A Filamentation Instability for Shock Accelerated Particles

B. Reville and A. R. Bell

UNIVERSITY OF OXFORD

Energetic particles drifting with respect to the local plasma generate large scale magnetic fields, leading to a self-focusing effect. This results in an inhomogeneous distribution of particles on large length scales. We present hybrid magnetohydrodynamic-particle in cell (MHD-PIC) simulations confirming this effect. This process may play a significant role in the growth of magnetic field in supernova remnants, but can be applied to other plasmas in which particles drift with respect to the background fluid.

A filamentary solution for cosmic-ray streaming

Within the diffusion approximation, the non-thermal particle distribution upstream of a non-relativistic shock front, as measured in the frame in which the fluid is at rest, is

$$F(x,\boldsymbol{p}) = f_0(x,p)\left(1+3\frac{u_{\rm sh}}{c}\cos\theta\right)$$

where $f_0(x,p)$ is the isotropic part of the spectrum. On average, the distribution is isotropic in the rest frame of the shock. In this frame, since $\boldsymbol{v}\times\partial f_0/\partial\boldsymbol{p}=0$, the magnetic field is unimportant in determining the particle distribution. Choosing the upstream in the half plane $x>0$, the electric field is $\boldsymbol{E}=-\boldsymbol{u}\times\boldsymbol{B}=u_{\rm sh}\boldsymbol{\nabla}A_\parallel$, where $A_\parallel$ is the component of the vector potential ($\boldsymbol{B}=\boldsymbol{\nabla}\times\boldsymbol{A}$) in the direction normal to the shock surface.
The Vlasov equation can thus be expressed as

$$\frac{\partial f}{\partial t}+c\frac{\boldsymbol{p}}{p}\cdot\boldsymbol{\nabla}f+e\boldsymbol{\nabla}\left(u_{\rm sh}A_\parallel\right)\cdot\frac{\partial f}{\partial\boldsymbol{p}}=0$$

which has an equilibrium solution

$$f=f(p-eu_{\rm sh}A_\parallel/c)\ .$$

Assuming $eu_{\rm sh}A_\parallel \ll pc$, the number density correlates with $A_\parallel$

$$n(y,z)=n_0+\frac{eu_{\rm sh}A_\parallel}{c}\int 8\pi p f_0 {\rm d}p$$

Transforming back to the upstream restframe, the cosmic-ray current is

$$j_{\rm cr}(y,z)=j_0+\chi(A_\parallel-\langle A_\parallel\rangle)\ \text{where}\ \chi=\frac{e^2n_0u_{\rm sh}^2}{p_{\rm min}c}$$

This correlation is confirmed using hybrid MHD-particle simulations.

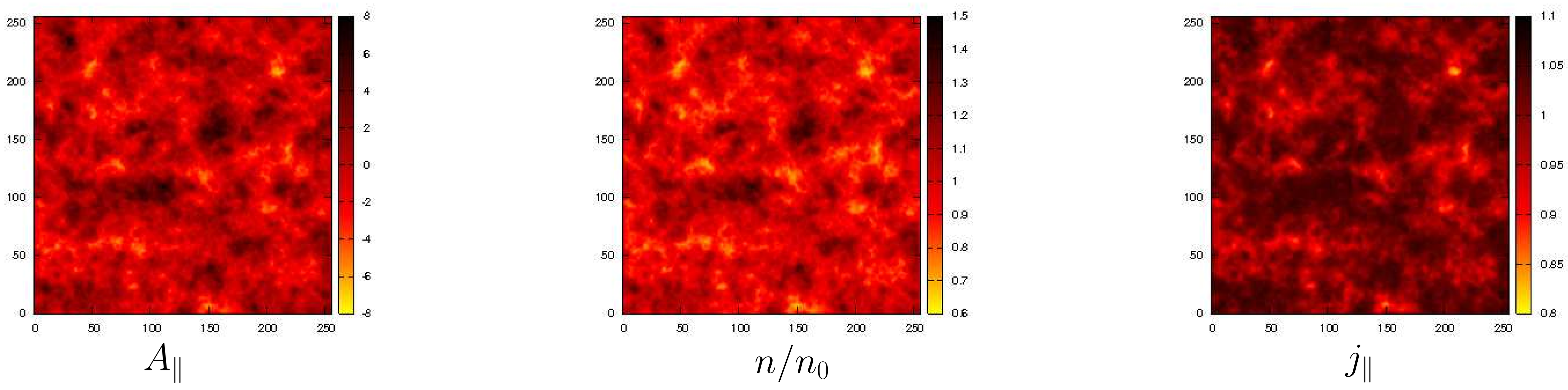

FIGURE 1: Results from hybrid MHD-particle simulations demonstrating the correlation between $A_\parallel$ and n and $j_\parallel$. The correlation with $j_\parallel$ is not as clear as that with n, due to the associated difficulty in capturing small (second order) departures from isotropy with a finite number of particles.

