Gyrokinetic Turbulence

arXiv:0704.0044; 0806.1069

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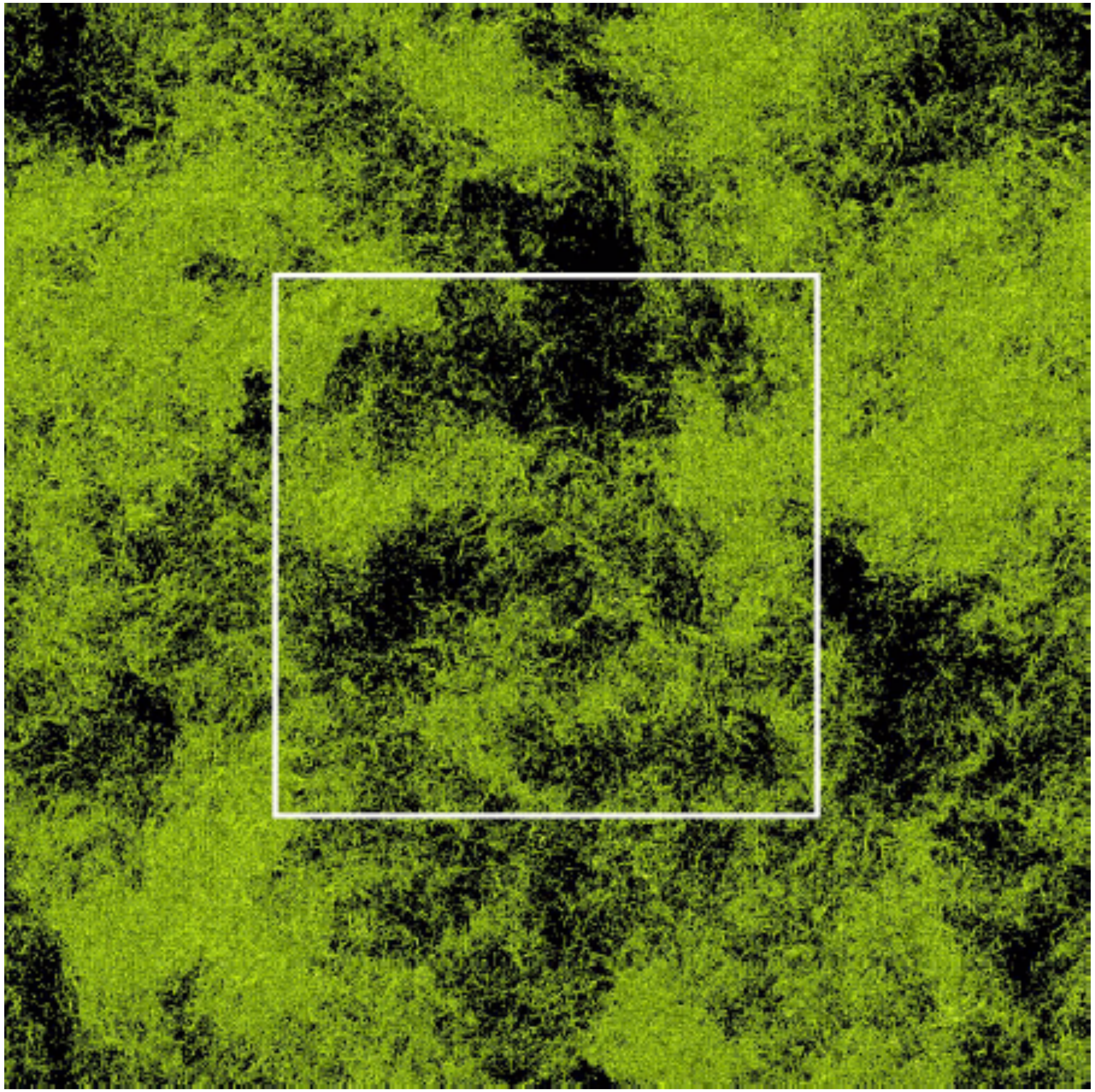
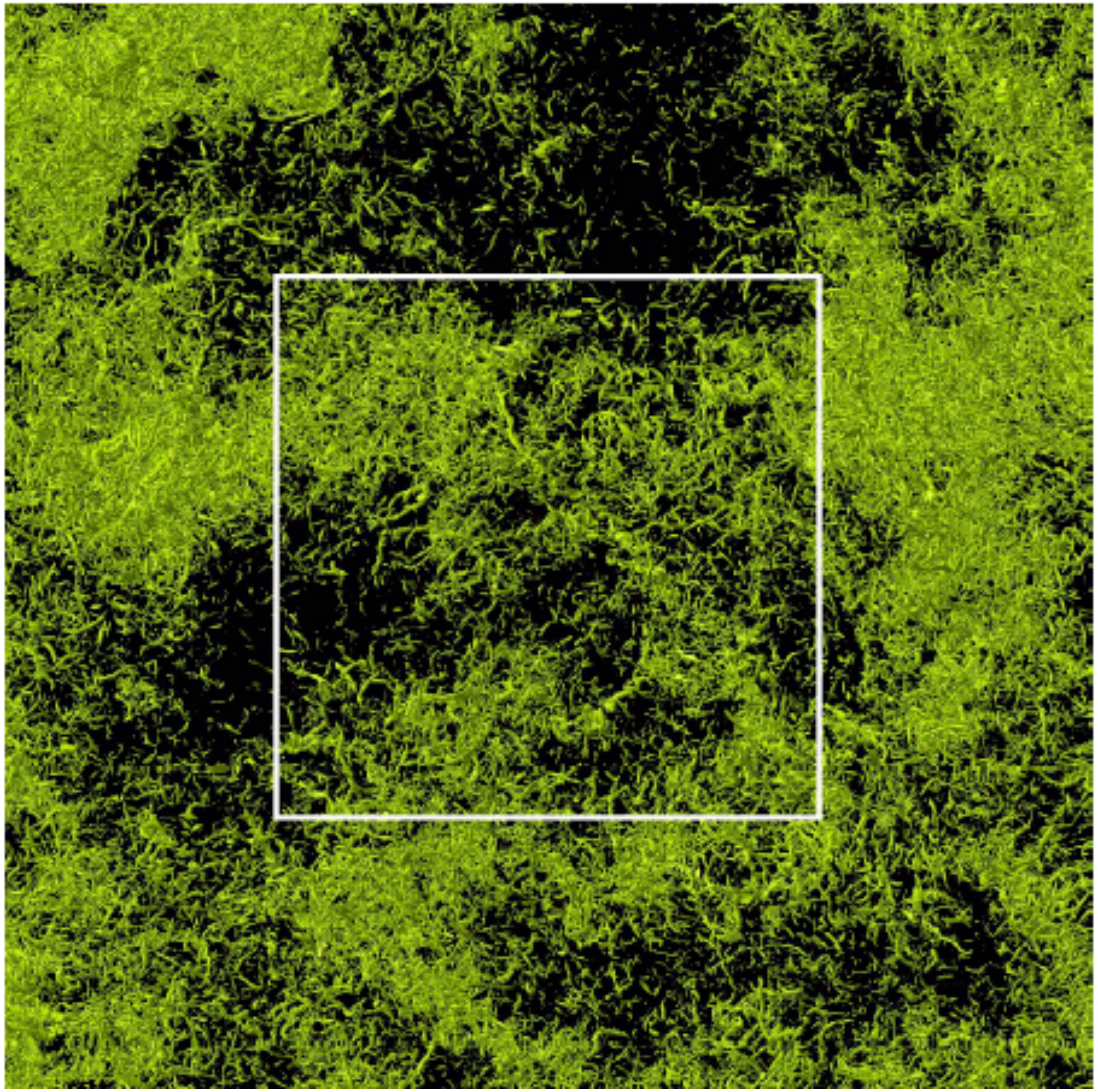
Turbulence is Multiscale Disorder



[Image: Y. Kaneda *et al*., **Earth Simulator, isovorticity surfaces, 4096^3**]

Turbulence is Multiscale Disorder



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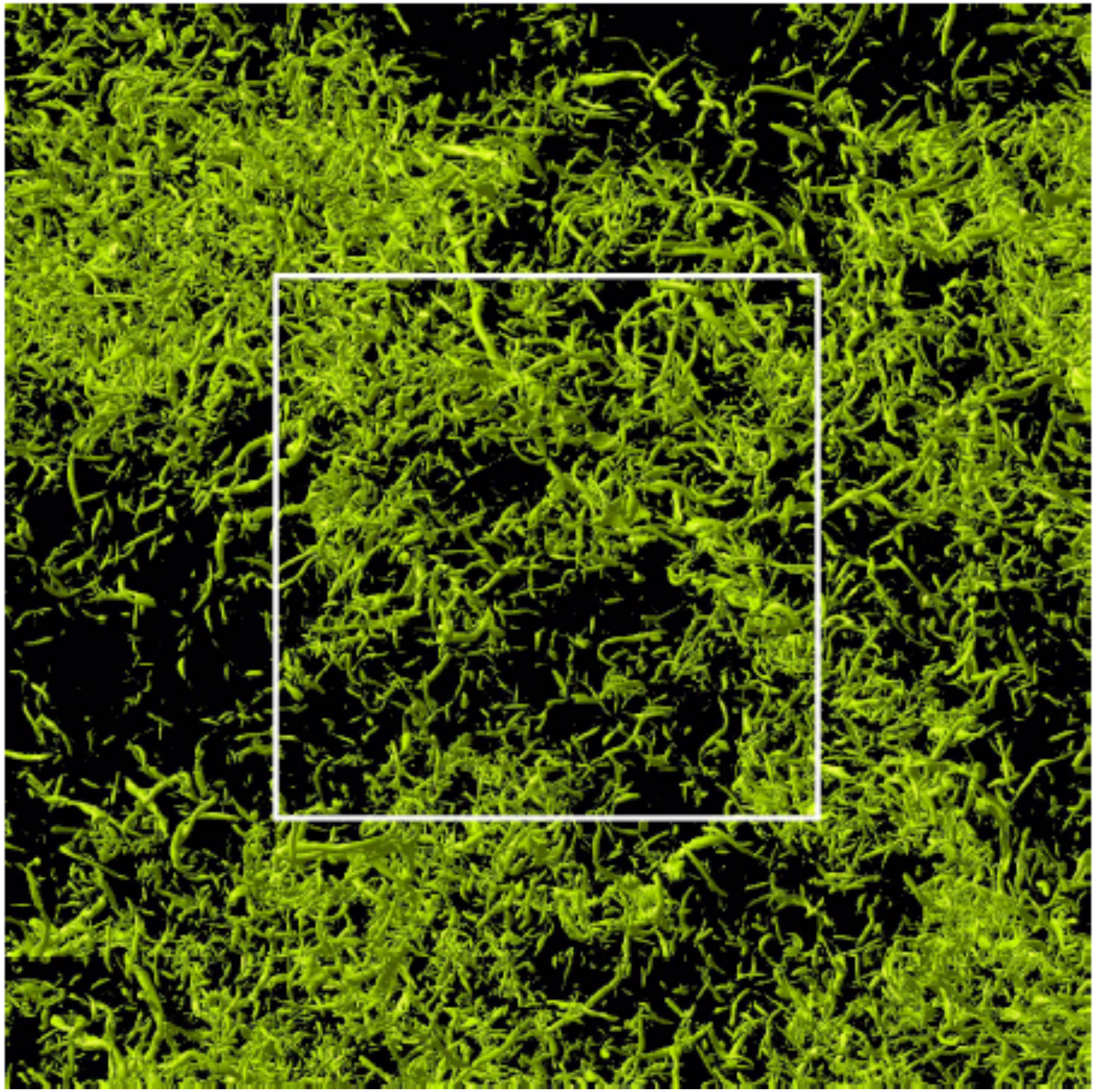

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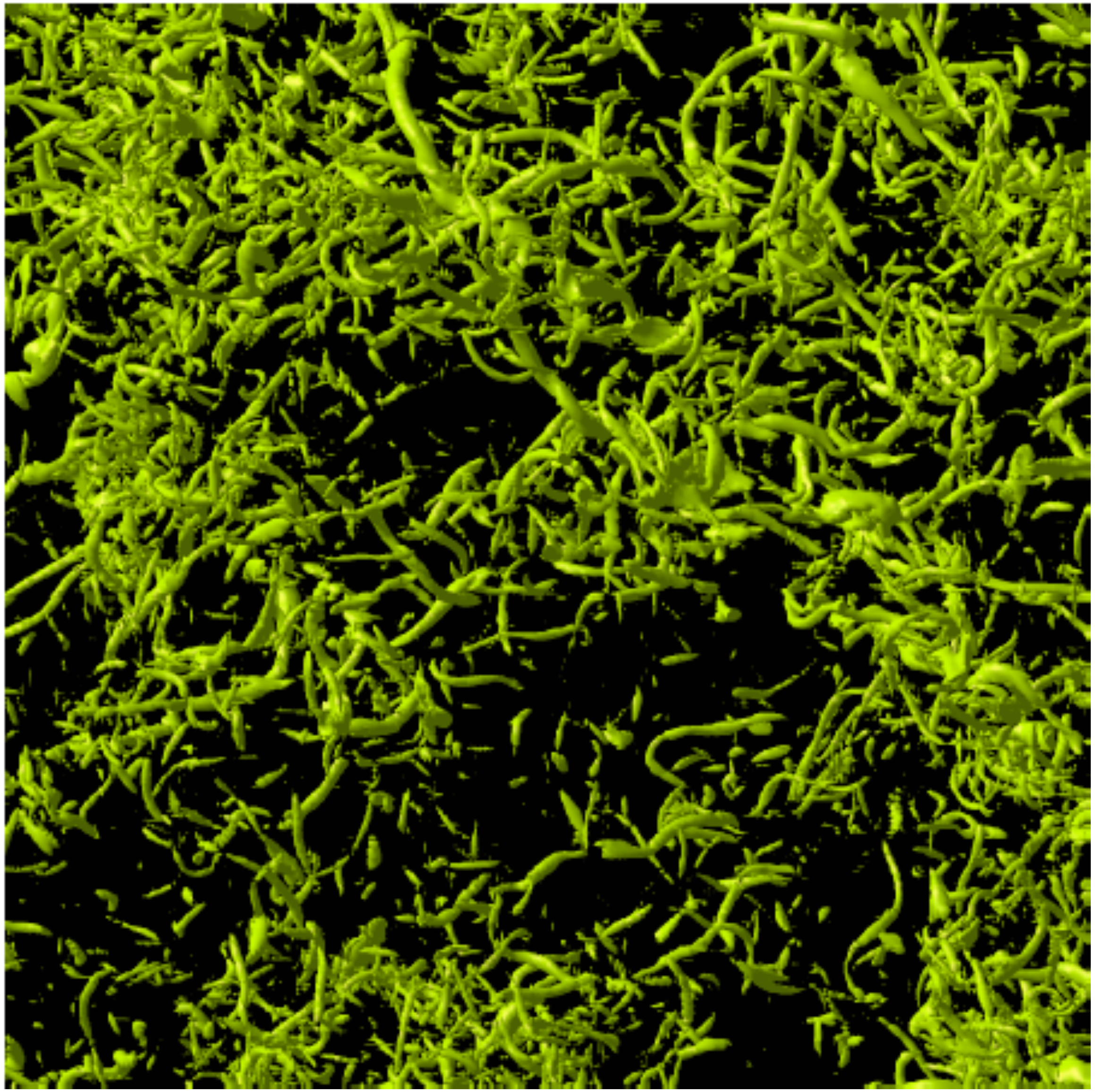

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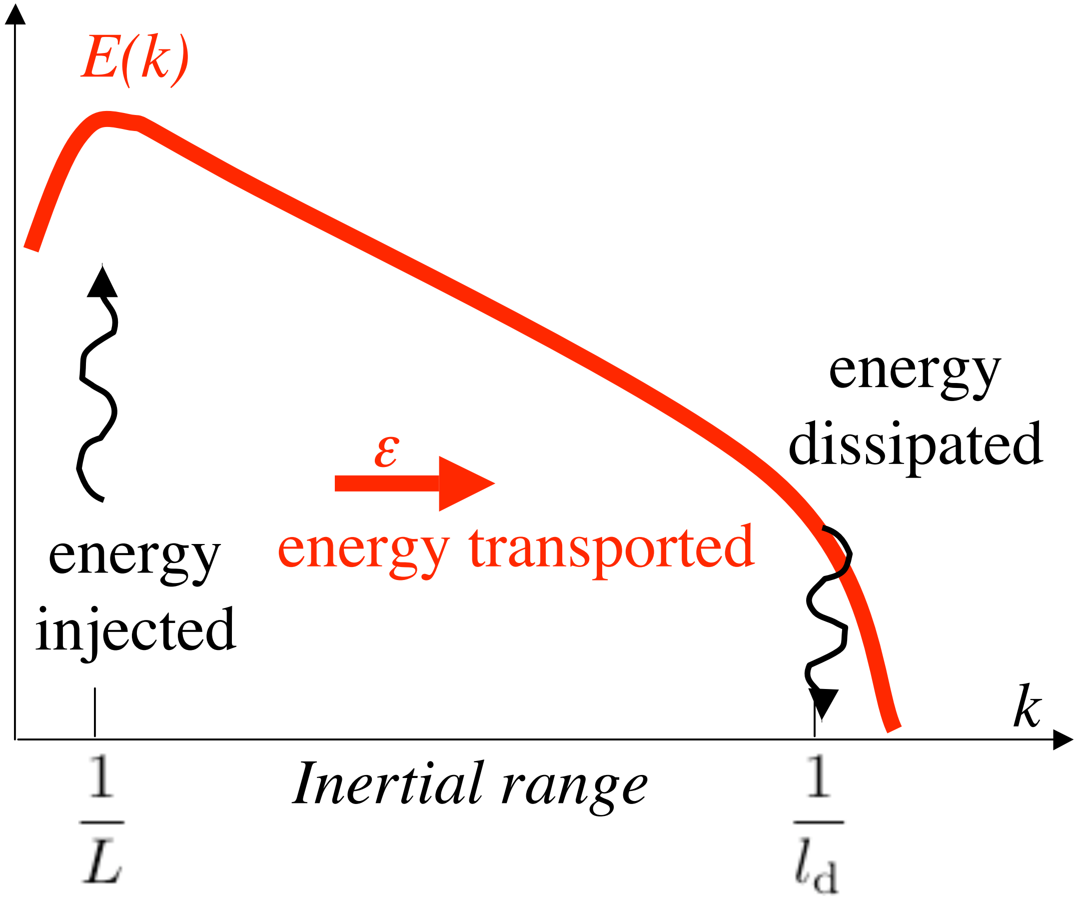
Turbulence: A Nonlinear Route to Dissipation

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \quad \boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \frac{\mathrm{d}^3\boldsymbol{r}}{V}\frac{u^2}{2} = \varepsilon - \nu \int \frac{\mathrm{d}^3\boldsymbol{r}}{V}|\boldsymbol{\nabla}\boldsymbol{u}|^2$$

$$l_\nu \sim (\nu^3/\varepsilon)^{1/4} \sim L\mathrm{Re}^{-3/4}$$

$$\varepsilon = (1/V)\int \mathrm{d}^3\boldsymbol{r}\,\boldsymbol{u}\cdot\boldsymbol{f}$$



Turbulence: A Nonlinear Route to Dissipation

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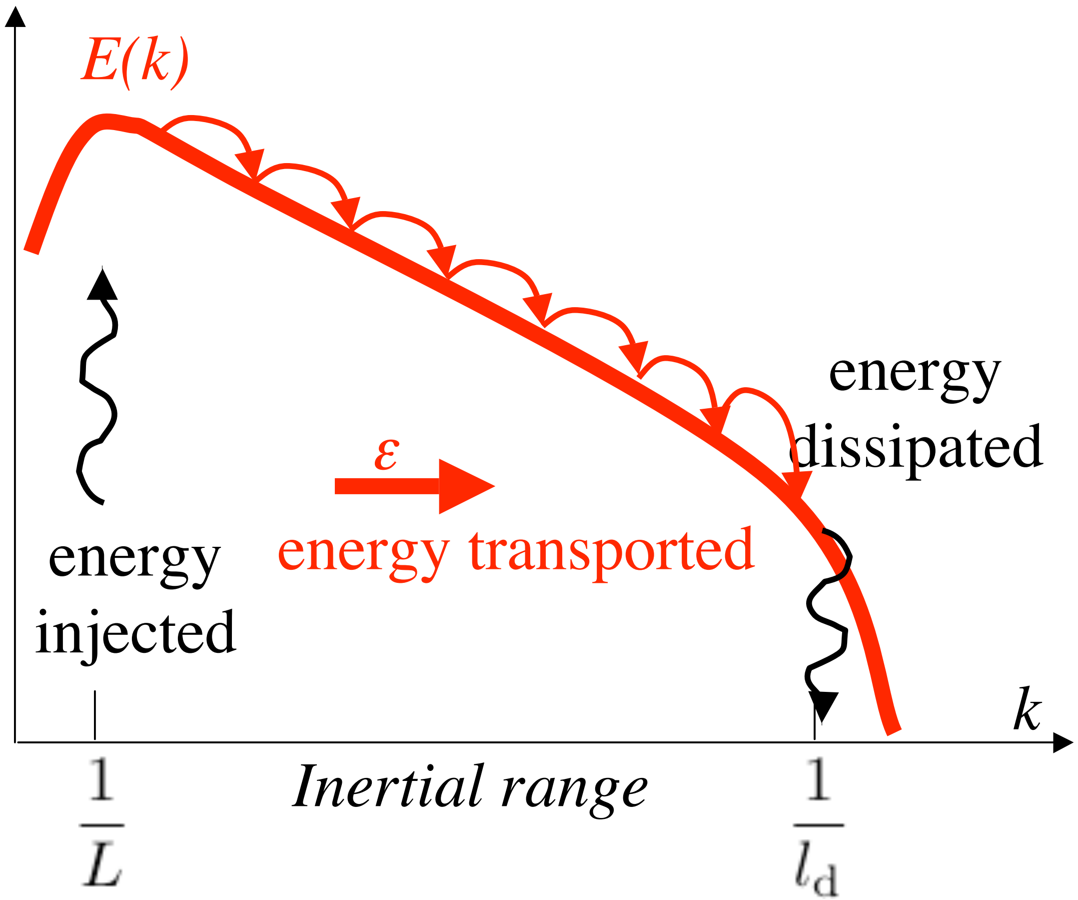
If cascade is local, intermediate scales fill up

