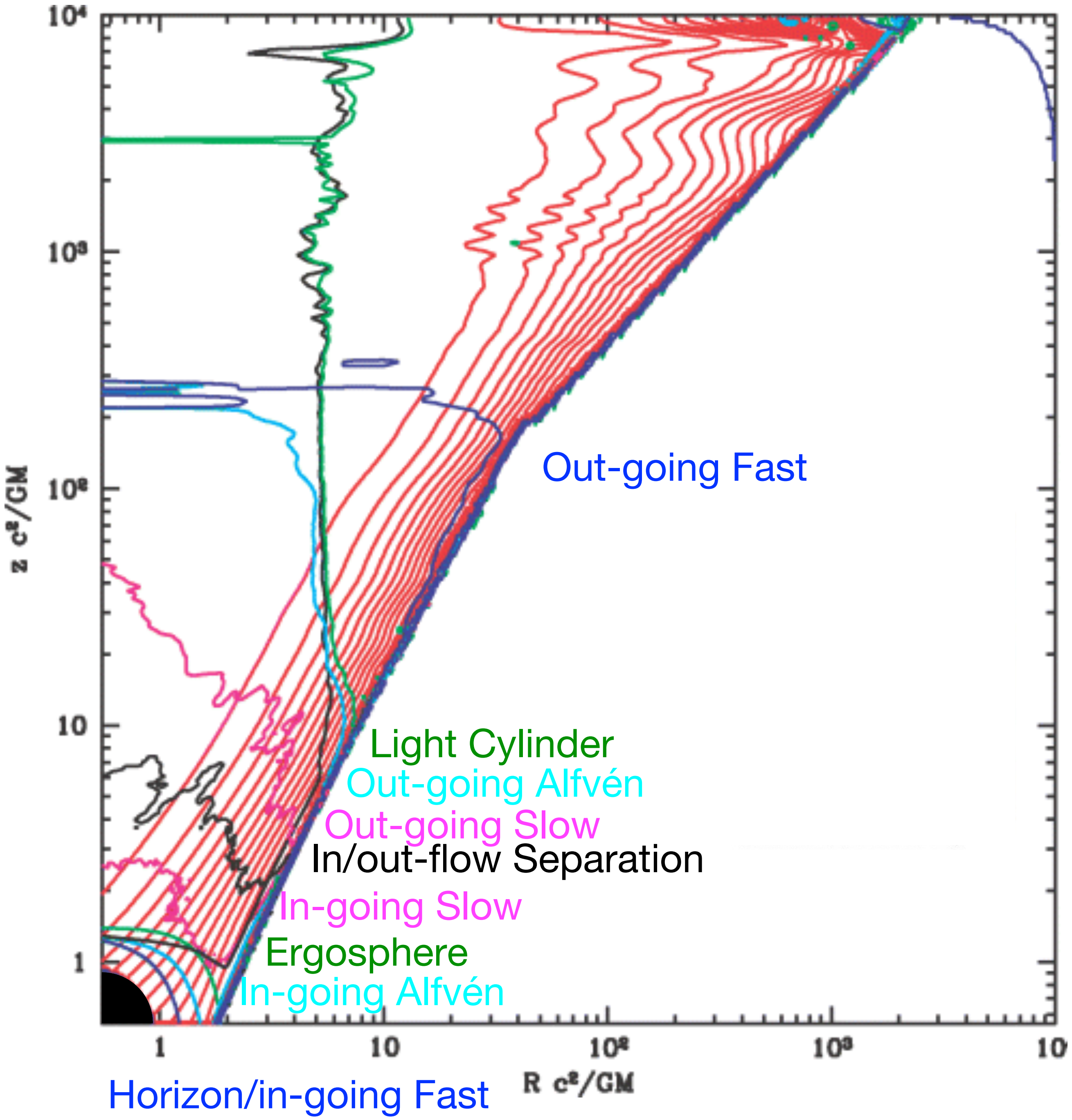

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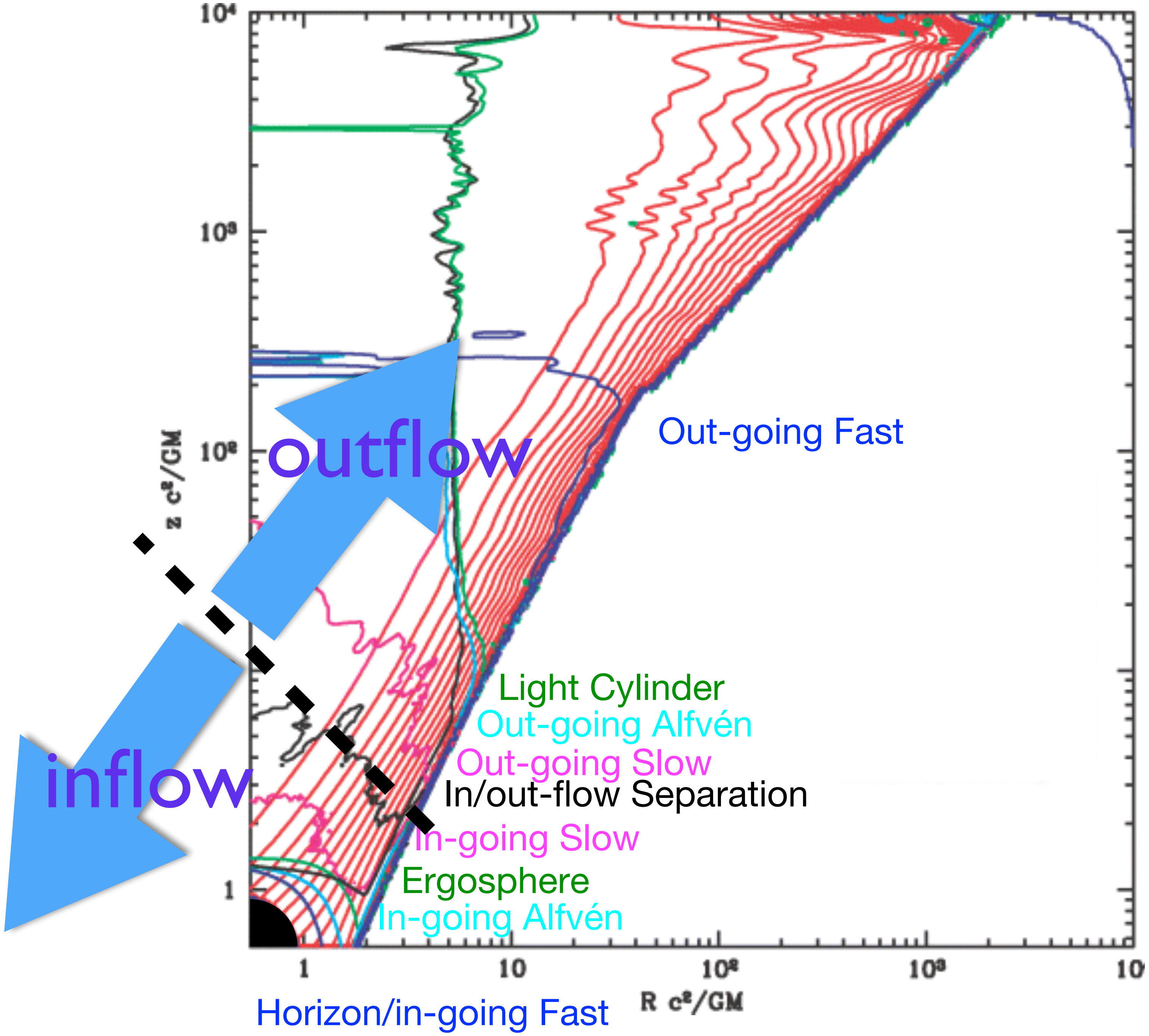

