A New Numerical Scheme for Relativistic Dissipative Hydrodynamics and Resistive Magnetohydrodynamics, and Application to Astrophysics

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abstract

In recent years, various high energy astrophysical phenomena are extensively studied by using the relativistic fluid approximation. However, there are only limited descriptions of the dissipative effect in relativistic regime, such as thermal conduction, viscosity, and resistivity. This is because a simple relativistic extension of the Navier-Stokes equation and resistive magnetohydrodynamic equation include unphysical exponentially growing modes originated from the acausal character of parabolic equation. In this poster, I present a new algorithm and numerical code that can treat viscosity, thermal conduction, and resistivity accurately and causally. Our new scheme solves the above problems, and can calculate relativistic phenomena stably and rigorously.

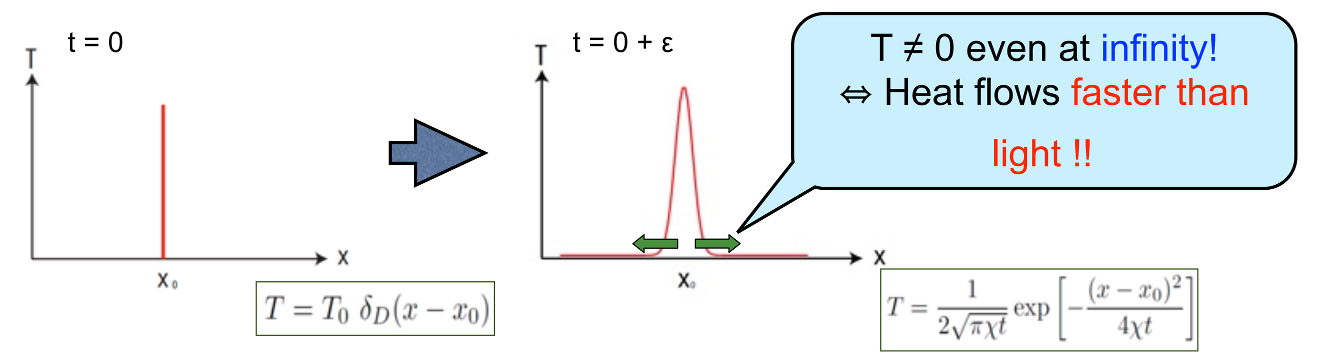
1. Introduction

・Acausality in dissipation theory

e.g.) energy equation (if relativistic extended heat flux is used)

$$nc_V \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} = \nabla \cdot (\kappa \nabla T)$$: parabolic partial differential equation

⇒ **characteristic velocity is infinite**



Perturbations **grow unphysically** in dissipative RHD because energy comes from acausal region unphysically!!

solution : consider the relaxation of dissipation !!

$$\begin{cases} \partial_t Q + \nabla \cdot \mathbf{F} = 0, \\ \partial_t \mathbf{F} = -\frac{1}{\tau}(\mathbf{F} + \eta \nabla Q). \end{cases} \Rightarrow \quad \partial_t^2 Q + \frac{1}{\tau}\partial_t Q - \frac{\eta}{\tau}\Delta Q = 0.$$

This is a telegrapher equation, and causal equation !

$$\begin{cases} \hat{D}\Pi = \frac{1}{\tau_\Pi}(\Pi_{NS} - \Pi) - I_\Pi, \\ \hat{D}\pi^{\mu\nu} = \frac{1}{\tau_\pi}(\pi^{\mu\nu}_{NS} - \pi^{\mu\nu}) - I^{\mu\nu}_\pi, \\ \hat{D}q^\mu = \frac{1}{\tau_q}(q^\mu_{NS} - q^\mu) - I^\mu_q, \end{cases}$$

Dissipative RHD

$$\begin{cases} \partial_t \mathbf{E} = \nabla \times \mathbf{B} - \mathbf{J}, \\ \mathbf{J} = \sigma\gamma[\mathbf{E} + \mathbf{v} \times \mathbf{B} - (\mathbf{E}\cdot\mathbf{v})\mathbf{v}] + q\mathbf{v} \end{cases}$$

Resistive RMHD

⇔ relaxaion timescale τ and $1/\sigma$ is very short compare to the characteristic timescale of hydrodynamics

⇒ equation is stiff and hard to solve numerically by using the ordinal explicit difference scheme !!

