

***Relativistic magnetized
outflows in AGNs, GBRs
and compacts'
mergers/collapse***

Maxim Lyutikov (Purdue)

The theme

The underlying assumption (simplification) is that large scale B-field play important dynamical and radiative role (in AGNs and GRBs). Obviously in pulsars.

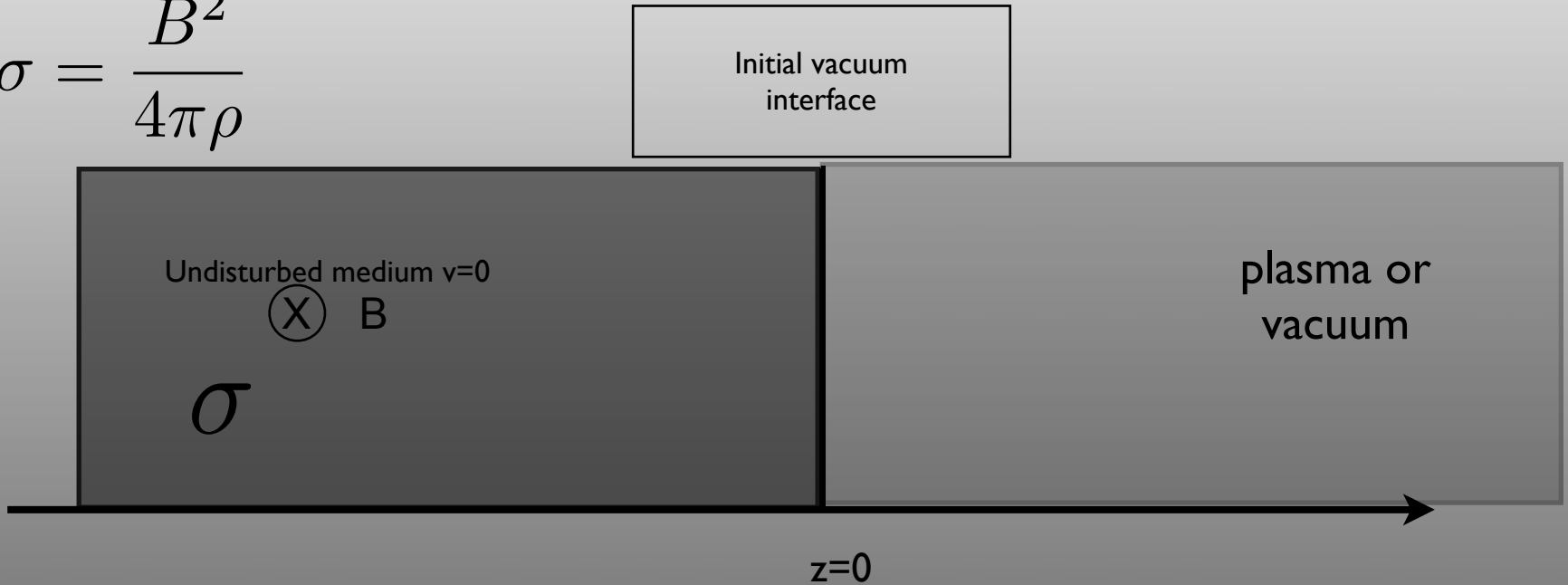
I. Exact 1D solutions in relativistic MHD

Needed for code testing

Often 1D is good enough as the first approximation

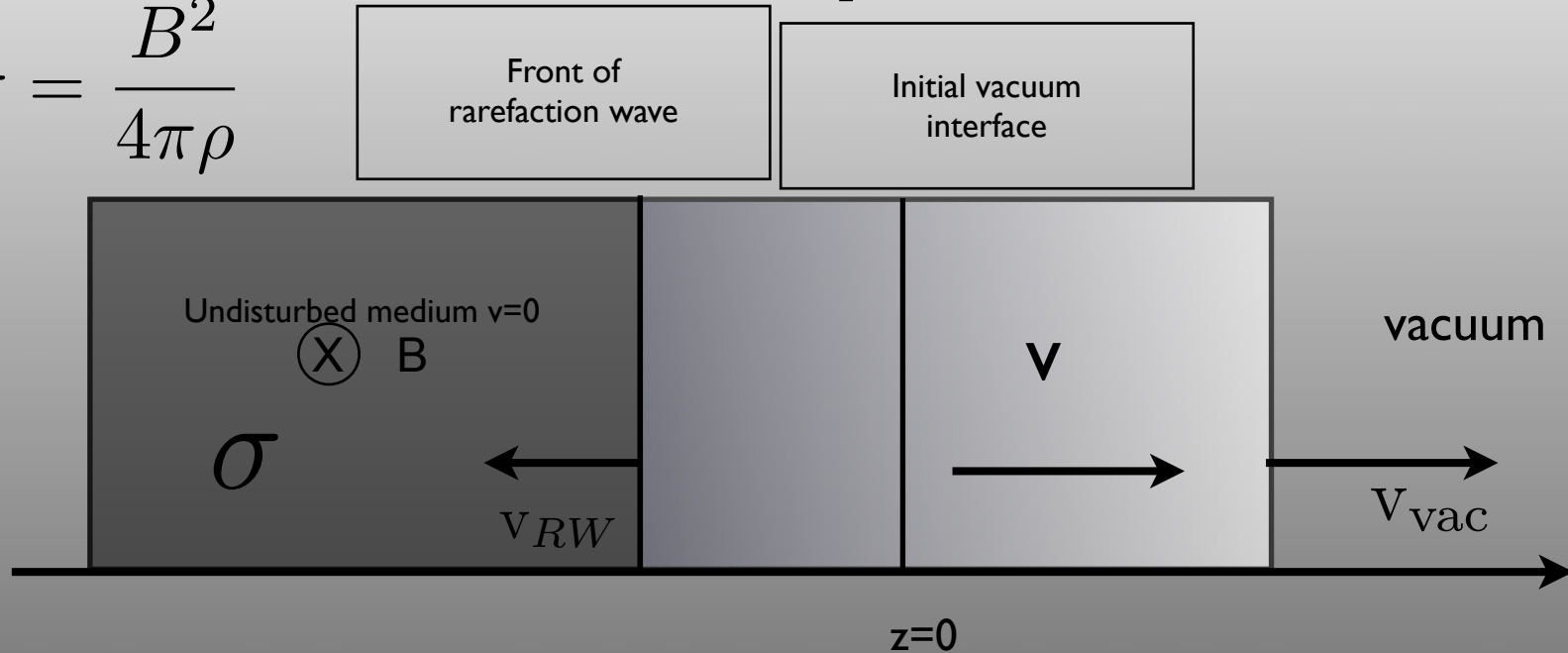
Relativistic simple waves & Riemann problem

$$\sigma = \frac{B^2}{4\pi\rho}$$



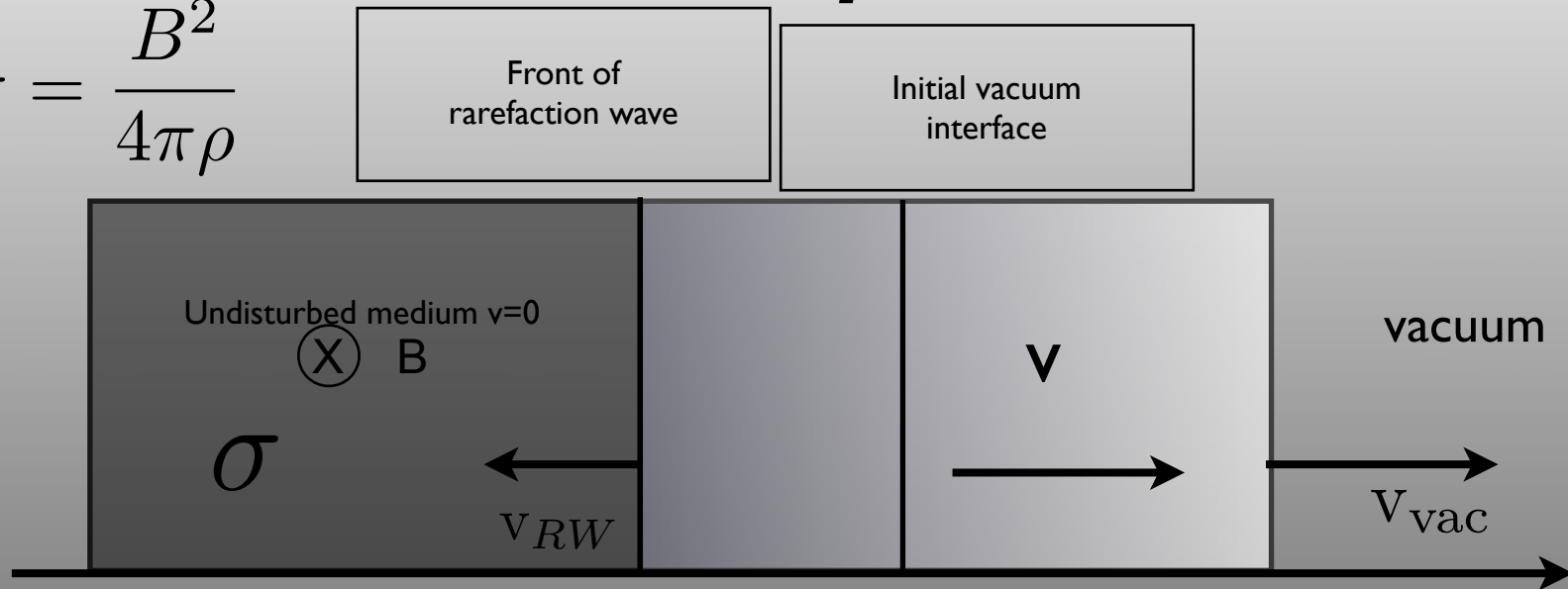
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$$(\partial_t + \beta\partial_z)\beta = -\frac{(\beta\partial_t + \partial_z)P}{(\mathcal{E} + \rho + P)\gamma^2}$$

$$(\partial_t + \beta\partial_z)P = -(\mathcal{E} + \rho + P)\gamma^2(\beta\partial_t + \partial_z)\beta$$

Modern shock-capturing numerical schemes (Godunov) are based on Riemann invariants. Exact non-linear solutions are rare, needed for code testing.

a. Self-similar expansion into vacuum of cold magnetized plasma

Lyutikov 2010a

$$(\partial_t + \beta \partial_z) \beta = - \frac{(\beta \partial_t + \partial_z) P}{(\mathcal{E} + \rho + P) \gamma^2}$$

$$(\partial_t + \beta \partial_z) P = -(\mathcal{E} + \rho + P) \gamma^2 (\beta \partial_t + \partial_z) \beta$$

$$\mathcal{E} = P = \frac{B^2}{8\pi}$$

$$J_{\pm} = \log \delta_{\beta} \delta_A^{\pm 2}$$

$$\delta_{\beta} = \delta_{\eta}^{2/3} \delta_{A,0}^{2/3}$$

$$\delta_A = \frac{\delta_{A,0}^{2/3}}{\delta_{\eta}^{1/3}}$$

$$\eta = z/t$$

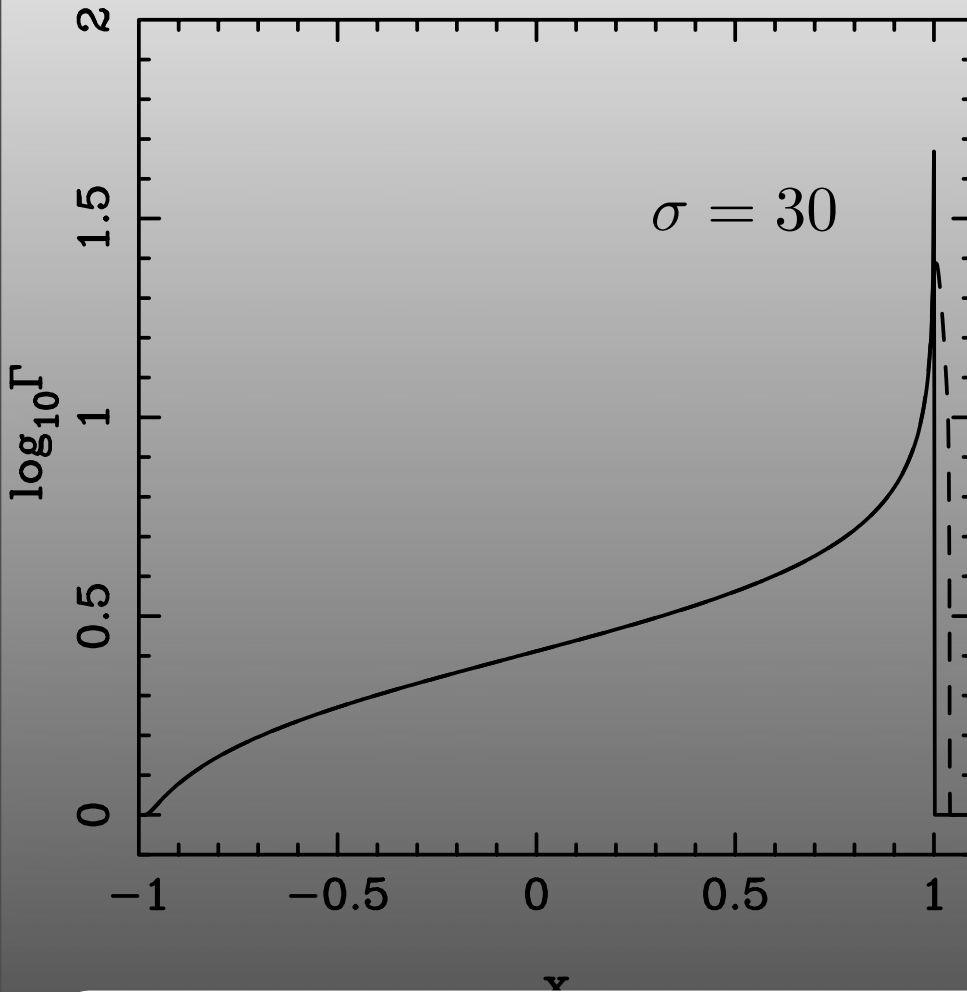
$$\delta = \sqrt{\frac{1 + \beta}{1 - \beta}}$$

$$\beta_{A,0} = \sqrt{\frac{\sigma}{1 + \sigma}}$$

Exact, fully non-linear solution for simple waves
(Riemann invariants and characteristics) in cold plasma

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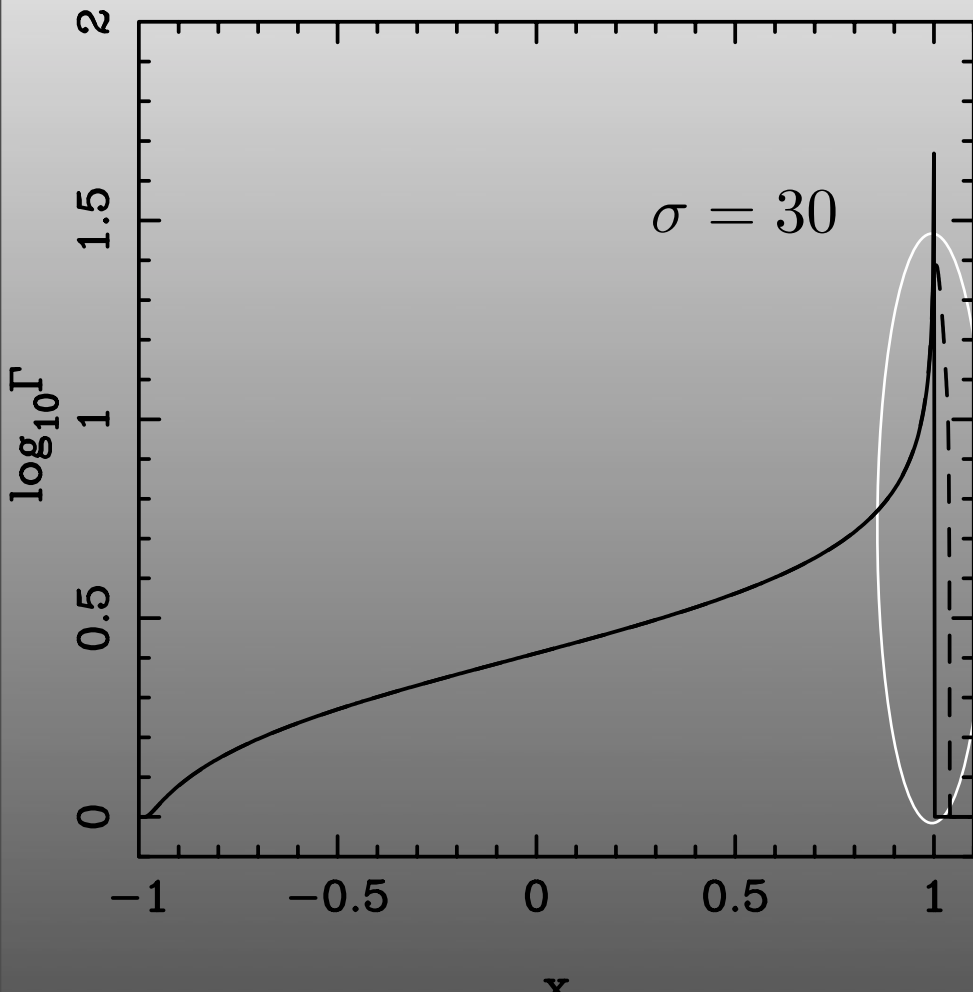
Testing theory and codes



Two (!) curves for density:
analytical (Lyutikov) and
simulations (Komissarov).
Codes can deal with high
magnetization, high Lorentz
factors, large density contrast.

The key results of this work were incorporated post-submission into appendices of Granot et al. 2010