Big whorls have little whorls
That feed on their velocity,
And little whorls have lesser whorls
And so on to viscosity.
L. F. Richardson 1922



Turbulence: A Nonlinear Route to Dissipation

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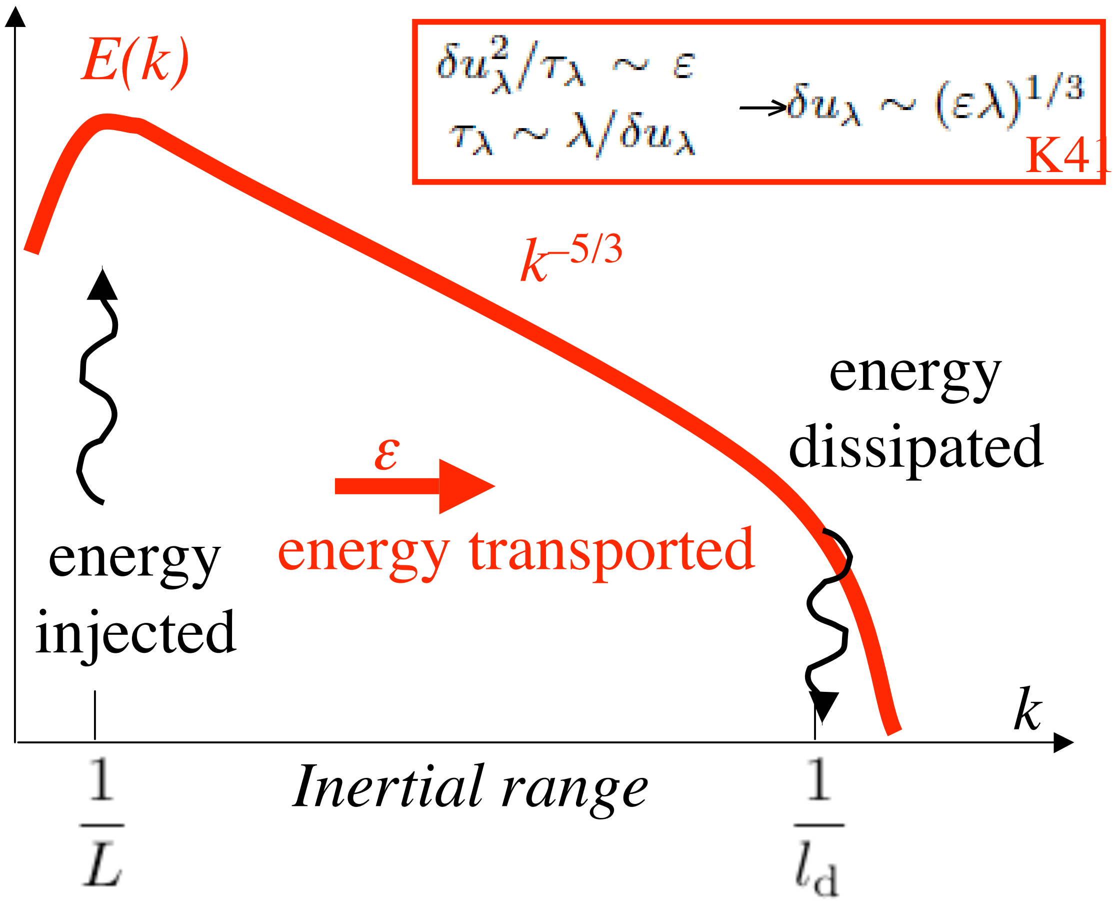
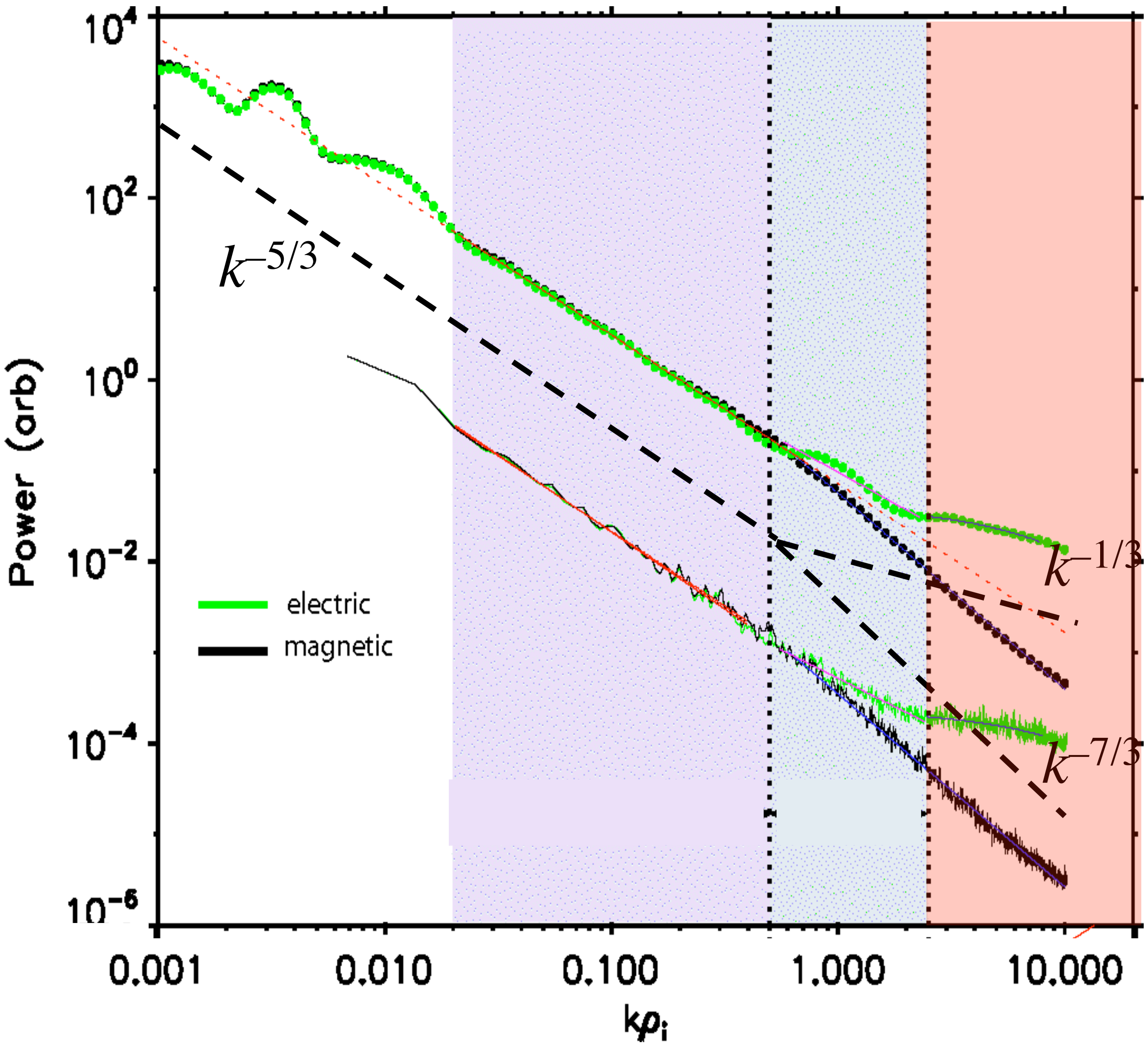


Plasma Turbulence: Analogous?

Turbulence in the solar wind

[Bale et al. 2005, *PRL* **94**, 215002]



Plasma Turbulence Extends to Collisionless Scales

$\lambda_{mfp} \sim 10^8$ km (~1 AU)
$\rho_i \sim 10^2$ km

Turbulence in the solar wind

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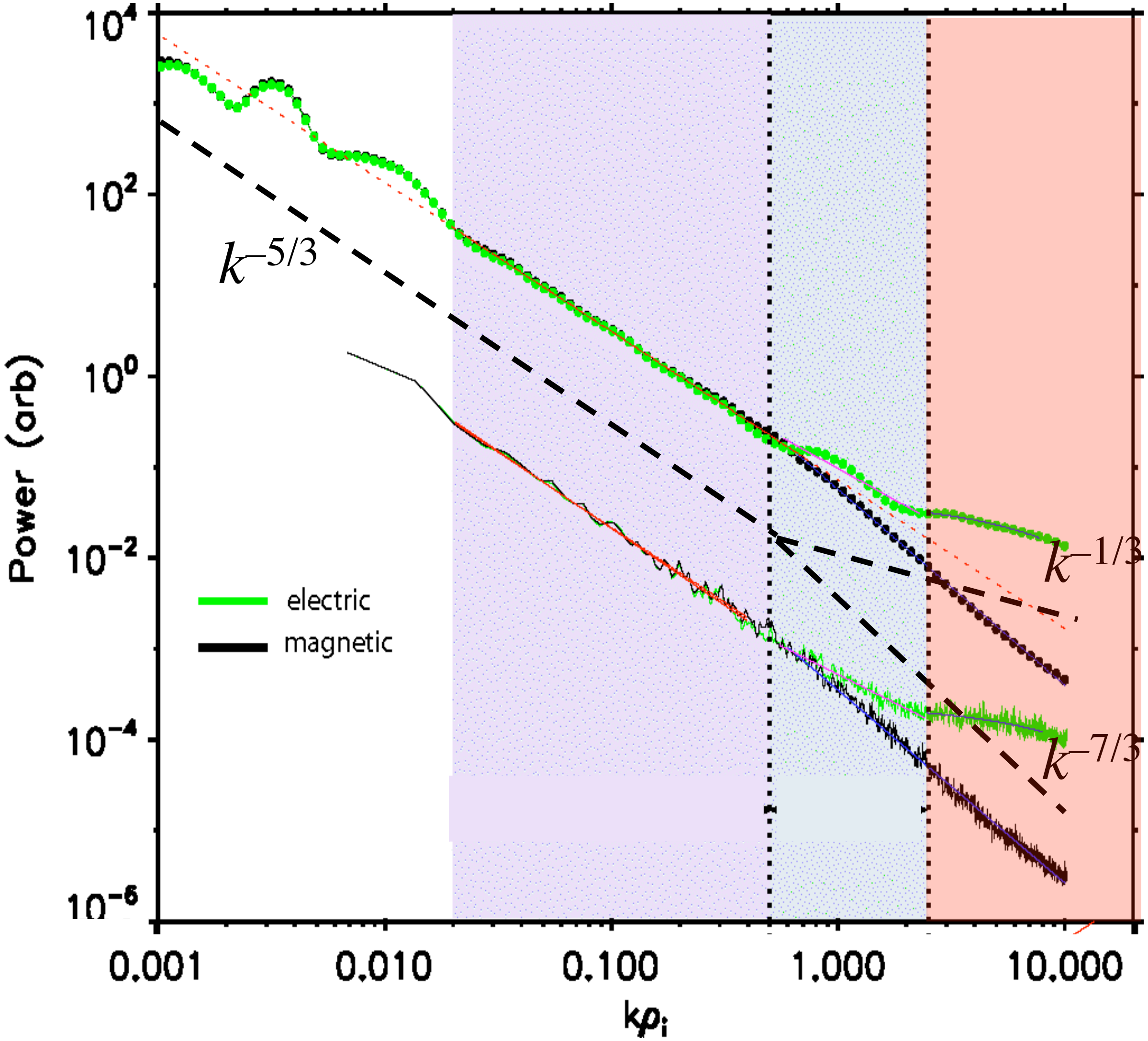


Plasma Turbulence Extends to Collisionless Scales

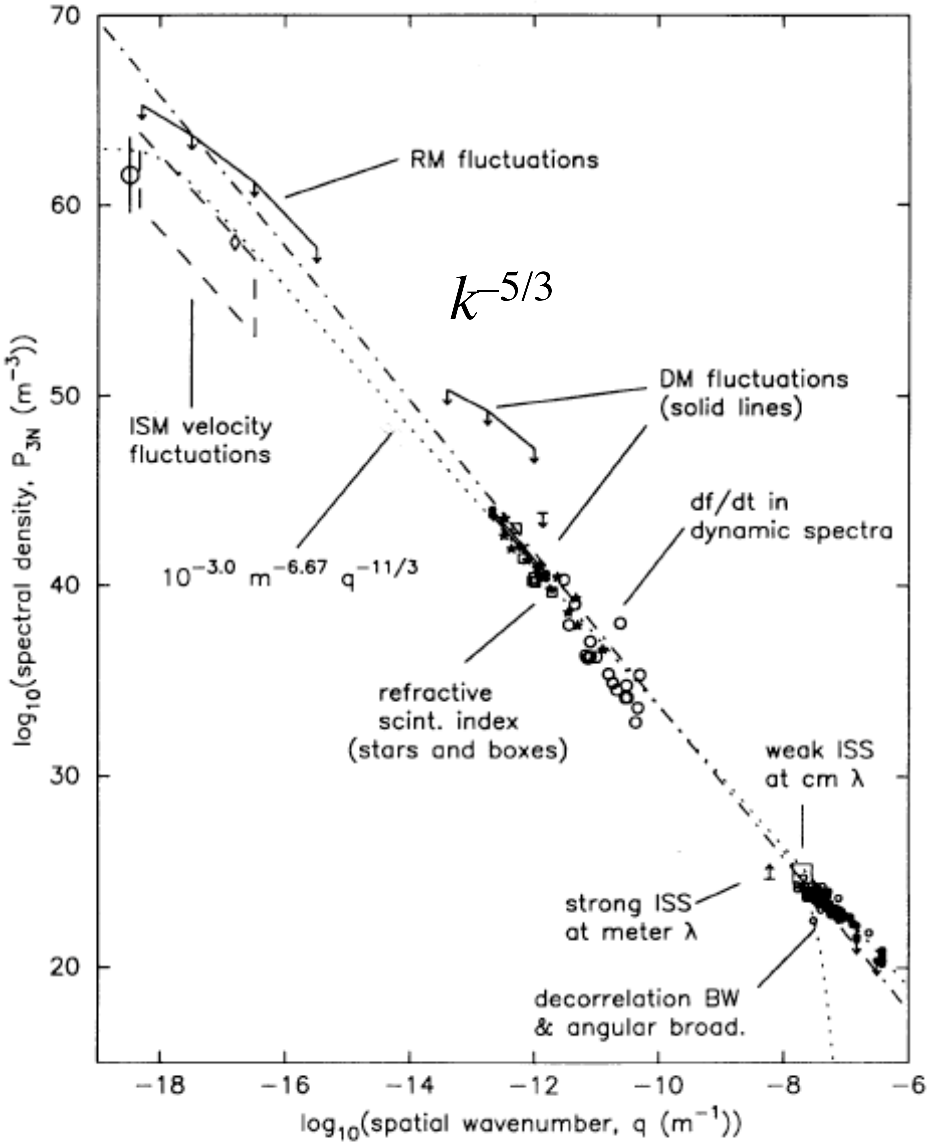
Interstellar medium: "Great Power Law in the Sky"

[Armstrong *et al*. 1995, *ApJ* **443**, 209]

$L \sim 10^{13}$ km (~100 pc)

$\lambda_{\mathrm{mfp}} \sim 10^7$ km

$\rho_i \sim 10^4$ km



Plasma Turbulence Extends to Collisionless Scales

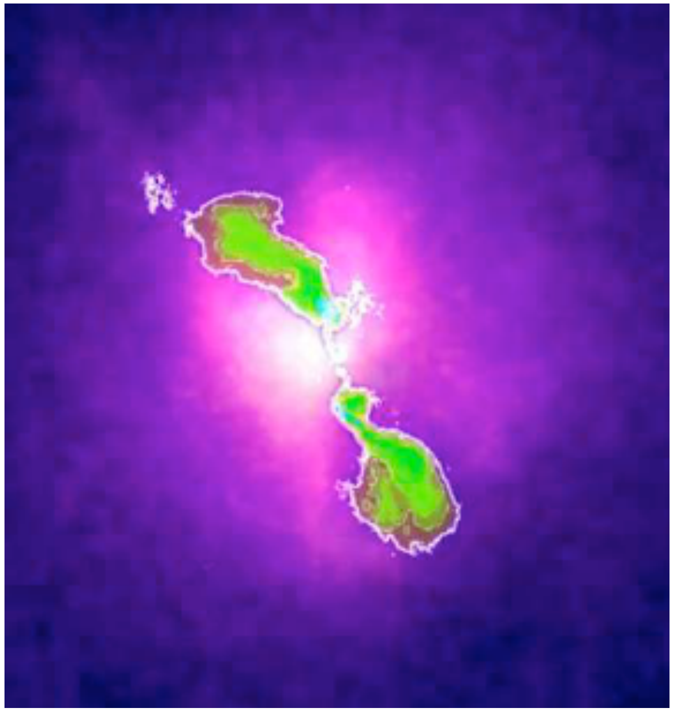
Intracluster (intergalactic) medium

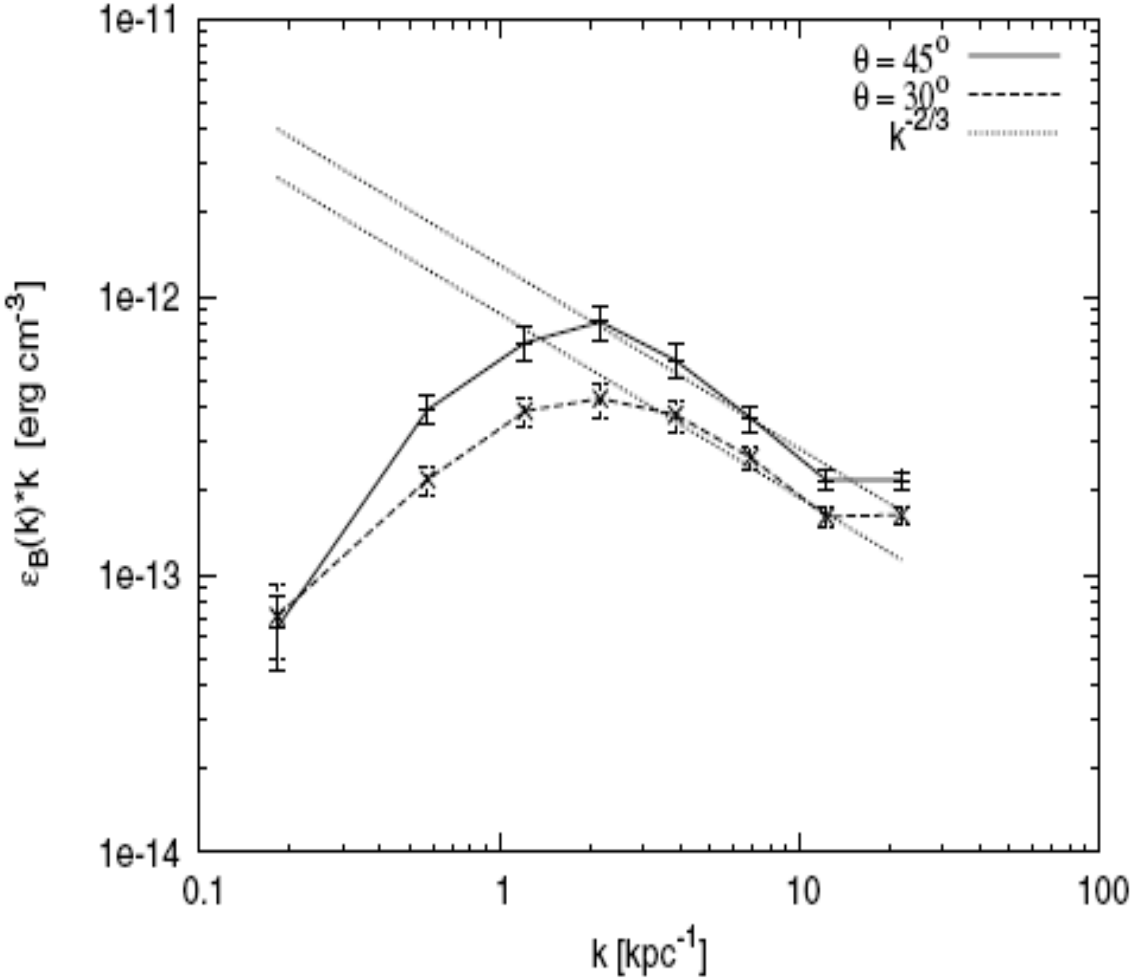
Hydra A cluster [Vogt & Enßlin 2005, *A&A* **434**, 67]

$L \sim 10^{19}$ km (~1 Mpc)

$\lambda_{\mathrm{mfp}} \sim 10^{16}$ km (~1 kpc)

$\rho_i \sim 10^4$ km



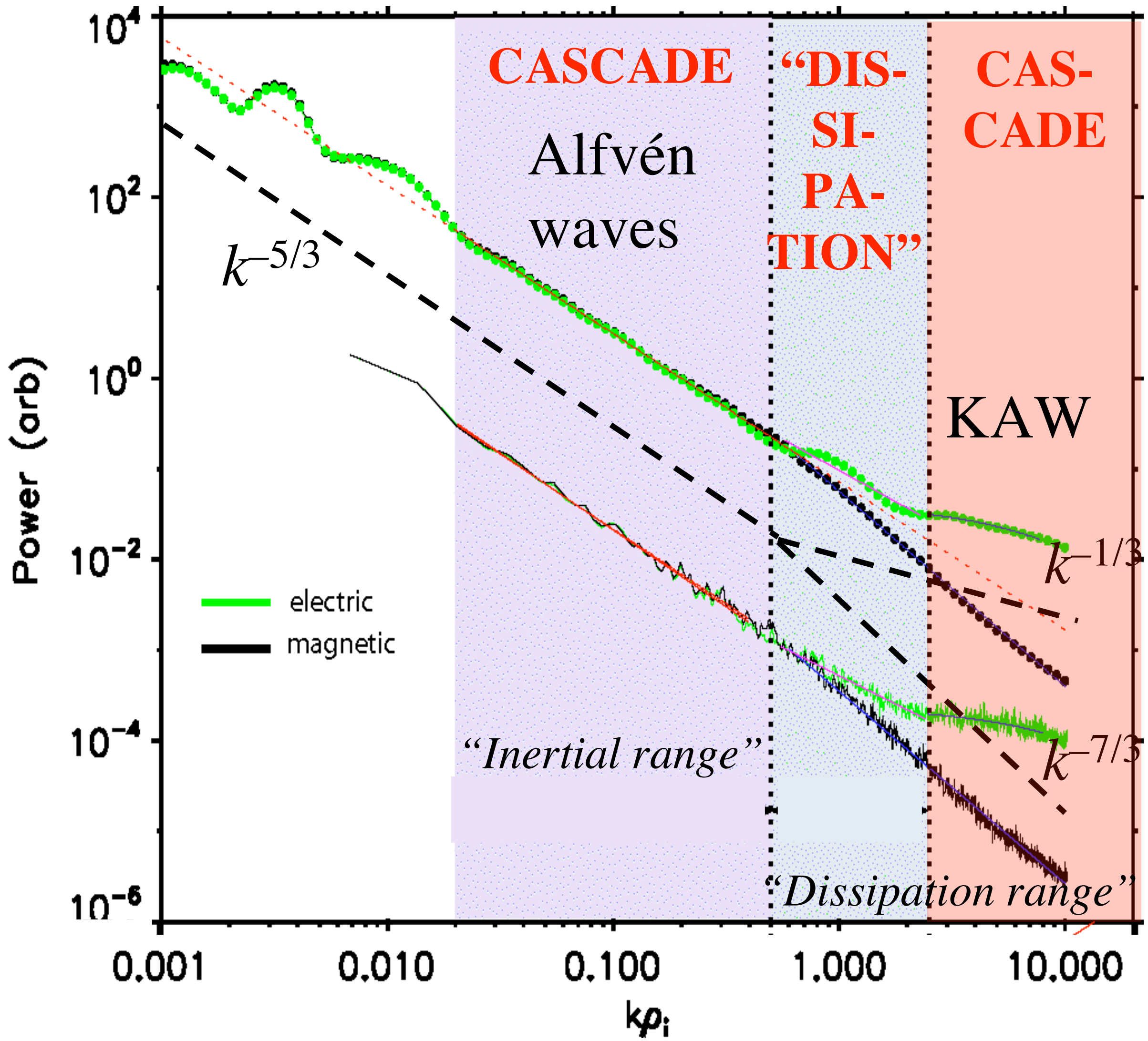


Plasma Turbulence Is Kinetic

- What is cascading in kinetic turbulence? (What is conserved?) What do the observed spectra tell us and how do we explain them?