⇒ **Piecewise Exact Solution (PES) method**

ref) T. Inoue, & S. Inutsuka, ApJ 687 (2008) 303
S. S. Komissarov, MNRAS 382 (2007) 995

$$\partial_t Q = -\alpha Q + P(t)$$: stiff equation

The evolution of Q(t) is much faster than that of P(t)

⇒ P(t) can be assumed to be constant

⇒ We can obtain the formal solution !!

$$Q(t^{n+1}) = \frac{P(t^{n+1/2})}{\alpha} + \left[Q(t^n) - \frac{P(t^{n+1/2})}{\alpha}\right] e^{-\alpha \Delta t}$$

2. dissipative RHD(Israel-Stewart theory)

ref) M. T & Shu-ichiro inutsuka, (2011), submitting to Journal of Computational Physics

$$\frac{\partial}{\partial t}\begin{pmatrix} \rho u^0 \\ \rho h u^0 u^i + q^0 u^i + q^i u^0 + \tau^{0i} \\ \rho h (u^0)^2 - p + 2q^0 u^0 + \tau^{00} \end{pmatrix} + \frac{\partial}{\partial x^j}\begin{pmatrix} \rho u^j \\ \rho h u^i u^j + pI^{ij} + q^i u^j + q^j u^i + \tau^{ij} \\ \rho h u^0 u^j + q^0 u^j + q^j u^0 + \tau^{0j} \end{pmatrix} = 0$$

$$\tau^{\mu\nu} = (\tau_0^{\mu\nu} - \tau_{NS}^{\mu\nu}) \exp\left[-\frac{t - t_0}{\tau_\tau}\right] + \tau_{NS}^{\mu\nu}$$

$$q^\mu = (q_0^\mu - q_{NS}^\mu) \exp\left[-\frac{t - t_0}{\tau_q}\right] + q_{NS}^\mu$$

: Piecewise Exact Solutions

・Numerical Scheme

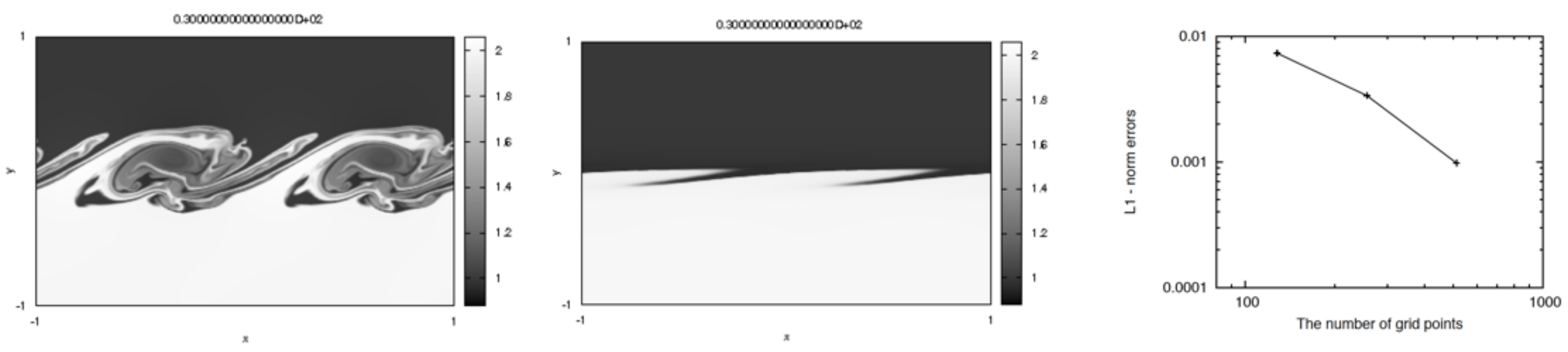
1. fluid equations = advection and diffusion equations

⇒ advection part + diffusion part

Solvable accurately by Riemann solver !!