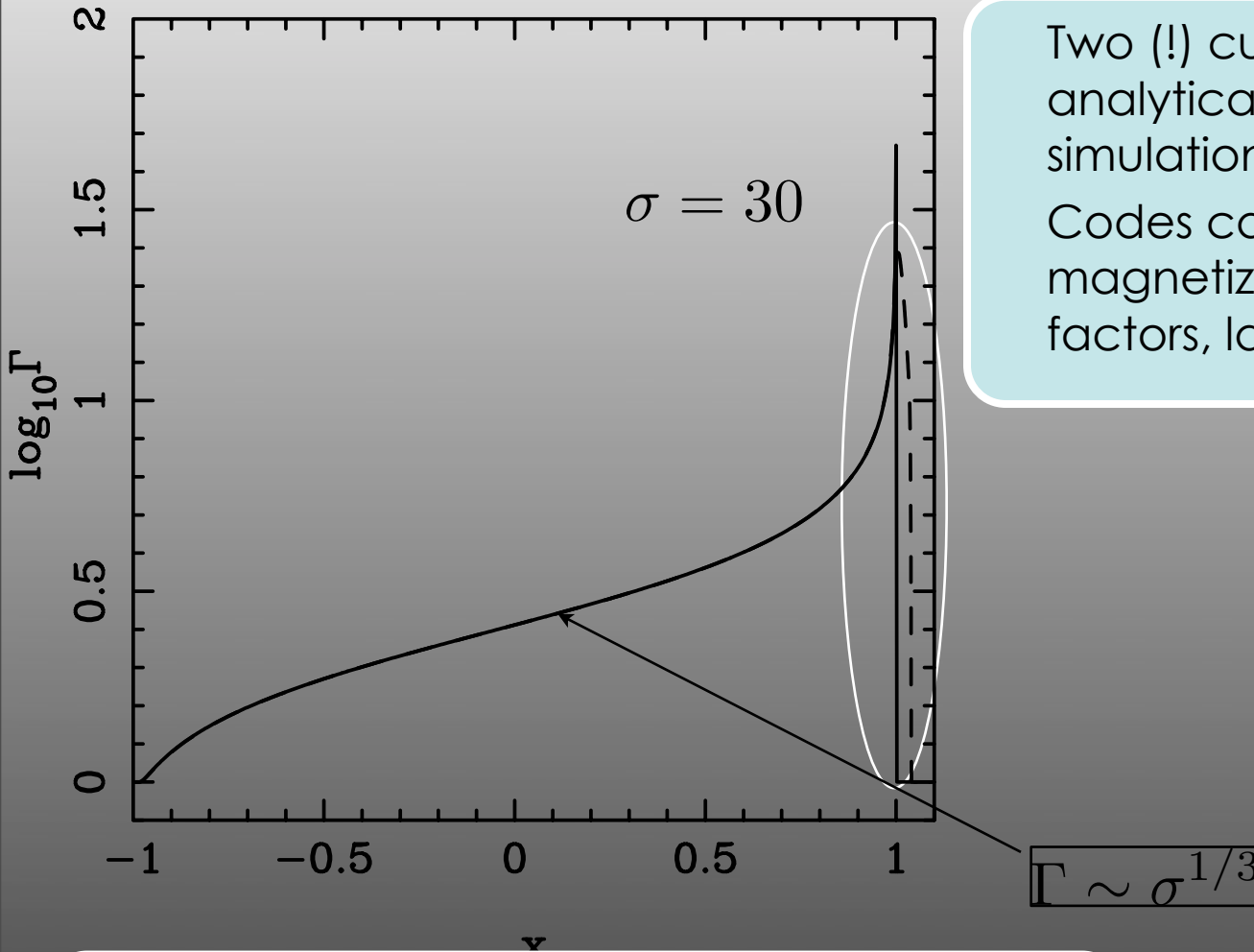
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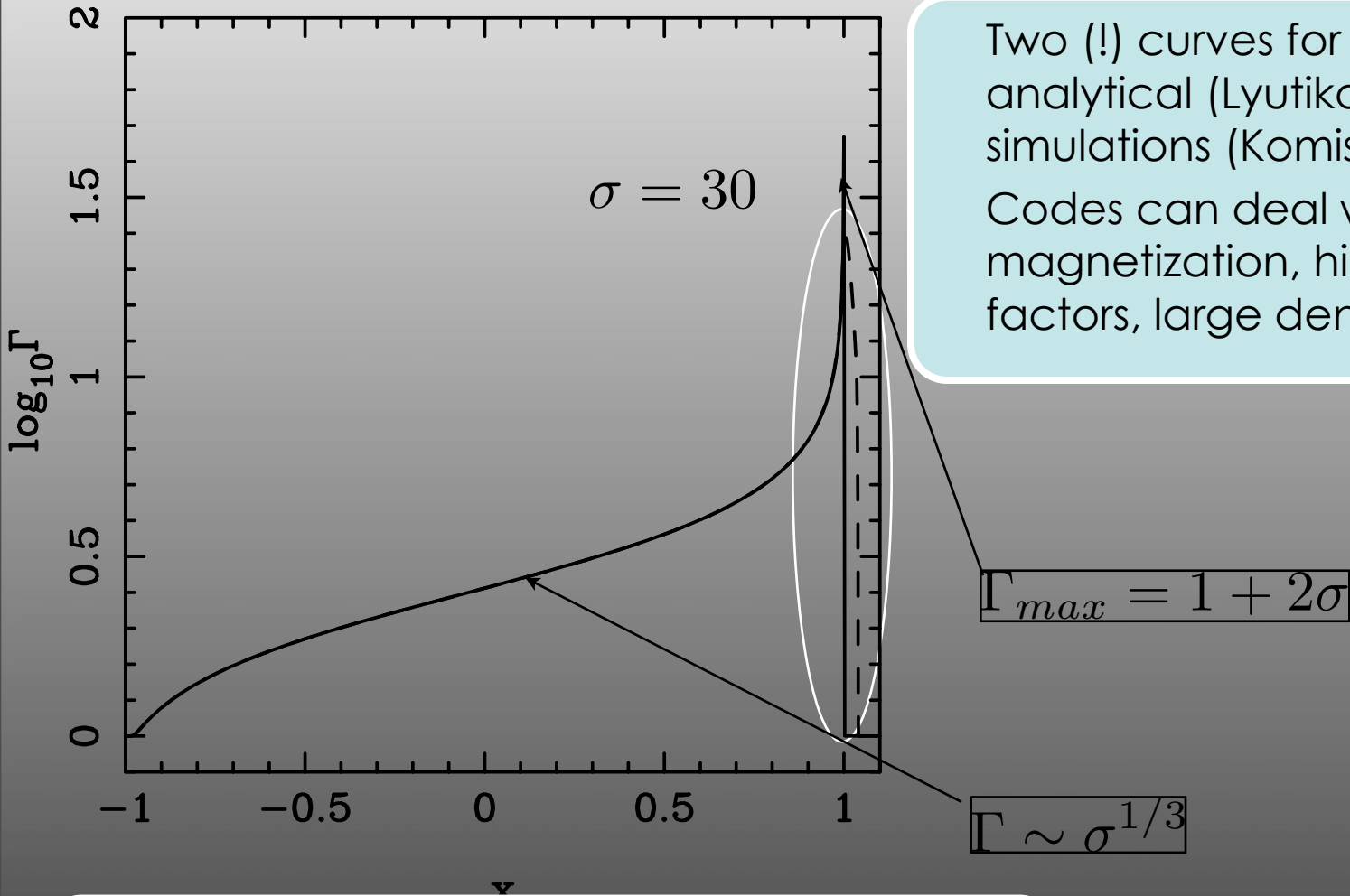
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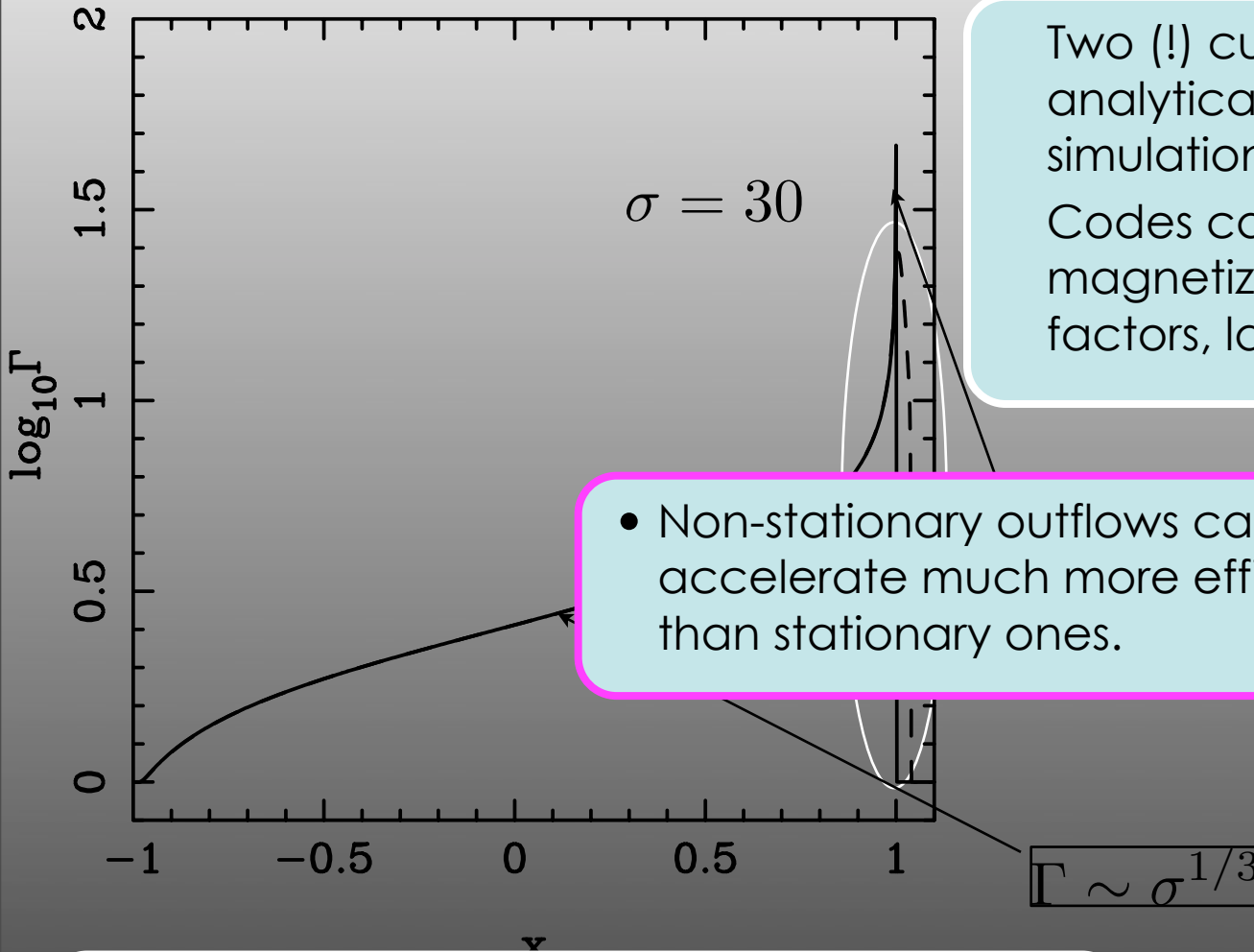
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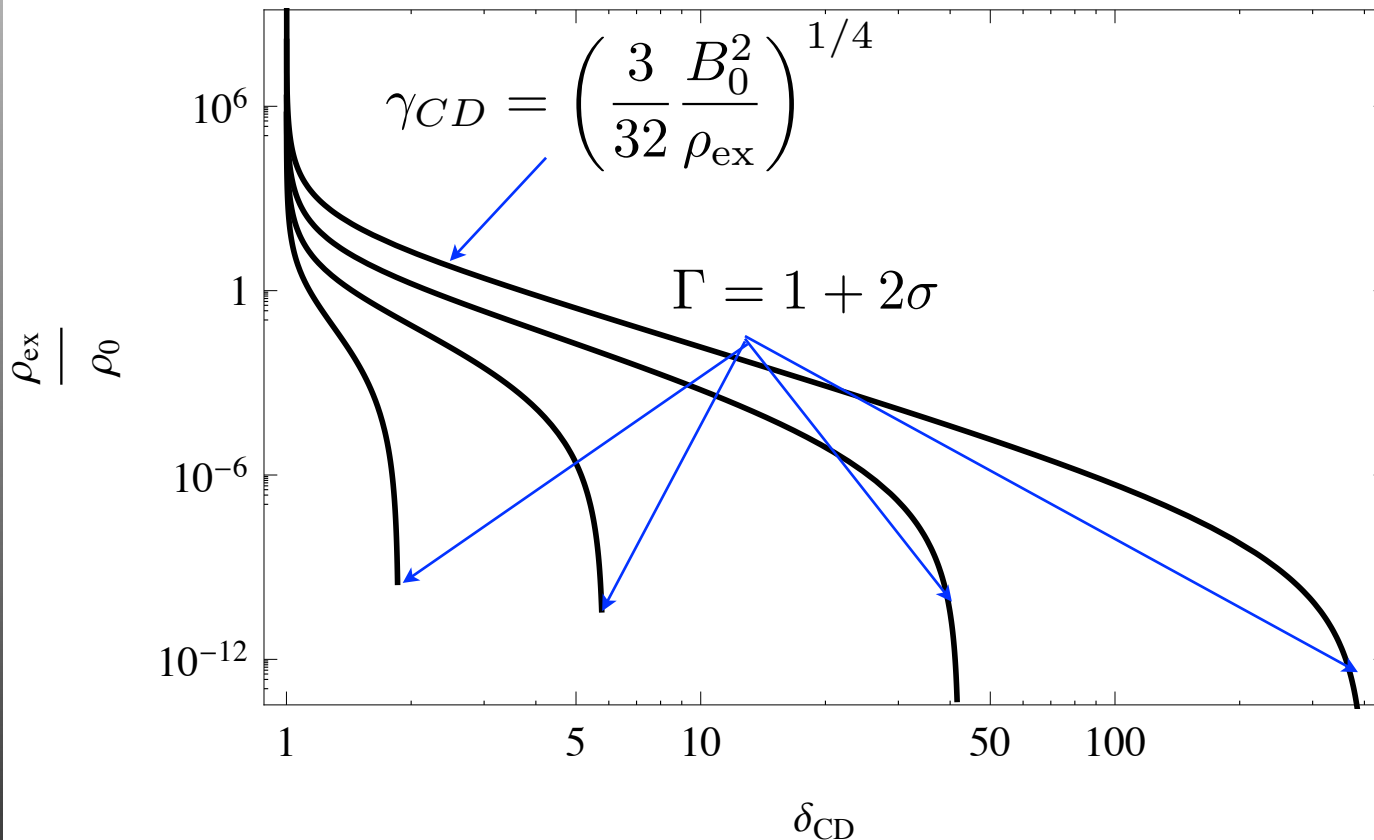
- Non-stationary outflows can accelerate much more efficiently than stationary ones.

The key results of this work were incorporated post-submission into appendices of Granot et al. 2010

b. Expansion into plasma: FS dynamics

Lyutikov 2010

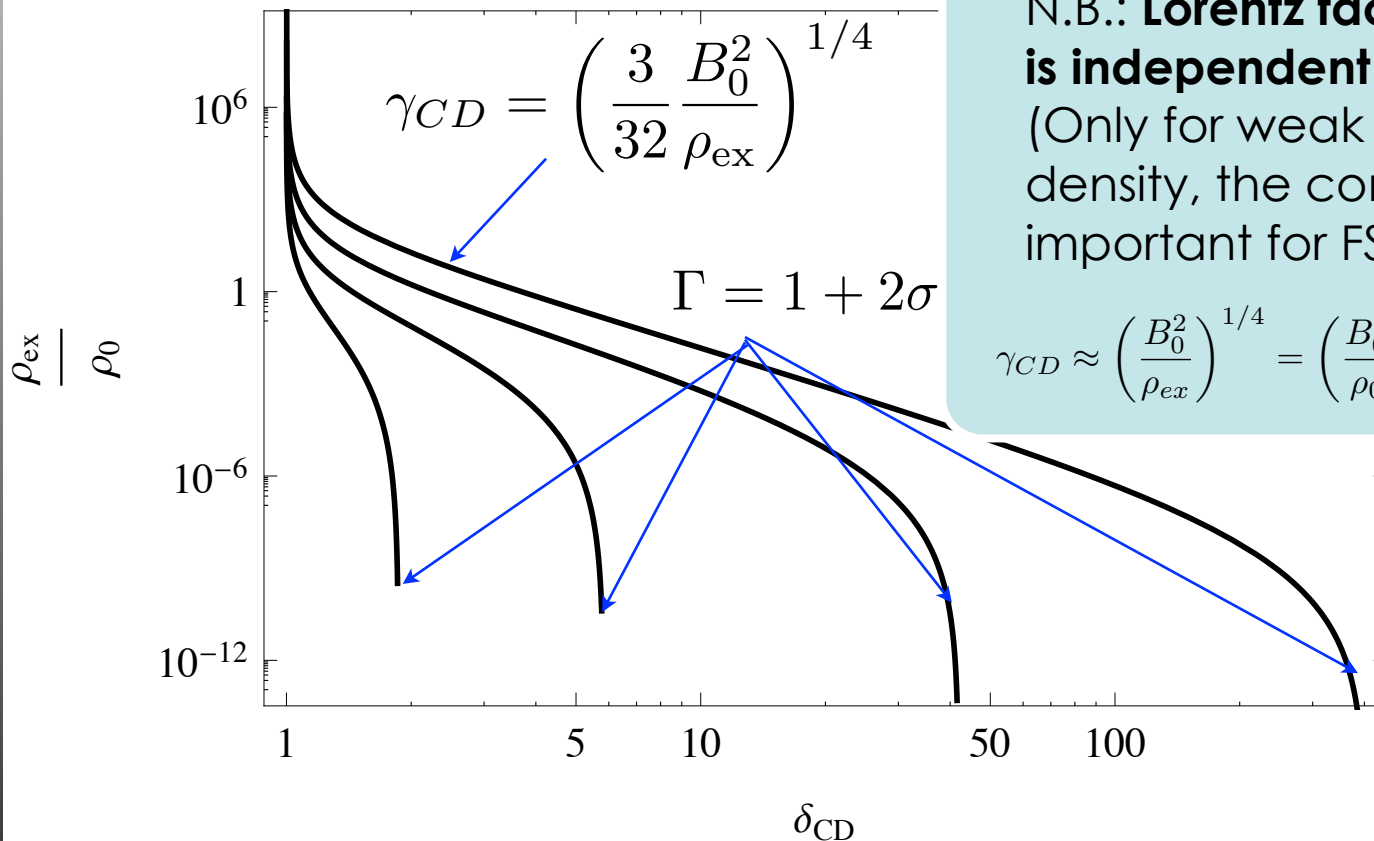
$$\frac{\rho_{\text{ex}}}{\rho_0} = - \frac{3(\delta_{A,0}^2 \delta_w - \delta_{\beta,CD,w})^4}{16\sigma \delta_{A,0}^4 \delta_w^2 (1 - \delta_{\beta,CD,w}^2)(2 + 3\delta_{\beta,CD,w} + 2\delta_{\beta,CD,w}^2)}$$



b. Expansion into plasma: FS dynamics

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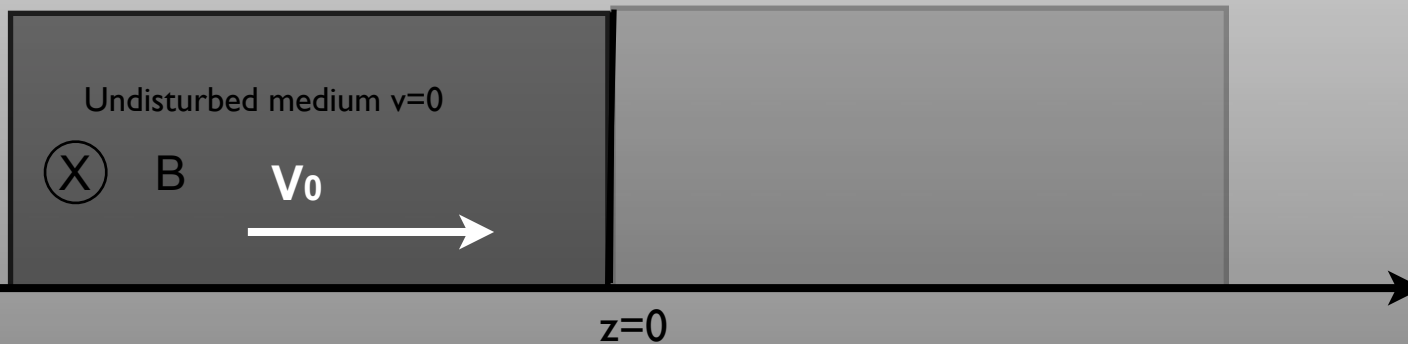
$$\frac{\rho_{ex}}{\rho_0} = - \frac{3(\delta_{A,0}^2 \delta_w - \delta_{\beta,CD,w})^4}{16\sigma \delta_{A,0}^4 \delta_w^2 (1 - \delta_{\beta,CD,w}^2)(2 + 3\delta_{\beta,CD,w} + 2\delta_{\beta,CD,w}^2)}$$



N.B.: Lorentz factor of the strong FS is independent of jet composition.
 (Only for weak FS, very low outside density, the composition is important for FS.)

$$\gamma_{CD} \approx \left(\frac{B_0^2}{\rho_{ex}} \right)^{1/4} = \left(\frac{B_0^2 \rho_0}{\rho_0 \rho_{ex}} \right)^{1/4} = \sigma^{1/4} \left(\frac{\rho_0}{\rho_{ex}} \right)^{1/4}$$

c. Moving piston: RS and rarefaction wave



Three regimes:

- Supersonic wind $\gamma_w > 2\gamma_{CD}\sqrt{\sigma}$, reverse shock
- Slow, high pressure wind, $\gamma_w < \gamma_{CD}$, rarefaction wave
- Intermediate case, fast subsonic wrt CD, $\gamma_{CD} < \gamma_w < 2\gamma_{CD}\sqrt{\sigma}$, compression wave, will turn into shock in 1D, not necessarily in 3D.

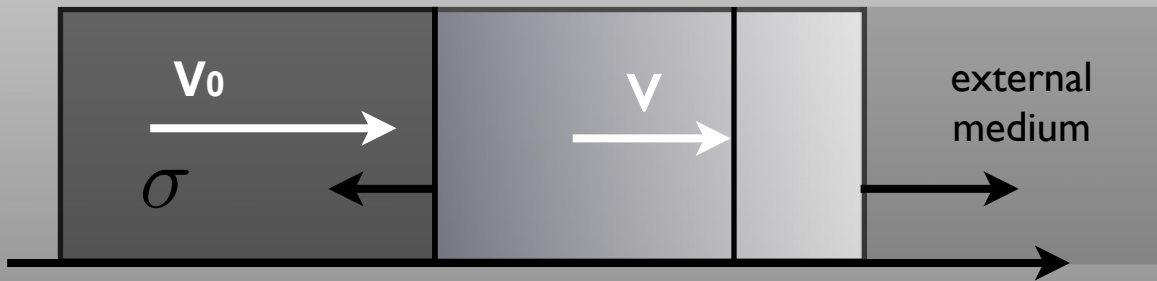
c. Moving piston: RS and rarefaction wave

reverse shock or
rarefaction wave

contact

FS

$$\gamma_{CD} = \left(\frac{3}{32} \frac{B_0^2}{\rho_{ex}} \right)^{1/4}$$

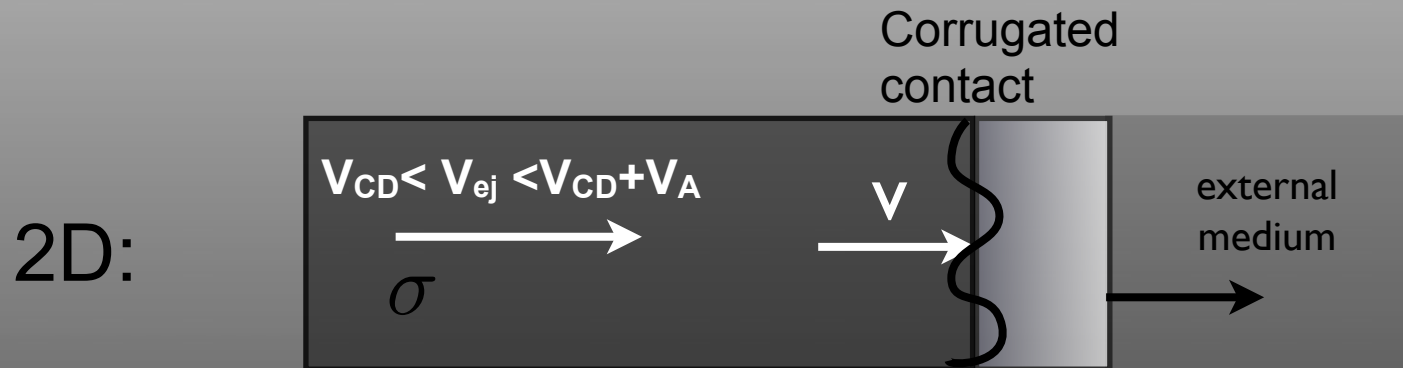
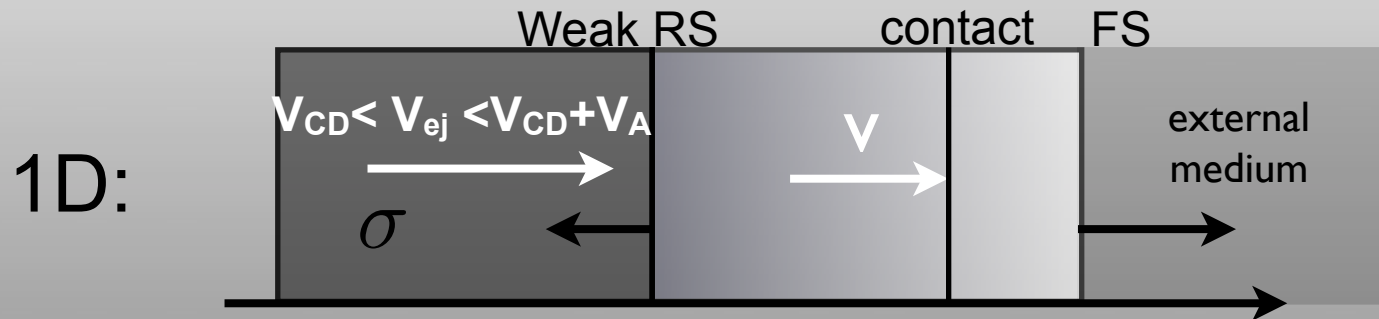


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Simulations to do: intermediate regime

Lyutikov 2010b



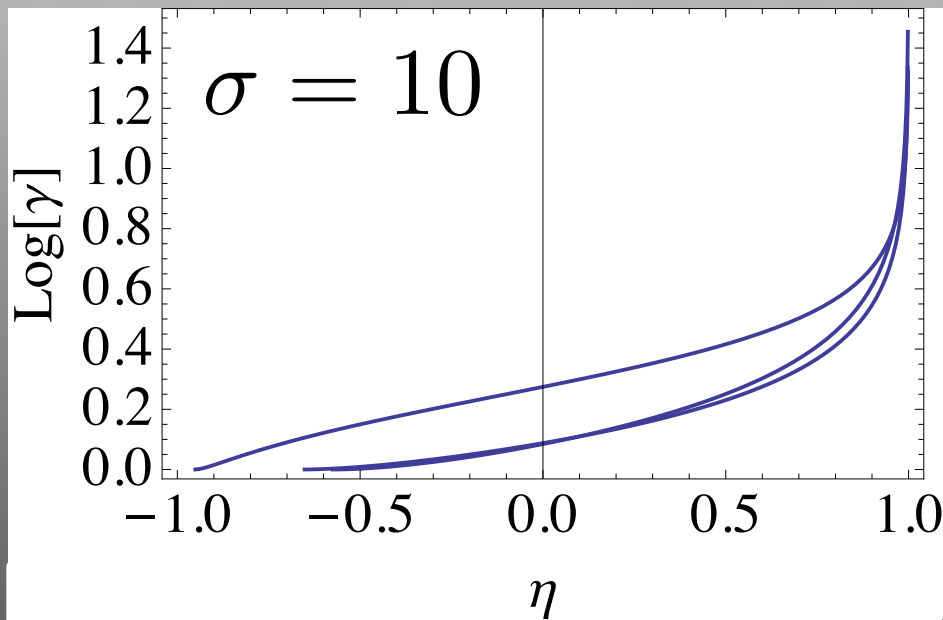
Launch a jet with parameters, so that jet is faster than the CD, but the relative velocity is subsonic:

- 1D case will form a shock
- 2D: not necessarily

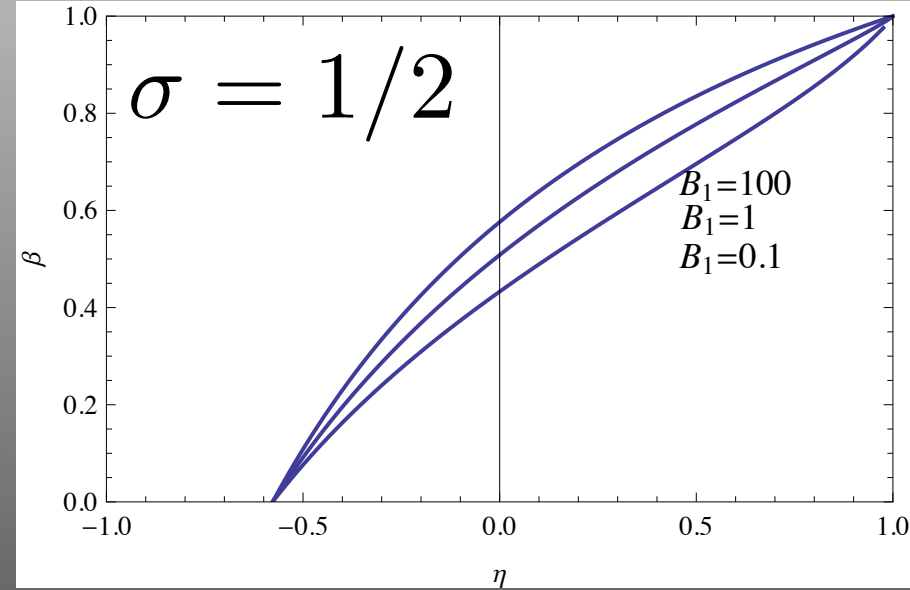
Only for relative supersonic velocity RS must form,

d. Hot magnetized plasma: exact solutions for simple waves

Hot magnetized plasma: mixture of fluids with different adiabatic indexes (4/3 for kinetic, 2 for magnetic pressures)