- Dissipation (as usually understood) is "collisionless" (Landau damping) How does that heat particles? (ions, electrons, minority ions)

Turbulence in the solar wind
[Bale et al. 2005, *PRL* **94**, 215002]



Plasma Turbulence *Ab Initio*

$$\frac{\partial f_s}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f_s + \frac{q_s}{m_s}\left(\boldsymbol{E} + \frac{\boldsymbol{v}\times\boldsymbol{B}}{c}\right)\cdot\frac{\partial f_s}{\partial \boldsymbol{v}} = \left(\frac{\partial f_s}{\partial t}\right)_{\rm c}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = 4\pi\sum_s q_s n_s, \qquad n_s = \int \mathrm{d}^3\boldsymbol{v}\, f_s,$$

$$\boldsymbol{\nabla}\times\boldsymbol{B} - \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} = \frac{4\pi}{c}\left(\boldsymbol{j} + \boldsymbol{j}_{\rm ext}\right), \qquad \boldsymbol{j} = \sum_s q_s \int \mathrm{d}^3\boldsymbol{v}\,\boldsymbol{v} f_s,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\boldsymbol{\nabla}\times\boldsymbol{E}, \qquad \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0.$$

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s = \int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j} = \varepsilon - \frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon = -(1/V)\int\mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

$$\nabla\cdot\boldsymbol{E} = 4\pi\sum_s q_s n_s, \qquad n_s = \int\mathrm{d}^3\boldsymbol{v}\,f_s,$$

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Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s=\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j}=\varepsilon-\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon=-(1/V)\int\mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

Entropy produced:

$$\frac{\mathrm{d}S_s}{\mathrm{d}t}\equiv\frac{d}{dt}\left[-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,f_s\ln f_s\right]=-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\ln f_s\left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{c}}\geq 0$$

Boltzmann 1872

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}f_s = \int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\boldsymbol{E}\cdot\boldsymbol{j} = \varepsilon - \frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon = -(1/V)\int \mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

Entropy produced:

$$T_{0s}\frac{\mathrm{d}S_s}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left[\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int \mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}(F_{0s}+\delta f_s) - \int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}\right]$$

$$= -\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}} - n_{0s}\nu_E^{ss'}(T_{0s}-T_{0s'})$$

$$f_s = F_{0s} + \delta f_s$$

$$F_{0s} = n_{0s}(\pi v_{\mathrm{th}s}^2)^{-3/2}\exp(-v^2/v_{\mathrm{th}s}^2)$$

$$v_{\mathrm{th}s} = (2T_{0s}/m_s)^{1/2}$$

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s=\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j}=\varepsilon-\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

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Entropy produced:

$$T_{0s}\frac{\mathrm{d}S_s}{\mathrm{d}t}=\frac{\mathrm{d}}{\mathrm{d}t}\left[\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,(F_{0s}+\delta f_s)-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}\right]$$

$$=-\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}-n_{0s}\nu_E^{ss'}(T_{0s}-T_{0s'})$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

Plasma Turbulence *Ab Initio*

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{m_s v^2}{2}\,f_s=\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\boldsymbol{E}\cdot\boldsymbol{j}=\varepsilon-\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\,\frac{E^2+B^2}{8\pi}$$

Work done

$$\varepsilon=-(1/V)\int\mathrm{d}^3\boldsymbol{r}\,\boldsymbol{E}\cdot\boldsymbol{j}_{\mathrm{ext}}$$

Heating:

$$\frac{3}{2}n_{0s}\frac{\mathrm{d}T_{0s}}{\mathrm{d}t}=-\overline{\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}}-n_{0s}\nu_E^{ss'}(T_{0s}-T_{0s'})$$

Fluctuation energy budget:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\underbrace{\frac{T_{0s}\delta f_s^2}{2F_{0s}}}_{-T\delta S}+\underbrace{\frac{E^2+B^2}{8\pi}}_{\text{energy}}\right]=\varepsilon+\underbrace{\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}}_{\text{heating}}$$

Plasma Turbulence: Generalised Energy Cascade

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\underbrace{\frac{T_{0s}\delta f_s^2}{2F_{0s}}}_{-T\delta S}+\underbrace{\frac{E^2+B^2}{8\pi}}_{\text{energy}}\right] = \varepsilon + \underbrace{\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}}_{\text{heating}}$$

Generalised energy = free energy of the particles + fields

Fowler 1968
Krommes & Hu 1994
Krommes 1999
Sugama et al. 1996
Hallatschek 2004
Howes et al. 2006
Schekochihin et al. 2007
Scott 2007

arXiv:0806.1069

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$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

–TδS energy heating

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

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Landau damping is a redistribution between e-m fluctuation energy and (negative) perturbed entropy (free energy). It was pointed out already by Landau 1946 that δf_s *does not decay: "ballistic response"* $\delta f_s \propto \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}t}$

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Plasma Turbulence: Analogous to Fluid, But…

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$-T\delta S$ energy

heating

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

small scales in 6D phase space

$$\partial_t\boldsymbol{u}+\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}=-\boldsymbol{\nabla}p+\nu\nabla^2\boldsymbol{u}+\boldsymbol{f}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\frac{u^2}{2}=\varepsilon-\nu\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}|\boldsymbol{\nabla}\boldsymbol{u}|^2$$

small scales in 3D physical space

arXiv:0806.1069

Plasma Turbulence: Analogous to Fluid, But…

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\underbrace{\frac{T_{0s}\delta f_s^2}{2F_{0s}}}_{-T\delta S}+\underbrace{\frac{E^2+B^2}{8\pi}}_{\text{energy}}\right]$$

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int \mathrm{d}^3\boldsymbol{v}\,\overbrace{\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}}^{\text{heating}}$$

small scales in 6D phase space

$$\nu_{ii}v_{\mathrm{th}i}^2\left(\frac{\partial}{\partial v}\right)^2\sim\omega\ \Rightarrow\ \frac{\delta v}{v_{\mathrm{th}i}}\sim\left(\frac{\nu_{ii}}{\omega}\right)^{1/2}\sim\frac{1}{\sqrt{k_\parallel\lambda_{\mathrm{mfp}}}}\ll 1$$

In gyrokinetic turbulence, the velocity-space and x-space cascades are intertwined, giving rise to a single phase-space cascade

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Plasma Turbulence: Analogous to Fluid, But…

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$-T\delta S$ energy heating

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_\mathrm{c}$$

SO, <u>IDEA #1</u>:

GENERALISED ENERGY CASCADE THROUGH PHASE SPACE

arXiv:0806.1069

IDEA #2:

CRITICAL BALANCE

Critical Balance

- **Strong anisotropy:** $$\frac{k_{\|}}{k_{\perp}} \ll 1$$

In magnetised plasma, confirmed by numerics (MHD) and observations (solar wind, ISM)

- **Strong nonlinearity:** $$\omega \sim k_{\|} v_A \sim k_{\perp} u_{\perp}$$

Critical balance as a physical principle proposed for Alfvénic turbulence by Goldreich & Sridhar 1995 [*ApJ* **438**, 763]

More generally, one might argue that ***in a magnetised plasma, parallel linear propagation scale and perpendicular nonlinear interaction scale will adjust to each other*** and the turbulent cascade route will be determined by this principle

- Weak turbulence drives itself into strong regime
- 2D turbulence ("overstrong") parallel-decorrelates and returns to critical balance

What Is Gyrokinetics?

- **Strong anisotropy:** $\epsilon \sim \frac{k_{\|}}{k_{\perp}} \ll 1$ *(this is the small parameter!)*
- **Strong nonlinearity:** $\omega \sim k_{\|} v_A \sim k_{\perp} u_{\perp}$
  (critical balance as an ordering assumption)

[Howes et al. 2006, *ApJ* **651**, 590]

What Is Gyrokinetics?

- **Strong anisotropy:** $\epsilon \sim \frac{k_\parallel}{k_\perp} \ll 1$ *(this is the small parameter!)*

- **Strong nonlinearity:** $\omega \sim k_\parallel v_A \sim k_\perp u_\perp$ → $\frac{\omega}{\Omega_i} \sim \frac{k_\parallel v_A}{\Omega_i} \sim \frac{k_\perp \rho_i}{\sqrt{\beta_i}}\epsilon$
  (critical balance as an ordering assumption)

- **Finite Larmor radius:** $k_\perp \rho_i \sim 1$ →

Low frequency

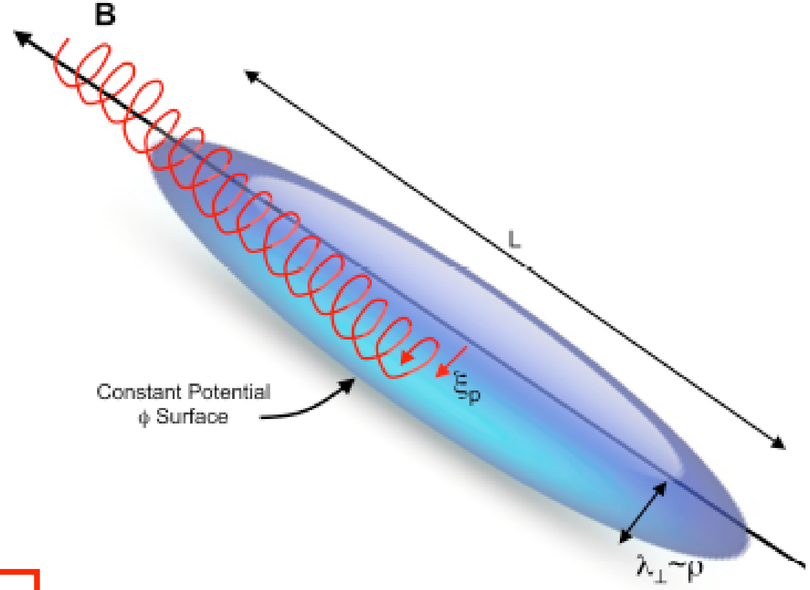
- **Weak collisions:** $\frac{\omega}{\nu_{ii}} \sim \frac{k_\parallel \lambda_{\rm mfp}}{\sqrt{\beta_i}} \sim 1$

GK ORDERING:

$$\frac{\omega}{\Omega_i} \sim \frac{e\phi}{T_e} \sim \frac{\delta B_\perp}{B_0} \sim \frac{\delta B_\parallel}{B_0} \sim \frac{\rho_i}{\lambda_{\rm mfp}} \sim \frac{k_\parallel}{k_\perp} \sim \epsilon$$

[Taylor & Hastie 1968, *Plasma Phys*. **10**, 479; Rutherford & Frieman 1968, *Phys. Fluids* **11**, 569;
Catto 1977, *Plasma Phys*. **20**, 719; Frieman & Chen 1982, *Phys. Fluids* **443**, 209;
for our derivation, notation, etc. see Howes et al. 2006, *ApJ* **651**, 590]

Gyrokinetics: Kinetics of Larmor Rings



Particle dynamics can be averaged over the Larmor orbits and everything reduces to kinetics of Larmor rings centered at

$$\mathbf{R}_s = \mathbf{r} + \frac{\mathbf{v} \times \hat{\mathbf{z}}}{\Omega_s}$$

Catto transformation

and interacting with the electromagnetic fluctuations.

$$\delta f_s = -q_s \varphi F_{0s}/T_{0s} + h_s(t, \boldsymbol{R}_s, v_\perp, v_\parallel)$$

only two velocity variables, i.e., 6D → 5D

[Howes et al. 2006, *ApJ* **651**, 590]

Gyrokinetics: Kinetics of Larmor Rings

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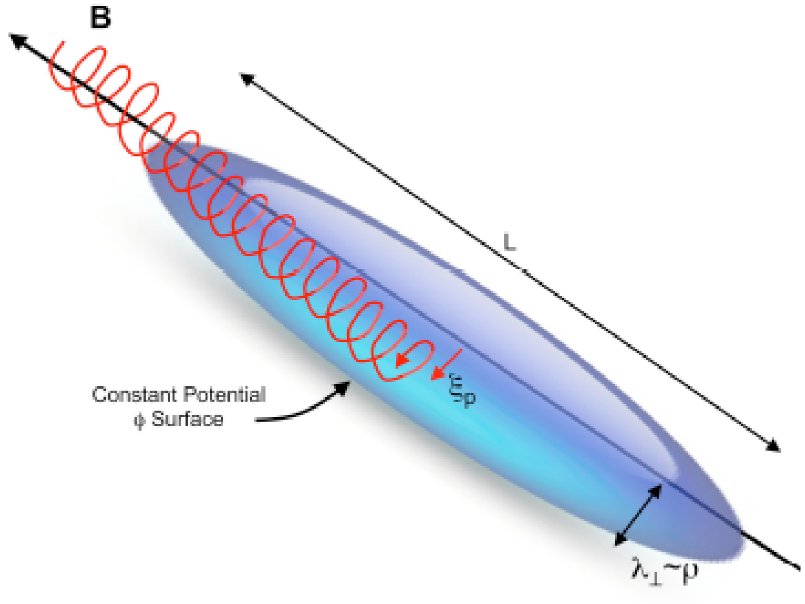


and interacting with the electromagnetic fluctuations.

$$\delta f_s = -q_s \varphi F_{0s}/T_{0s} + h_s(t, \boldsymbol{R}_s, v_\perp, v_\parallel)$$

$$\frac{\partial h_s}{\partial t} + v_\parallel \frac{\partial h_s}{\partial z} + \frac{c}{B_0}\{\langle\chi\rangle_{\boldsymbol{R}_s}, h_s\} = \frac{q_s F_{0s}}{T_{0s}} \frac{\partial \langle\chi\rangle_{\boldsymbol{R}_s}}{\partial t} + \left(\frac{\partial h_s}{\partial t}\right)_{\mathrm{c}}$$

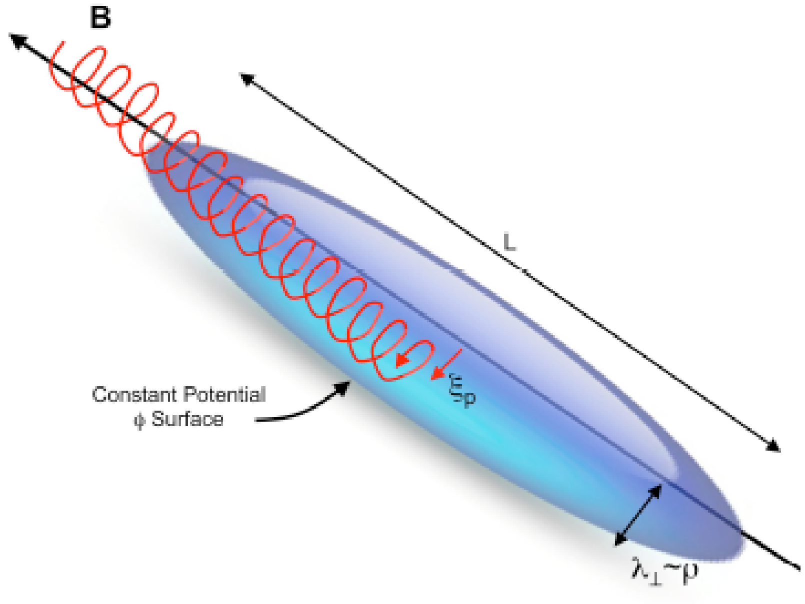
$$\chi = \varphi - \boldsymbol{v}\cdot\boldsymbol{A}/c, \ \boldsymbol{B} = B_0\hat{z} + \delta\boldsymbol{B}, \ \delta\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$$

+ Maxwell's equations (quasineutrality and Ampère's law)

$$\langle\chi(t, \mathbf{r}, \mathbf{v})\rangle_{\mathbf{R}_s} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, \chi\left(t, \mathbf{R}_s - \frac{\mathbf{v}\times\hat{\mathbf{z}}}{\Omega_s}, \mathbf{v}\right)$$

[Howes et al. 2006, *ApJ* **651**, 590]

Gyrokinetics: Kinetics of Larmor Rings



Averaged gyrocentre drifts:

- $\mathbf{E}\times\mathbf{B}_0$ drift
- ∇B drift
- motion along perturbed fieldline

$$\left\langle \frac{d\mathbf{R}_s}{dt} \right\rangle_{\mathbf{R}_s} \cdot \frac{\partial h_s}{\partial \mathbf{R}_s}$$

Averaged wave-ring interaction

$$-\left\langle \frac{d\mathcal{E}_s}{dt} \frac{\partial f_s}{\partial \mathcal{E}_s} \right\rangle_{\mathbf{R}_s}$$

$$\delta f_s = -q_s \varphi F_{0s}/T_{0s} + h_s(t, \boldsymbol{R}_s, v_\perp, v_\parallel)$$

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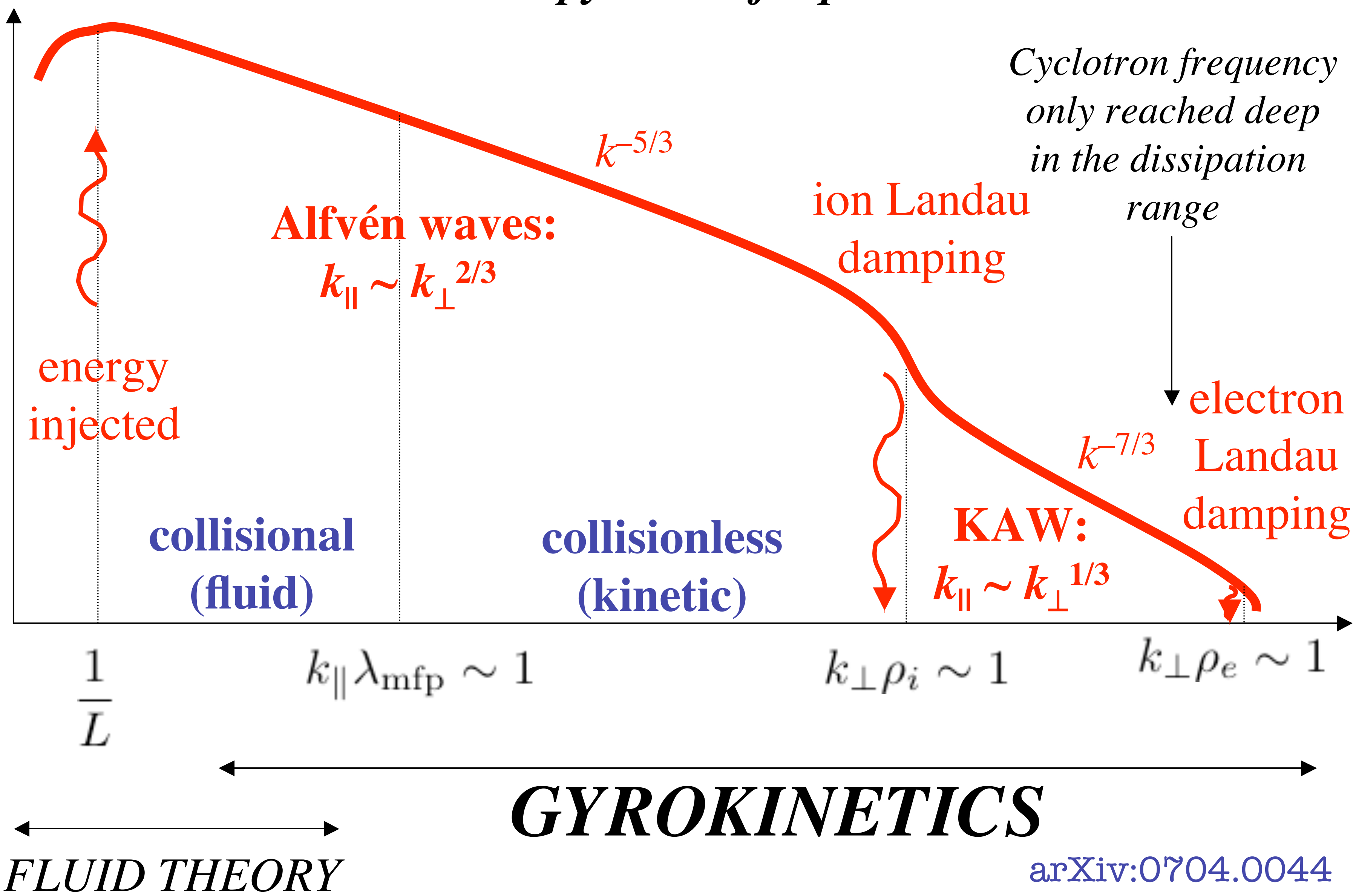
[Howes et al. 2006, *ApJ* **651**, 590]

Why is Gyrokinetics Valid?