Magnetic field growth

The currents resulting from the streaming particles are well-known to lead to the growth of hydromagnetic instabilities. The non-resonant mode described in Bell (2005) is expected to have the largest growth rate

$$\gamma_{\rm NR}=\sqrt{\frac{j_\parallel B_\perp}{r_0\rho}}$$

The filamentation of the streaming particles can lead to an enhancement of the growth of small scale fields, but also influences the longwavelength magnetic field growth. This can be seen from the MHD equations, when coupled with the correlation described above. The dispersion relation reads

$$\frac{\partial^2 j_{\rm cr}}{\partial t^2}=\frac{\chi B_\perp^2}{\rho}j_{\rm cr}+\left(\left[(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right]\cdot\boldsymbol{\nabla}\right)j_{\rm cr}-(\boldsymbol{u}\cdot\boldsymbol{\nabla})\frac{\partial j_{\rm cr}}{\partial t}$$

where the linear growth on long wavelengths is

$$\gamma_{\rm Fil}=\sqrt{\frac{\chi B_\perp^2}{\rho}}$$

Since the growth is scale independent it will dominate over the non-resonant mode on scales

$$r_0>\frac{c}{u_{\rm sh}}\frac{p_0}{e|B_\perp|}$$

Hence, if the perpendicular field is amplified to large values ($B_\perp^2>B_0^2$) the filamentation instability will dominate the growth of the large scale field.
We attempt to verify this result using hybrid MHD-particle simulations.

Numerical simulations

The non-linear evolution of the system is investigated using 2D MHD-PIC simulations. Particles are initialised such that they are approximately gyrotropic with a net drift normal to the plane of the simulation. The field is initialised as $\boldsymbol{B}=B_0\hat{\boldsymbol{x}}+\boldsymbol{\nabla}_\perp\times A_\parallel(y,z)$ where $A_\parallel$ is generated from a sum of discrete Fourier modes with a user-defined power-spectrum.
The energetic particle current expands/contracts loops of magnetic field, depending on the handedness of the loop. This can be seen in the Fourier analysis of the perpendicular magnetic field shown in Figure. 1. For the parameters used, the filamentation instability is expected to dominate at wavelengths $2\pi/k>L/8$. After an initial inertial phase, the low k modes grow exponentially, with growth rate $\gamma\approx0.25\gamma_{\rm Fil}$.
Due to large scale nature of the simulations, the magnetic tension cannot compete with the $\boldsymbol{j}\times\boldsymbol{B}_\perp$ force, and ultimately results in vacuum solutions on the small scales.