Plasma beta = 0, 1, 10



e. Arbitrary 1D motion of magnetized plasma

- Ideal 1D fluid motion can be reduced to **linear** equation using **hodograph** transformation $(c_s, v) = f(x, t) \rightarrow (x, t) = f(c_s, v)$.

$$\partial_r^2 \chi - w \partial_w \chi + (1 - w) w \partial_w^2 \chi = 0$$

w is enthalpy, for cold plasma $w = \frac{\rho + B^2}{\rho}$
 r is rapidity

χ is Khalatnikov potential

~ Bernoulli potential +
no vorticity condition

$$t = \gamma \frac{\partial \chi}{\partial w} - \frac{\beta}{\gamma w} \frac{\partial \chi}{\partial \beta}$$

$$x = \beta t - \frac{1}{\gamma^3 w} \frac{\partial \chi}{\partial \beta}$$

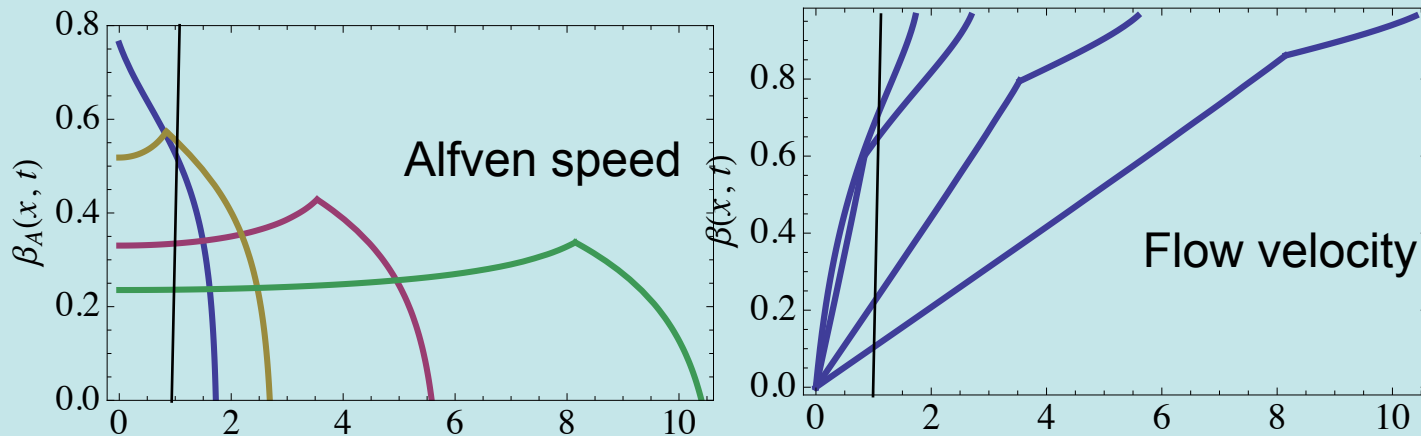
Solve for χ , find $t(v, v_A)$,
 $x(v, v_A)$, invert

Also **relativistic Darboux equation** (Riemann invariants as independent variables)

$$\partial_{J_1} \partial_{J_2} \chi + \frac{1}{4} \frac{\partial_{J_1} \chi + \partial_{J_2} \chi}{\sinh \frac{J_1 + J_2}{2}} = 0$$

Non-self-similar problem: expansion of magnetized slab

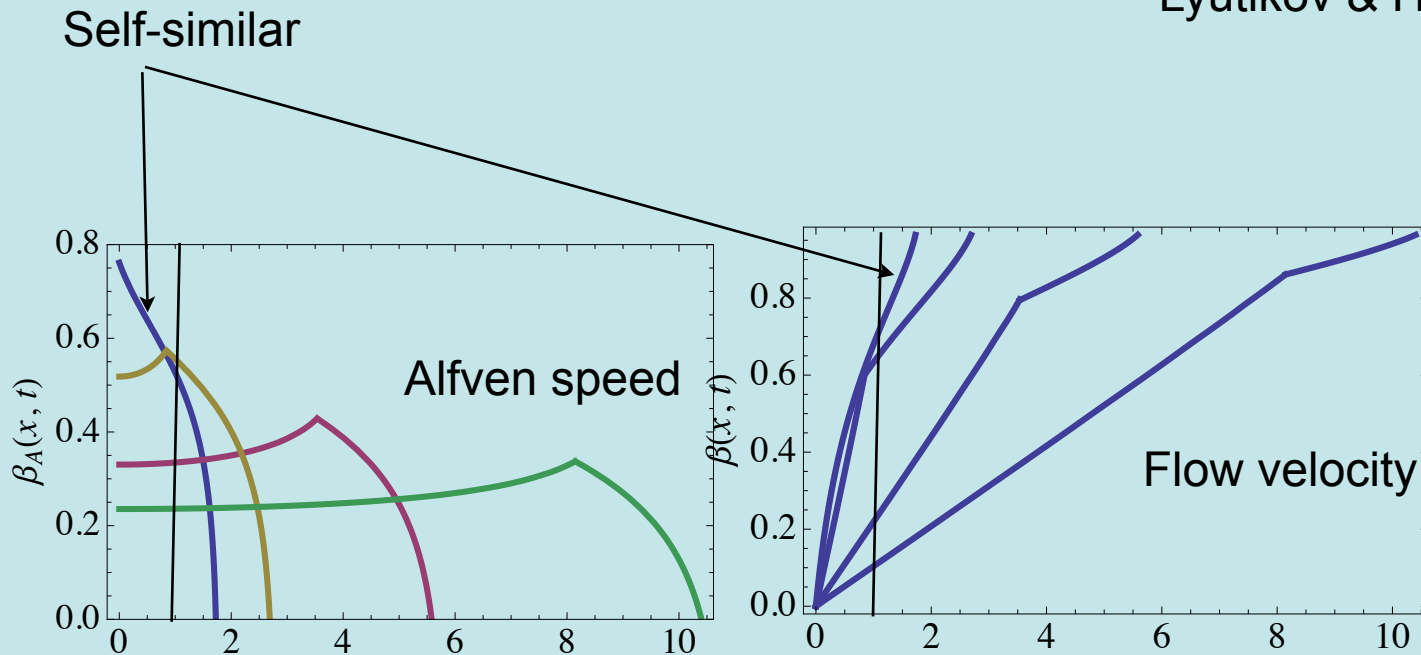
Lyutikov & Hadden, in prep.



Second rarefaction^x wave slows the flow down (contrary to the initial claim in Granot et al. 2010 that the wave “pushes” against the wall).

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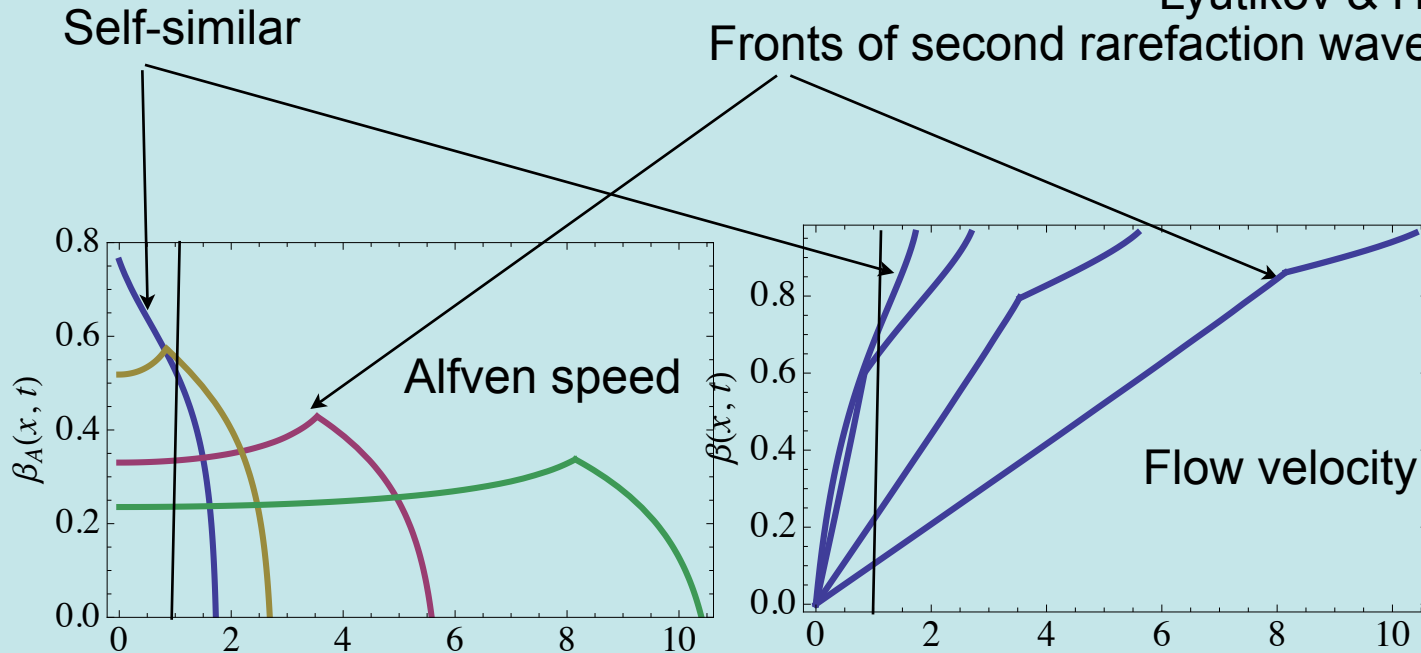
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II. Applications of exact solutions of relativistic MHD

a. TeV flares and Doppler factor crisis in AGNs

Henri & Sauge 2006

- Radiative modeling of TeV flares requires $\delta_{\text{TeV}} \geq 100$
 - Fast variability $\Delta t \rightarrow \Delta t' / \Gamma^2$
 - Compactness parameter $\tau \rightarrow \tau' / \Gamma^6$
 - (I will mix up a bit TeV-GeV data. At least both IC.)
- Direct observations of superluminal radio knots imply $\delta_{\text{knot}} \leq 10$
 - MOJAVE: blobs motion reflects underlying flow (bidirectional motions, no inward moving features, multiple blobs in the same jet with the same speed, correlations of jet speeds with other properties)
- Somewhat similarly (?) GeV photons in GRB 080916C \rightarrow Gamma ~ 2000 .

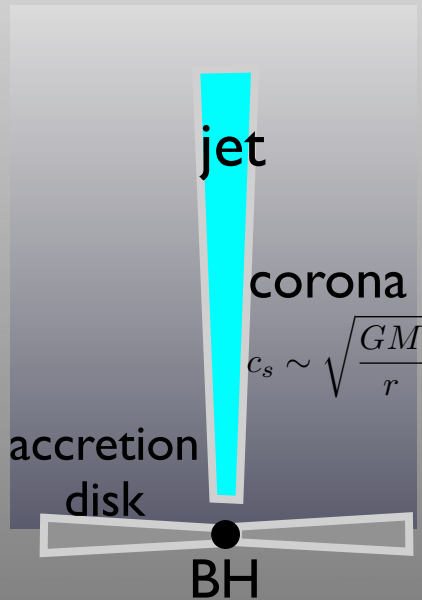
Non-stationary acceleration is more efficient,

$$\Gamma_{max} = 1 + 2\sigma$$

Steady state: $\Gamma \sim \sigma^{1/3}$

Non-stationary jet injection in static corona

Lyutikov & Lister 2010



Dynamic time across the jet

$$t_{dyn} \sim r\Theta_j/c_s$$

Variations in disk/launching

$$t_j \sim \xi r_{BH}/c$$

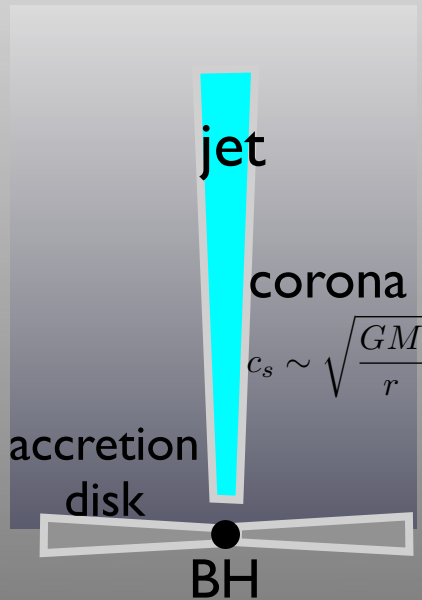
At sufficiently large radii,

$$r_{\text{breakout}} \geq \left(\frac{\xi}{\theta_j}\right)^{2/3} r_{BH} = 2 \times 10^{16} \text{ cm} M_{\odot,9} \xi_2^{2/3} \theta_{j,-1}^{-2/3}$$

variations of launching proceed on time scales shorter than the dynamical time scale across the jet,

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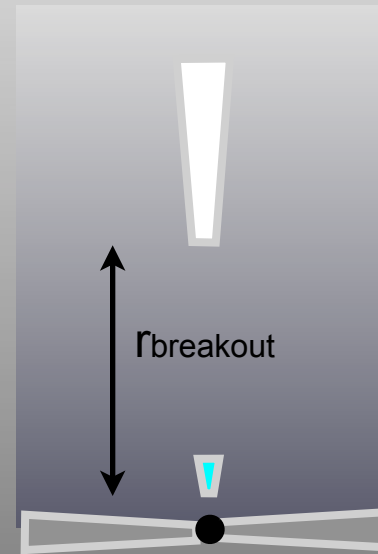


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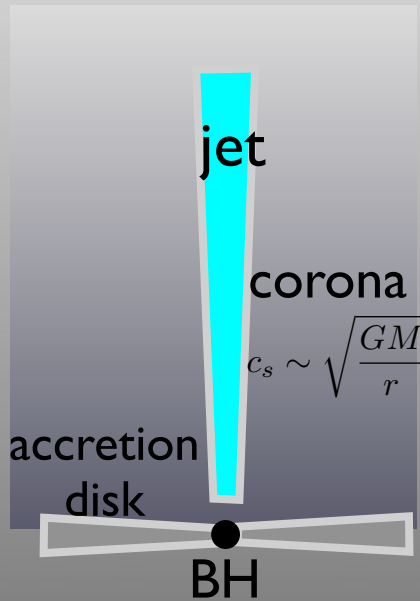
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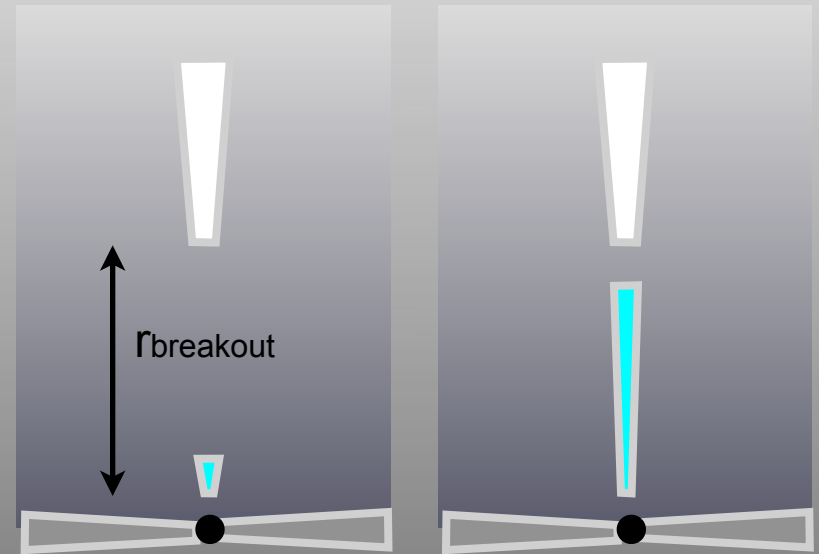


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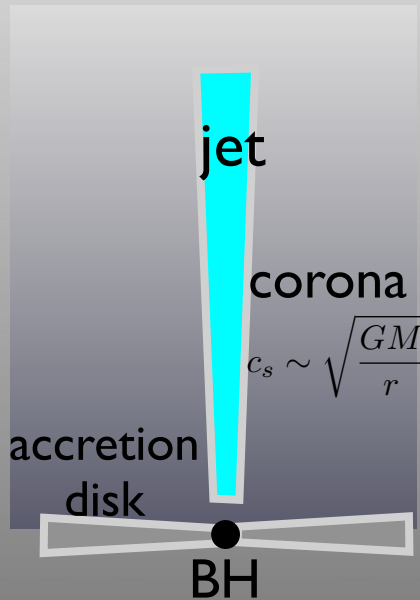
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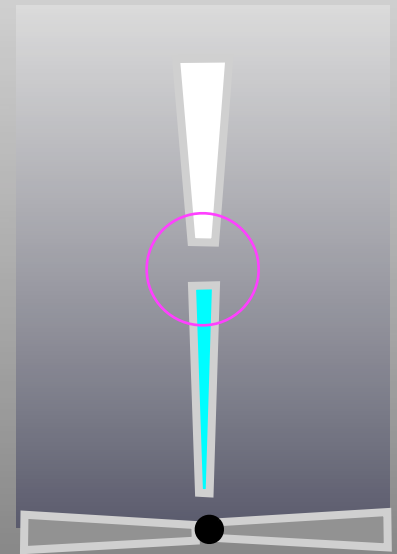
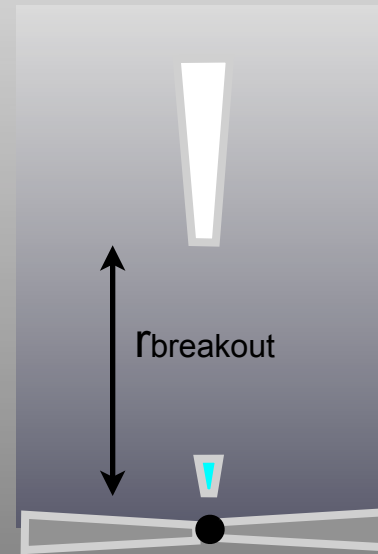


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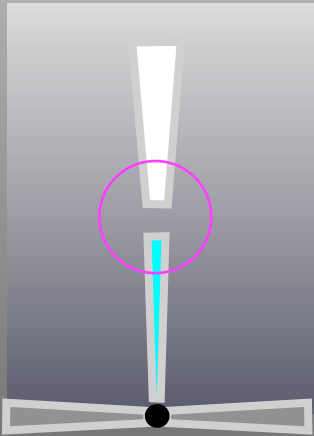
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TeV/GeV flares and radio blobs

TeV and GeV emission in blazars is produced in the leading expansion edge moving with $\Gamma \sim 100$, while the observed velocities of the radio blobs correspond to the bulk motion with $\Gamma \sim 10$



Before breakout

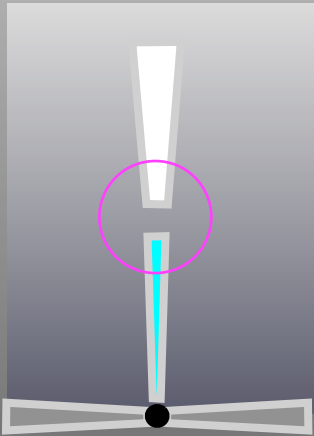
$$\gamma_w = \left(\frac{L}{\rho_{\text{ex}} c^3} \right)^{1/4} r^{-1/2} \sim 10$$

After breakout:

$$\begin{aligned} \text{leading edge } \gamma &\sim 4\gamma_w \sigma \sim 100 \\ \text{bulk: } \gamma &\sim 2\gamma_w \sigma^{1/3} \sim 10 \end{aligned}$$

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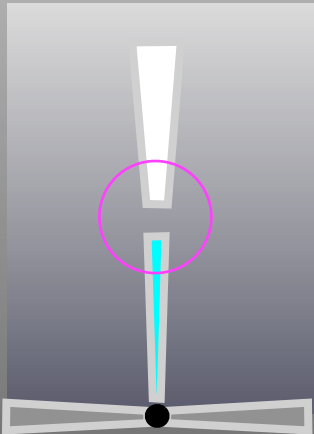
Can accommodate short variability and compactness

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Radio blobs

Predicted correlations

- Cores are optically thick at r_{gamma} , typically $r_c > r_{\text{breakout}}$:

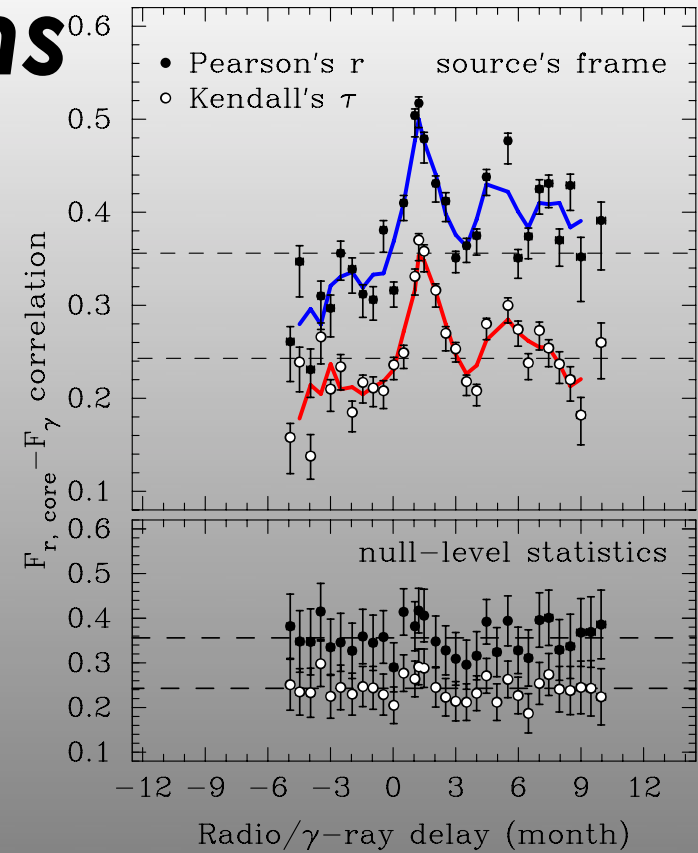
$$r_{\text{core}} \approx 1.4 \text{pc} \zeta_R^{2/3} L_{46}^{2/3} \gamma_{w,1}^{-1/3} \nu_9^{-1}$$

$$r_{\text{breakout}} \geq \left(\frac{\xi}{\theta_j} \right)^{2/3} r_{\text{BH}} = 2 \times 10^{16} \text{cm} M_{\odot,9} \xi_2^{2/3} \theta_{j,-1}^{-2/3}$$

- Jet breakout will occur while the jet is still optically thick in radio.

$$\Delta t_{\gamma-R} \sim \frac{r_{\text{core}}/c}{2\gamma_w^2} \sim \text{weeks} - \text{months}$$

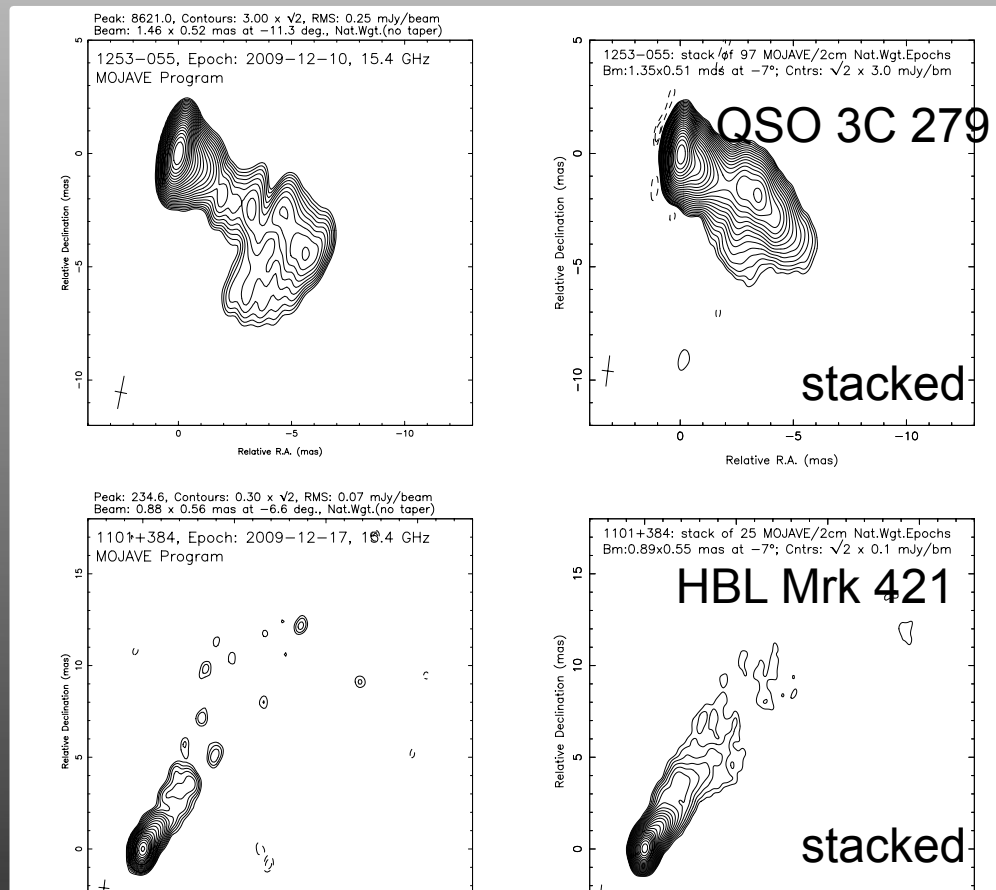
- **Gamma-rays correlate with radio, leading by ~ weeks**
- Better correlated (shorter delay) at higher radio frequencies
- Acceleration at large r : avoid Compton drag near BH.