Because anisotropy makes frequencies low.



arXiv:0704.0044

Why is Gyrokinetics Useful?

- Because it is a simplifying analytical step that is a natural staring point for further theory
  [Howes et al. 2006, *ApJ* **651**, 690
  Schekochihin et al.,
  arXiv:0704.0044]

- Because it reduces the kinetic problem to 5D, making it numerically tractable
  (publicly available codes developed in fusion research: e.g., GS2, GENE, GYRO…)

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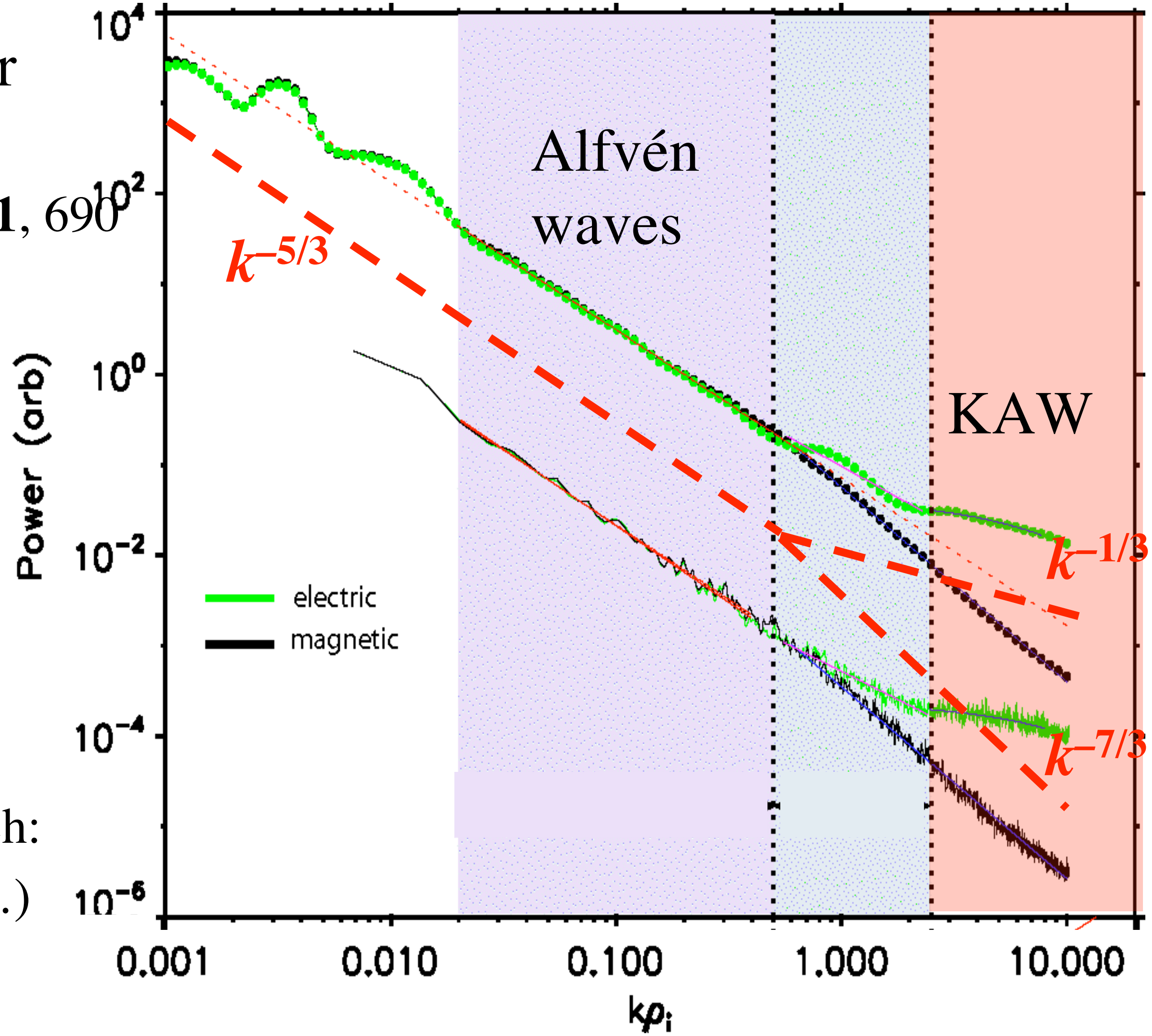
Alfvén-wave turbulence in the SW

[by Bale et al. 2005, *PRL* **94**, 215002]



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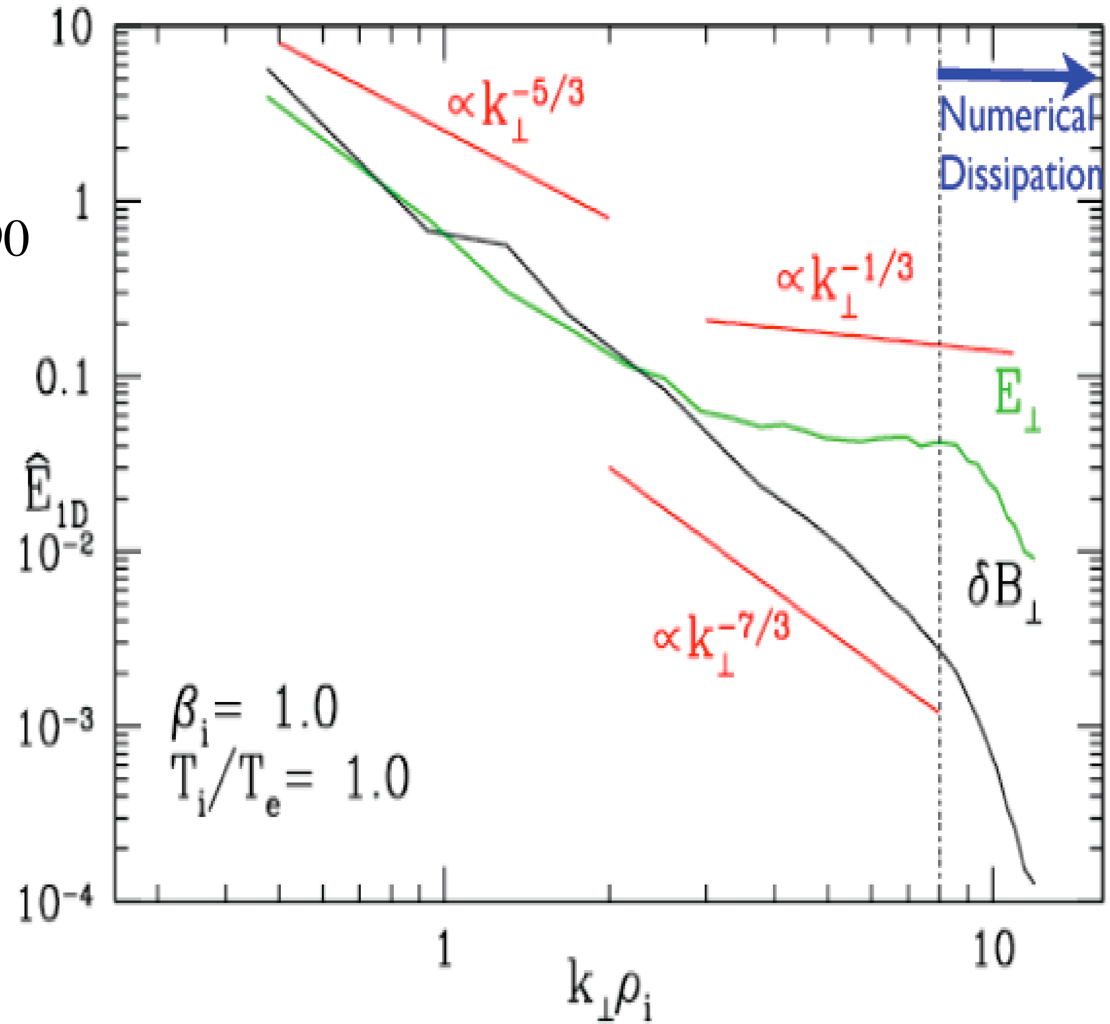
Alfvén-wave turbulence using GS2 (by Greg Howes)



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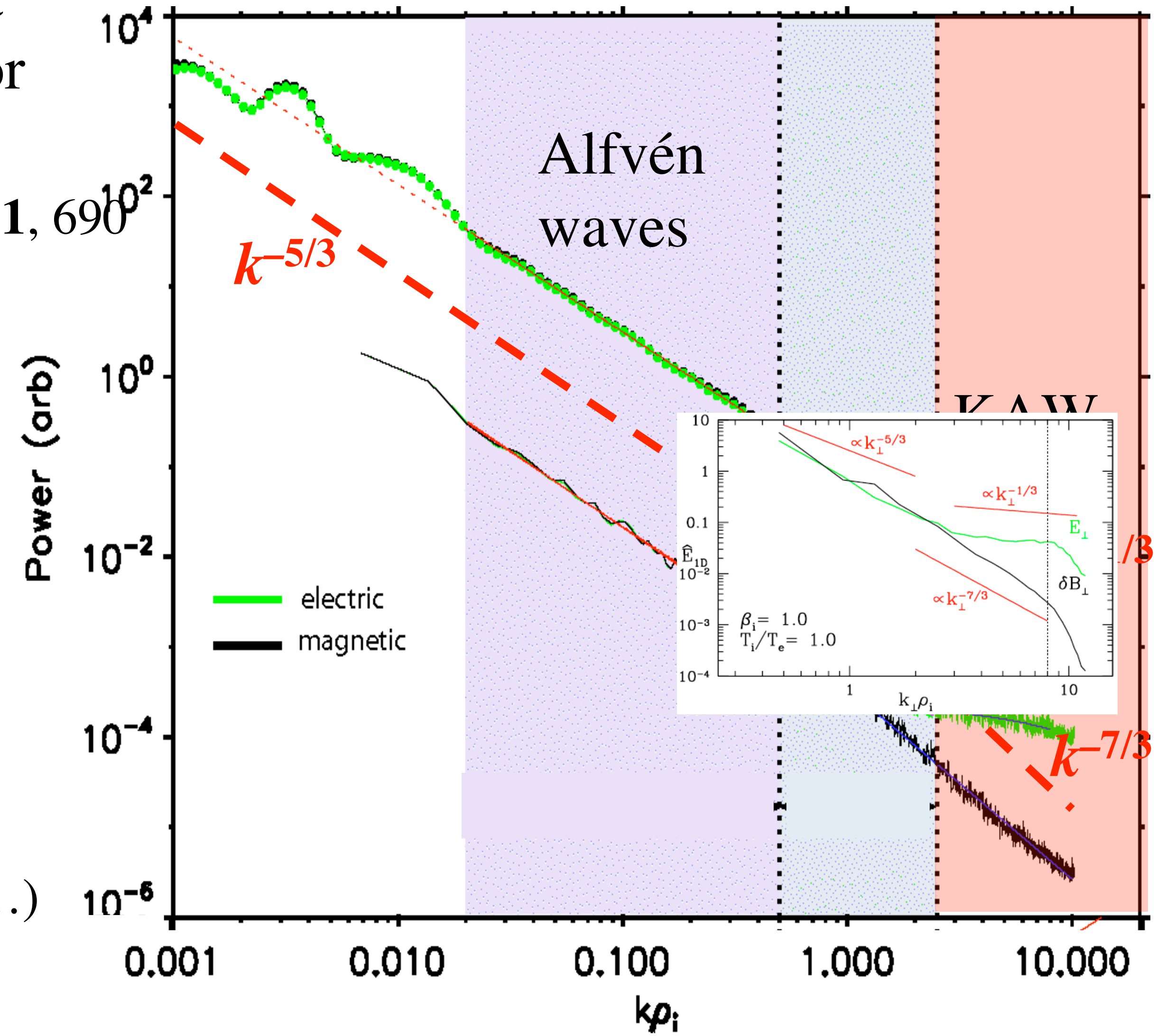
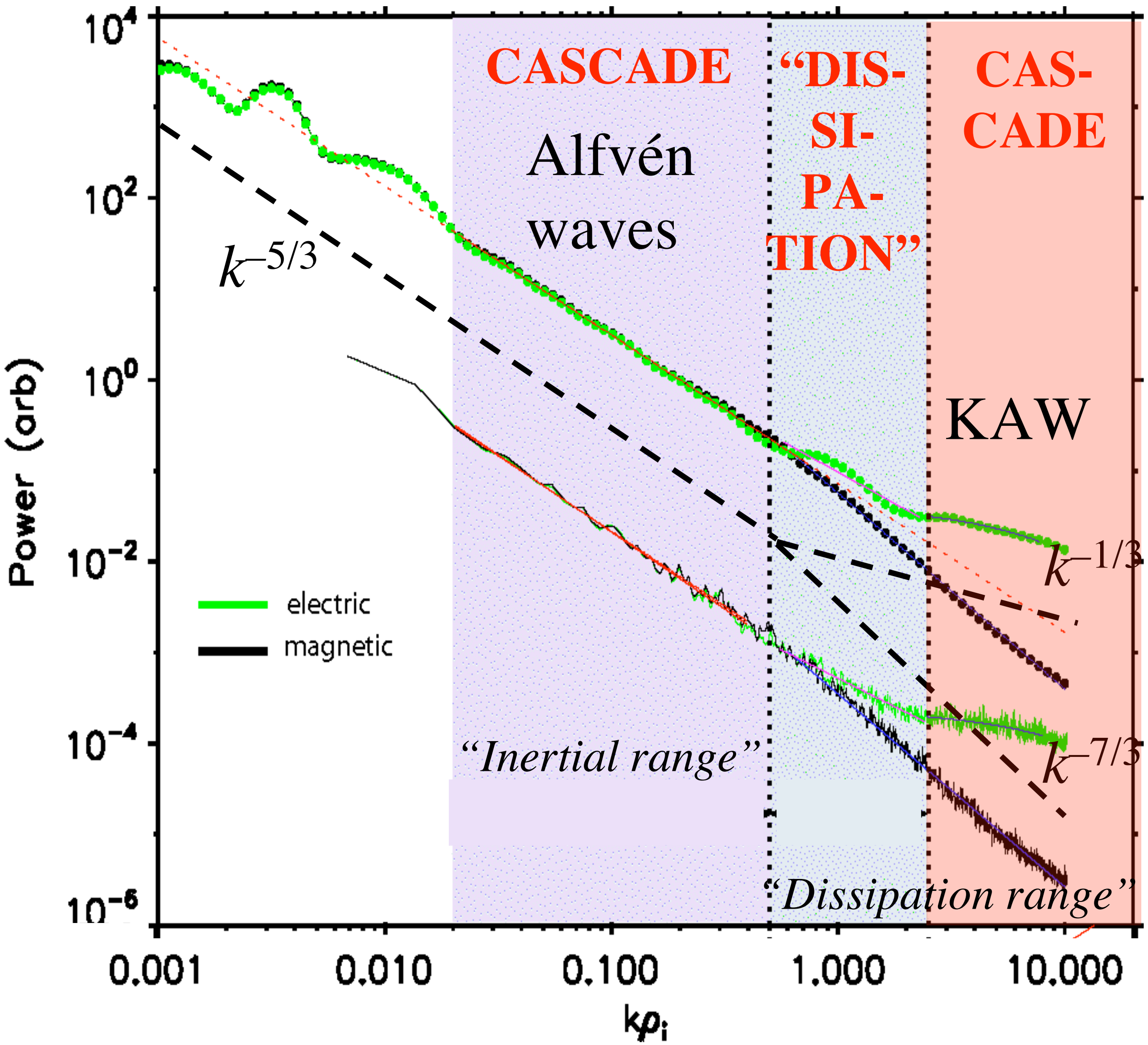


Kinetics vs. Fluid Models: What Is New?

- What is cascading in kinetic turbulence? (What is conserved?) What do the observed spectra tell us and how do we explain them?

- Dissipation (as usually understood) is “collisionless” (Landau damping) How does that heat particles? (ions, electrons, minority ions)

Alfvén-wave turbulence in the SW [by Bale et al. 2005, *PRL* **94**, 215002]



Gyrokinetics: Kinetics of Larmor Rings

SO, <u>IDEA #3</u>:

GYROAVEARGED KINETIC THEORY AT LOW FREQUENCIES

- Only two velocity variables, i.e., 6D → 5D
- All high-frequency stuff averaged out

$$\delta f_s = -q_s \varphi F_{0s}/T_{0s} + h_s(t, \boldsymbol{R}_s, v_\perp, v_\parallel) \qquad \boldsymbol{R}_s = \boldsymbol{r} + \boldsymbol{v}_\perp \times \hat{\boldsymbol{z}}/\Omega_s$$

$$\frac{\partial h_s}{\partial t} + v_\parallel \frac{\partial h_s}{\partial z} + \frac{c}{B_0}\{\langle\chi\rangle_{\boldsymbol{R}_s}, h_s\} = \frac{q_s F_{0s}}{T_{0s}} \frac{\partial \langle\chi\rangle_{\boldsymbol{R}_s}}{\partial t} + \left(\frac{\partial h_s}{\partial t}\right)_c$$

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+ Maxwell's equations (quasineutrality and Ampère's law)

$$\langle\chi(t,\mathbf{r},\mathbf{v})\rangle_{\mathbf{R}_s} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\,\chi\left(t, \mathbf{R}_s - \frac{\mathbf{v}\times\hat{\mathbf{z}}}{\Omega_s}, \mathbf{v}\right)$$

[Howes et al. 2006, *ApJ* **651**, 590]

Generalised Energy in Gyrokinetics

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s^2}{2F_{0s}}+\frac{E^2+B^2}{8\pi}\right]$$

$-T\delta S$ energy heating

$$=\varepsilon+\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\sum_s\int\mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\delta f_s}{F_{0s}}\left(\frac{\partial\delta f_s}{\partial t}\right)_{\mathrm{c}}$$

$$\delta f_s = -q_s\varphi F_{0s}/T_{0s} + h_s(t,\boldsymbol{R}_s,v_\perp,v_\parallel) \qquad \boldsymbol{R}_s = \boldsymbol{r}+\boldsymbol{v}_\perp\times\hat{\boldsymbol{z}}/\Omega_s$$

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Generalised Energy in Gyrokinetics

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\int\frac{\mathrm{d}^3\boldsymbol{r}}{V}\left[\sum_s\left(\int \mathrm{d}^3\boldsymbol{v}\,\frac{T_{0s}\langle h_s^2\rangle_{\boldsymbol{r}}}{2F_{0s}} - \frac{q_s^2\varphi^2 n_{0s}}{2T_{0s}}\right) + \frac{|\delta\boldsymbol{B}|^2}{8\pi}\right]$$

$-T\delta S$ energy

$$= \varepsilon + \sum_s\int \mathrm{d}^3\boldsymbol{v}\int\frac{\mathrm{d}^3\boldsymbol{R}_s}{V}\frac{T_{0s}h_s}{F_{0s}}\left(\frac{\partial h_s}{\partial t}\right)_{\mathrm{c}}$$

arXiv:0704.0044

$$\delta f_s = -q_s\varphi F_{0s}/T_{0s} + h_s(t, \boldsymbol{R}_s, v_\perp, v_\parallel) \qquad \boldsymbol{R}_s = \boldsymbol{r} + \boldsymbol{v}_\perp \times \hat{\boldsymbol{z}}/\Omega_s$$