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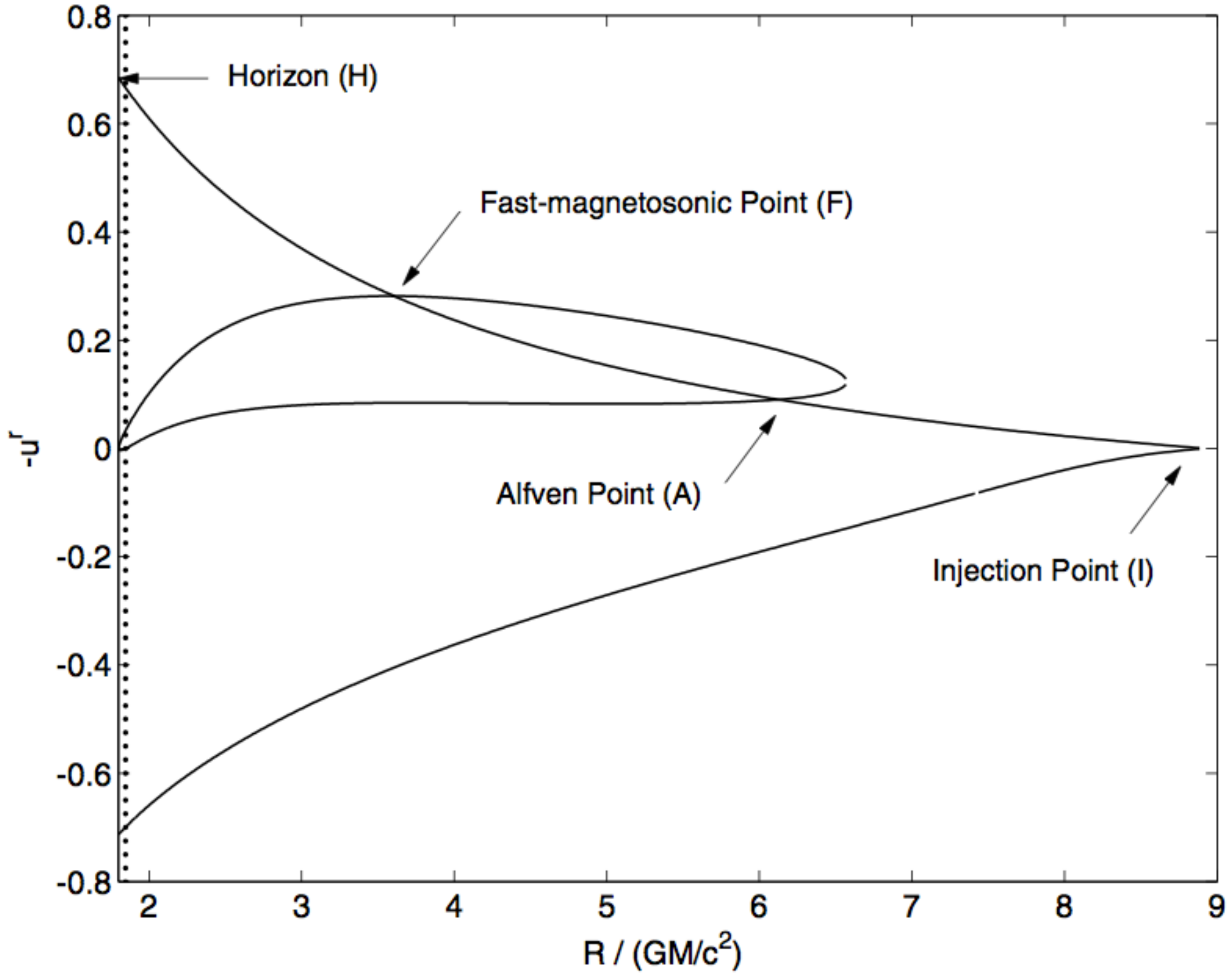
Model Setup

- stationary, axisymmetry magnetic dominated flow in Kerr spacetime
- cold flow; velocity=0 at separation surface (determined by a/M, field geometry, and field angular velocity)
- Four conserved quantities along field line:
(E, L, mass loading, field angular velocity)
= (separation surface, Alfven surface, magnetizaton, field angular velocity)
- remaining two condition can be determined by:
passing fast surface + matching condition

passing fast surface (I)

- inflow: pass fast surface always exist (required by causality)



Example of a physical GRMHD inflow solution along a hole-threading field line
(from H.-Y. Pu et al. 2012)

passing fast surface (II)

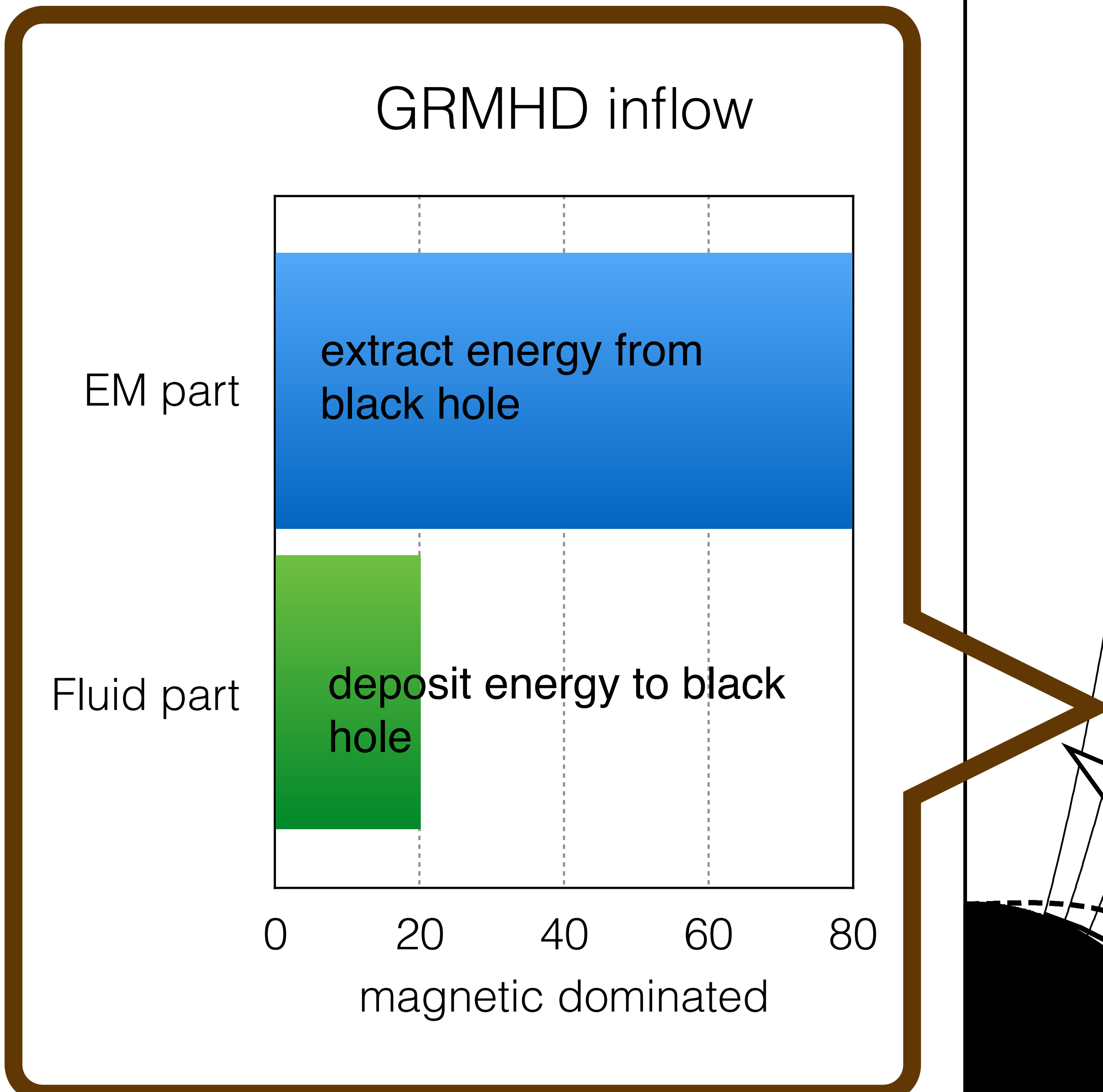
- outflow: no fast surface exist when a force-free field line is considered
- MHD perturbed of force-free field is essential for the existence of fast surface (Beskin and Nokhrina 2006)
- prescribed field: $\Psi = \Psi_0 + \epsilon f$