2. relaxation equation ⇒ evolve by using PES method

・Results of test simulations (2D Kelvin-Helmholtz instability)



inviscid case — 1.5 fluid crossing time; η = 0.005; convergence rate

3. Resistive RMHD

ref) M. T & T. Inoue , The Astrophysical Journal, 734, 1, (2011).

$$\partial_t \begin{pmatrix} \rho\gamma \\ \rho h \gamma^2 \mathbf{v} + \mathbf{E} \times \mathbf{B} \\ \rho h \gamma^2 - p + \frac{1}{2}(E^2 + B^2) \end{pmatrix} + \partial_x \begin{pmatrix} \rho\gamma v^x \\ \rho h \gamma^2 v^i v^x + p\eta^{ix} - E^i E^x - B^i B^x + \left[\frac{1}{2}(E^2 + B^2)\right]\eta^{ix} \\ \rho h \gamma^2 v^x + (\mathbf{E} \times \mathbf{B})^x \end{pmatrix} = 0$$

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = \mathbf{0}, \qquad \mathbf{E}_\parallel = \mathbf{E}_\parallel^0 \exp\left[-\frac{\sigma}{\gamma}t\right],$$

$$\partial_t \mathbf{E} - \nabla \times \mathbf{B} = -q\mathbf{v}, \qquad \mathbf{E}_\perp = \mathbf{E}_\perp^* + (\mathbf{E}_\perp^0 - \mathbf{E}_\perp^*) \exp[-\sigma\gamma t],$$

・Numerical Scheme

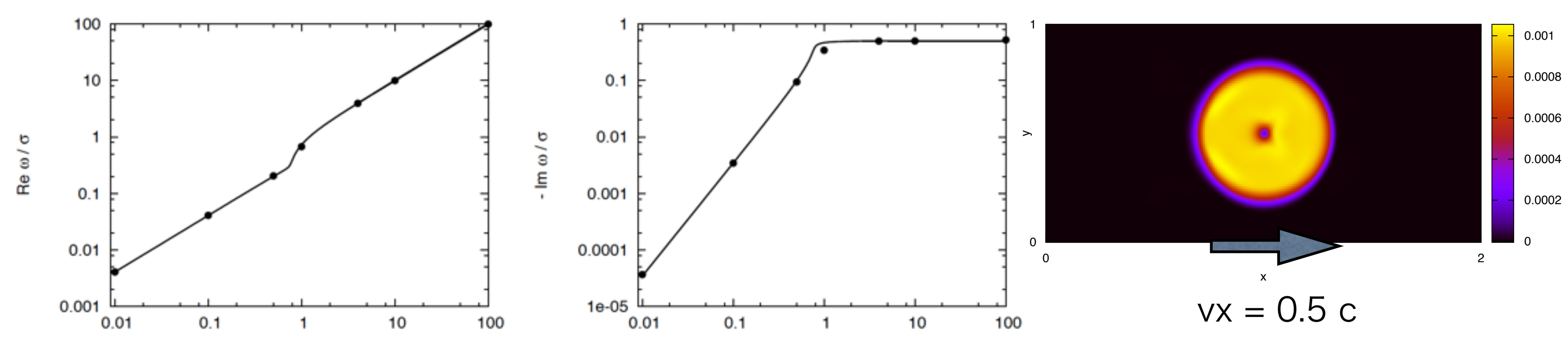
1. Electromagnetohydrodynamics equations

⇒ fluid part + electromagnetic part

・fluid part = Riemann solver
・electromagnetic part = method of characteristics
+ Constraint transport (CT)

2. stiff equation of E ⇒ evolve by using PES method

・Results of test simulations



The dispersion relation of the propagation of Alfven wave with resistivity

magnetic loop advection after 1 fluid crossing time

Summery:
・We have developed new numerical scheme for **dissipative RHD** and **resistive RMHD**
・If one considers the relativistic dissipation, one has to deal with stiff equation, and it is hard to solve numerically.
We have solved this problem by using Piecewise Exact Solution.
・Our method based on the splitting the hydrodynamic equation into advection part, EM part, and dissipation part, and calculate advection and EM by using Riemann solver and MOC.
⇒ We can calculate accurately problems with various characteristic velocity.

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