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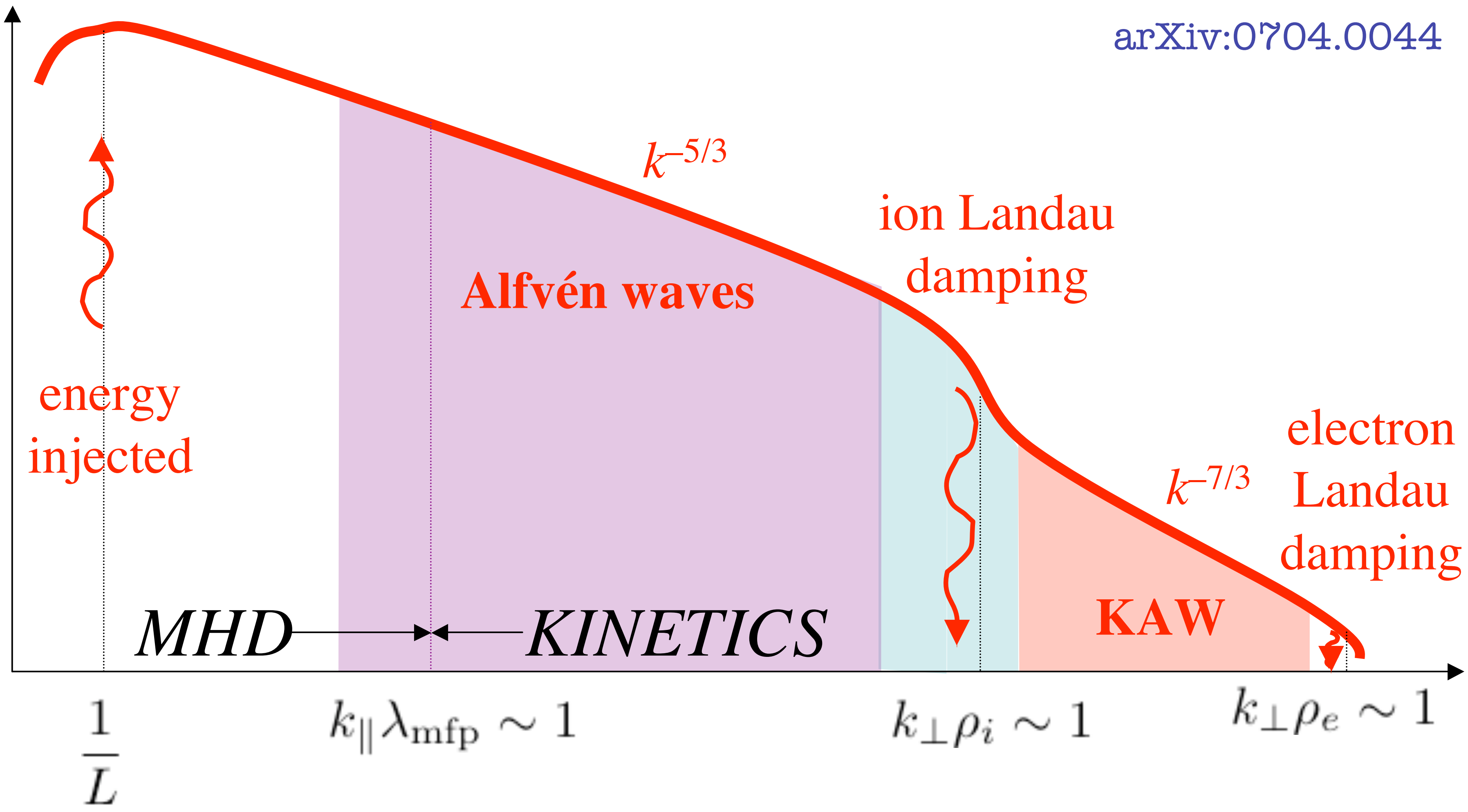
[Howes et al. 2006, *ApJ* **651**, 590]

The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \left(\sum_s \int d^3\mathbf{v} \frac{T_{0s}\delta f_s^2}{2F_{0s}} + \frac{|\delta \mathbf{B}|^2}{8\pi} \right)$$

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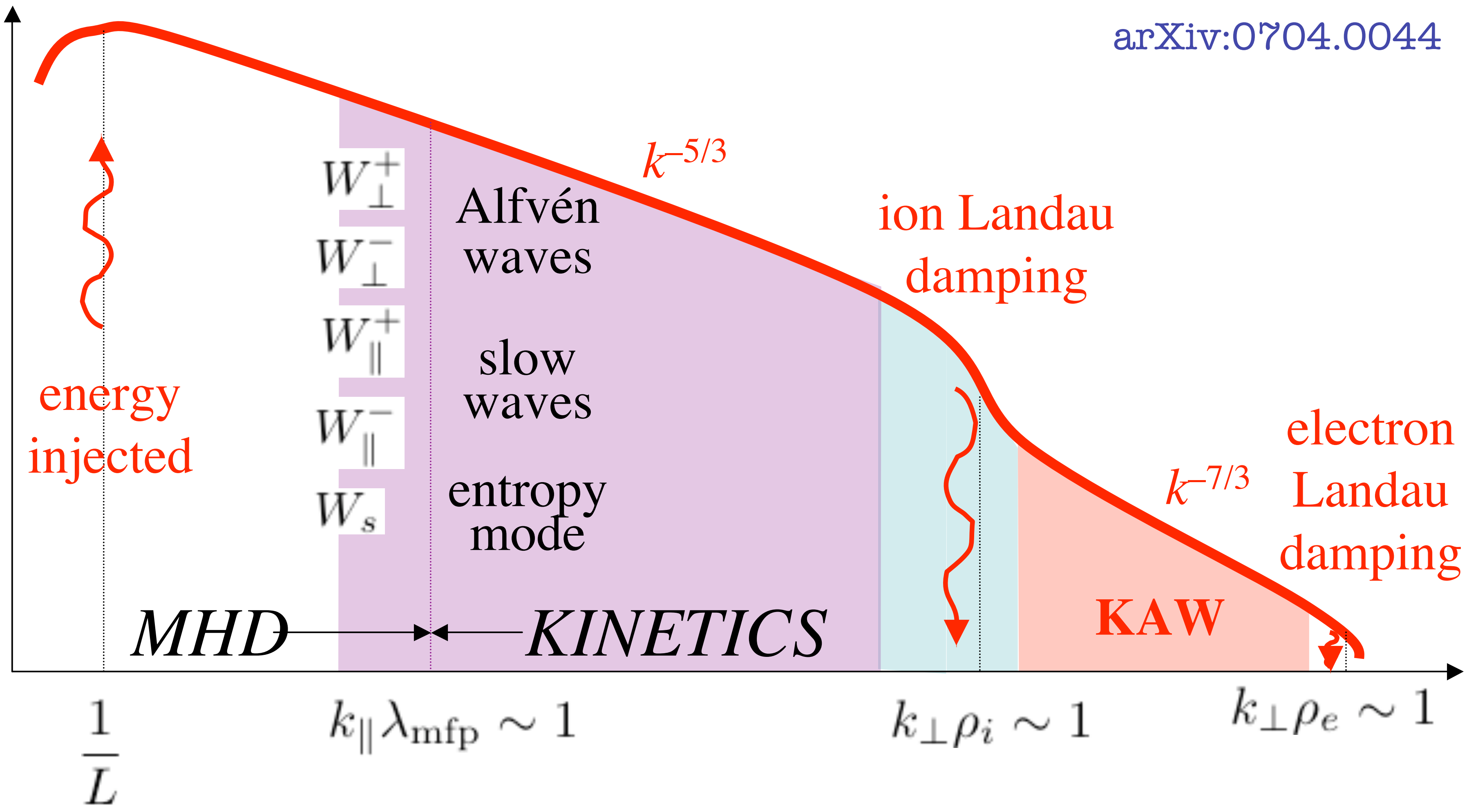
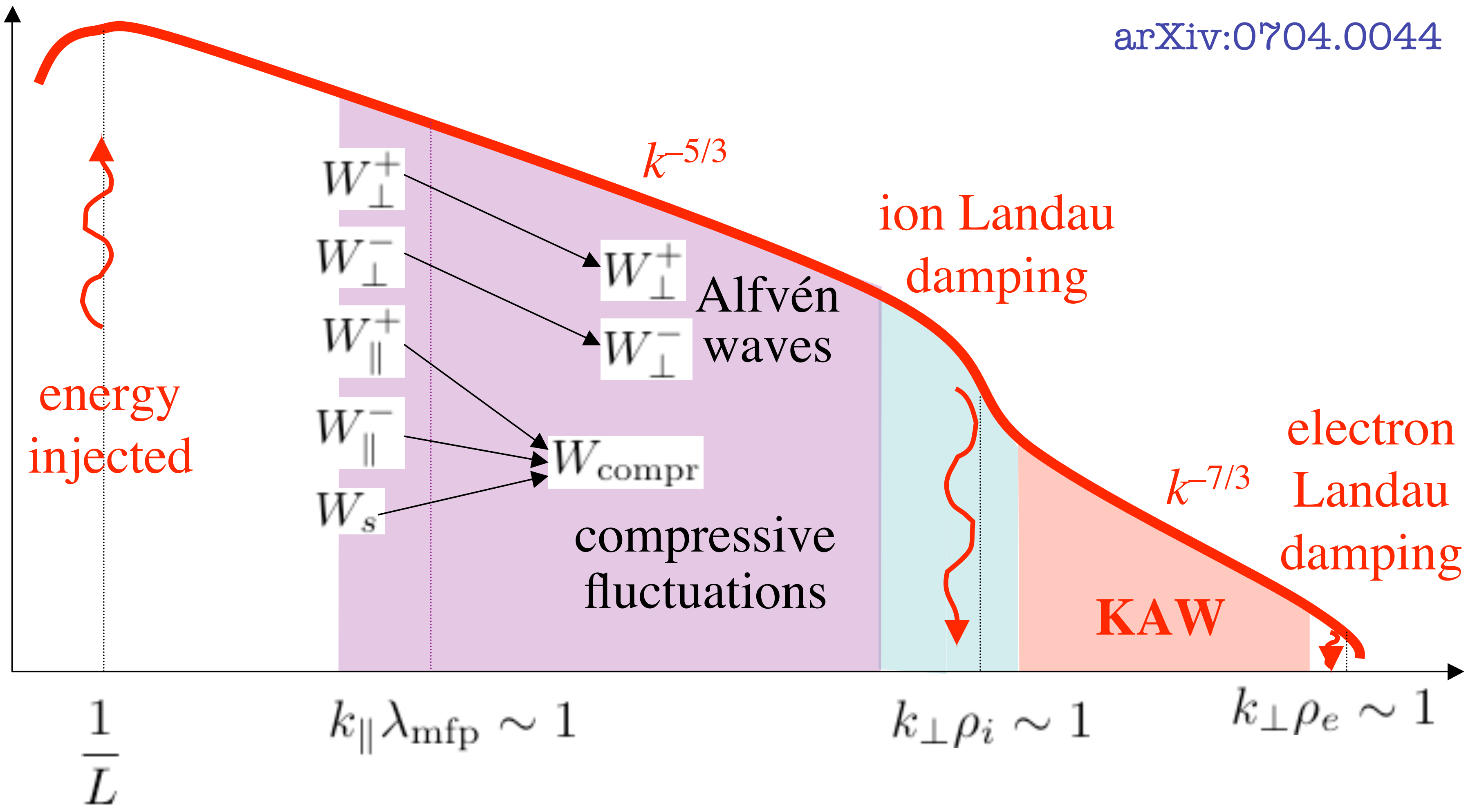


$$W = \int d^3\mathbf{r} \left[\frac{m_i n_{0i}}{2} \left(|\nabla \zeta^+|^2 + |\nabla \zeta^-|^2 \right) + \frac{m_i n_{0i}}{2} \left(|z_\parallel^+|^2 + |z_\parallel^-|^2 \right) + \frac{3}{4} n_{0i} T_{0i} \frac{1 + Z/\tau}{5/3 + Z/\tau} \frac{\delta s^2}{s_0^2} \right]$$

Alfvén waves **slow waves** **entropy fluctuations**

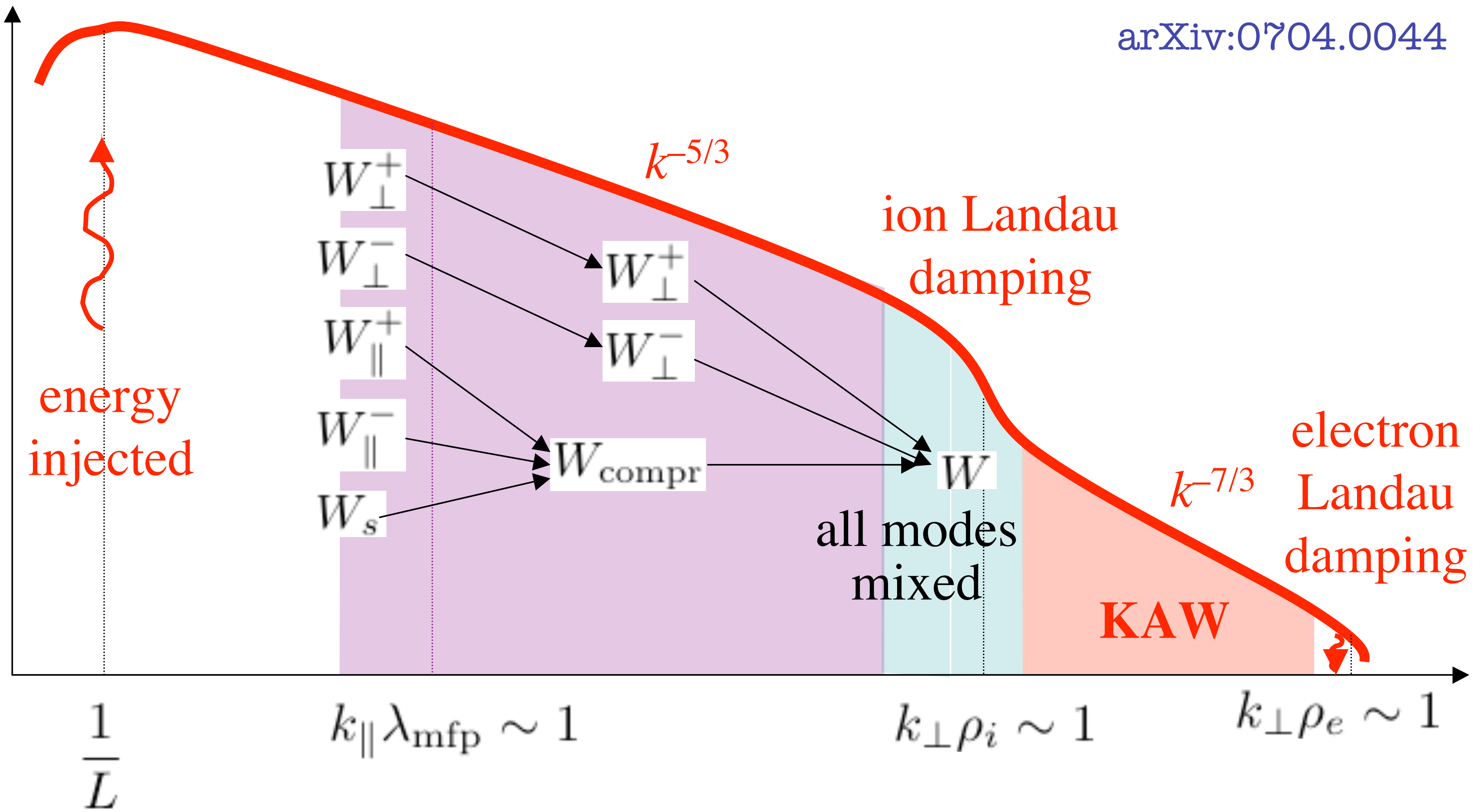
The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \left[\frac{m_i n_{0i}}{2} \left(|\nabla \zeta^+|^2 + |\nabla \zeta^-|^2 \right) + \frac{n_{0i} T_{0i}}{2} \left(\frac{Z}{\tau} \frac{\delta n_e^2}{n_{0e}^2} + \frac{2}{\beta_i} \frac{\delta B_\parallel^2}{B_0^2} + \frac{1}{n_{0i}} \int d^3\mathbf{v} \frac{T_{0i} \delta \tilde{f}_i^2}{2F_{0i}} \right) \right]$$

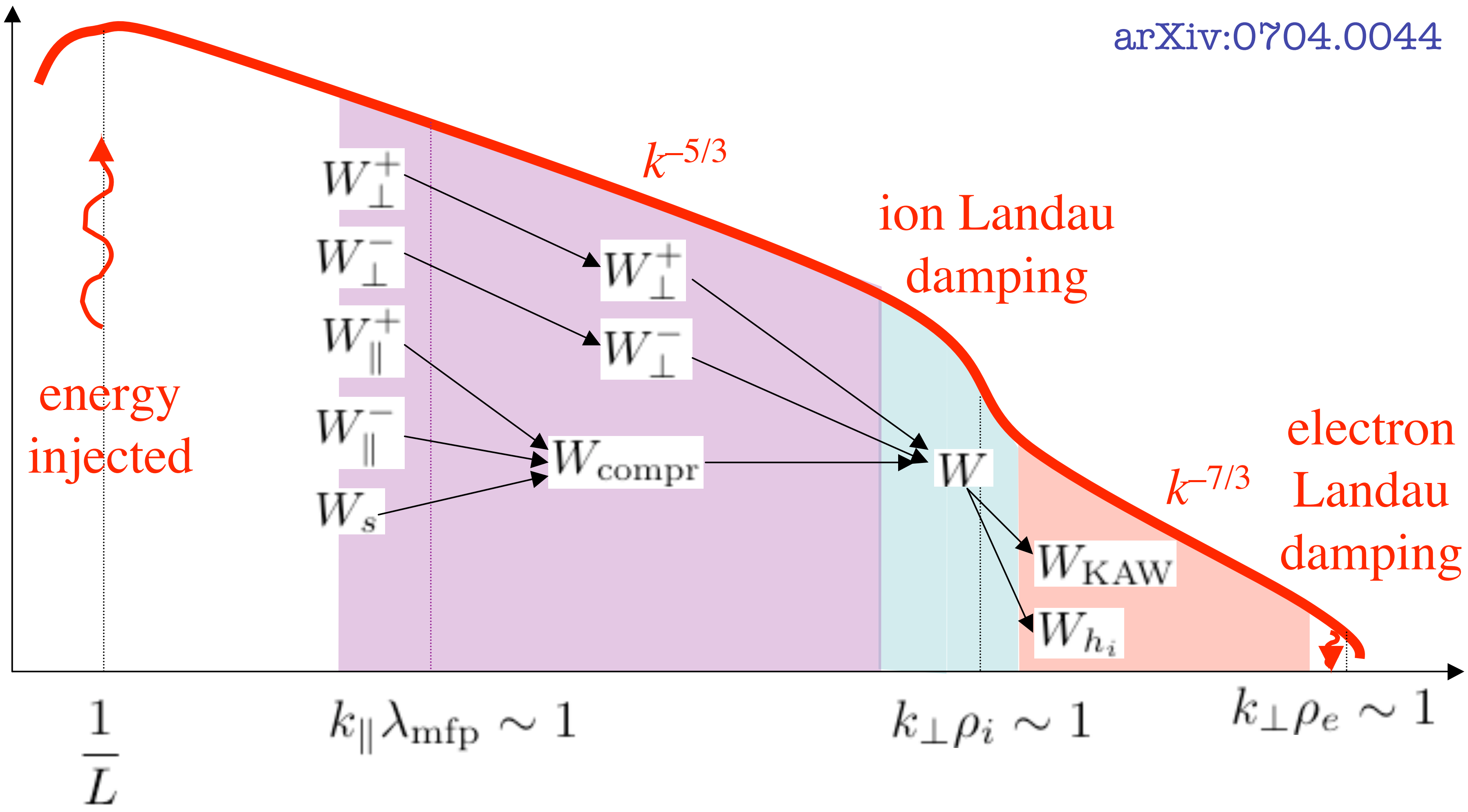
Alfvén waves **compressive fluctuations**

The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \left(\int d^3\mathbf{v} \frac{T_{0i}\delta f_i^2}{2F_{0i}} + \frac{n_{0e}T_{0e}}{2} \frac{\delta n_e^2}{n_{0e}^2} + \frac{|\delta \mathbf{B}|^2}{8\pi} \right)$$