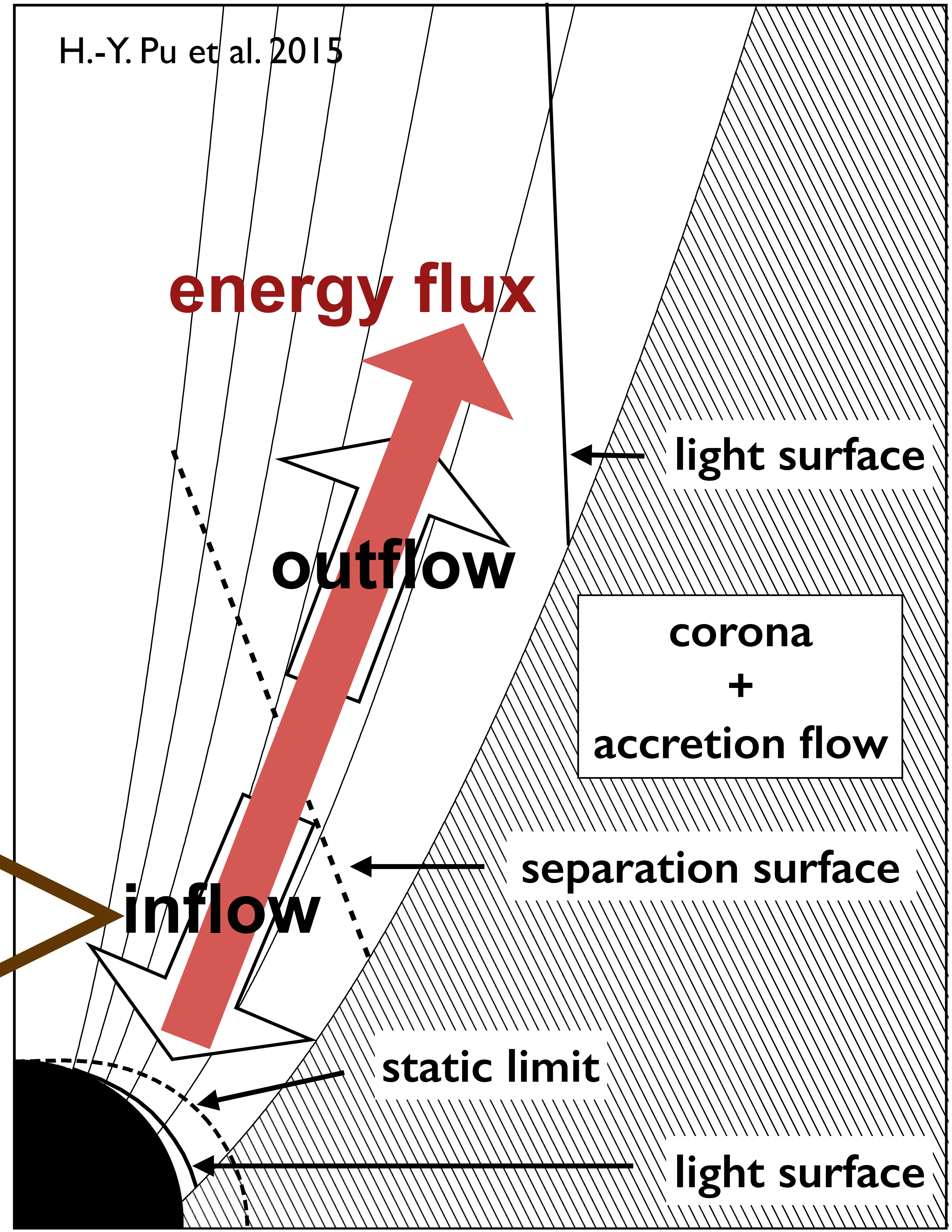
$$\Psi_0 = \frac{\pi C}{\Omega_F} \sinh^{-1}\left(\Omega_F r (1 - cos\,\theta)\right)$$

$$\epsilon\, f = \epsilon\ \pi C \Omega_F r \sin\theta, \qquad \epsilon \ll 1$$

Outward energy flux

$$\begin{aligned}\mathcal{E}^r &= \mathcal{E}^r_{\rm FL} + \mathcal{E}^r_{\rm EM} \\ &= nE_{\rm FL}u^r + nE_{\rm EM}u^r \\ &= -n\mu u_t u^r - \frac{\Omega_F}{4\pi}\frac{B_\phi}{\Sigma \sin\theta}A_{\phi,\theta}\end{aligned}$$





The matching condition single out a unique outflow solution for a given perturbation!

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$$\frac{\Omega_F}{4\pi}\left(B_\phi\right)^- = \frac{\Omega_F}{4\pi}\left(B_\phi\right)^+$$

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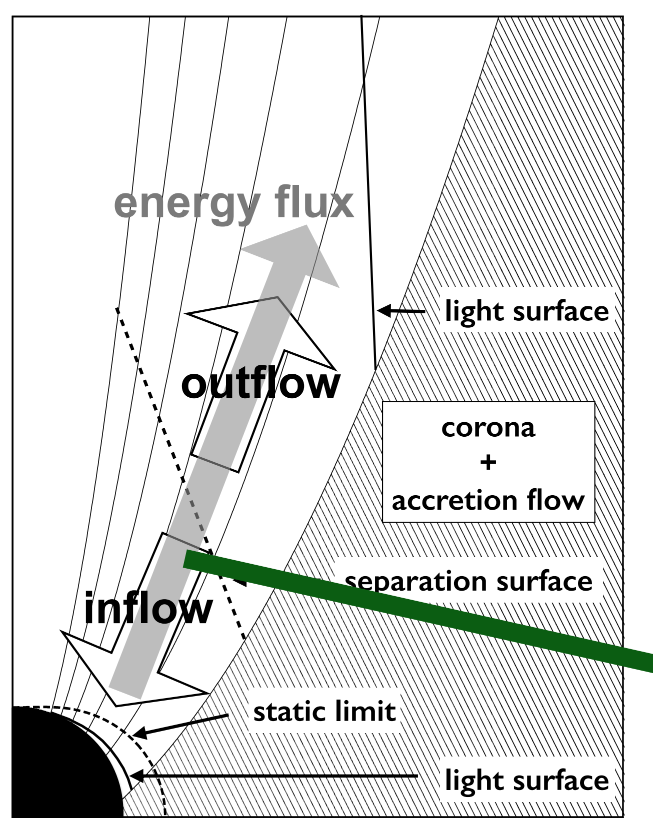