Gamma-rays-radio correlation with ~ months delay (Pushkarev et al 2010), radio 15 GHz trailing.

Morphologies

- Jet morphology: higher gamma blobs merge later (e.g. variable jets in FSRQ); low gamma: smooth jets in LBLs).

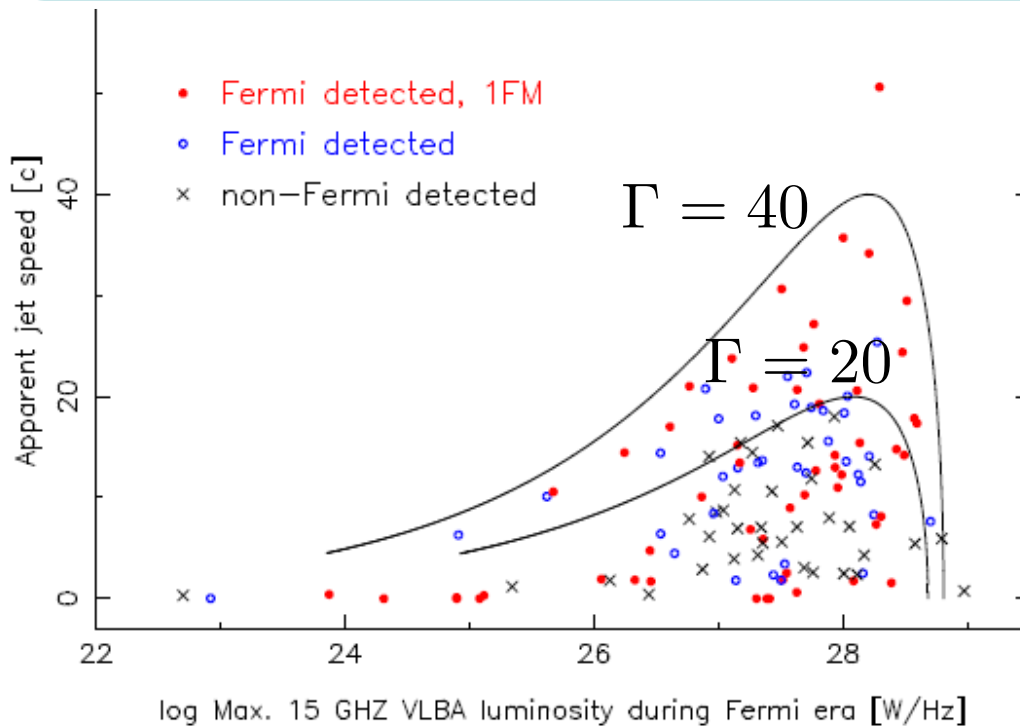


High Gamma,
late merging,
knotty jet

Low Gamma,
early merging,
smooth jets

Predicted correlations

- GeV photons associated with fast beak-out parts: Fermi-detected AGNs have higher Gamma
- jets of gamma-ray-selected AGNs are more aligned than those in radio-selected (but: mini-jets?)
- Gamma-ray emission is more boosted than radio, shorter variability times



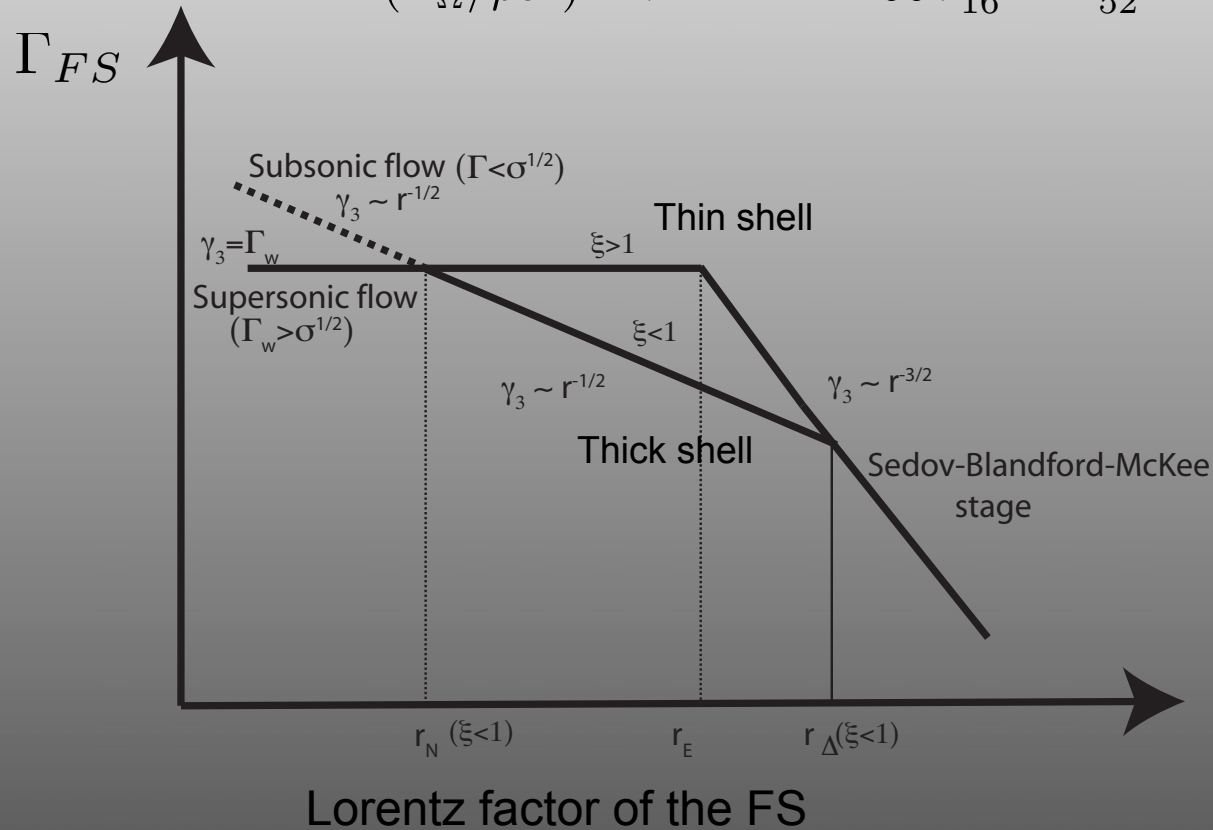
Acceleration on 1-10 pc
- observed? (Lobanov)

b. Magnetized GRB outflows: FS dynamics

Lyutikov 2010c

$$\Gamma \sim (L_{\Omega}/\rho c^3)^{1/4} r^{-1/2} = 1200 r_{16}^{-1/2} L_{52}^{1/4}$$

Fermi GeV?

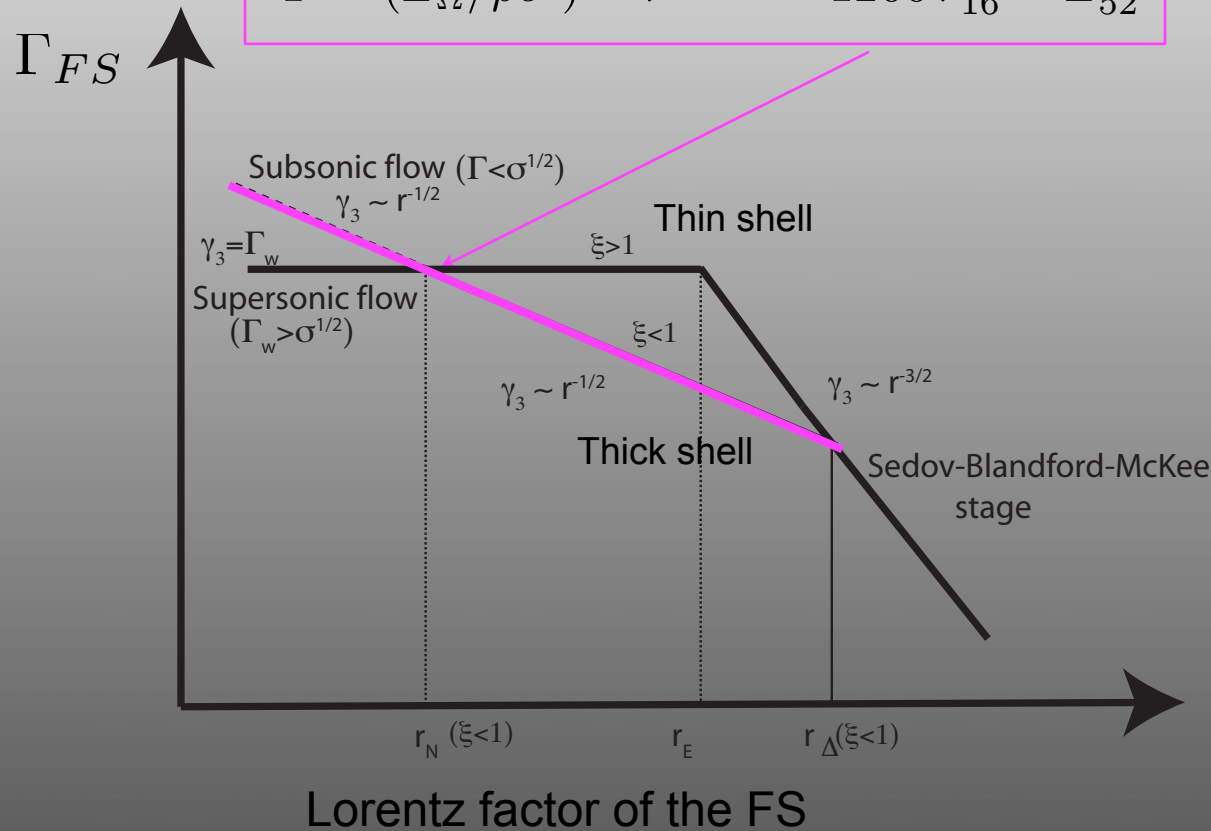


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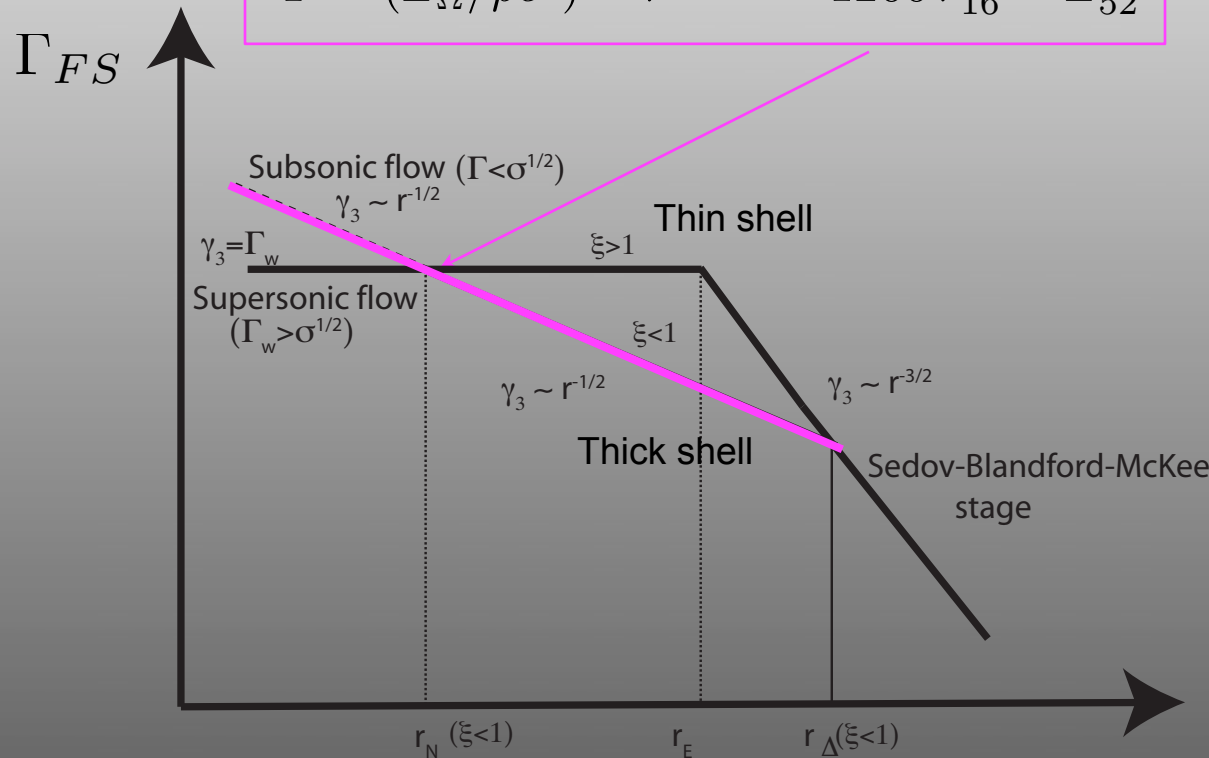


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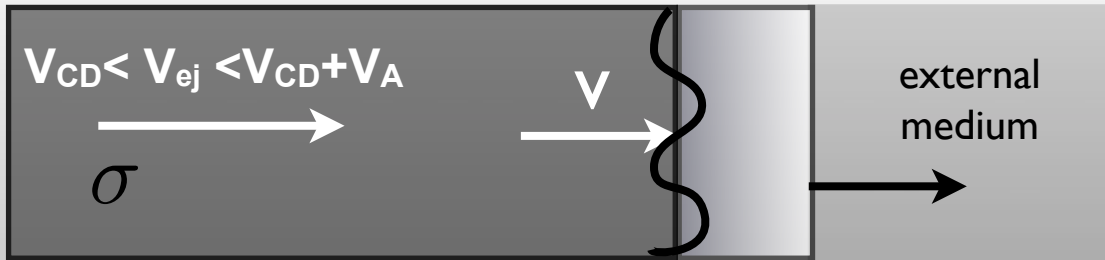
$$\Gamma \sim (L_{\Omega}/\rho c^3)^{1/4} r^{-1/2} = 1200 r_{16}^{-1/2} L_{52}^{1/4}$$

Fermi GeV?



Zhang & Kobayashi, 2005: “only the kinetic energy of the baryonic component (E_k) defines the energy that interacts with the ambient medium.[...]One should define the deceleration radius using E_k along, [...] at the deceleration radius, the Poynting energy is not yet transferred to the ISM”. This is incorrect (Lyutikov 2005).

Corrugated
contact



c. RS in GRBs

Lyutikov 2010c

- Reverse shock forms at a **finite** distance, $\sim 10^{16}$ cm for $\sigma \sim 1$.
- Two conditions for reverse shock: weak and strong (in 1D compression wave always turns into shock, not necessarily in multi-D)

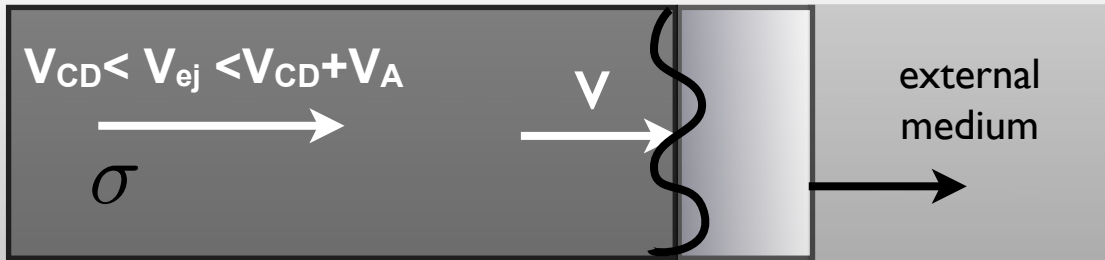
$$\gamma_w > \sqrt{\frac{3}{8}} \sqrt{\frac{\rho_0}{\rho_{\text{ex}}}} \sqrt{\sigma}, \quad r_{RS, \text{weak}} = \frac{1}{\gamma_w^2} \sqrt{\frac{3\sigma L}{2\pi\rho_{ISM}c^3}} = 10^{16} \text{ cm } n^{-1/2}$$

$$\gamma_w > \sqrt{6} \sqrt{\frac{\rho_0}{\rho_{\text{ex}}}} \sigma^{3/2}, \quad r_{RS, \text{strong}} = \sigma r_{RS, \text{weak}}$$

In GRBs prompt optical is rare, highly variable

- Is highly variable optical emission related to nontrivial 2+D dynamics of magnetized RS?

Corrugated
contact



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- Reverse shock forms at a **finite** distance, $\sim 10^{16}$ cm for $\sigma \sim 1$.
- Two conditions for reverse shock: weak and strong (in 1D compression wave always turns into shock, not necessarily in multi-D)

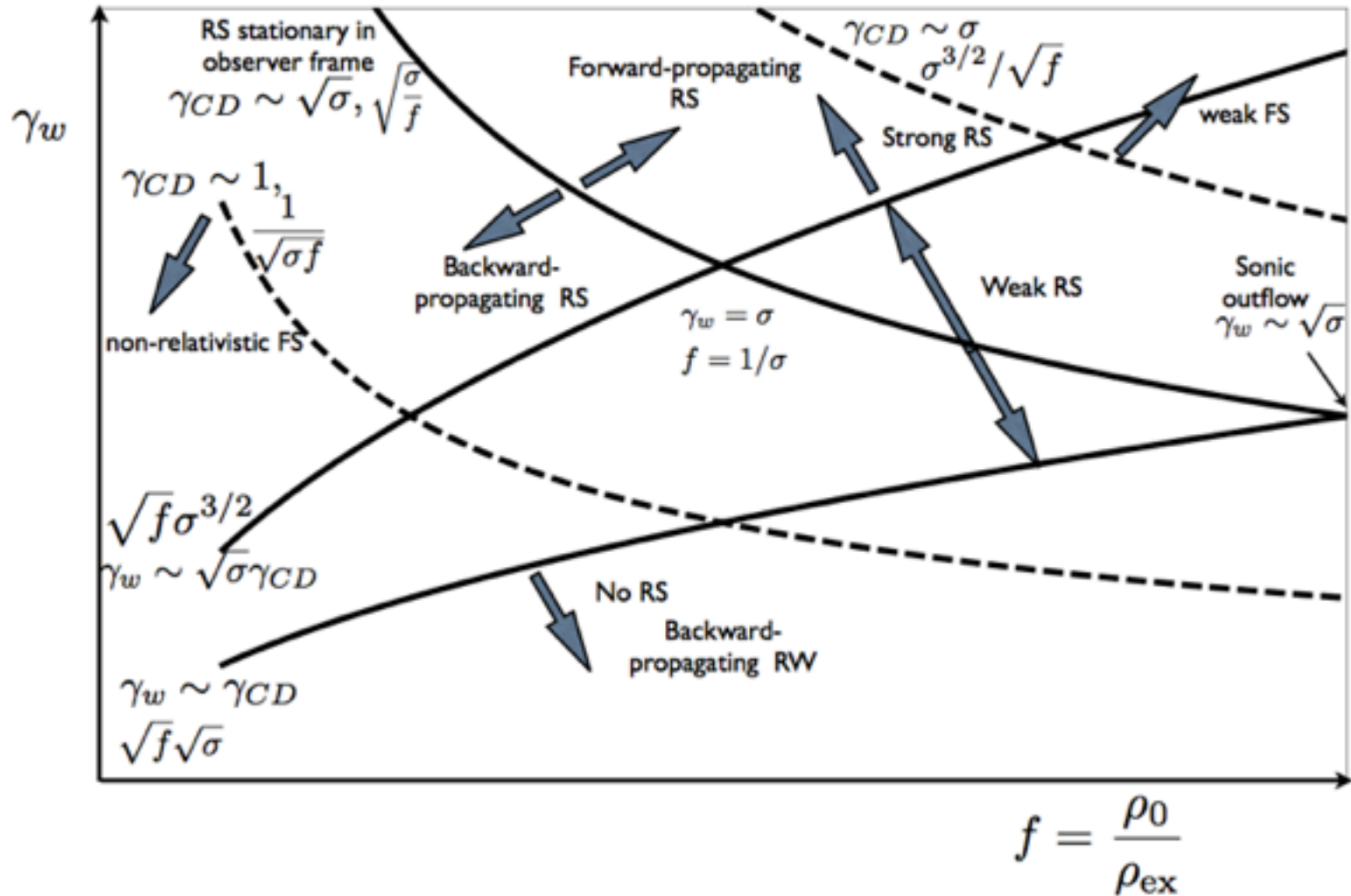
$$\gamma_w > \sqrt{\frac{3}{8}} \sqrt{\frac{\rho_0}{\rho_{ex}}} \sqrt{\sigma}, \quad r_{RS,weak} = \frac{1}{\gamma_w^2} \sqrt{\frac{3\sigma L}{2\pi\rho_{ISM}c^3}} = 10^{16} \text{ cm } n^{-1/2}$$

$$\gamma_w > \sqrt{6} \sqrt{\frac{\rho_0}{\rho_{ex}}} \sigma^{3/2}, \quad r_{RS,strong} = \sigma r_{RS,weak}$$

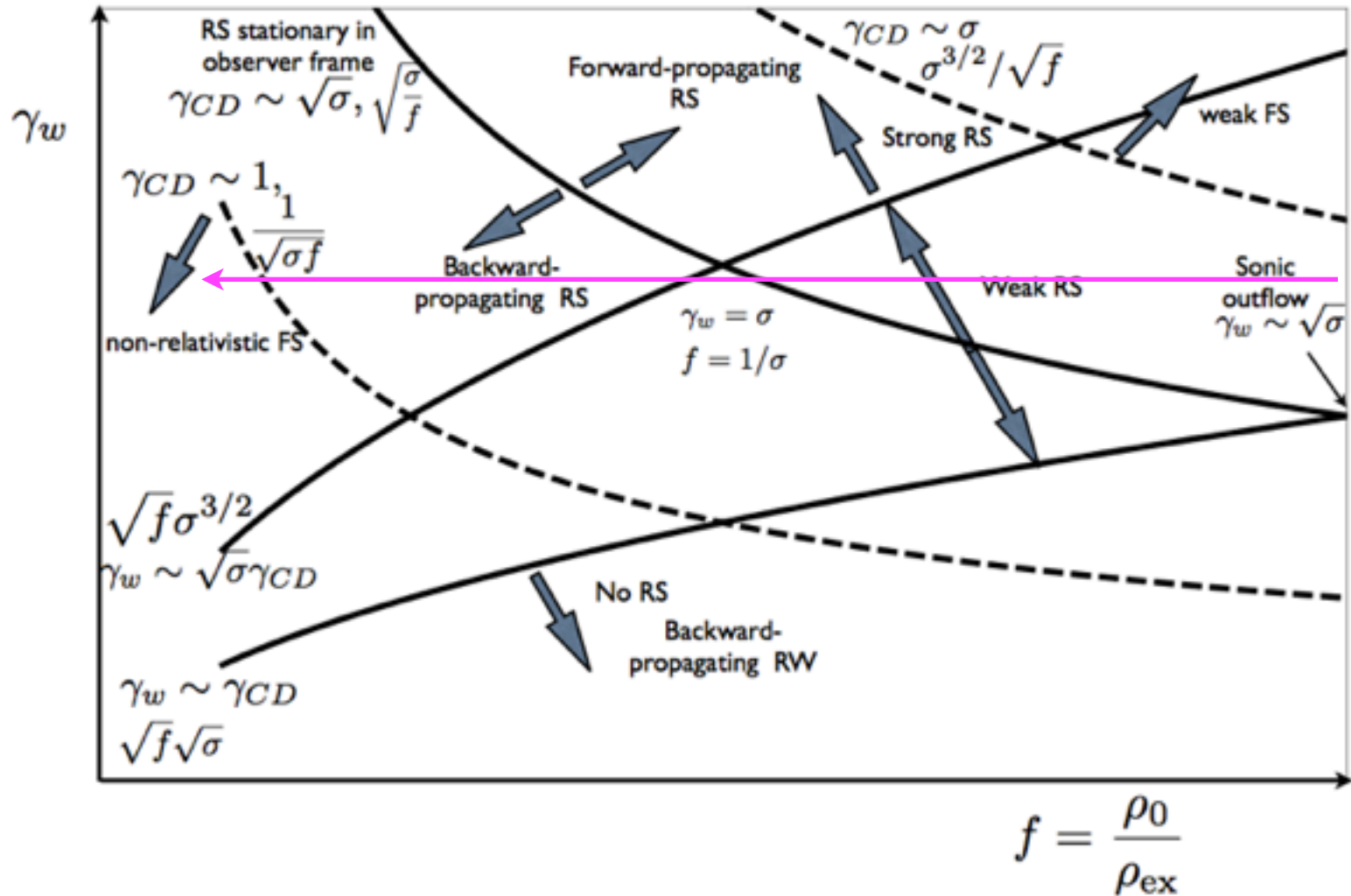
In GRBs prompt optical is rare, highly variable

- Is highly variable optical emission related to nontrivial 2+D dynamics of magnetized RS?