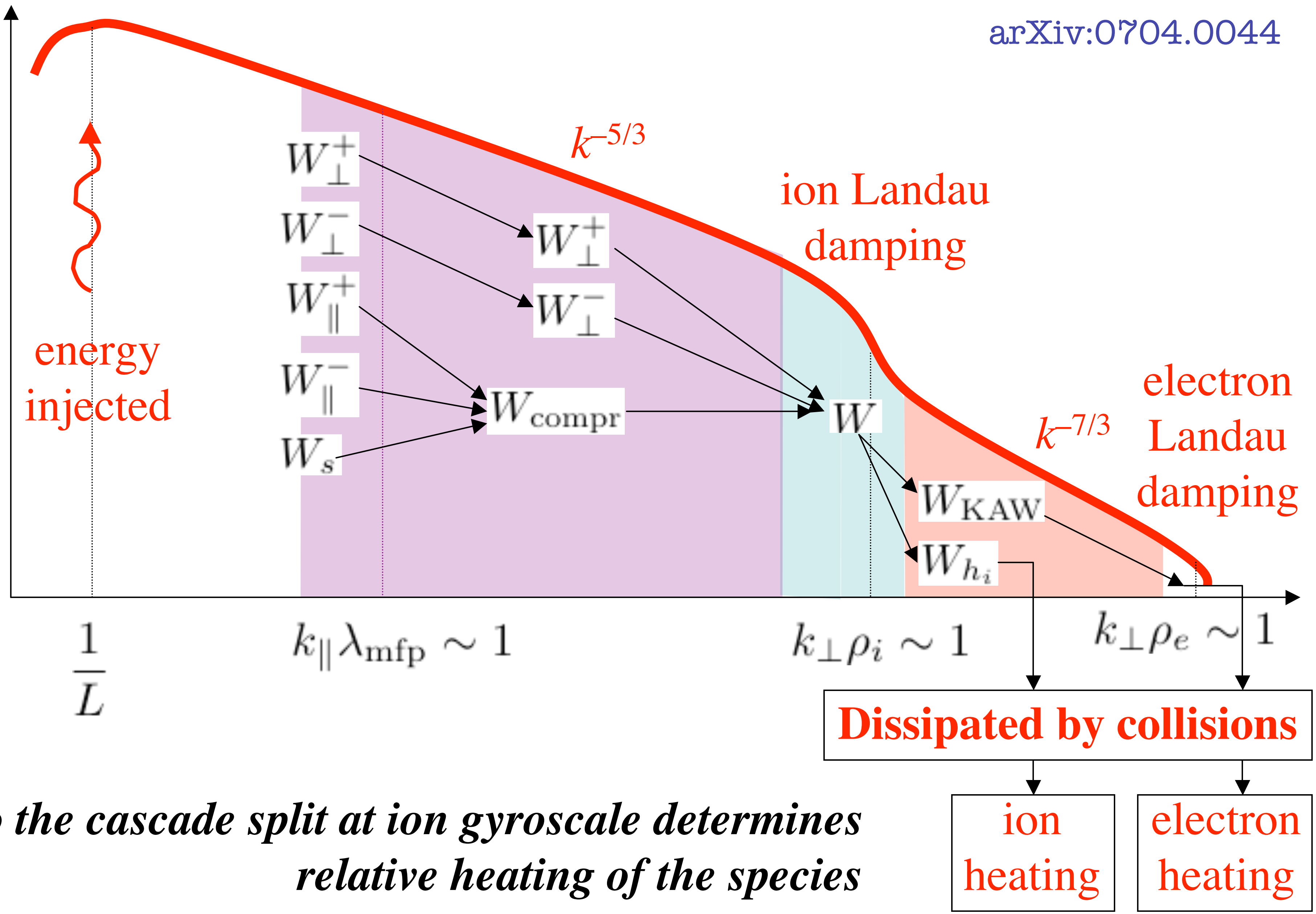
The Grand Kinetic Cascade



$$W = \int d^3\mathbf{r} \int d^3\mathbf{v}\, \frac{T_{0i} h_i^2}{2F_{0i}} + \int d^3\mathbf{r} \left\{ \frac{\delta B_\perp^2}{8\pi} + \frac{n_{0i} T_{0i}}{2} \left(1 + \frac{Z}{\tau}\right) \left[1 + \frac{\beta_i}{2}\left(1 + \frac{Z}{\tau}\right)\right] \left(\frac{Ze\phi}{T_{0i}}\right)^2 \right\}$$

entropy cascade **kinetic Alfvén waves**

The Grand Kinetic Cascade



So the cascade split at ion gyroscale determines relative heating of the species

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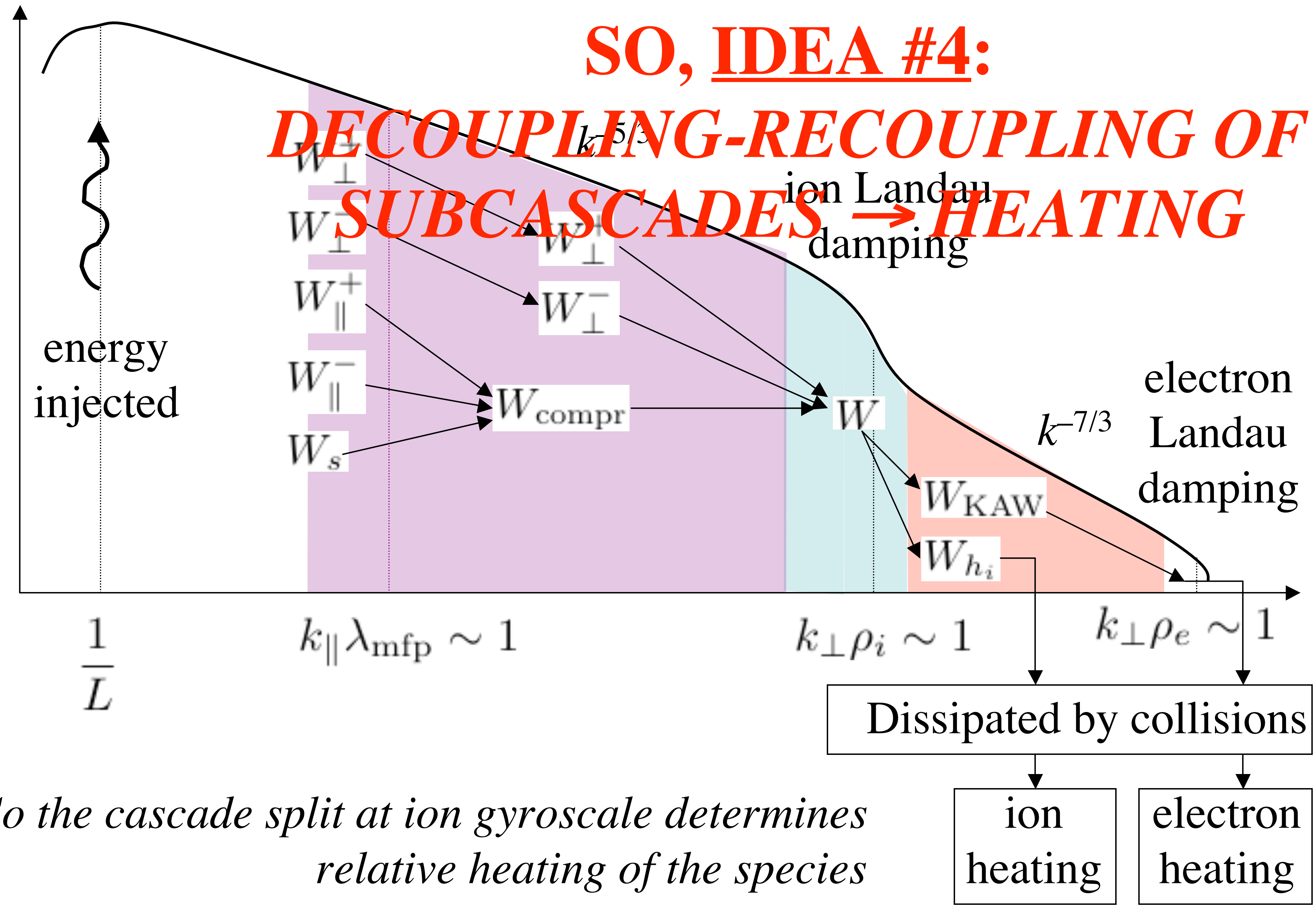

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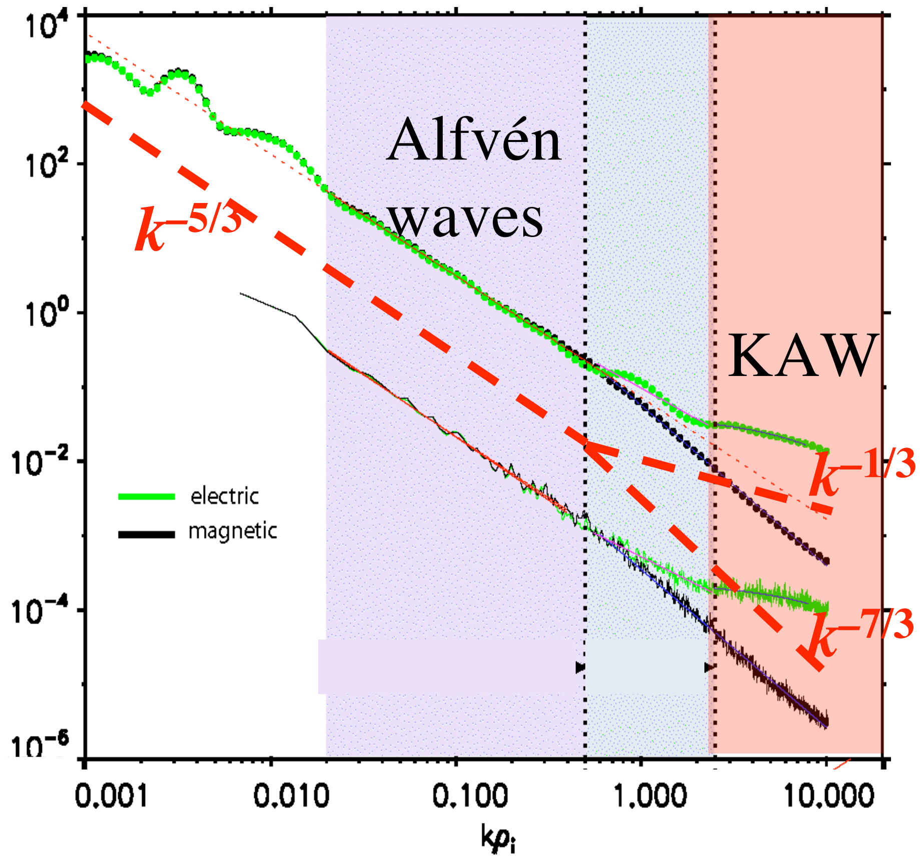
Ion Gyroscale Transition: GK DNS by G. Howes

Alfvén-wave turbulence in the solar wind
[by Bale et al. 2005, *PRL* **94**, 215002]



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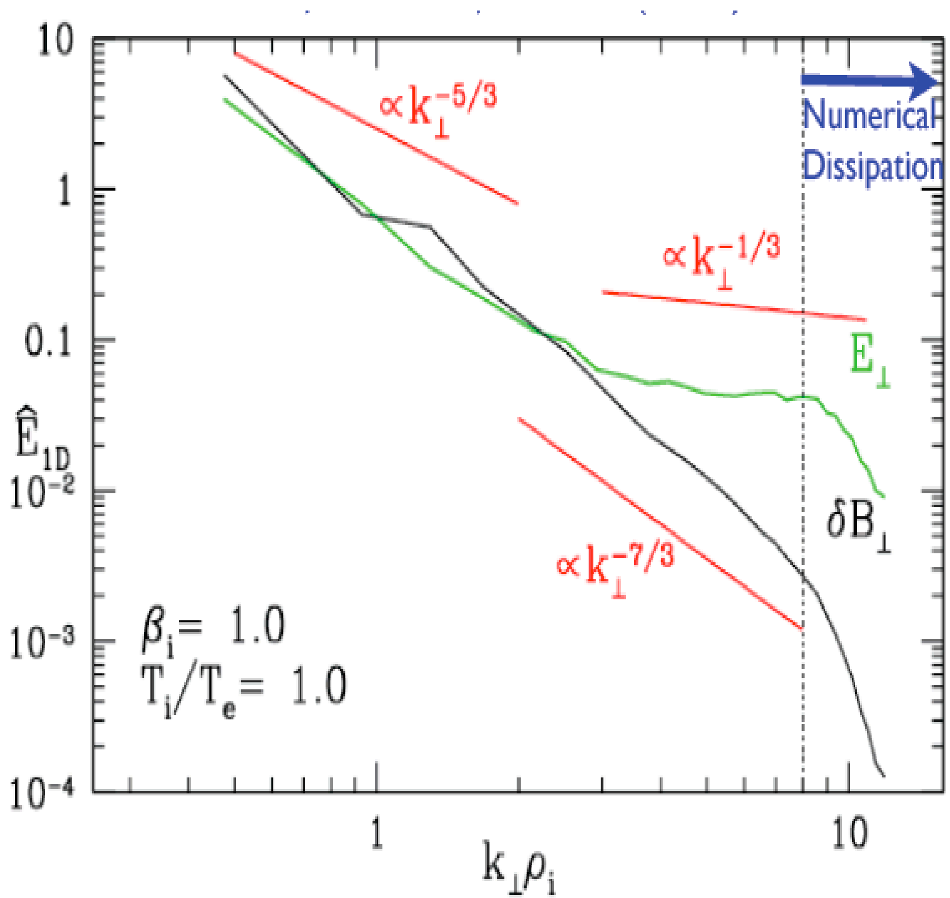

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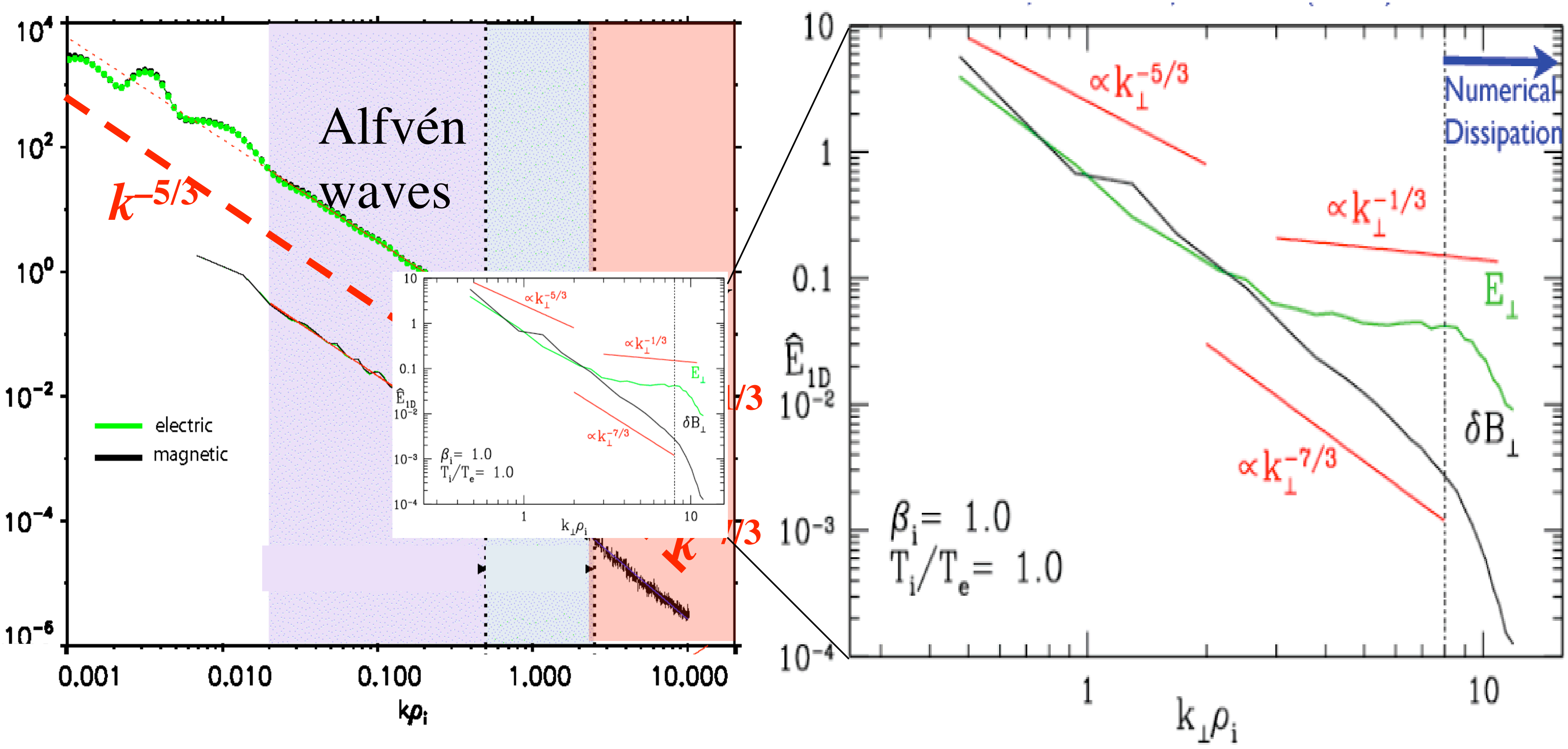

Main Points So Far

- **IDEA #1: Kinetic turbulence is a generalised energy cascade in phase space towards collisional scales**
- **IDEA #2: Cascade is anisotropic and critically balanced (linear parallel propagation scale = nonlinear perpendicular interaction scale)**
- **IDEA #3: Can be described by gyrokinetics — gyroangle averaged low frequency kinetics of Larmor rings**
- **IDEA #4: Cascade splits into various non-energy-exchanging channels in different ways, depending on scales (some of these described by fluid/hybrid models); mixing and resplitting of these subcascades at ion gyroscale determines relative heating of the two species**

Details are in these preprints: arXiv:0704.0044, 0806.1069

Further Topics

- **Alfvénic turbulence and passive compressive fluctuations in the inertial range**
- **Energetic minority ions and their heating**
- **Kinetic Alfvén wave turbulence in the "dissipation range"**
- **Entropy cascade in phase space and nonlinear phase mixing**
- **Pressure anisotropies and resulting instabilities**
- **Magnetogenesis**

- ***The answer to the general question about life, universe, and everything…***

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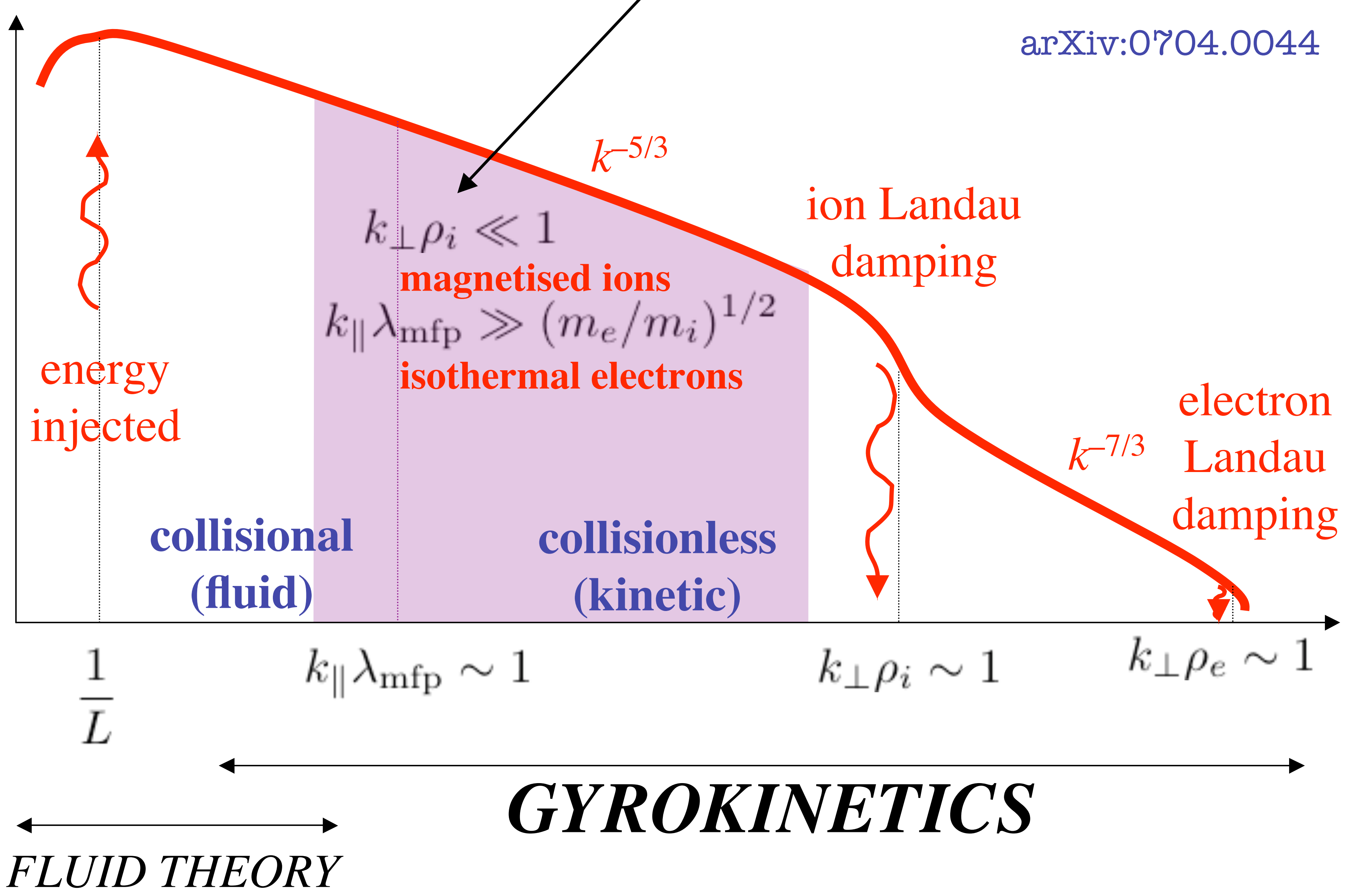
$$\sqrt{\frac{m_i}{m_e}} \approx 42$$

Details are in these preprints: arXiv:0704.0044, 0806.1069

Kinetic Reduced MHD



KRMHD: Alfvén Waves

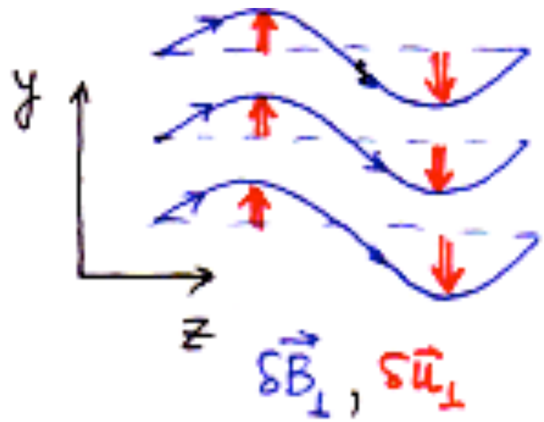
- ***Alfvénic fluctuations*** $\mathbf{u}_\perp = \hat{\mathbf{z}} \times \nabla_\perp \Phi, \quad \Phi = \frac{c}{B_0}\phi$ and $\frac{\delta \mathbf{B}_\perp}{\sqrt{4\pi\rho_0}} = \hat{\mathbf{z}} \times \nabla_\perp \Psi$

  rigourously satisfy *Reduced MHD Equations:*

$$\frac{\partial}{\partial t}\nabla_\perp^2 \Phi + \left\{\Phi, \nabla_\perp^2 \Phi\right\} = v_A \frac{\partial}{\partial z}\nabla_\perp^2 \Psi + \left\{\Psi, \nabla_\perp^2 \Psi\right\}$$

$$\frac{\partial \Psi}{\partial t} + \left\{\Phi, \Psi\right\} = v_A \frac{\partial \Phi}{\partial z}$$

[Strauss 1976, *Phys. Fluids* **19**, 134]



[Schekochihin et al., arXiv:0704.0044
cf. Higdon 1984, *ApJ* **285**, 109; Lithwick & Goldreich 2001, *ApJ* **562**, 279]