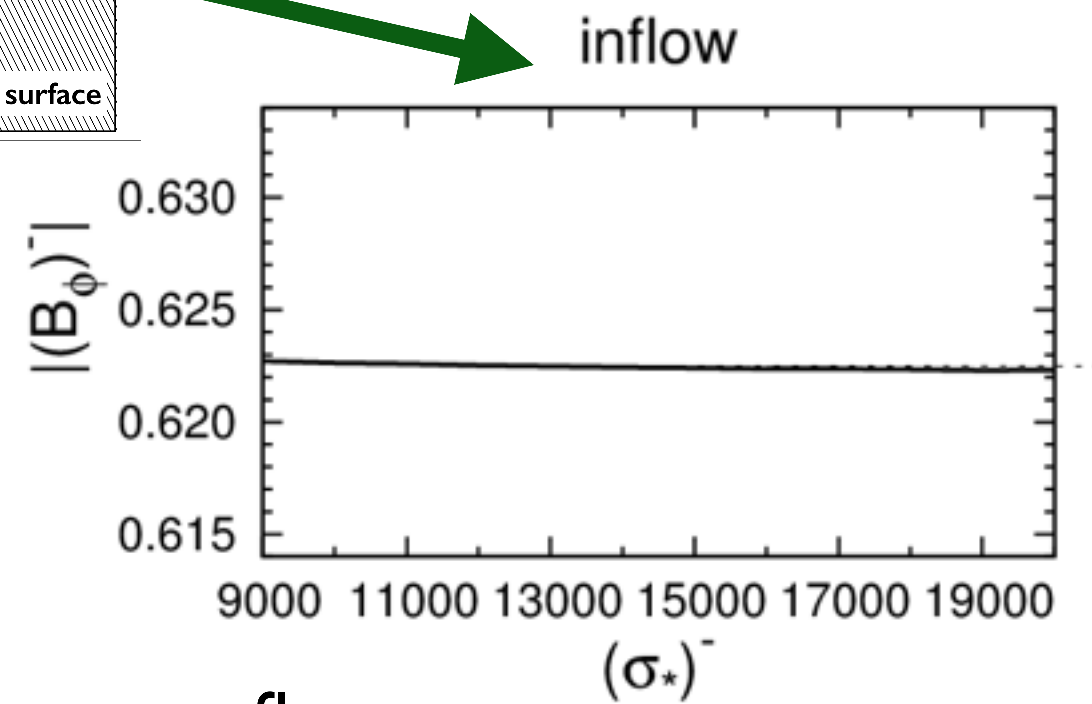

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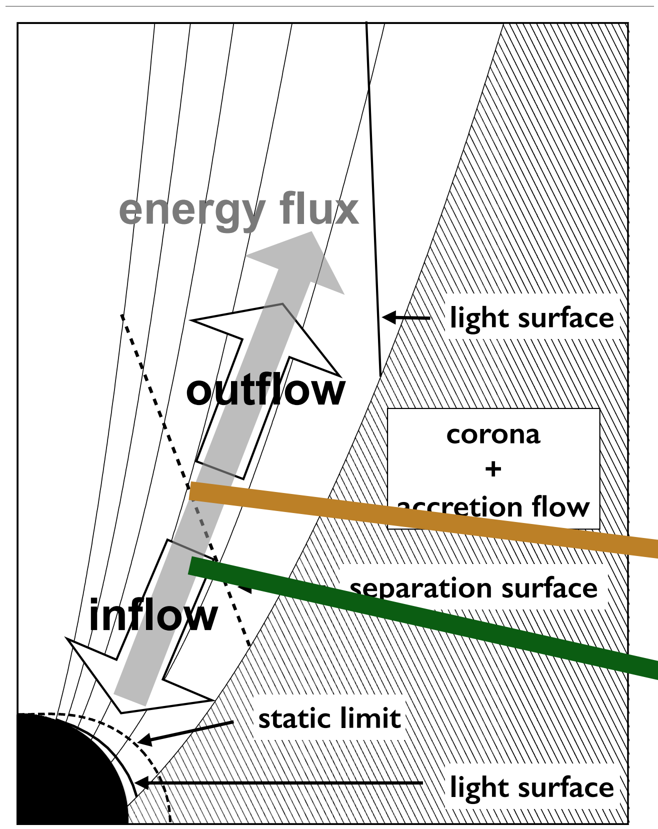

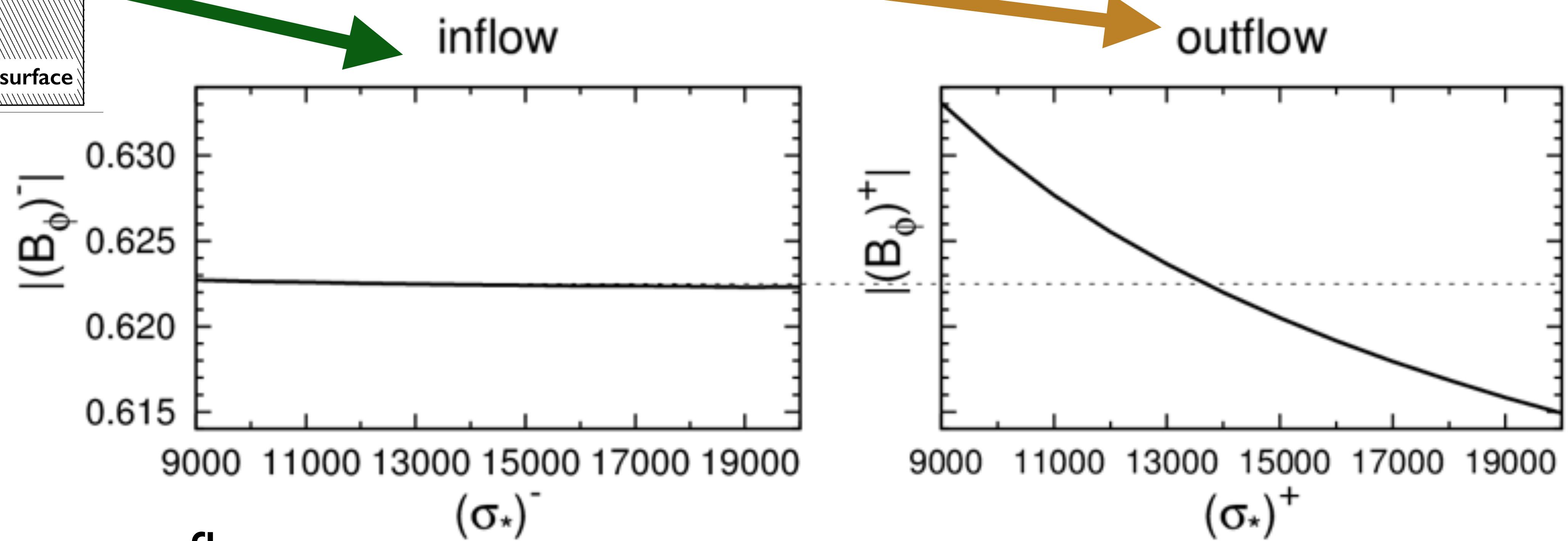

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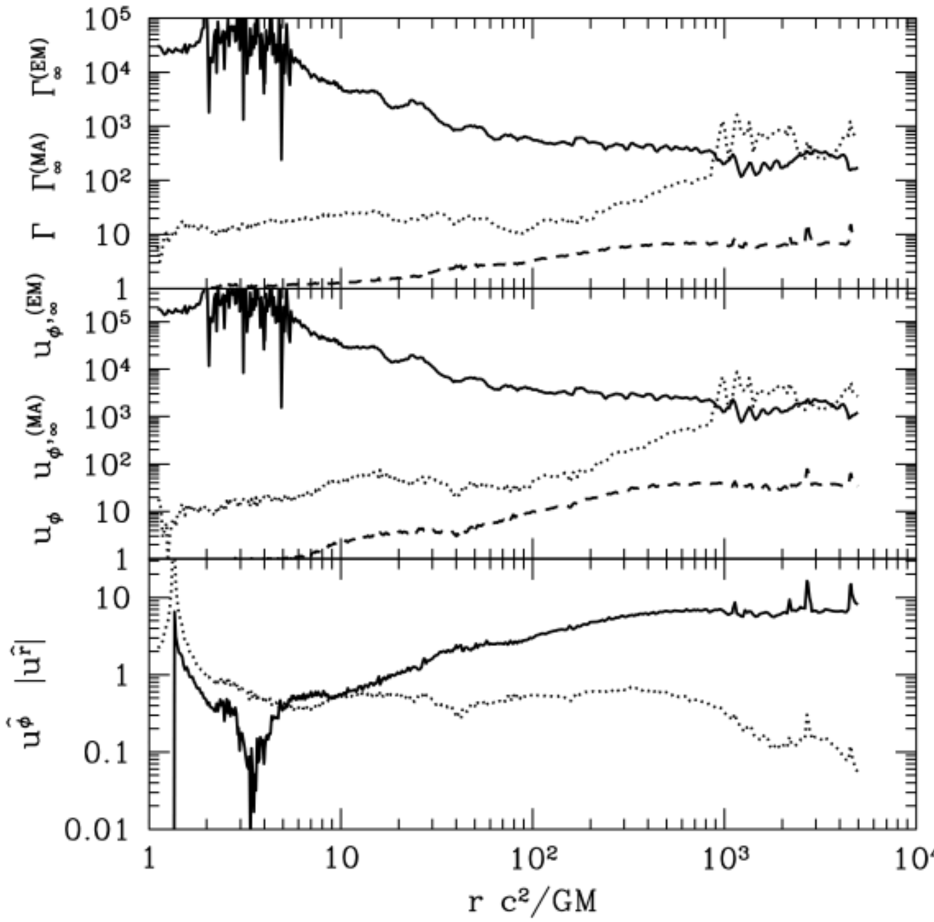
Results