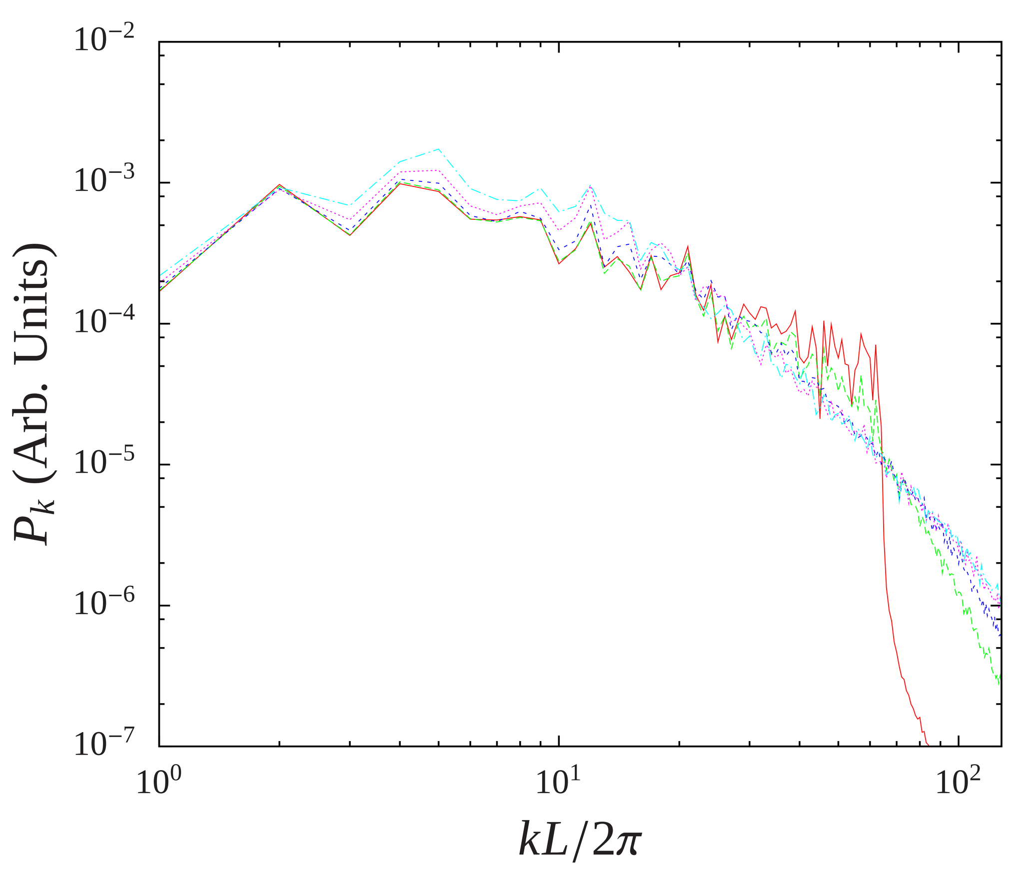

FIGURE 2: Evolution of the power spectrum for $B_\perp^2$ binned in intervals of $\Delta k=1$ (at $t=0,2,4,6,8$ in numerical units). The power in low k modes grows exponentially, with growth rate $\gamma\sim0.25\gamma_{\rm Fil}$.

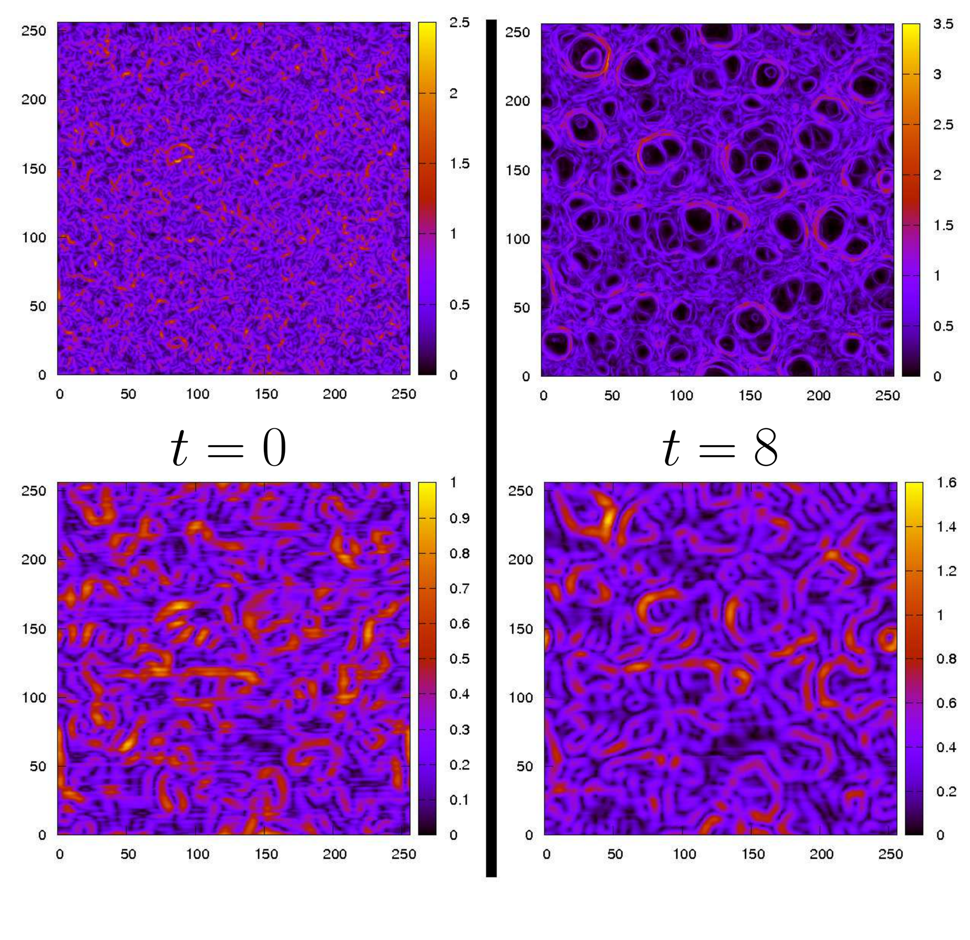

FIGURE 3: Plot of $|B_\perp|$ at beginning (left) and end (right) of simulation. The top plots show the output of the simulation, while the bottom plots show the Fourier filtered fields, with all modes $k/2\pi>L/16$ removed.

Conclusions

The self generation of magnetic fields at shock waves is essential for the first order Fermi process to occur. While numerical simulations have established the growth of magnetic fields on small lengthscales (Bell, 2004), this is not sufficient to explain the acceleration of cosmic-rays beyond the Lagage-Cesarsky limit (e.g. Reville et al., 2008). We have identified an instability which couples the growth on small scales to longer wavelength structures. The analysis presented here is limited to two-dimensions, but the effect should persist in three dimensions provided the fields are self-generated.

References

Bell A. R., 2004, MNRAS, 353, 550

Bell A. R., 2005, MNRAS, 358, 181

Reville B., O'Sullivan S., Duffy P., Kirk J. G., 2008, MNRAS, 386, 509

The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement no. 247039.

Oxford Physics

erc