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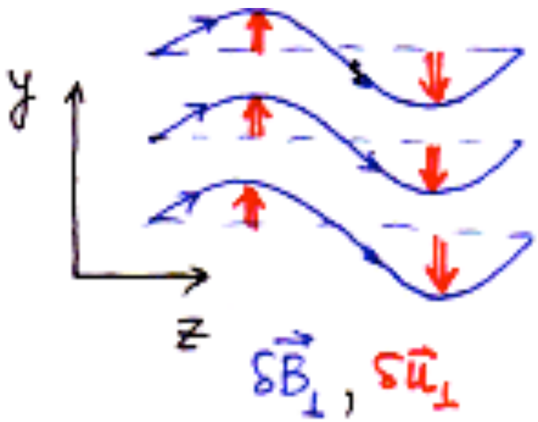
[Kadomtsev & Pogutse 1974, *Sov. Phys. JETP* 38, 283]



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[Kadomtsev & Pogutse 1974, *Sov. Phys. JETP* **38**, 283
Strauss 1976, *Phys. Fluids* **19**, 134]

- Alfvén-wave cascade is indifferent to collisions and damped only at the ion gyroscale
- The GS95 theory describes this part of the turbulence
- Alfvén waves are decoupled from density and magnetic-field-strength fluctuations (slow waves and entropy mode in the fluid limit)

[Schekochihin et al., arXiv:0704.0044
cf. Higdon 1984, *ApJ* **285**, 109; Lithwick & Goldreich 2001, *ApJ* **562**, 279]

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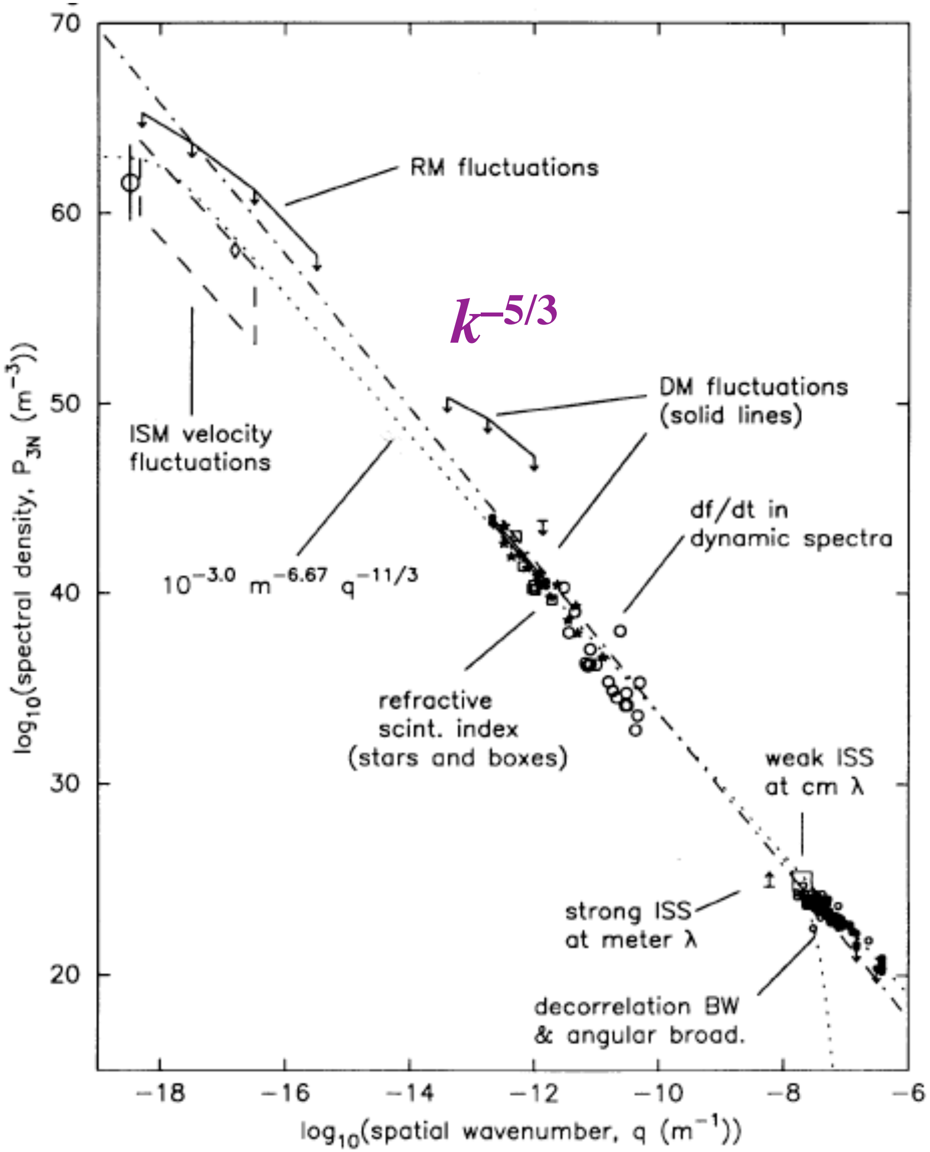
[Kadomtsev & Pogutse 1974, *Sov. Phys. JETP* **38**, 283
Strauss 1976, *Phys. Fluids* **19**, 134]

SO, <u>IDEA #5</u>:

DECOUPLED RMHD ALFVENIC CASCADE IN THE INERTIAL RANGE

[Schekochihin et al., arXiv:0704.0044
cf. Higdon 1984, *ApJ* **285**, 109; Lithwick & Goldreich 2001, *ApJ* **562**, 279]

ISM: Density Fluctuations



Electron-density fluctuations in the interstellar medium

[Armstrong *et al*. 1995, *ApJ* **443**, 209]

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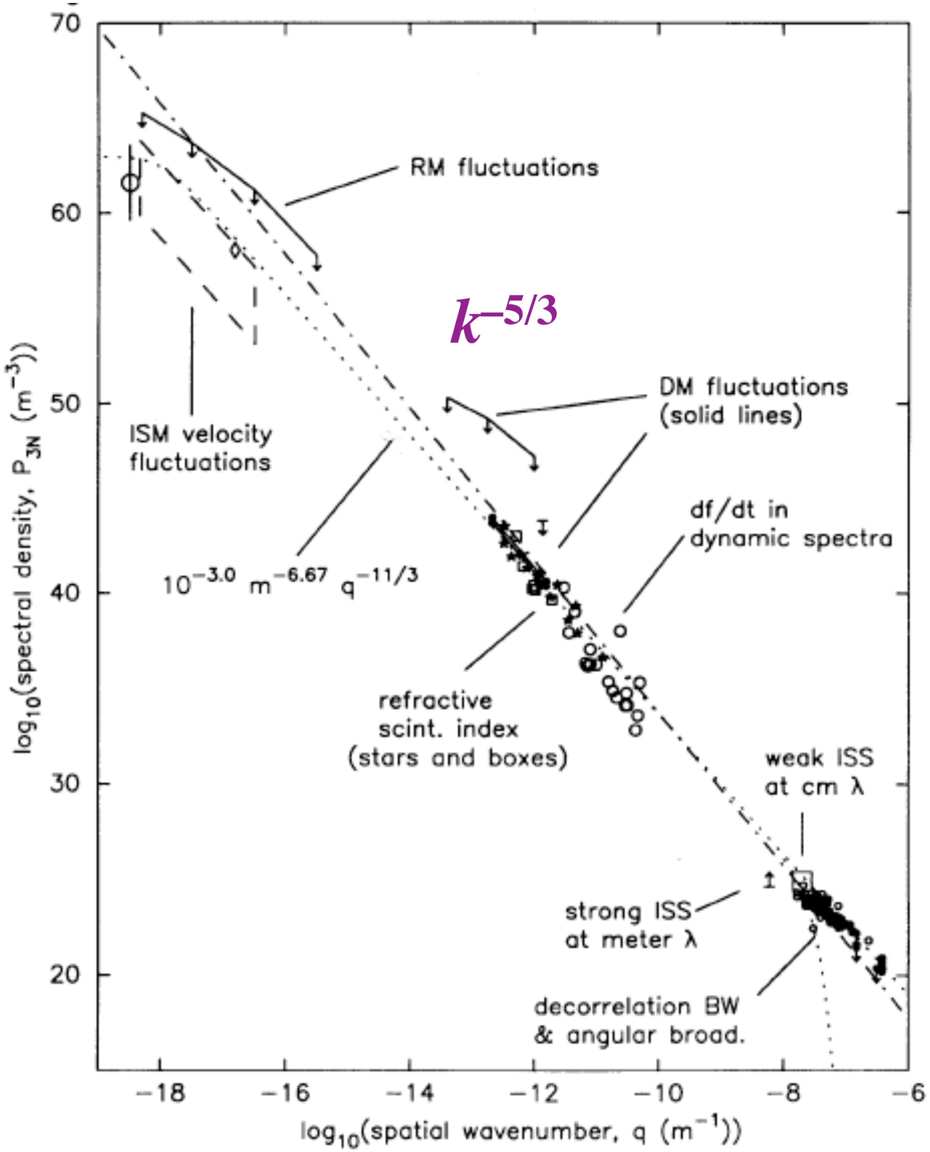

"Great Power Law In the Sky"

… coined by Steve Spangler

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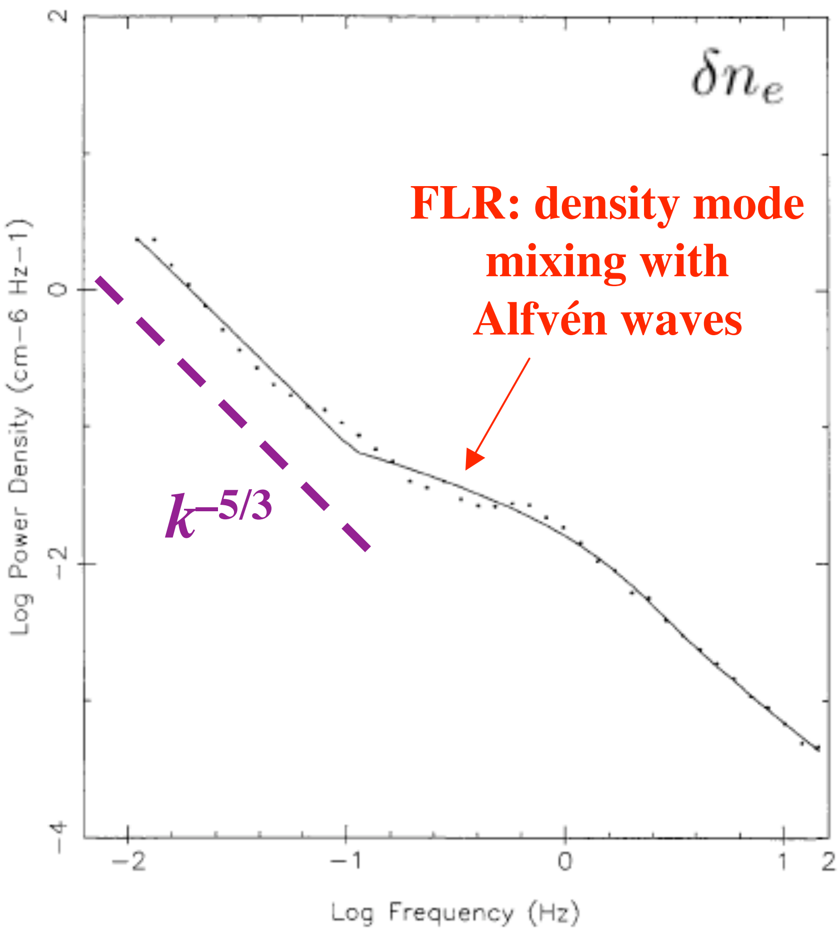
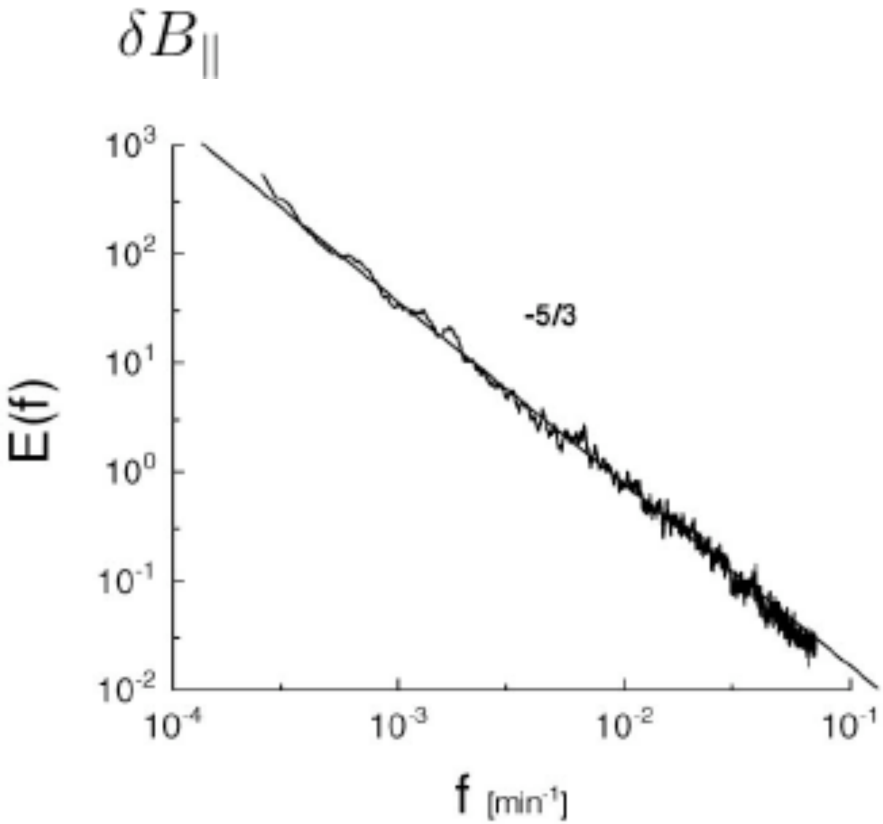
[Armstrong *et al*. 1995, *ApJ* **443**, 209]

SW: Density and Field-Strength Fluctuations



Density fluctuations in the solar wind at ~1 AU (31 Aug. 1981)
[Celnikier, Muschietti & Goldman1987, *A&A* **181**, 138]



Spectrum of magnetic-field strength in the solar wind at ~1 AU (1998)
[Bershadskii & Sreenivasan 2004, *PRL* **93**, 064501]

KRMHD: Density and Magnetic-Field Strength

δn_e and $\delta B_\parallel$ require kinetic description: our expansion gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\parallel}{B_0} f_{0i}\right) + v_\parallel \hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla}\left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0.$$

$$\frac{\delta n_e}{n_{0e}} = \frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\,\delta f_i$$

$$\frac{\delta B_\parallel}{B_0} = -\frac{\beta_i}{2}\frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\left(1 + \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\right)\delta f_i$$

Maxwellian equilibrium

KRMHD

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{u}_\perp\cdot\boldsymbol{\nabla}_\perp = \frac{\partial}{\partial t} + \{\Phi,\cdots\}$$

$$\hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla} = \frac{\partial}{\partial z} + \frac{\delta\boldsymbol{B}_\perp}{B_0}\cdot\boldsymbol{\nabla}_\perp = \frac{\partial}{\partial z} + \frac{1}{v_A}\{\Psi,\cdots\}$$

Density and field-strength fluctuations are passively mixed by Alfvén waves

[Schekochihin et al., arXiv:0704.0044
cf. Higdon 1984, *ApJ* **285**, 109; Lithwick & Goldreich 2001, *ApJ* **562**, 279]

KRMHD: Density and Magnetic-Field Strength

δn_e and $\delta B_\|$ require kinetic description: our expansion gives

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\|}{B_0} f_{0i}\right) + v_\| \hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla}\left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0,$$

$$\frac{\delta n_e}{n_{0e}} = \frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\,\delta f_i$$

$$\frac{\delta B_\|}{B_0} = -\frac{\beta_i}{2}\frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\left(1 + \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\right)\delta f_i$$

In the Lagrangian frame of the Alfvén waves…

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{u}_\perp\cdot\boldsymbol{\nabla}_\perp \to \frac{\partial}{\partial t}$$

$$\hat{\boldsymbol{b}}\cdot\boldsymbol{\nabla} = \frac{\partial}{\partial z} + \frac{\delta \boldsymbol{B}_\perp}{B_0}\cdot\boldsymbol{\nabla}_\perp \to \frac{\partial}{\partial l_0}$$

[Schekochihin et al., arXiv:0704.0044]

KRMHD: Density and Magnetic-Field Strength

δn_e and $\delta B_\parallel$ require kinetic description: our expansion gives

$$\frac{\partial}{\partial t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\parallel}{B_0} f_{0i}\right) + v_\parallel \frac{\partial}{\partial l_0}\left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0.$$

$$\frac{\delta n_e}{n_{0e}} = \frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\,\delta f_i$$

equation is linear!

$$\frac{\delta B_\parallel}{B_0} = -\frac{\beta_i}{2}\frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\left(1 + \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\right)\delta f_i$$

In the Lagrangian frame of the Alfvén waves…

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \boldsymbol{u}_\perp \cdot \boldsymbol{\nabla}_\perp \to \frac{\partial}{\partial t}$$

$$\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} = \frac{\partial}{\partial z} + \frac{\delta \boldsymbol{B}_\perp}{B_0} \cdot \boldsymbol{\nabla}_\perp \to \frac{\partial}{\partial l_0}$$

[Schekochihin et al., arXiv:0704.0044]

KRMHD: Density and Magnetic-Field Strength

δn_e and $\delta B_\parallel$ require kinetic description: our expansion gives

$$\frac{\partial}{\partial t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\parallel}{B_0} f_{0i}\right) + v_\parallel \frac{\partial}{\partial l_0}\left(\delta f_i + \frac{\delta n_e}{n_{0e}} f_{0i}\right) = 0.$$

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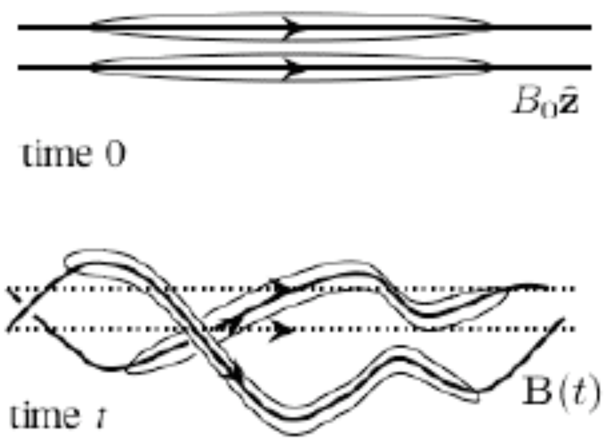
In the Lagrangian frame of the Alfvén waves…



No refinement of scale along perturbed magnetic field

(but there is along the guide field, i.e. k_z grows)

Collisionless Damping

δn_e and $\delta B_\|$ require kinetic description: our expansion gives

$$\frac{\partial}{\partial t}\left(\delta f_i - \frac{v_\perp^2}{v_{\mathrm{th}i}^2}\frac{\delta B_\|}{B_0}f_{0i}\right) + v_\| \frac{\partial}{\partial l_0}\left(\delta f_i + \frac{\delta n_e}{n_{0e}}f_{0i}\right) = 0.$$

$$\frac{\delta n_e}{n_{0e}} = \frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\,\delta f_i$$

equation is linear!

$$\frac{\delta B_\|}{B_0} = -\frac{\beta_i}{2}\frac{1}{n_{0i}}\int \mathrm{d}^3\boldsymbol{v}\left(1+\frac{v_\perp^2}{v_{\mathrm{th}i}^2}\right)\delta f_i$$

For $\beta_i \sim 1$, $\gamma \sim k_{\|0}v_{\mathrm{th}i} \sim k_{\|0}v_A \ll k_\| v_A$

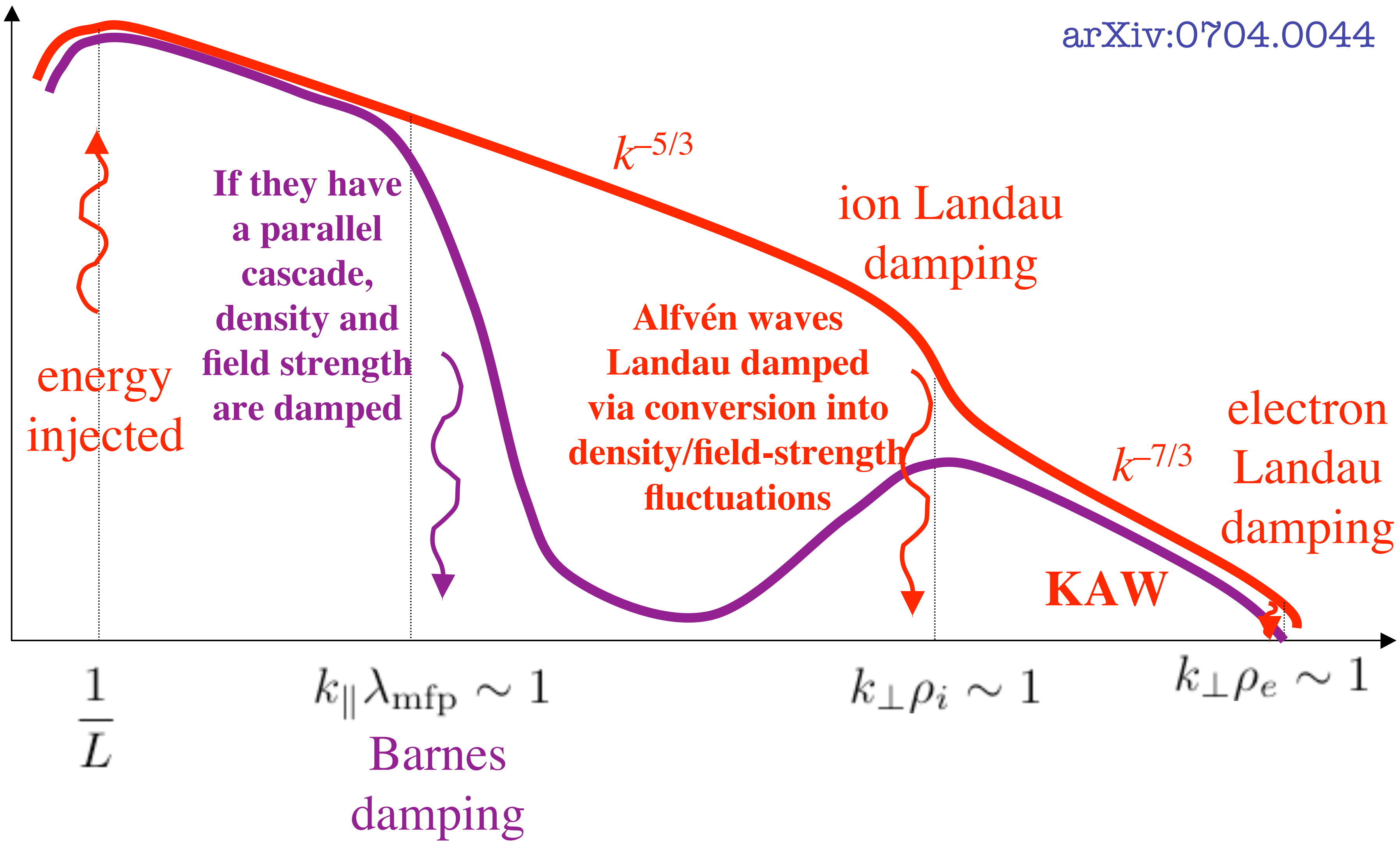
[Barnes 1966, *Phys. Fluids* **9**, 1483]

time to be cascaded in $k_\perp$ by Alfvén waves, for which $k_\| \sim k_\perp^{2/3}$