McKinney 2006 (GRMHD simulation)

Location of characteristic surfaces

conversion of energy

$$\Gamma = \sqrt{-g_{tt}}\,u^t$$

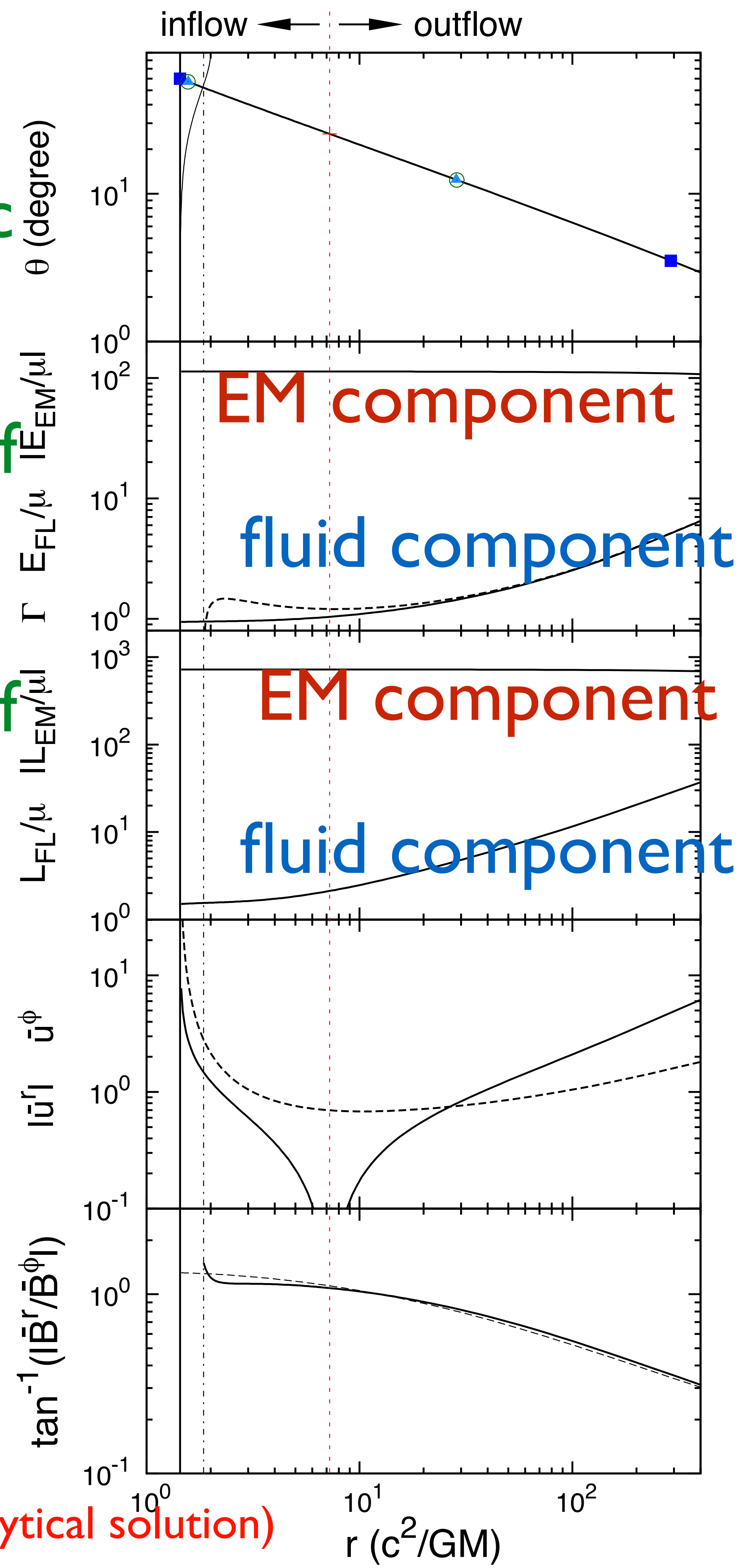
conversion of angular momentum

dynamical structure

pitch angle

$$\bar{B}^r = \sqrt{g_{rr}}\,B^r,$$

$$\bar{B}^\phi = \sqrt{g_{\phi\phi}}\,B^\phi.$$



Pu et al. 2015 (semi-analytical solution)

Next Step

- consider super-fast magnetosonic regime
- consider multi field lines
    - mass loading is different for different field line

Trial and preliminary results

q=0 : force-free case; fast surface exist when q is not zero

fixed stream line geometry

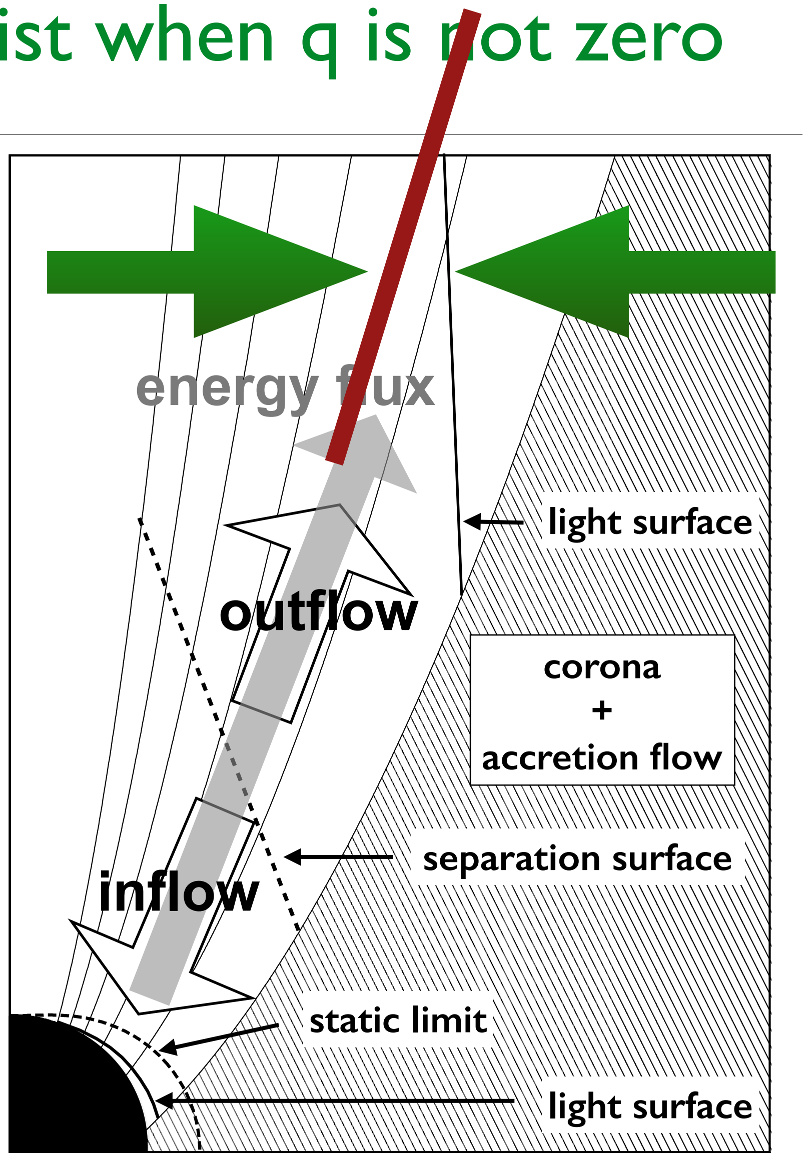
(BZ parabolic field)

$$\Psi(r,\theta) = \frac{\mathcal{C}}{2}\left\{r(1-\cos\theta) + 2(1+\cos\theta)\left[1-\ln(1+\cos\theta)\right]\right\}$$

modified flux function
(related to neighbour field lines)

$$\Phi = \sqrt{-g}\,\tilde{B}_{\rm p} = \sqrt{-g}\left[B_{\rm p}^2 \Pi(q)\right]^{1/2}$$