Conditions in GRBs: FS & RS



Conditions in GRBs: FS & RS



Upshot: GRBs

Dynamics of FS and RS even in mildly magnetized outflows is considerably different from the fluid case.

III. Structure of magnetized jets

Grad-Shafranov equation

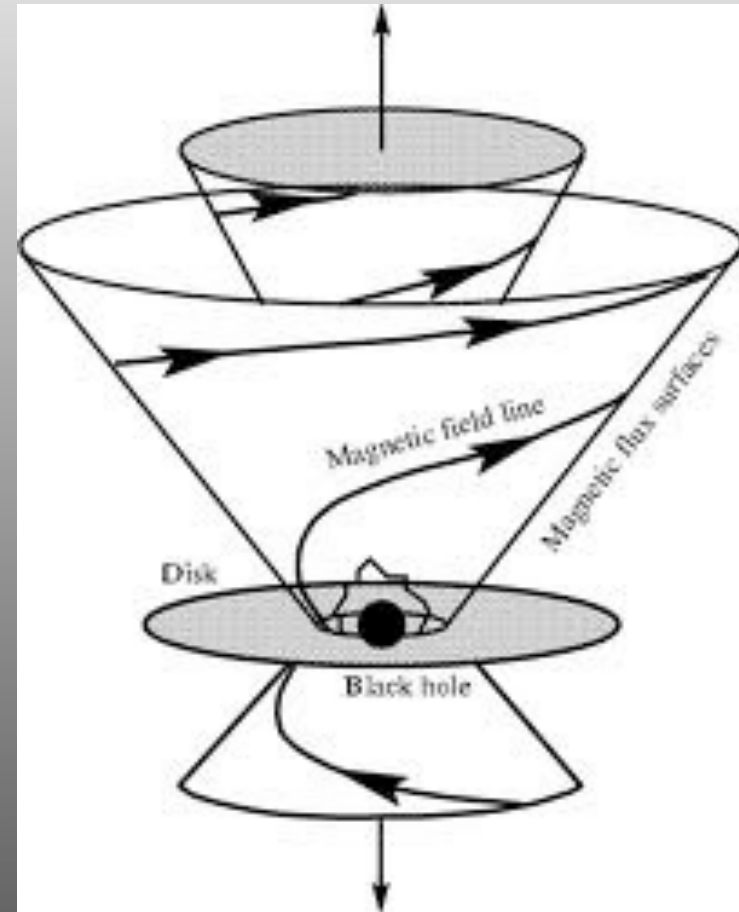
Stationary axisymmetric B-field.

Shape of flux surface $\Psi(r, \theta)$

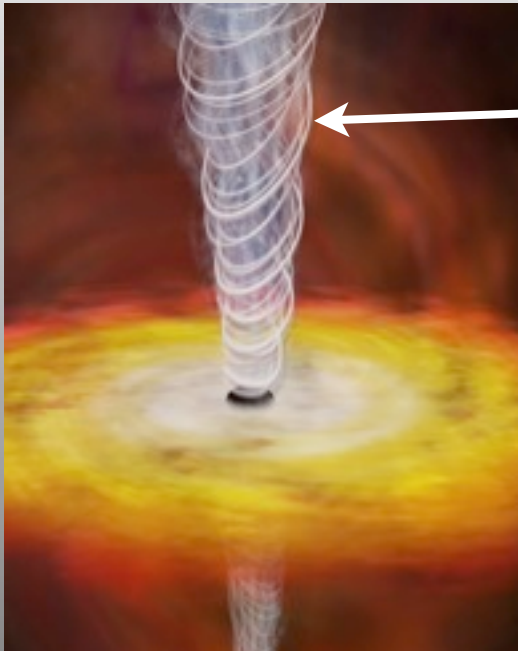
Current enclosed by the flux surface $I(\Psi)$

Flux surface is at same pressure $P(\Psi)$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right) \right) \Psi = P(\Psi) r^2 \sin^2 \theta + 2 \partial_{\Psi} I^2(\Psi)$$



Structure of magnetized jets.



- On the jet boundary, both poloidal and toroidal B-field should be zero
- Force-Free Lundquist fields: **must** have current sheet
- Not clear if evolution is intrinsic or driven by resistive dissipation of the current sheet.

In Grad-Shafranov formalism

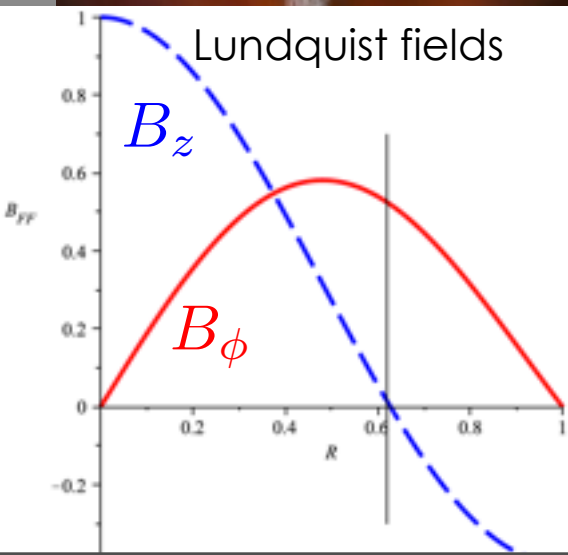
$$B_\phi \propto \Psi$$

$$B_p \propto \Psi'$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{\sin \theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right) \right) \Psi = P(\Psi) r^2 \sin^2 \theta + 2 \partial_\Psi I^2(\Psi)$$

But pressure and current are not known a priori: Need to find equation and its solution that satisfied the overdetermined boundary conditions!

Can be done



Jet with no current sheet

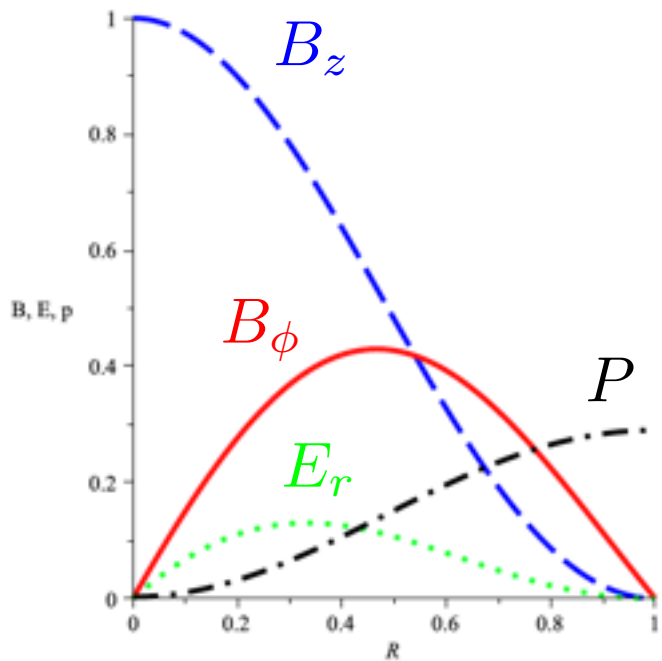
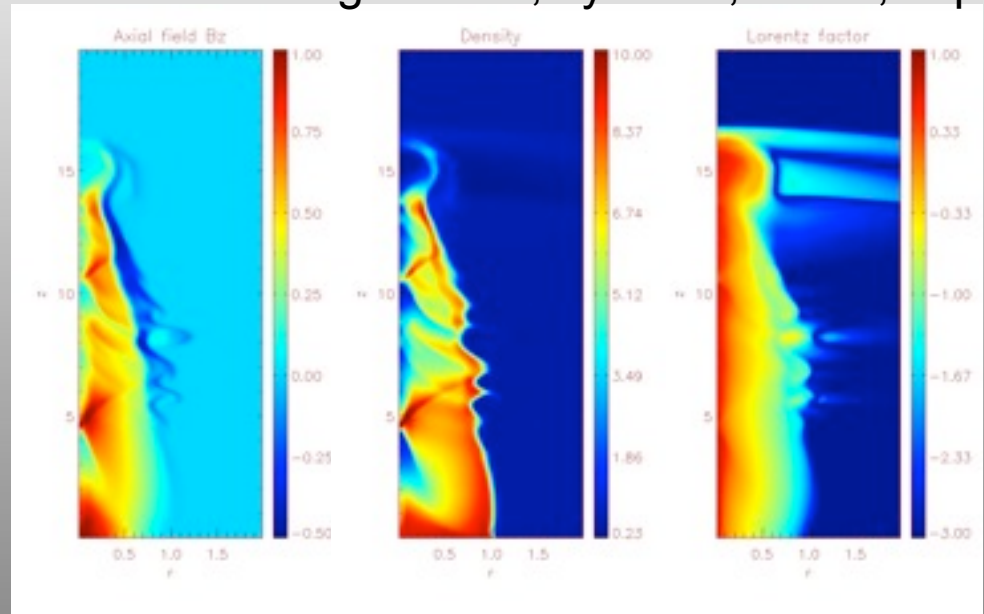
Gourgouliatos, Lyutikov, Fendt, in prep

$$E_R = \frac{v_z}{(1 - v_z^2)^{1/2}} c_t \alpha_t J_1(\alpha_t R),$$

$$B_\phi = \frac{1}{(1 - v_z^2)^{1/2}} c_t \alpha_t J_1(\alpha_t R),$$

$$B_z = \alpha_t \left(c_t J_0(\alpha_t R) - \frac{F_t}{\alpha_t^2} \right),$$

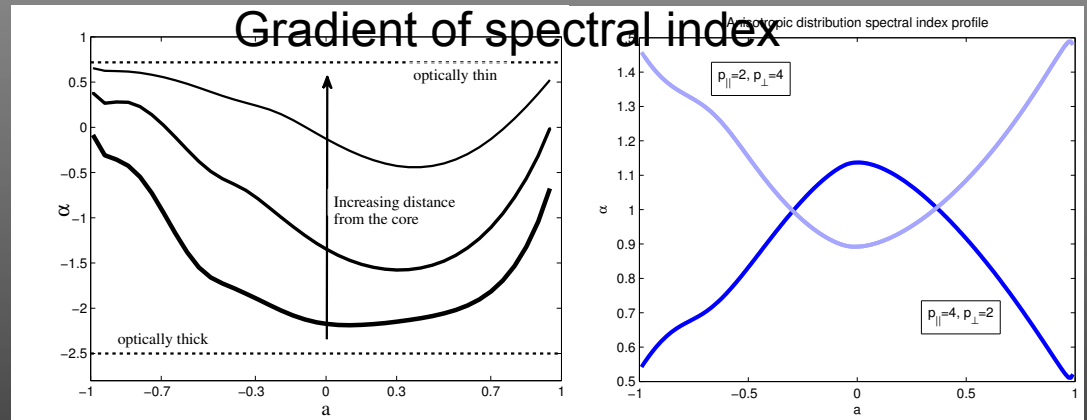
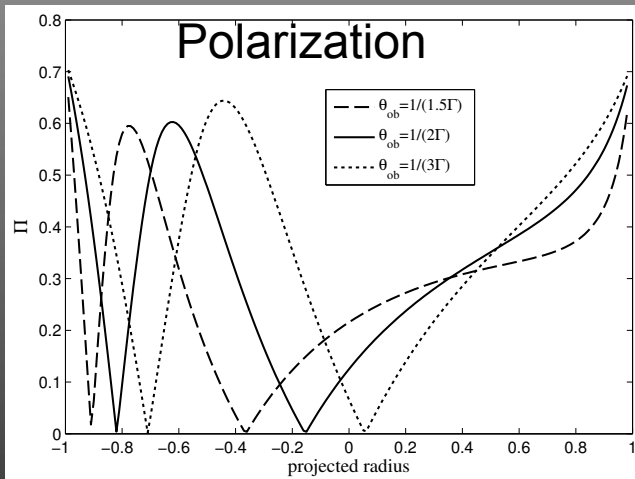
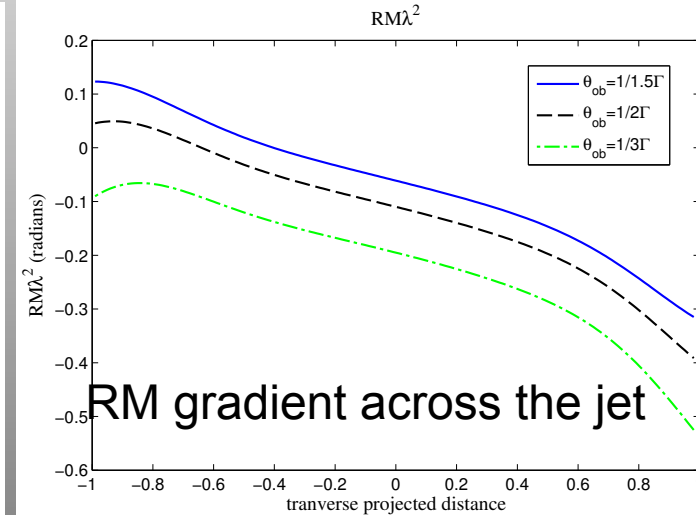
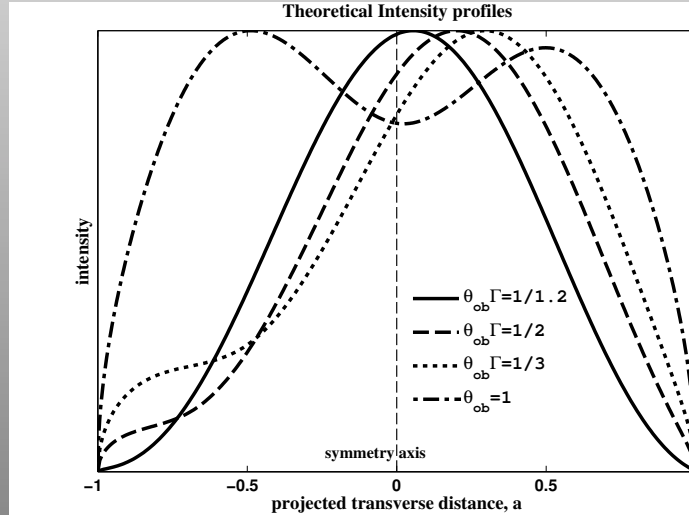
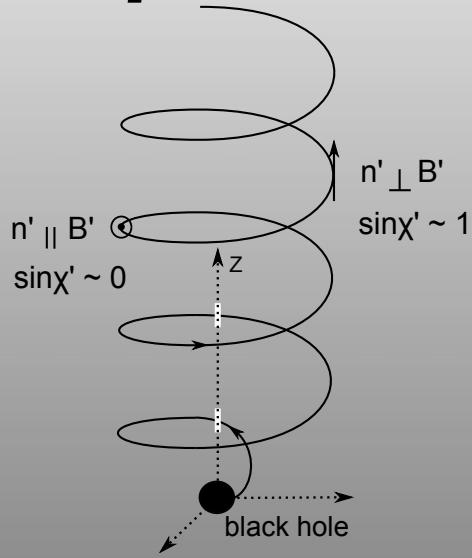
$$p = \frac{1}{4\pi} F_t \left(c_t J_0(\alpha_t R) - \frac{F_t}{\alpha_t^2} \right) + p_{t,0}$$



- Simple force-free-like solutions, some pressure gradient (but not on the surface)
- internally confined by external medium.
- Jets are more stable
- Sheared & rotating
- Also: **expanding magnetic clouds**

Axisymmetric jets with B-field produce non-symmetric profiles

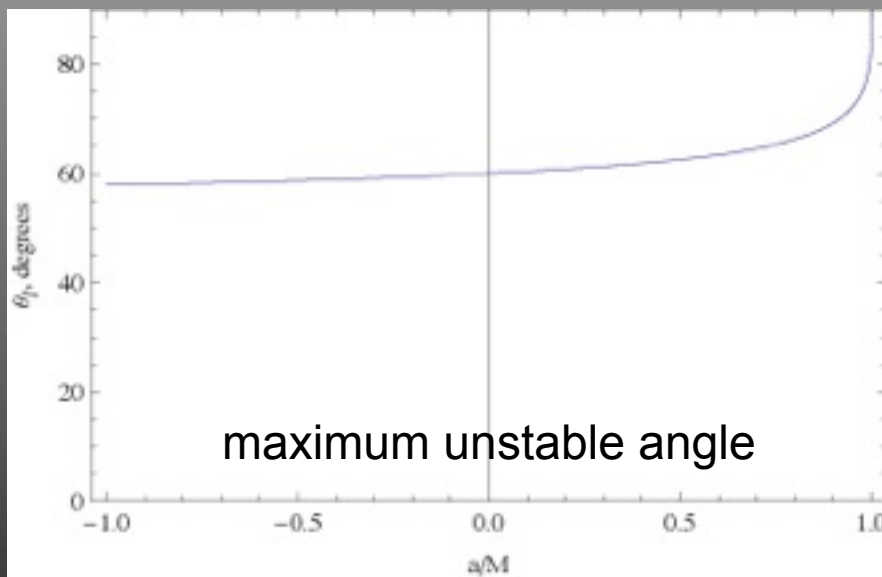
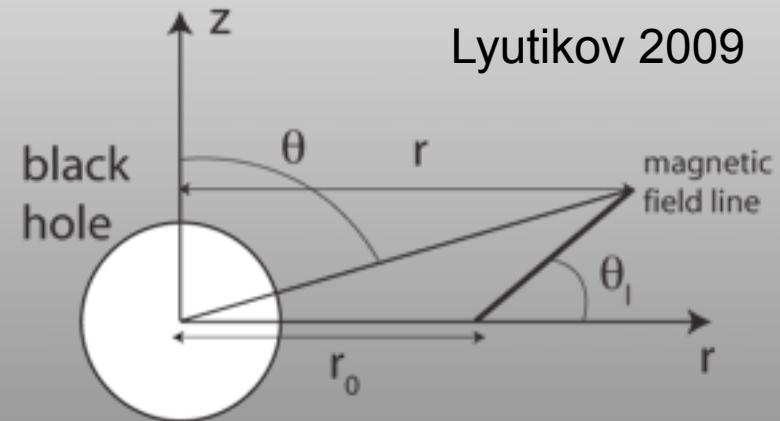
Clausen-Brown, Lyutikov & Kharb, 2011



Anisotropic distribution spectral index profile

IV: Jet launching from disk: Blandford-Payne mechanism in Kerr metric

- Centrifugal launching: particle in B-field as bead on wire
- Non-relativistic and Schwarzschild: 60°
- critically spinning Kerr black hole can launch a jet along the rotation axis of the black hole.

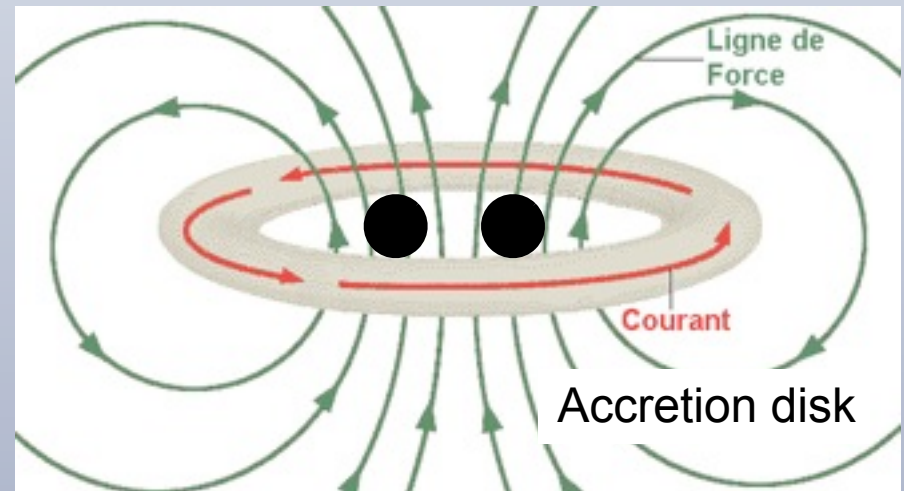
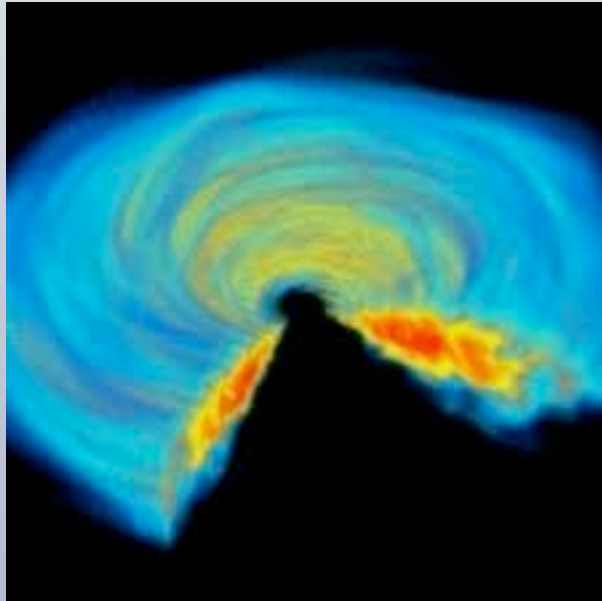


***V. Jets in mergers of compact objects
(NS-NS as short GRBs engine?)***

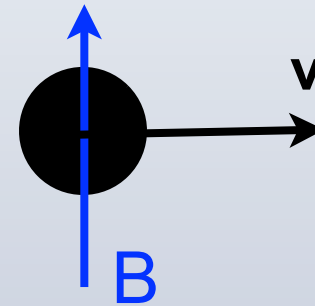
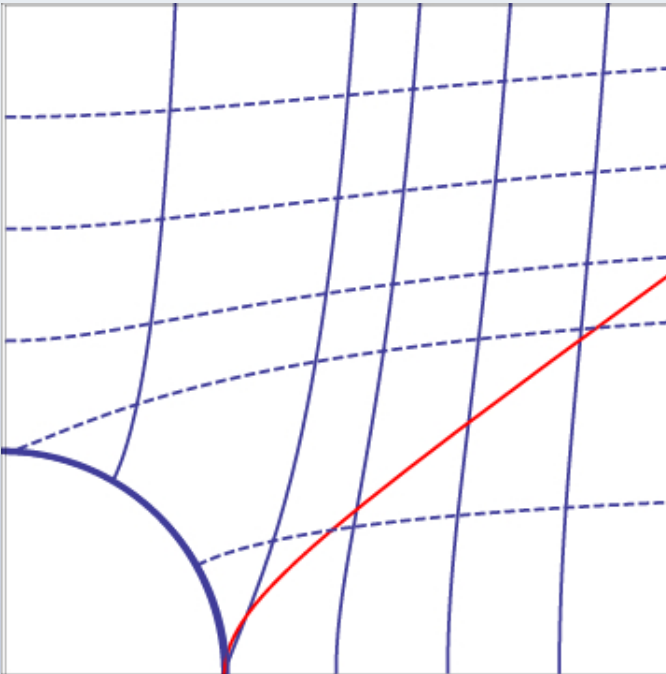
Merging BHs + accretion disk