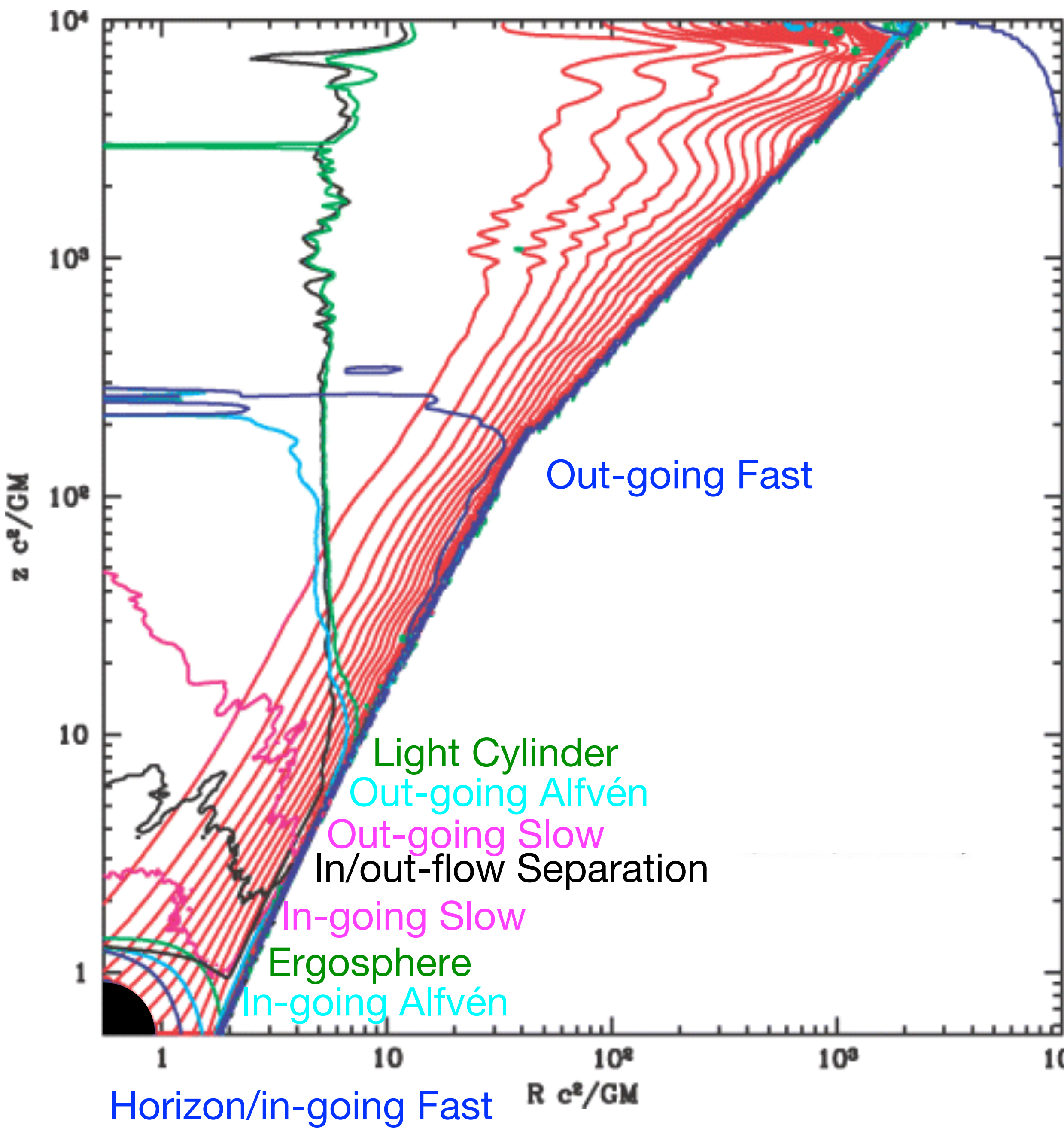
$$\Pi(q) = \left(\frac{r}{r_{\rm c}}\right)^{-q}$$



r_c is chosen at the outer light cylinder

GRMHD Simulation (a/M=0.9375)

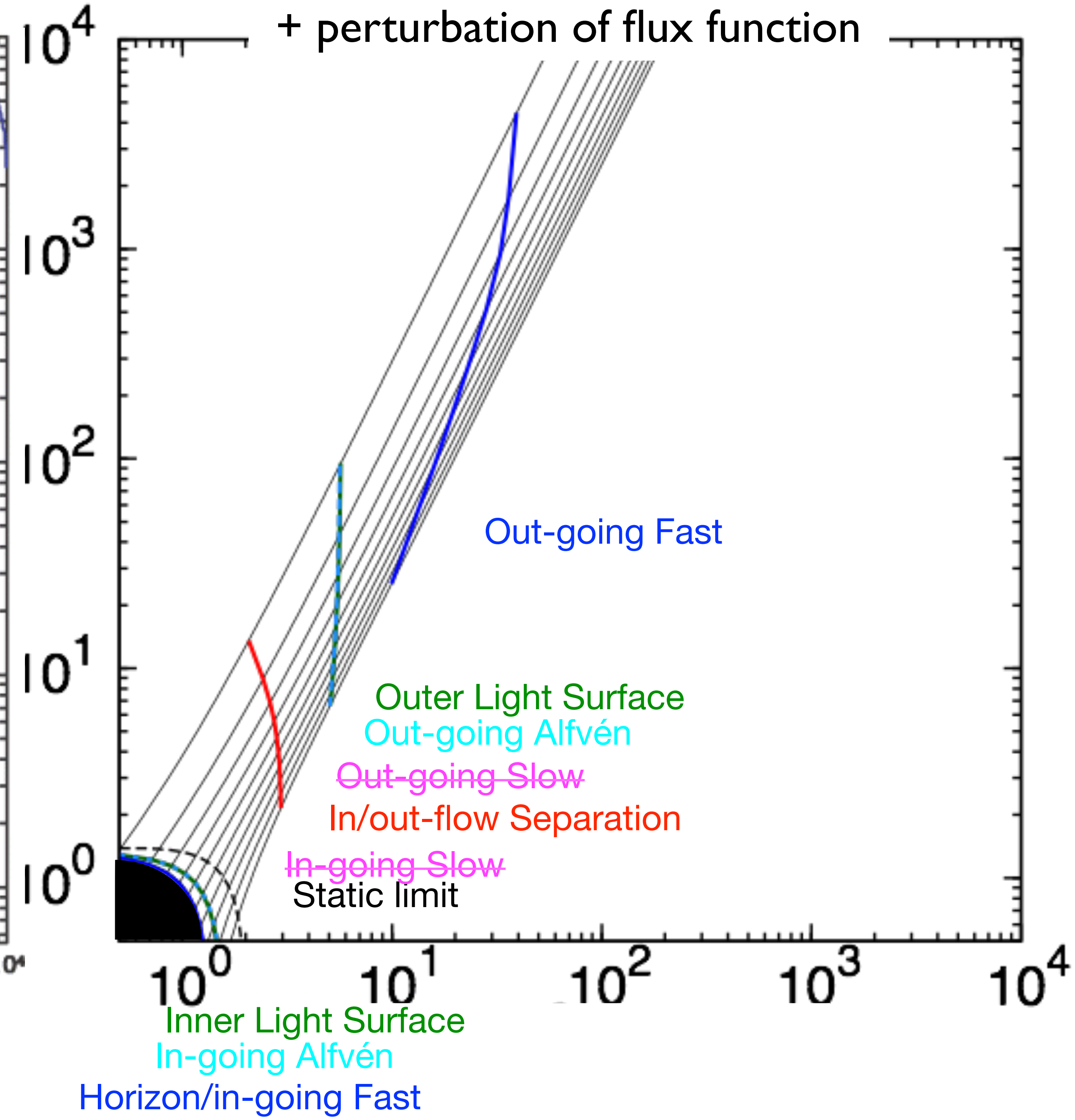
B_p field lines and characteristic surfaces