- $R < 100 R_{\text{SH}}$, viscous time-scale shorter than GW. Disk stays at $\sim 100 R_{\text{SH}}$, hard to excite fluid motions in a far-away disk

Milosavljevic & Phinney

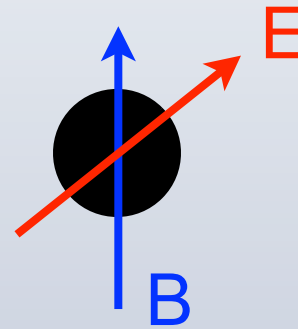
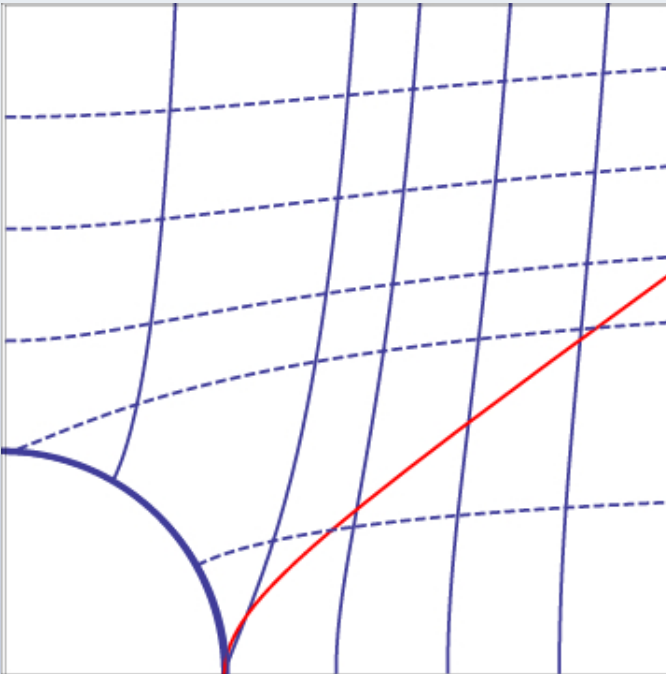


BH moving across B-field in vacuum generates a non-zero E_{\parallel}



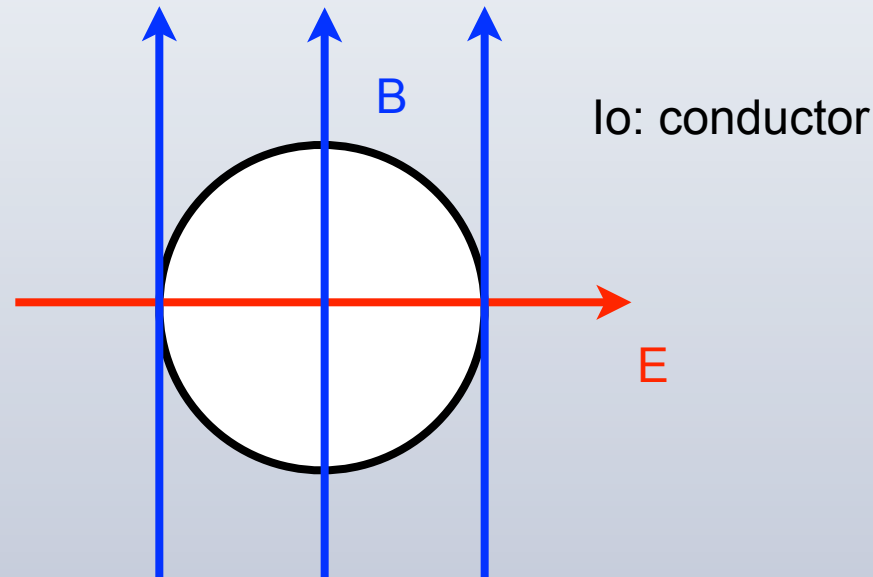
In presence of BH, in vacuum,
for E- & B- fields orthogonal at
infinity, a non-zero parallel E-
field

BH moving across B-field in vacuum generates a non-zero E_{\parallel}



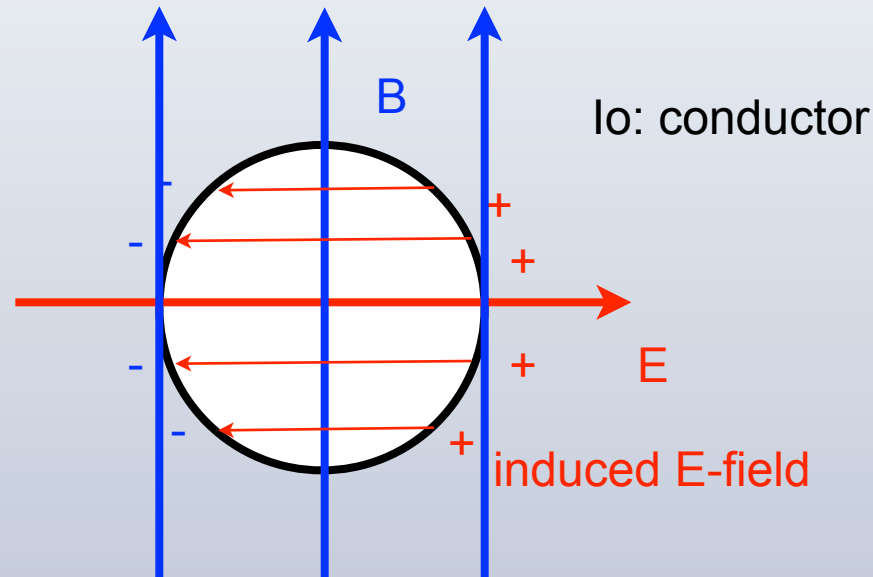
In presence of BH, in vacuum,
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Recall how Io and pulsars produce parallel E-field



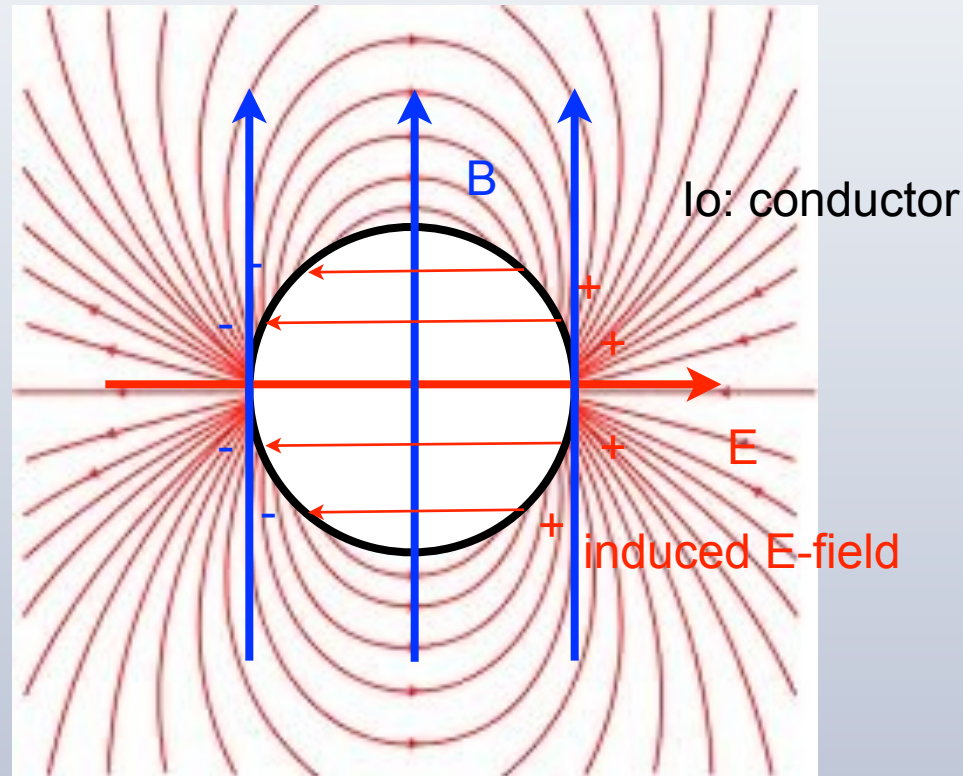
- Parallel E-field is generated by **real** charges
- (Same in pulsar, real charges kill inductive E-field)

Recall how Io and pulsars produce parallel E-field



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Recall how Io and pulsars produce parallel E-field



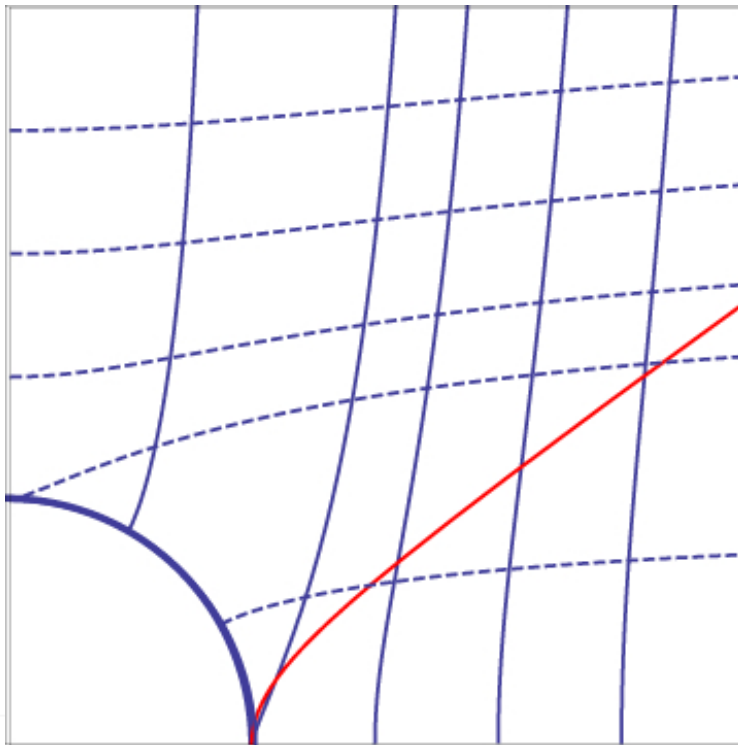
- Parallel E-field is generated by **real** charges
- (Same in pulsar, real charges kill inductive E-field)

Non-zero second EM invariant

Lyutikov 2011a

- In presence of BH, parallel E-field is generated **in vacuum**
- Non-zero second EM invariant

$$\mathbf{E} \cdot \mathbf{B} = -\cos \phi \sin 2\theta \beta_0 B_0^2 \frac{M}{r}$$



NOT what one would guess using the membrane paradigm

Parallel E-field: vacuum breakdown

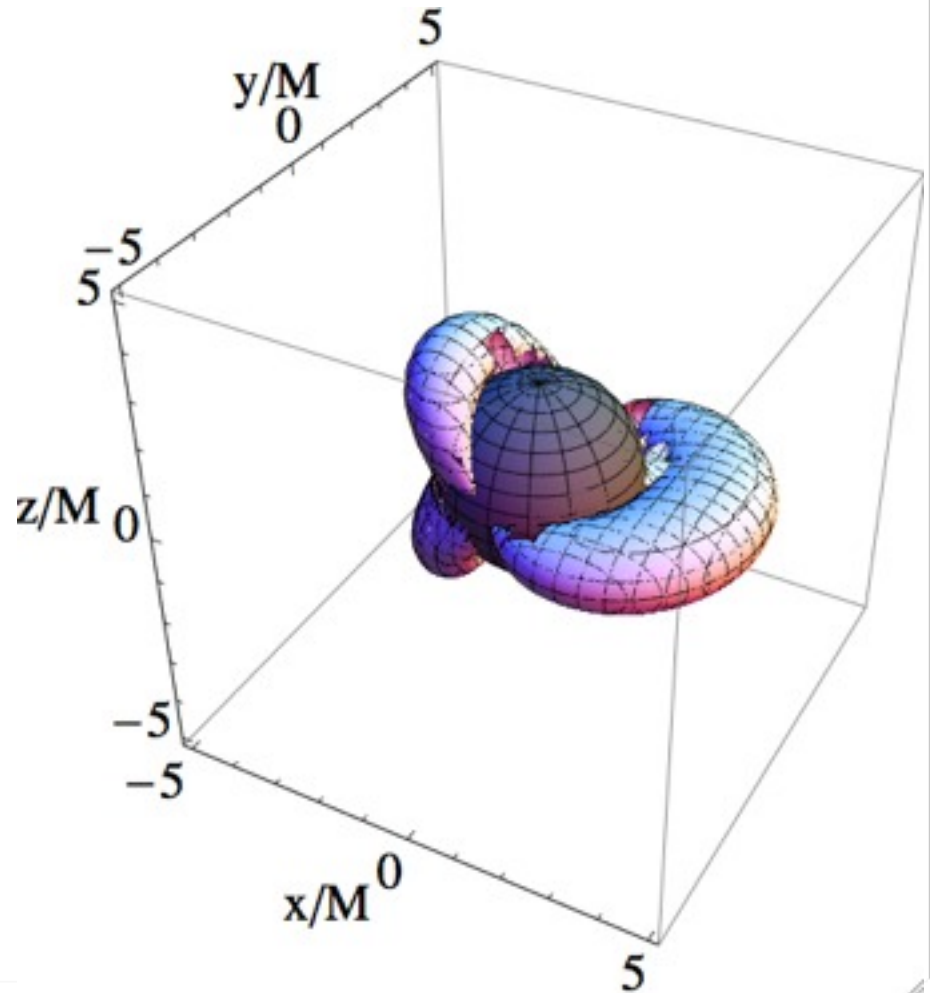
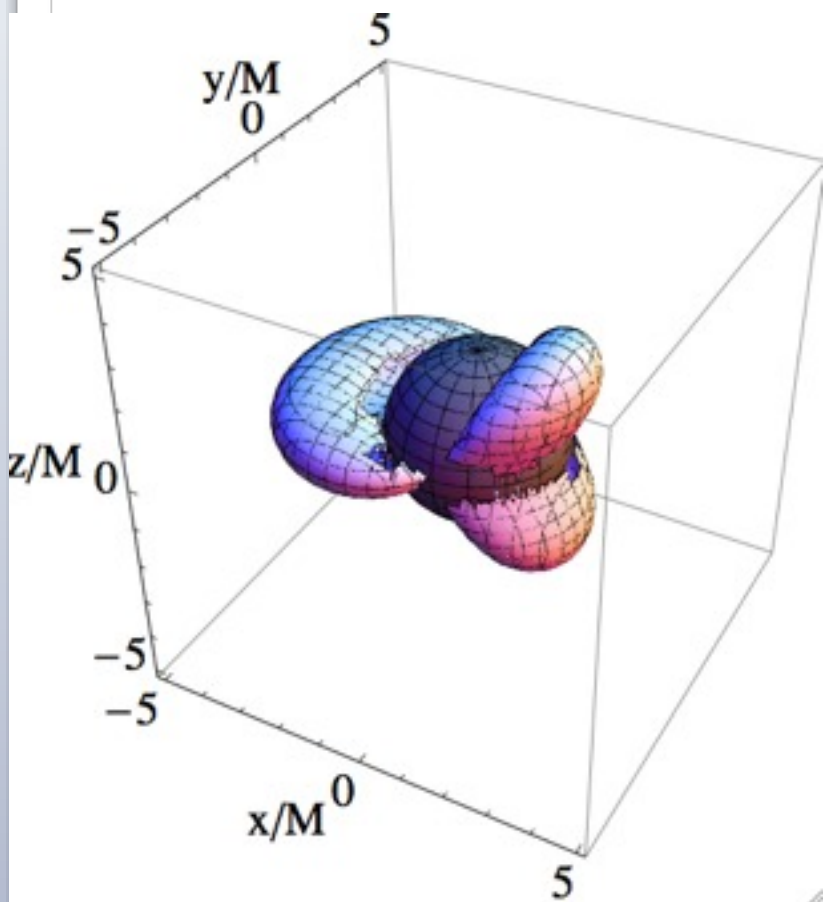
- Total potential drop $\Delta\Phi \approx \beta_0 r_G B_0 \sim 10^{14} \text{V}$
- Any stray particle will break vacuum, typically, after $\sim \text{GeV}$.
- Via emission of photon (eg., IC) and ensuing two photon pair production
- Plasma will generate charge density, trying to kill parallel E-field.

$$\rho_{\text{ind}} = \frac{1}{4\pi} \nabla \cdot \mathbf{E}_{\parallel}$$
$$\rho_0 = \frac{B_0 (v_0 / R_G)}{2\pi c}$$

- Analogue of Goldreich-Julian density, $v_0 / R_G \rightarrow \Omega_{\text{eff}}$

Induced charge density

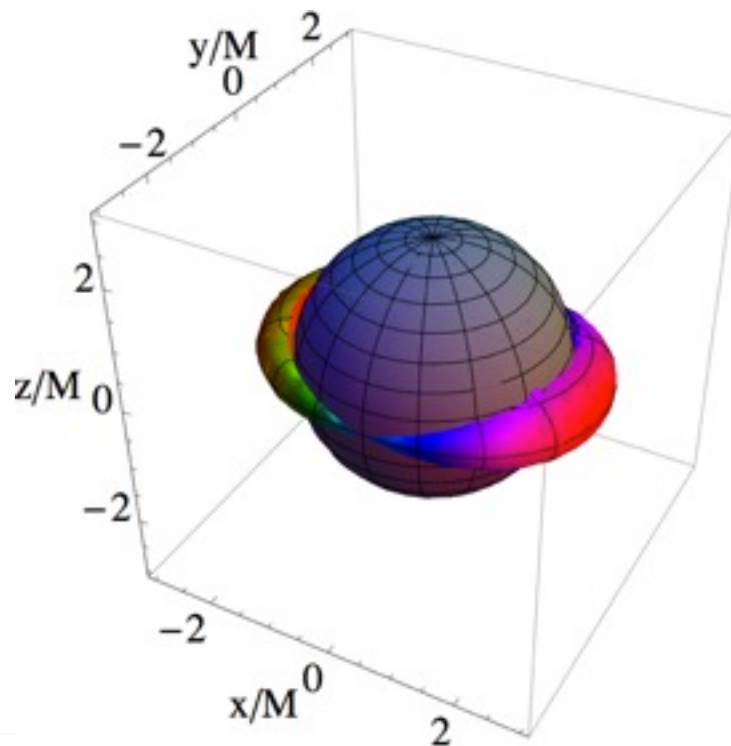
- Contours of constant charge density



First EM invariant changes sign

- First EM invariant, $B^2 - E^2$, changes sign at

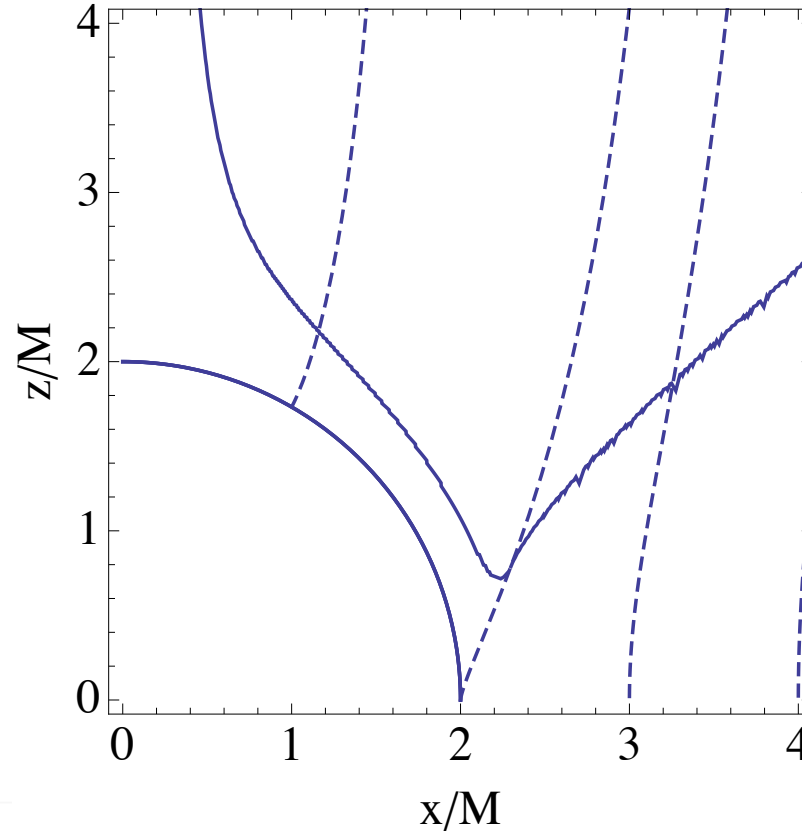
$$r = \frac{2M (\sin^2 \theta - \beta_0^2 (\cos^2 \theta \cos^2 \phi - \sin^2 \phi))}{1 - \beta_0^2}$$



Pair formation front

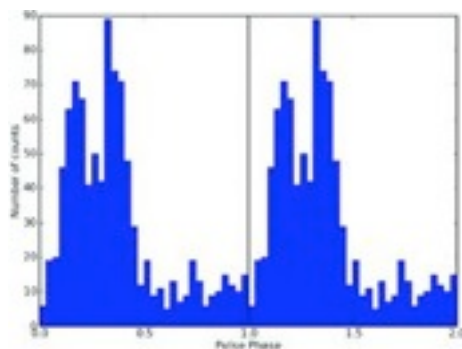
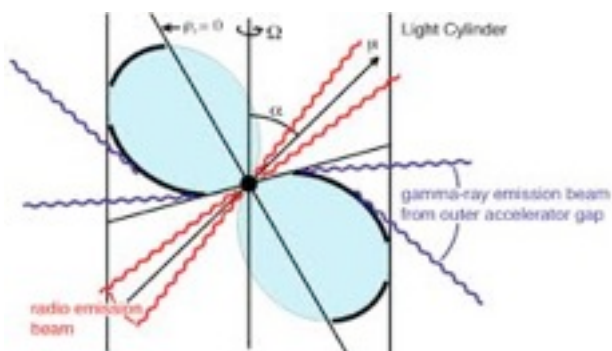
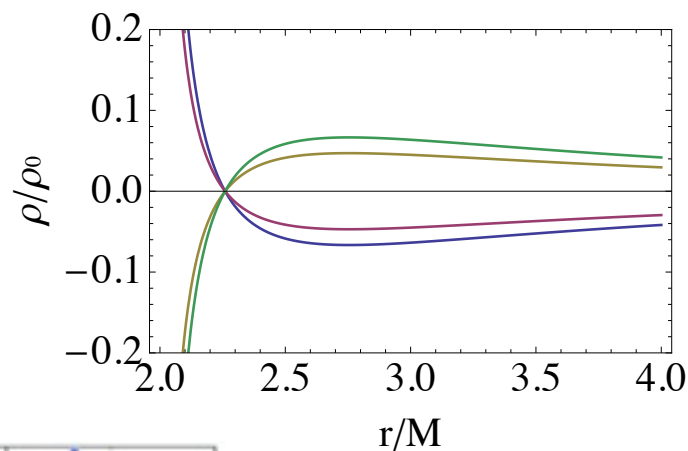
- Structure of the equipotential surface