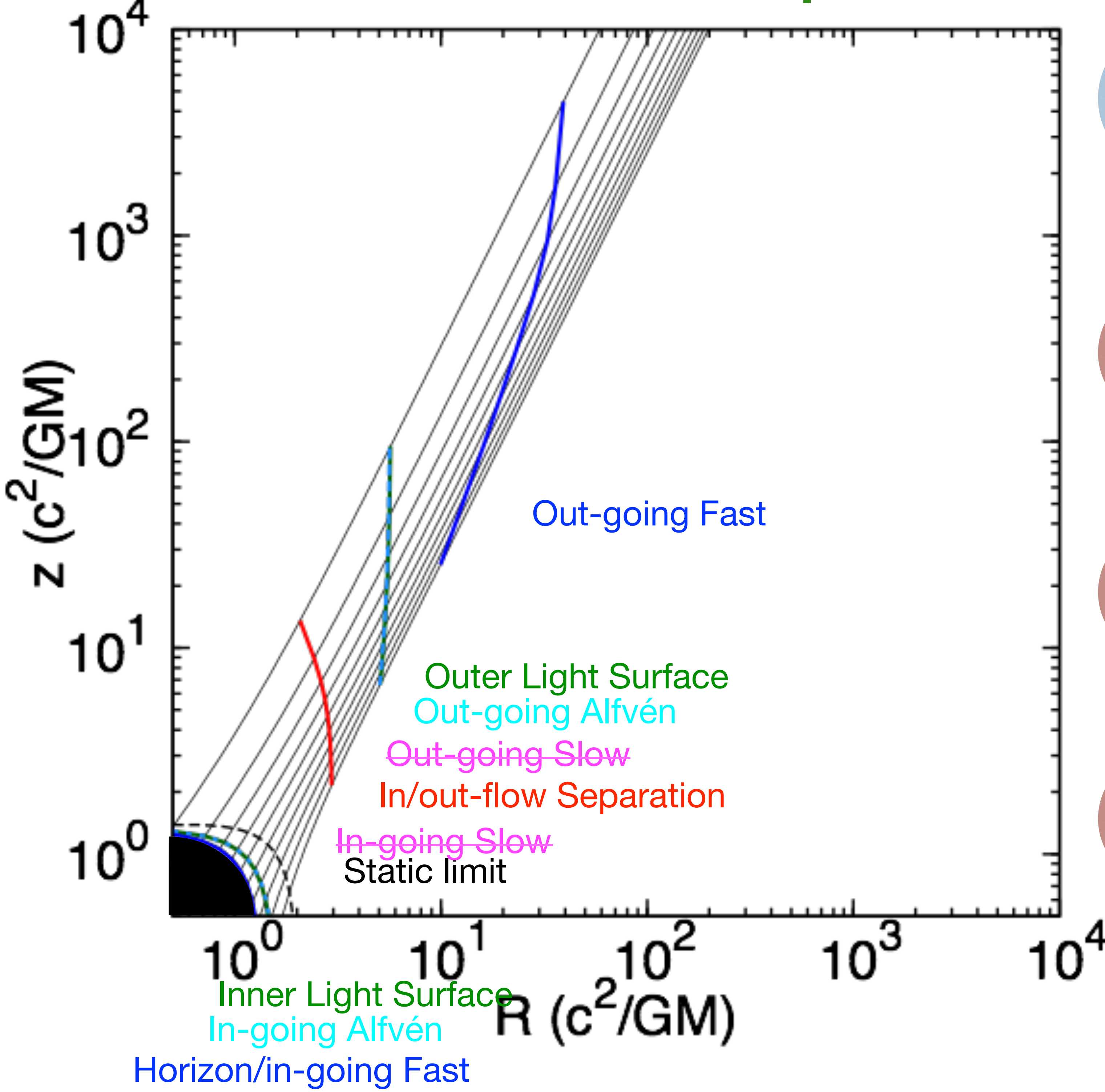
McKinney (2006)

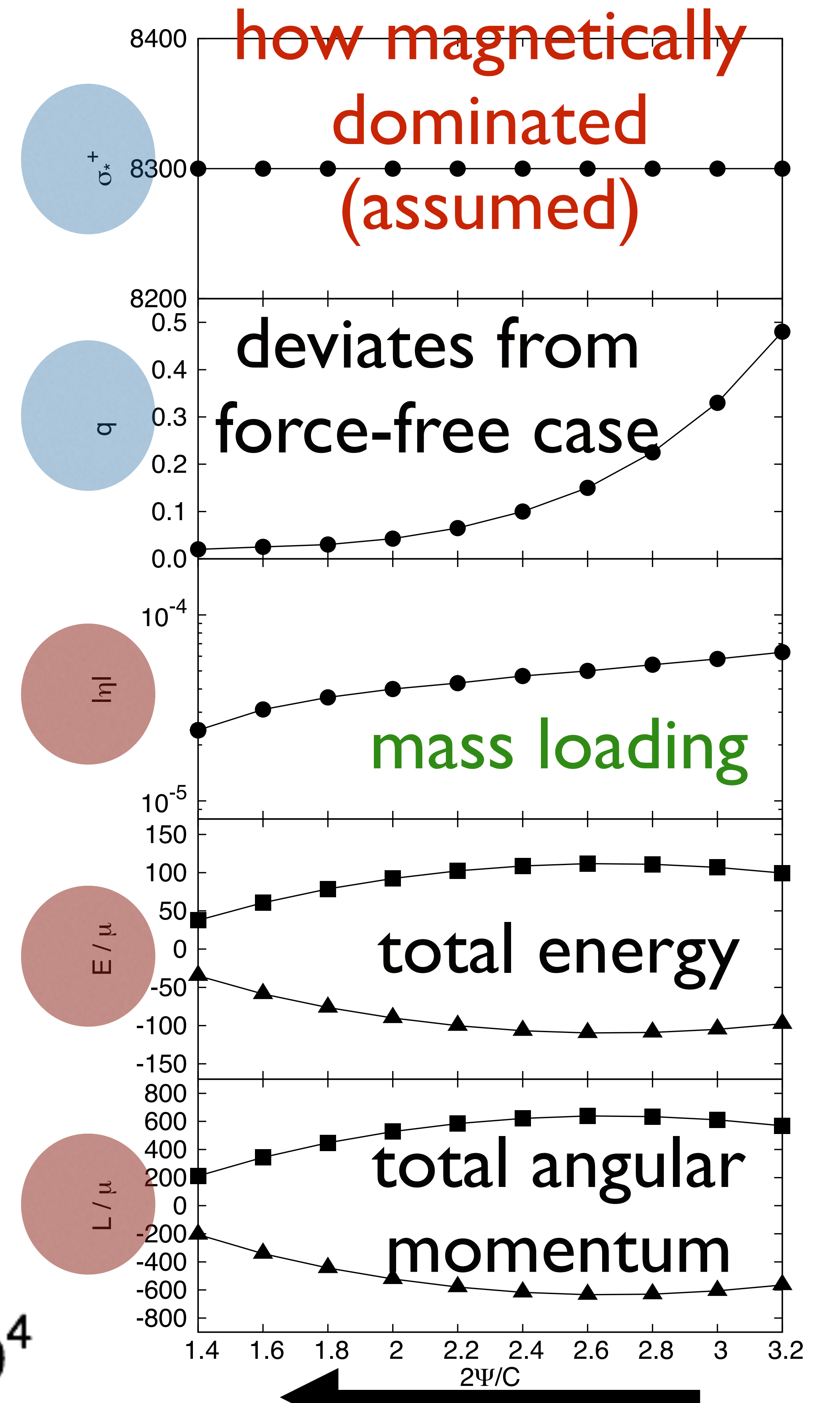
Steady GRMHD (cold) solution (a/M=0.9375)

B_p field: parabolic solution (Blandford & Znajek 1977)



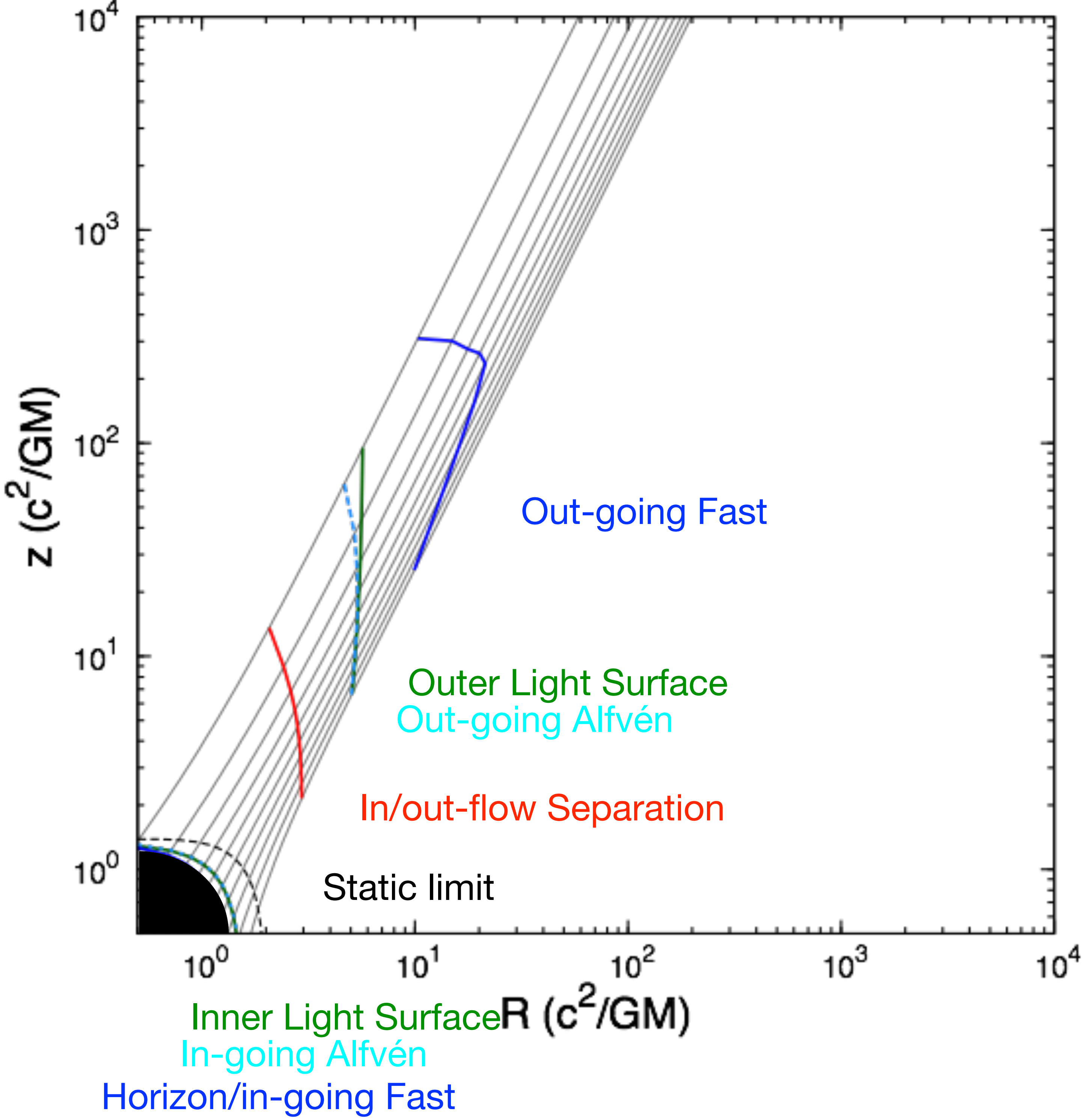
mass loading decreasing towards to the pole

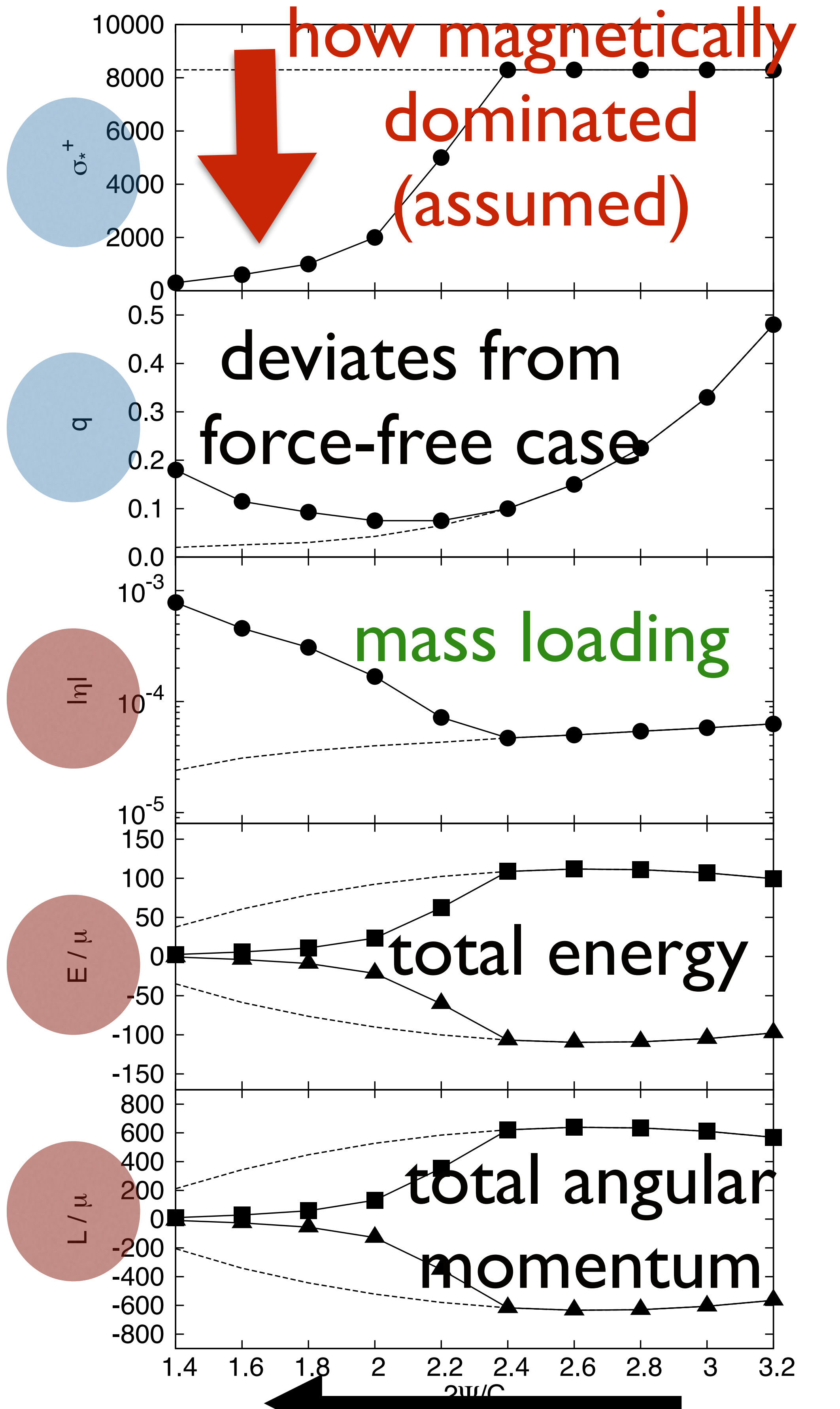




field line close to the pole

a turn-over feature shows up if adding mass loading near the pole region





Summary (I)

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  - Poynting flux dominated GRMHD flow
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- **Parabolic field line** (e.g. observation of M87; Asada & Nakamura 2012)

Summary (II)

- MHD perturbation is essential for the existence of the fast magnetosonic surface
- Matching condition: the outward energy flux is continuously propagate outward
- The matching condition single out a unique outflow solution
- Next step: exploring solutions in super-fast regime, and solutions of global field configuration