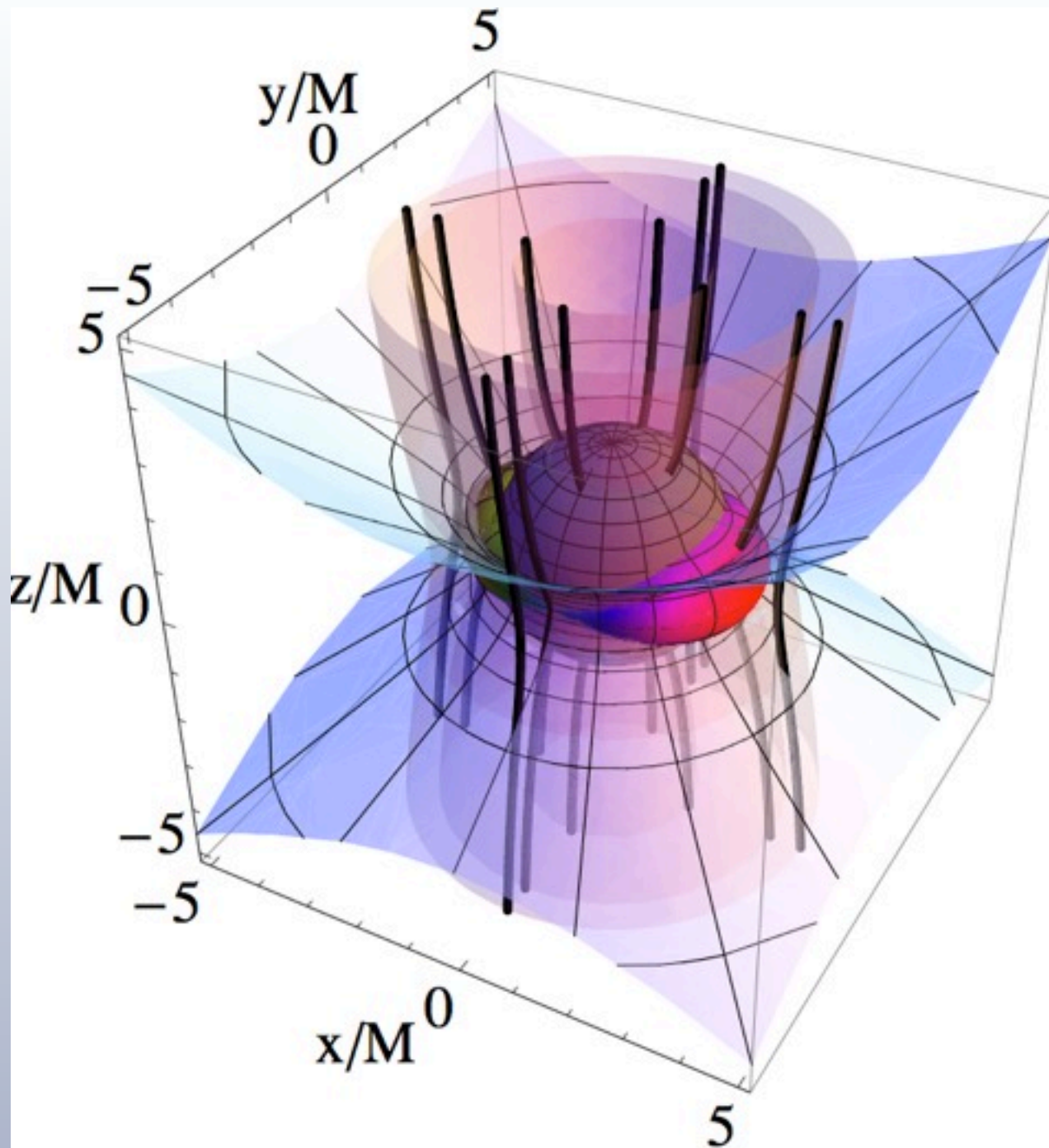
$$\int_{\text{along B-field}} E_{\parallel} ds = \text{Const}$$



Outer gap

- Charge density along B-field lines starting at equator at different azimuthal angles
- Pulsar-like nonthermal?
 - Coherent radio
 - GeV: at outer gaps
 - May be beamed

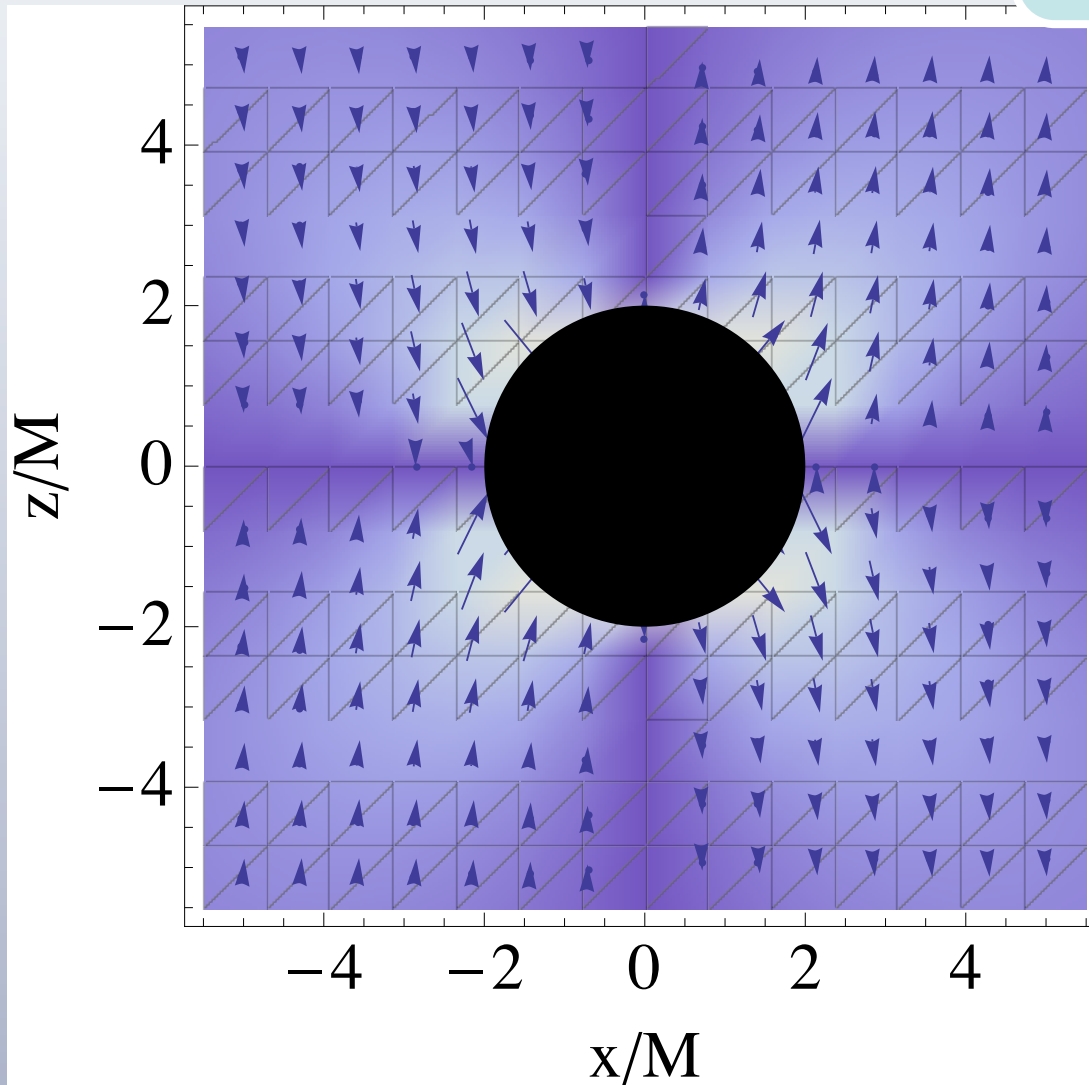




Large scale currents

$$I \sim \beta_0 M B_0$$

A BH moving through B-field will produce jets.



Poynting outflow

$$L_{EM,u} \approx M^2 E_0^2 = M^2 B_0^2 \beta_0^2$$

$$\approx B_\phi^2 r^2 c$$

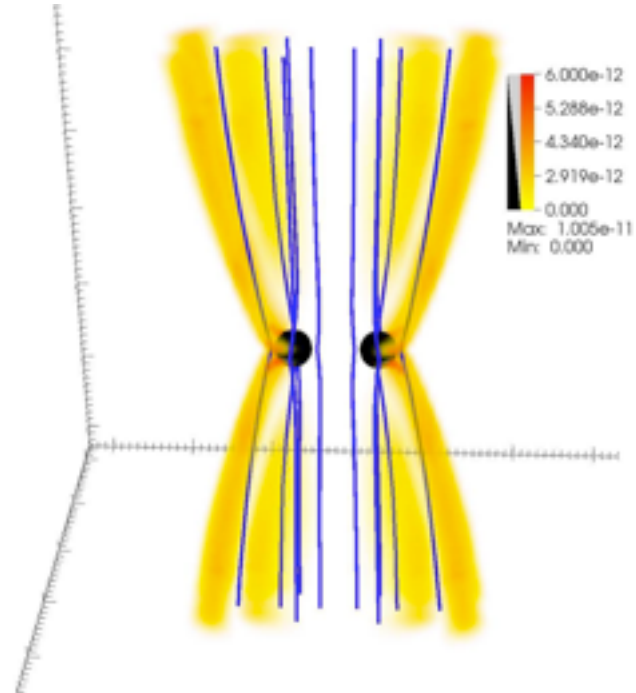
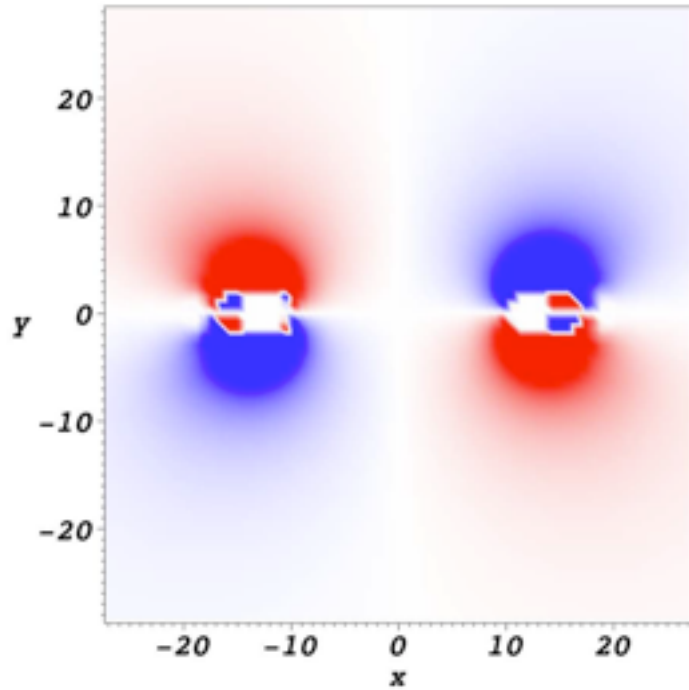
$$\int dS \mathbf{E} \times \mathbf{B}_\phi$$

$$\approx \Delta \Phi^2$$

- For Keplerian velocity

$$L_{EM,u} = \frac{(GM)^3 B_0^2}{c^5 R}$$

Simulations



Charge density for head-on
collision of two BH
Palenzuela et al

VI. EM emission in mergers of compact objects

EM precursors in mergers

- NS-NS (Unipolar induction over R_{NS})

$$L \sim \beta^2 R_{NS}^2 B^2$$

(Precursors in short GRBs - Hansen & Lyutikov 2001)

- NS-BH, BH-BH (Unipolar induction over R_{sc})

$$L \sim \beta^2 R_G^2 B^2$$

Need magnetar field to get to 10^{51} erg/s

Collapse of a NS into BH: Poynting flux (and jets?) from isolated Kerr BH

Time-dependent Grad-Shafranov equation

Lyutikov 2011b
(thanks to Lehner, Beskin, Komissarov, Tchekhovskoy)

- Two types of time-dependent:

- **variable current** for given shape of flux surfaces

$$\varpi^2 \nabla \left(\frac{1 - \varpi^2 \Omega^2}{\varpi^2} \nabla P \right) + \frac{4I(\nabla P \cdot \nabla I)}{(\nabla P)^2} + \varpi^2 \Omega (\nabla P \cdot \nabla \Omega) = 0$$

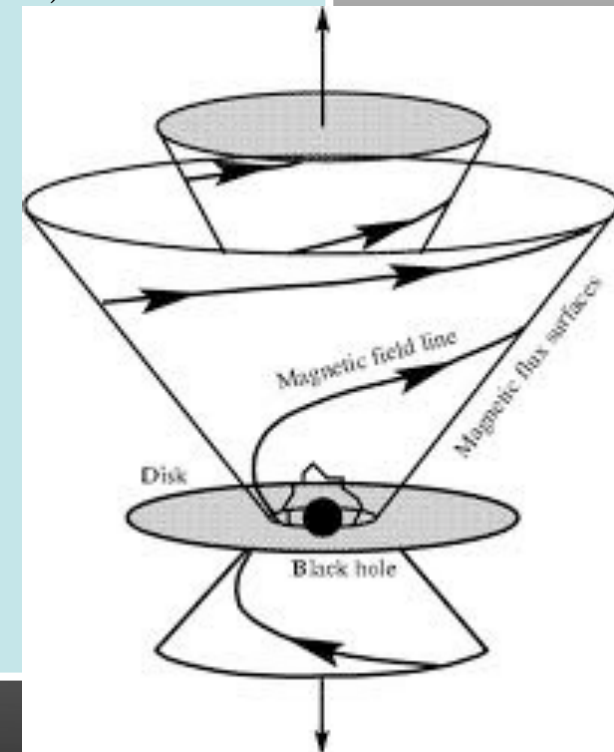
$$\partial_t^2 \Omega = \frac{\mathbf{B} \cdot \nabla (\mathbf{B} \cdot \nabla \Omega)}{B_p^2}$$

- **motion of flux surfaces**

$$\Delta^* P - \partial_t^2 P + \frac{4I(\nabla P \cdot \nabla I)}{(\nabla P)^2} - 2\partial_t \left(\frac{I^2 \partial_t P}{(\nabla P)^2} \right) = 0$$

$$F'(\nabla P)^2 = 2I\partial_t P$$

$$\partial_t I = \frac{1}{2} \Delta^* F$$



Time-dependent Michel's solution in Schwarzschild metric

- Rotating NSs generate plasma out of vacuum, no external currents needed

- Magnetosphere of collapsing NS:

$$B_\phi = -\frac{R_s^2 \Omega \sin \theta}{\alpha r} B_s, \quad B_r = \left(\frac{R_s}{r}\right)^2 B_s,$$

$$E_\theta = B_\phi, \quad j_r = -2 \left(\frac{R_s}{r}\right)^2 \frac{\cos \theta \Omega B_s}{\alpha}$$

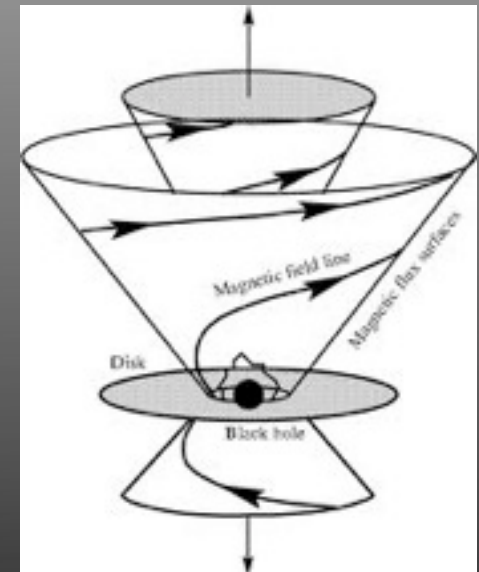
$$\Omega \equiv \Omega (r - t + r(1 - \alpha^2) \ln(r\alpha^2)) \quad \alpha = \sqrt{1 - 2M/r}$$

$$B_s R_s^2 = \text{const}$$

BH rotates with finite

$$\Omega_H \approx \frac{\chi}{5} \frac{c^4 R_{\text{NS}}^2}{(GM_{\text{NS}})^2} \Omega_{\text{NS}} = 2.9 \times 10^3 \text{rads}^{-1} \chi_{-1} P_{\text{NS},-3}^{-1}$$

($\alpha = 0.04$ for a ms NS, slows down!)



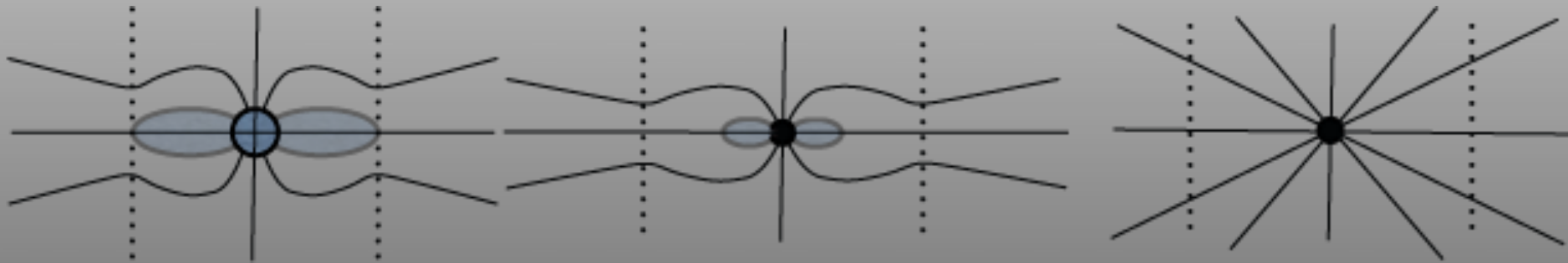
Hold on: “No hair” theorem?

- NS surface never crosses the BH horizon.
- Horizon locking condition: finite spin \rightarrow Spinning magnetized BH????

No hair theorem not applicable: high plasma conductivity introduces topological constraint (frozen-in B-field).

Conserved number: magnetic flux through the surface:

$$N_B = \frac{eBR^2}{\pi\hbar c} = 4.8 \times 10^{30} B_{12} R_6^2$$



As long as BH can produce pairs, open B-field does not slide off.

Field structure relaxes to split monopole

Isolated BH acts as a pulsar, spins down electromagnetically, generates Poynting wind (jets?).

One malfunction (global reconnection at the equatorial current sheet) will break the engine forever.

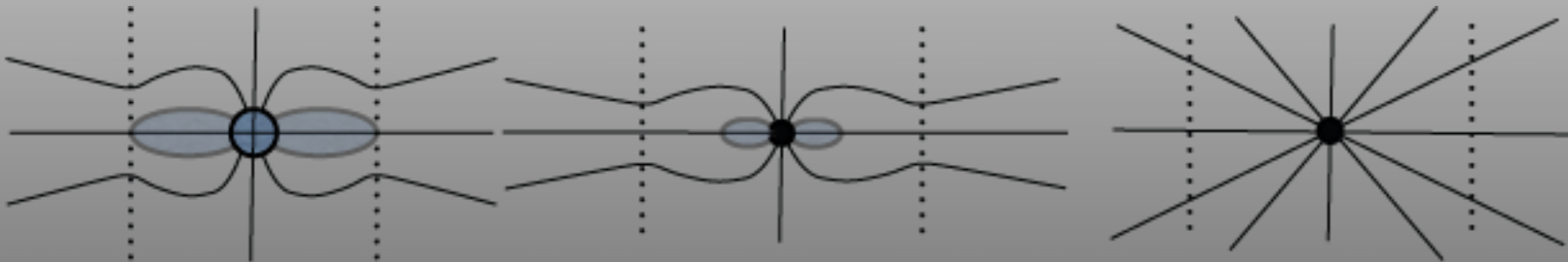
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BH's hair!

Simulations to be done: NS collapse into BH assuming conducting (e.g., force-free) outside plasma. B-field will remain attached (even non-rotating, like Baumgarte & Shapiro)

As long as BH can produce pairs, open B-field does not slide off.

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Application to GRBs

Shorts and Longs are very similar, even though the progenitors are very different.

Late times ($t > 10^5$ sec)- FS dominated -OK

But prompt and early afterglows? (Plateaus, flares)

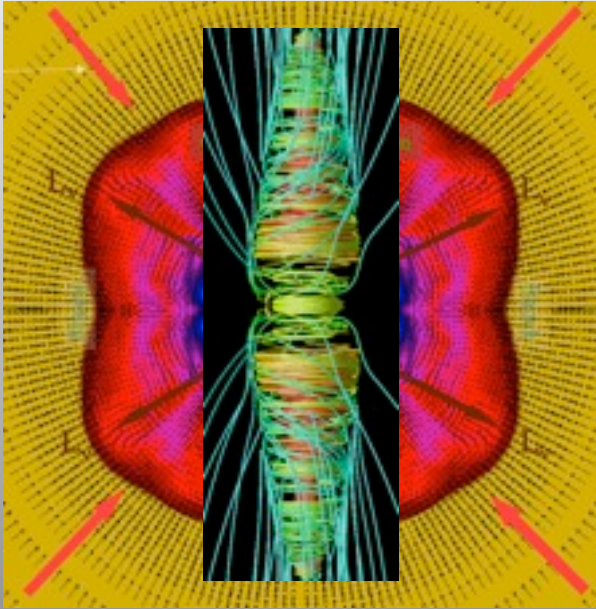
Formation of magnetized BH that retains its B-field for a long time and spins-down electromagnetically

Millisecond magnetar (but: monopolar spindown is more efficient than dipolar). Need dynamo to bring $B \sim 10^{14}$ G.

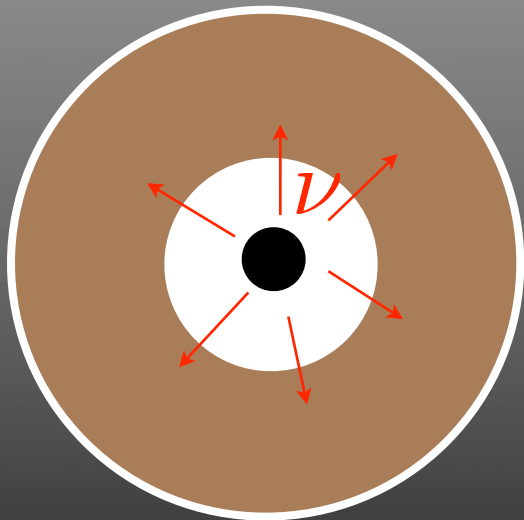
Early afterglows from internal dissipation in the wind (Lyutikov 2009)

***N+1. Double explosions in GRBs:
jet formation***

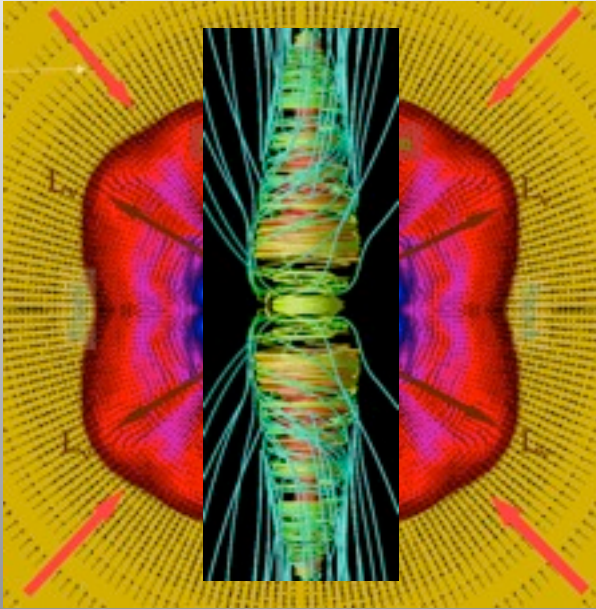
Double explosions in Long GRBs



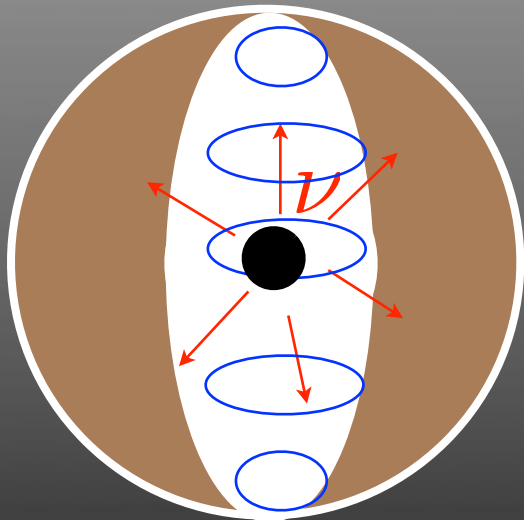
- In collapsars one may image nearly equal contributions from ν and B-field, each not sufficiently powerful, but when combined, jet make explosion along the axis, not along equator - **failed SN**, but successful GRB.
- Jet just needs to make a hole to escape.



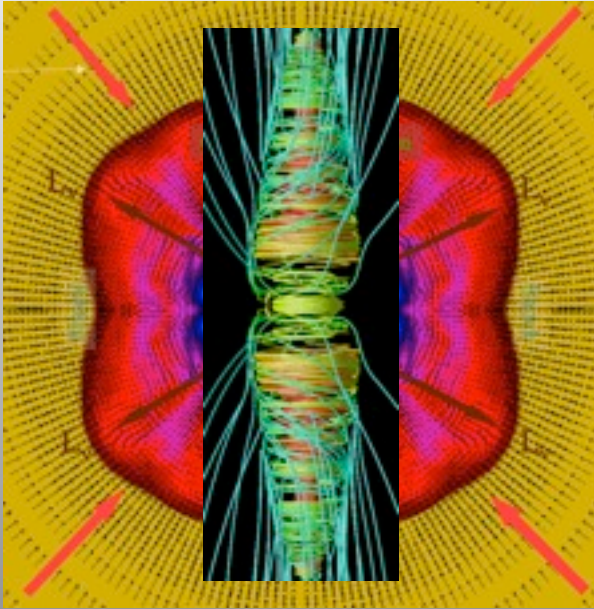
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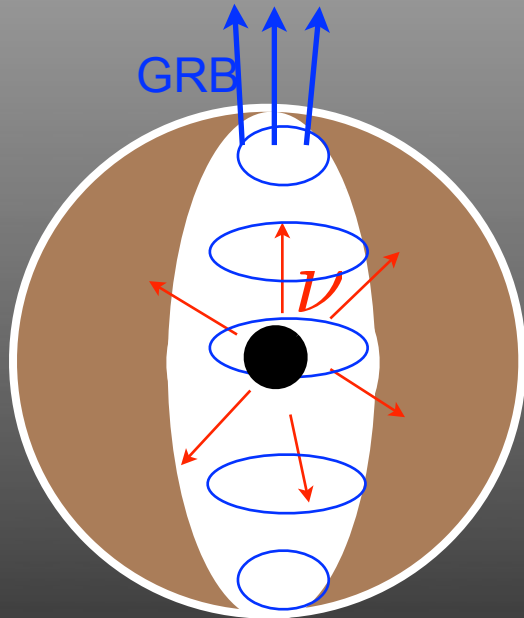
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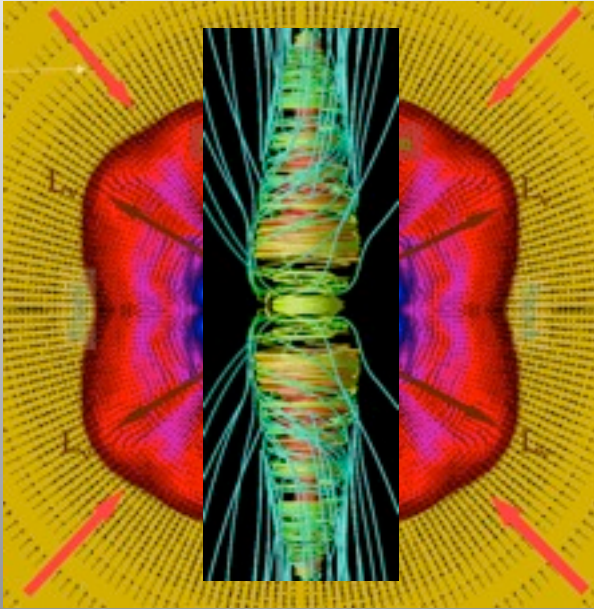
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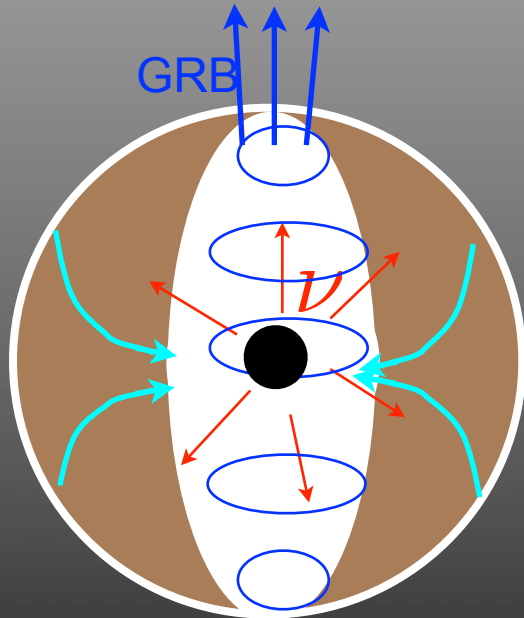
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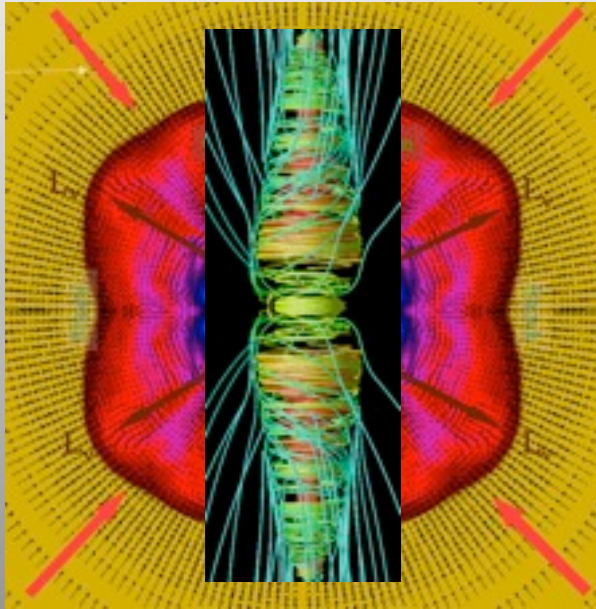
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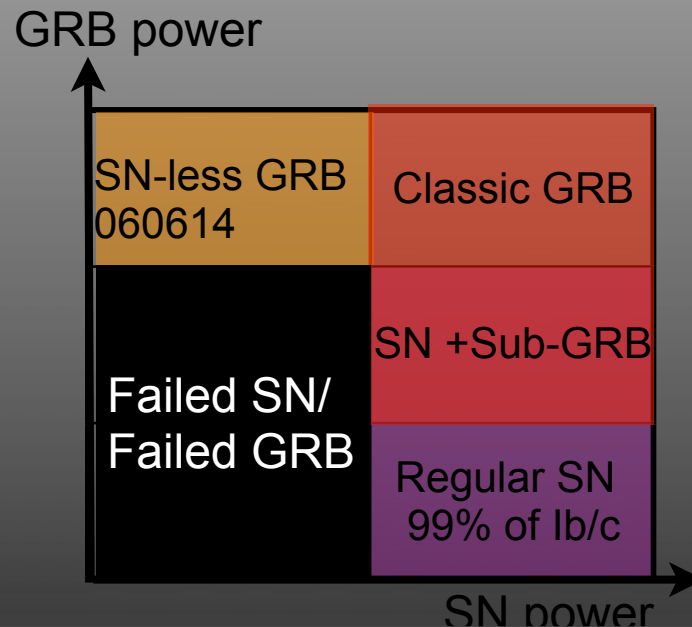
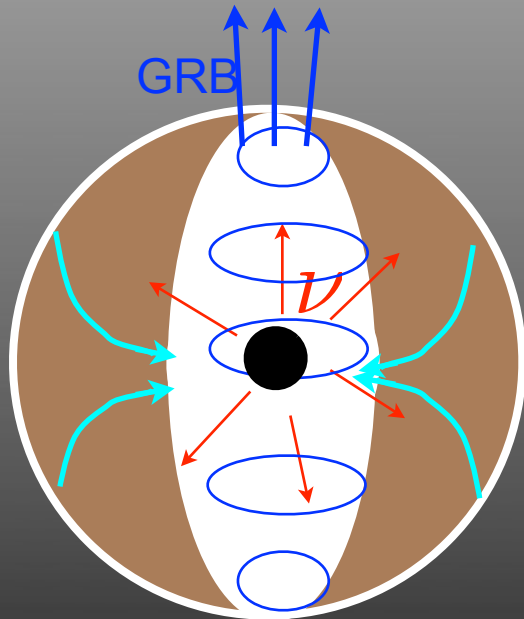
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Double explosions in Long GRBs



- In collapsars one may image nearly equal contributions from ν and B-field, each not sufficiently powerful, but when combined, jet make explosion along the axis, not along equator - **failed SN**, but successful GRB.
- Jet just needs to make a hole to escape.

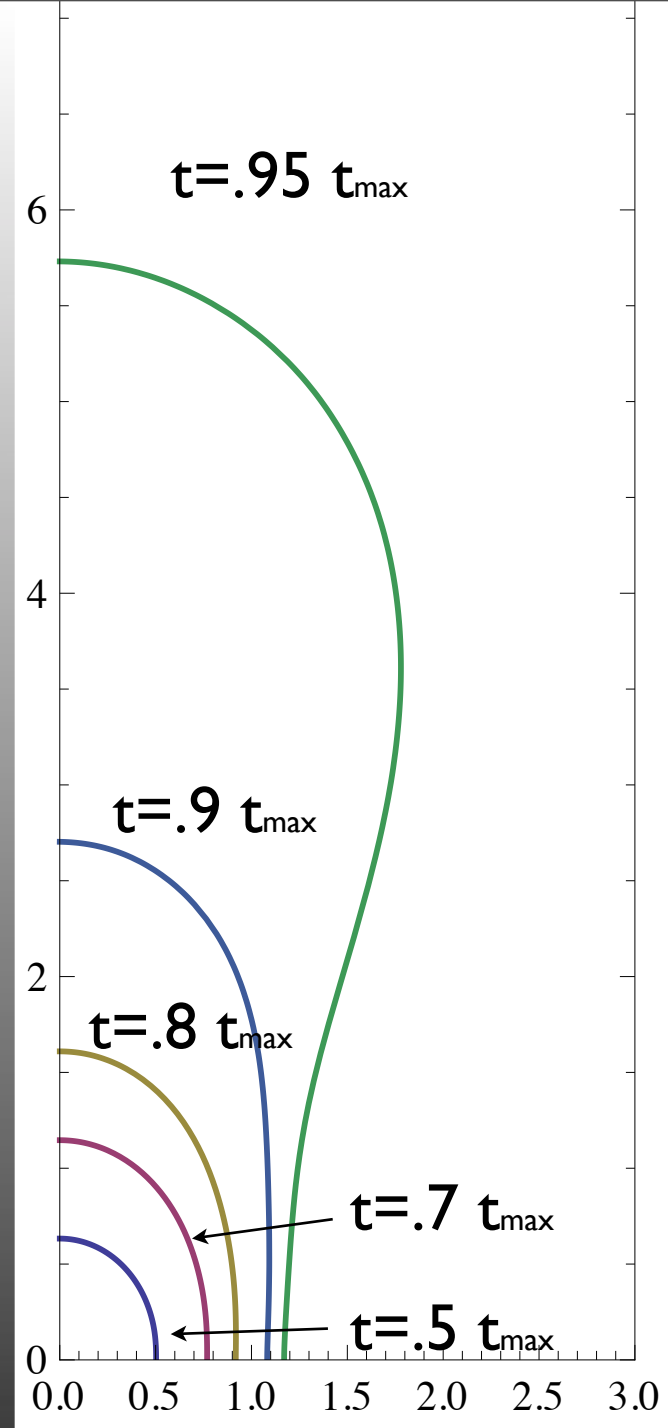


Double explosions in GRBs: jet formation

- Nu-explosion launched the envelope, created steep density profile.
- GRB-engine is **weakly anisotropic**, creates a second shock, which propagates in steep density gradient: accelerating, RT unstable
- “Chimney” is formed, for

$$\rho \propto r^{-m}, m > 4$$

Second nearly-spherical explosion in steep density gradient can create a collimated jet.



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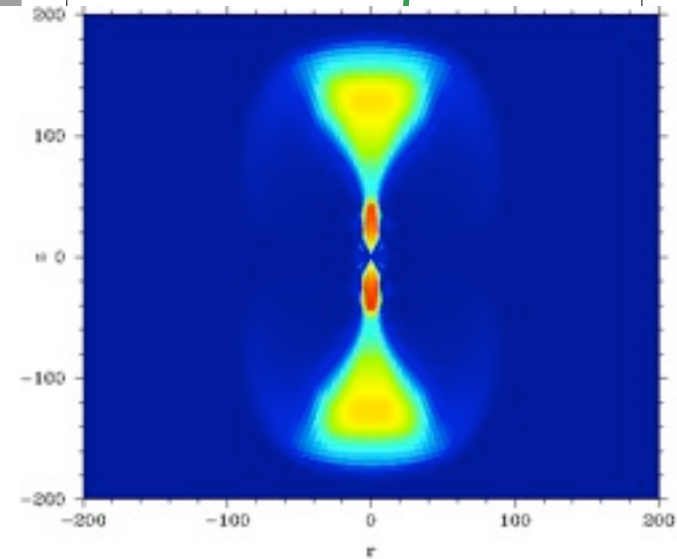
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$t = .95 t_{\max}$

6

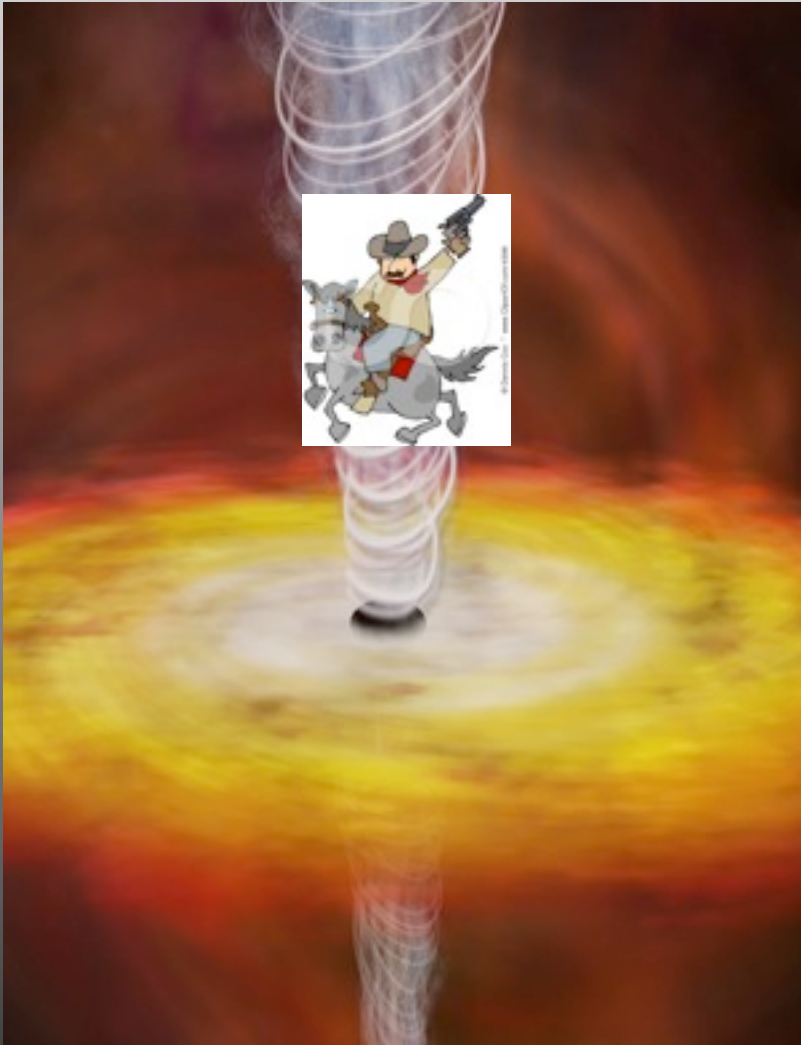
4

Komissarov & Lyutikov, in prep

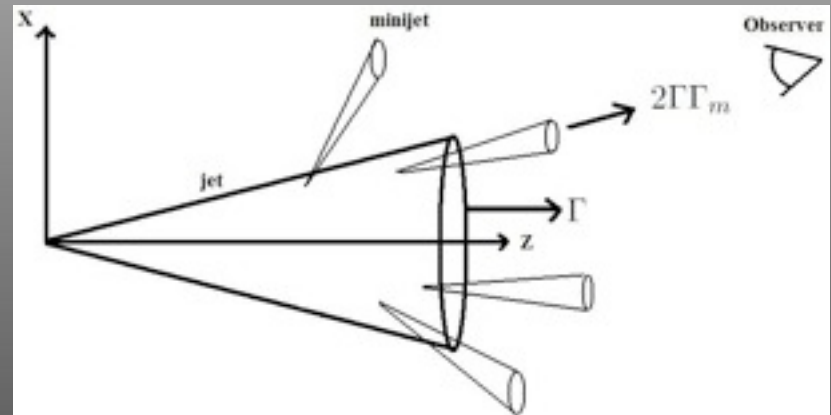


$m=7$

N+2. Mini-jets (drunk cowboy)



- Emission beamed in jet frame (Blandford & Lyutikov 2003, Lyutikov 2006, Ghisellini et al. 2008, Lazar et al. 2009, Giannios et al. 2009, Narayan & Kumar 2009)



Fast variability from large radii,

$$R_{em} \sim 10^{15} - 10^{16} \text{ cm}$$

(Lyutikov & Blandford 2003)



➤ Emission is beamed in outflow frame

– really beamed $\Delta\theta_{em} \ll 1$

– random internal motion of emitters, $\Delta\theta_{em} \sim 1/\gamma_{rand}$

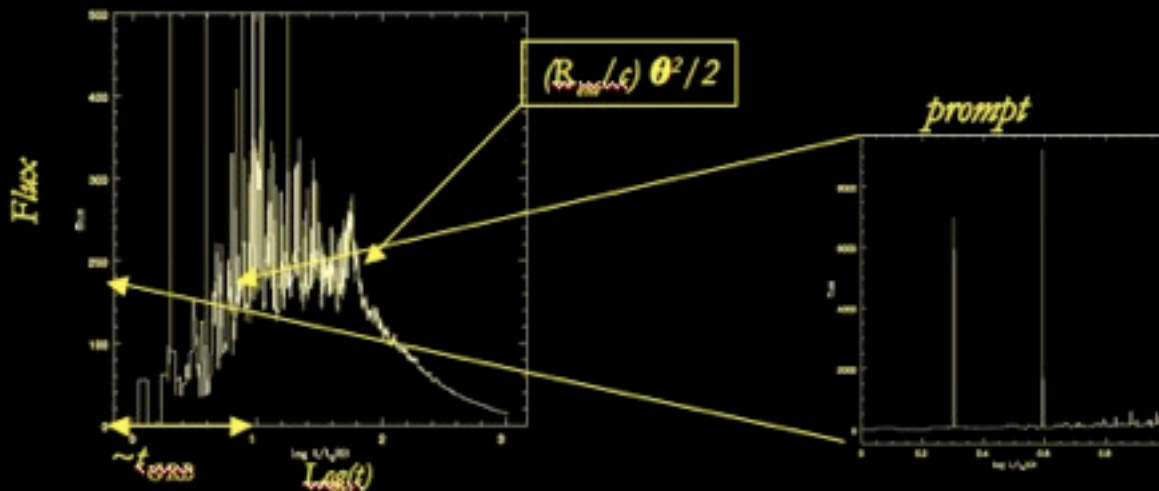
➤ X-flares and breaks are tails of prompt

➤ fast variability

➤ no need for long central engine activity

➤ softening with time, harder spikes

➤ These are preliminary results: alternatives need to be investigated



$$\begin{aligned} \Gamma_{bulk} &= 50, \gamma_{rand} = 5 \\ \theta_{ob} &= \pi/10, \\ \theta_{jet} &= 3\theta_{ob} \\ \text{efficiency} &= 10\% \end{aligned}$$

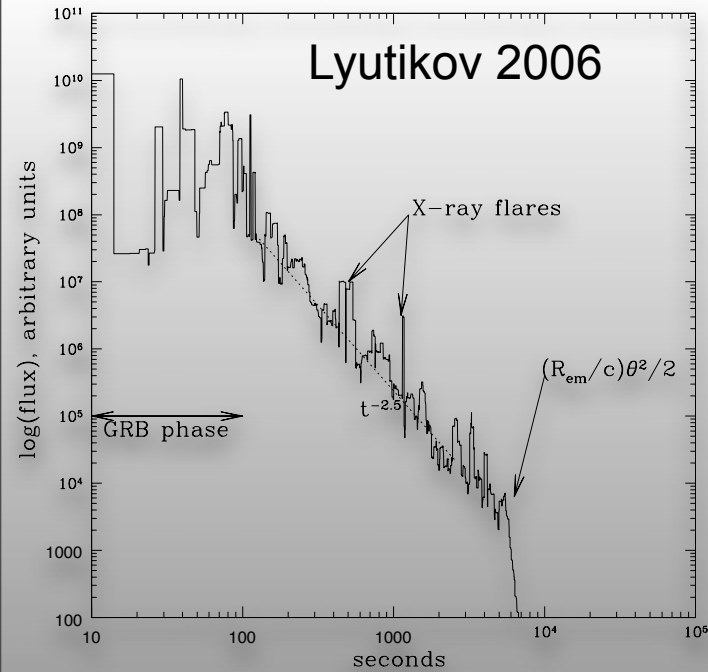
(Lyutikov in prog.)

Also: Ghisellini et al. 2008, Lazar et al. 2009, Giannios et al. 2009, Narayan & Kumar 2009

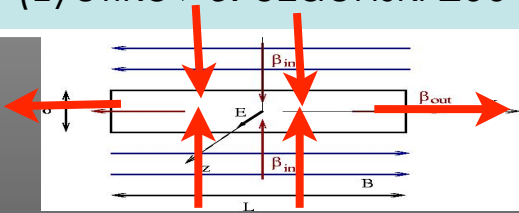
$$\Gamma_{eff} = 2\Gamma\gamma_{rand}$$

$$\Delta t \sim \frac{c}{R} \frac{1}{8\Gamma^2\gamma_{rand}^2}$$

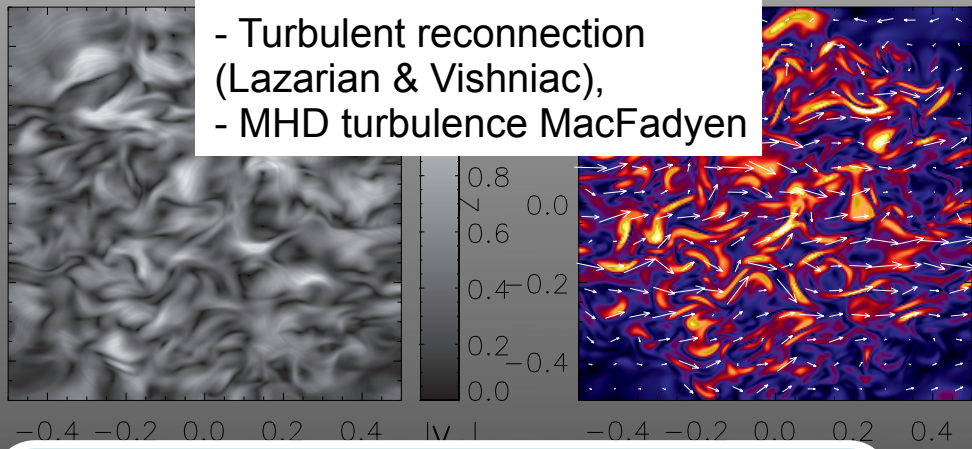
Observed emission can be highly variable and with high efficiency (tapping into most of the proper volume)



- Relativistic reconnection: jets with $\gamma_{out} \sim \sigma \gg 1$ (Lyutikov & Uzdenski 2004)



- Not fluid "turbulence", $\gamma_{rand} \sim \sqrt{9/8} = 1.06$
 - RM & RT instabilities will produce $vT \ll c$ turbulence



Flux may be dominated by rare "bull's eye" shots. Crab flares? (Clausen-Brown & Lyutikov, in prep.)