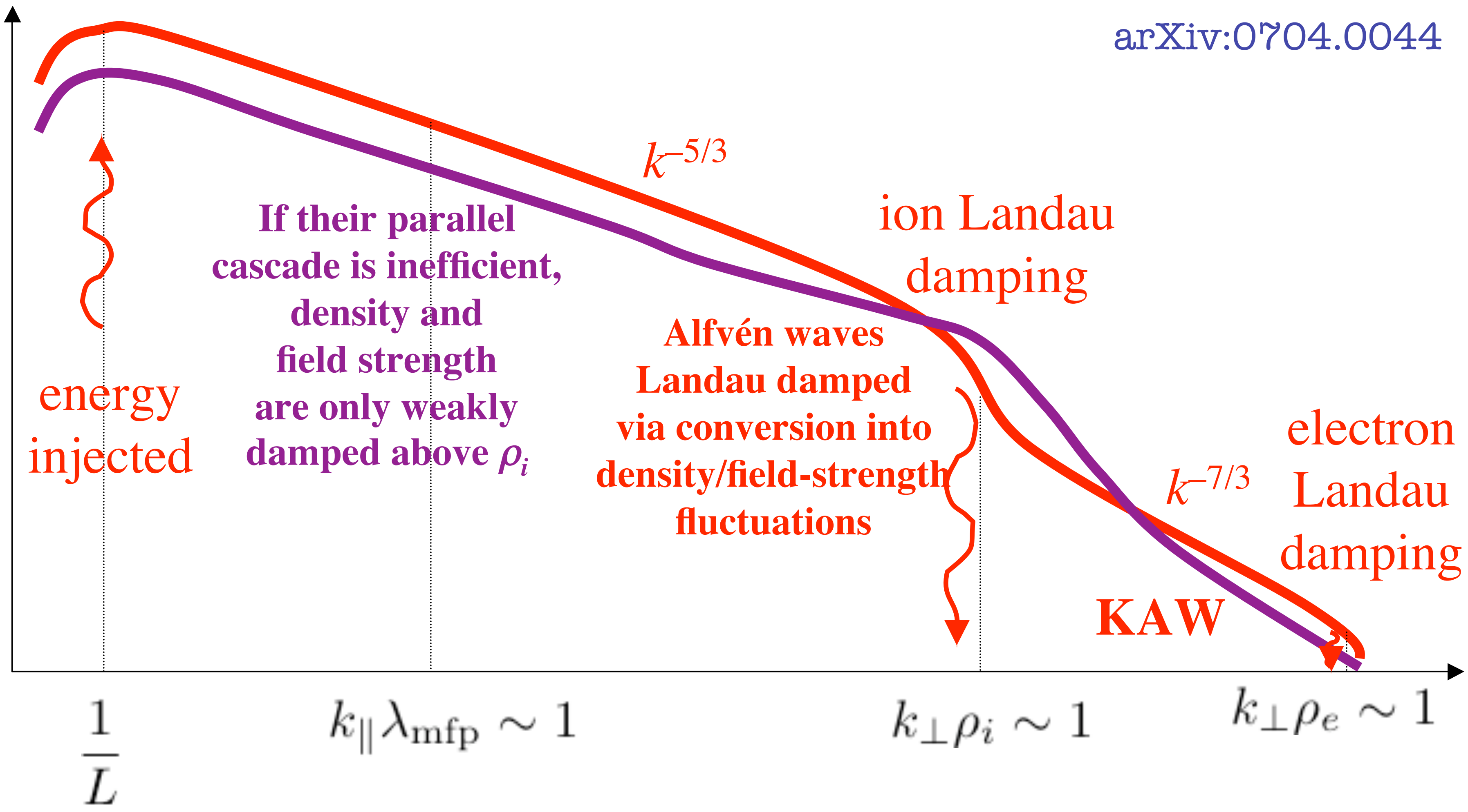
Cascades of density and field strength fluctuations are undamped above ion gyroscale

… but parallel cascade might be induced due to dissipation
[Lithwick & Goldreich 2001, *ApJ* **562**, 279]

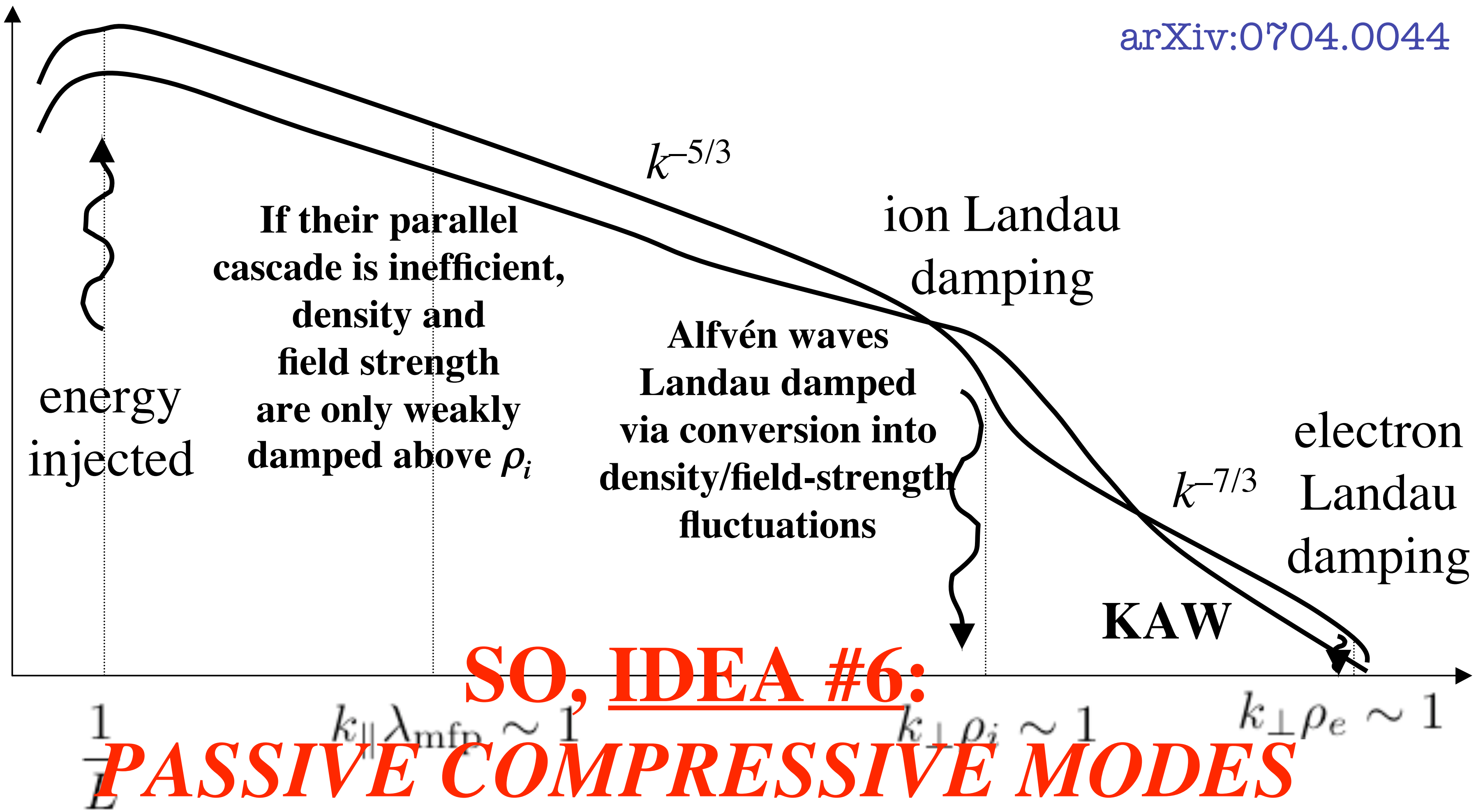
Damping of Cascades



Damping of Cascades



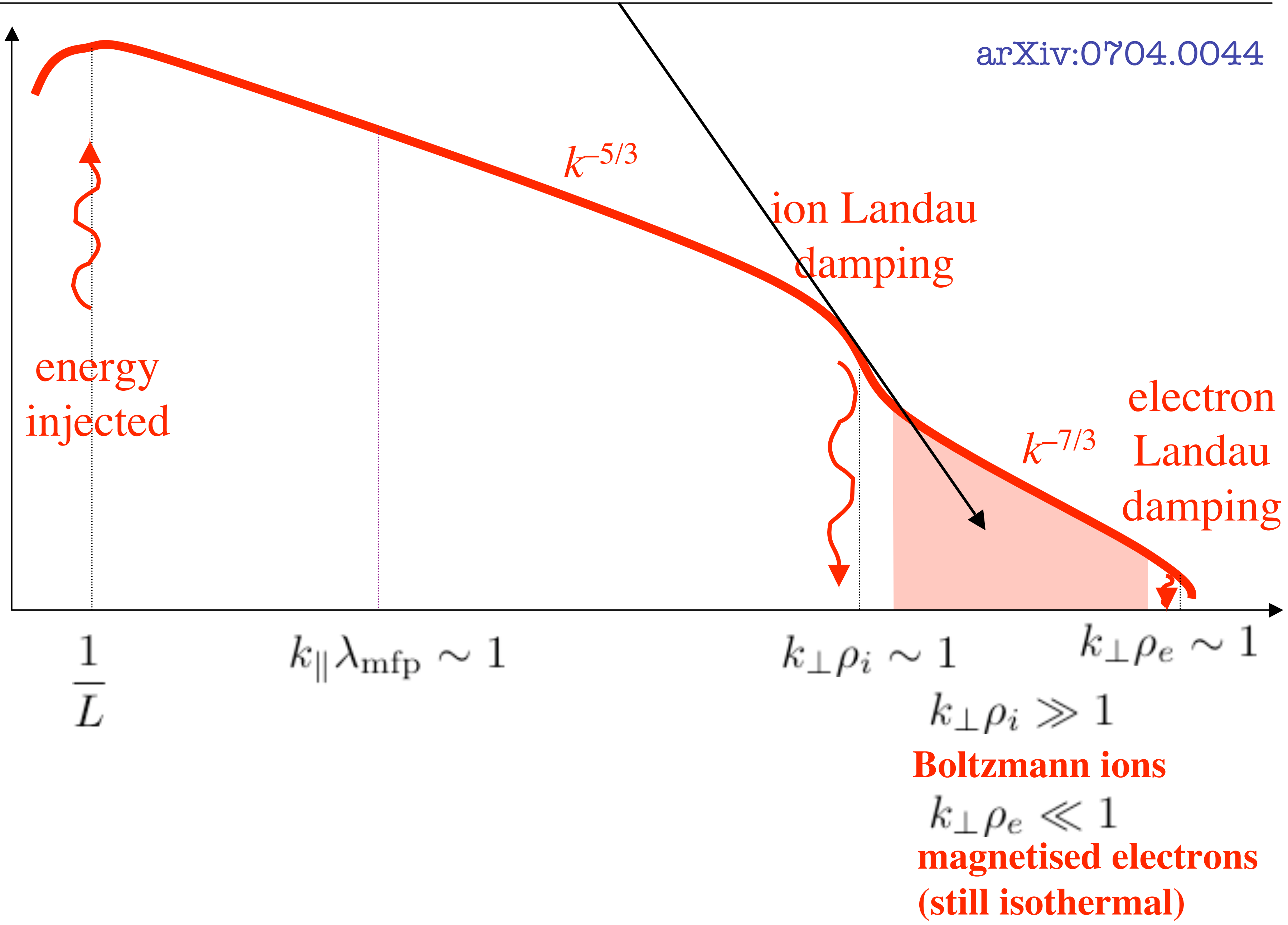
Damping of Cascades



SO, <u>IDEA #6</u>:

PASSIVE COMPRESSIVE MODES IN THE INERTIAL RANGE WITH NO PARALLEL CASCADE?

Electron Reduced MHD



Electron Reduced MHD

Start with GK, consider the scales such that $k_\perp \rho_i \gg 1,\; k_\perp \rho_e \ll 1$

$$\frac{\partial \Psi}{\partial t} = v_A \left(1+Z/\tau\right) \hat{\mathbf{b}}\cdot\nabla\Phi,$$

$$\frac{\partial \Phi}{\partial t} = -\frac{v_A}{2+\beta_i\left(1+Z/\tau\right)} \hat{\mathbf{b}}\cdot\nabla\left(\rho_i^2\nabla_\perp^2\Psi\right)$$

$$\frac{\delta n_e}{n_{0e}} = -\frac{Ze\phi}{T_{0i}} = -\frac{2}{\sqrt{\beta_i}}\frac{\Phi}{\rho_i v_A},$$

$$\frac{\delta B_\parallel}{B_0} = \frac{\beta_i}{2}\left(1+\frac{Z}{\tau}\right)\frac{Ze\phi}{T_{0i}} = \sqrt{\beta_i}\left(1+\frac{Z}{\tau}\right)\frac{\Phi}{\rho_i v_A}$$

$$u_{\parallel e} = \frac{c}{4\pi e n_{0e}}\nabla_\perp^2 A_\parallel = -\frac{\rho_i \nabla_\perp^2\Psi}{\sqrt{\beta_i}}$$

This is the anisotropic version of EMHD [Kingsep *et al*. 1990, *Rev. Plasma Phys*. **16**, 243], which is derived (for β_I>>1) by assuming magnetic field frozen into electron fluid and doing a RMHD-style anisotropic expansion:

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c}{4\pi e n_{0e}}\nabla\times\left[(\nabla\times\mathbf{B})\times\mathbf{B}\right]$$

$$\frac{\delta\mathbf{B}}{B_0} = \frac{1}{v_A}\hat{\mathbf{z}}\times\nabla_\perp\Psi + \hat{\mathbf{z}}\frac{\delta B_\parallel}{B_0}$$

arXiv:0704.0044

Kinetic Alfvén Waves

Start with GK, consider the scales such that $k_\perp \rho_i \gg 1,\ k_\perp \rho_e \ll 1$

$$\frac{\partial \Psi}{\partial t} = v_A \left(1+Z/\tau\right) \hat{\mathbf{b}} \cdot \nabla \Phi,$$
$$\frac{\partial \Phi}{\partial t} = -\frac{v_A}{2+\beta_i\left(1+Z/\tau\right)} \hat{\mathbf{b}} \cdot \nabla \left(\rho_i^2 \nabla_\perp^2 \Psi\right)$$

Linear wave solutions:

$$\omega = \pm \sqrt{\frac{1+Z/\tau}{2+\beta_i\left(1+Z/\tau\right)}}\, k_\perp \rho_i k_\parallel v_A$$

Eigenfunctions:

$$\Theta_{\mathbf{k}}^{\pm} = \sqrt{\left(1+Z/\tau\right)\left[2+\beta_i\left(1+Z/\tau\right)\right]}\, \Phi_{\mathbf{k}} \mp k_\perp \rho_i \Psi_{\mathbf{k}}$$

- There is a cascade of KAW,

$$\delta B_\parallel / B_0 \sim \Phi / \rho_i v_A \sim k_\perp \Psi / v_A \sim \delta B_\perp / B_0$$

- Critical balance + constant flux argument à la K41/GS95 give $k_\perp^{-7/3}$ spectrum of magnetic field with anisotropy $k_\parallel \sim k_\perp^{1/3}$

[Biskamp et al. 1996, *PRL* **76**, 1264; Cho & Lazarian 2004, *ApJ* **615**, L41]

- Electric field has $k_\perp^{-1/3}$ spectrum: $\delta E \sim k_\perp \phi \sim k_\perp \rho_i (v_A/c) \delta B$

arXiv:0704.0044

Kinetic Alfvén Waves

Start with GK, consider the scales such that $k_\perp \rho_i \gg 1,\ k_\perp \rho_e \ll 1$

$$\frac{\partial \Psi}{\partial t} = v_A \left(1 + Z/\tau\right) \hat{\mathbf{b}} \cdot \nabla \Phi,$$
$$\frac{\partial \Phi}{\partial t} = -\frac{v_A}{2 + \beta_i \left(1 + Z/\tau\right)} \hat{\mathbf{b}} \cdot \nabla \left(\rho_i^2 \nabla_\perp^2 \Psi\right)$$

Linear wave solutions:

$$\omega = \pm \sqrt{\frac{1 + Z/\tau}{2 + \beta_i \left(1 + Z/\tau\right)}}\, k_\perp \rho_i k_\parallel v_A$$

Eigenfunctions:

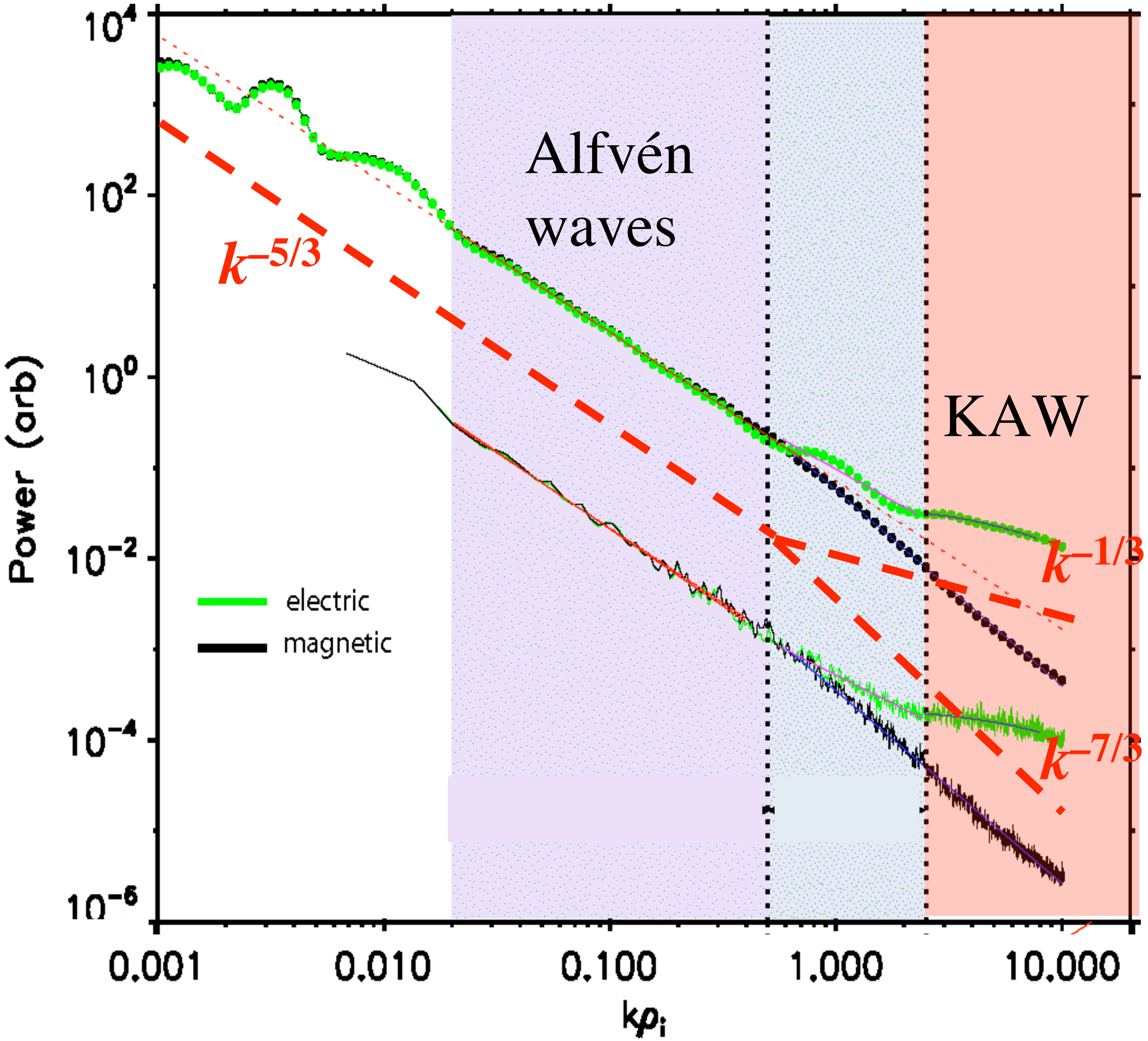
$$\Theta_{\mathbf{k}}^{\pm} = \sqrt{\left(1 + Z/\tau\right) \left[2 + \beta_i \left(1 + Z/\tau\right)\right]}\, \Phi_{\mathbf{k}} \mp k_\perp \rho_i \Psi_{\mathbf{k}}$$

SO, IDEA #7:

CRITICALLY BALANCED KAW CASCADE IN THE DISSIPATION RANGE

arXiv:0704.0044

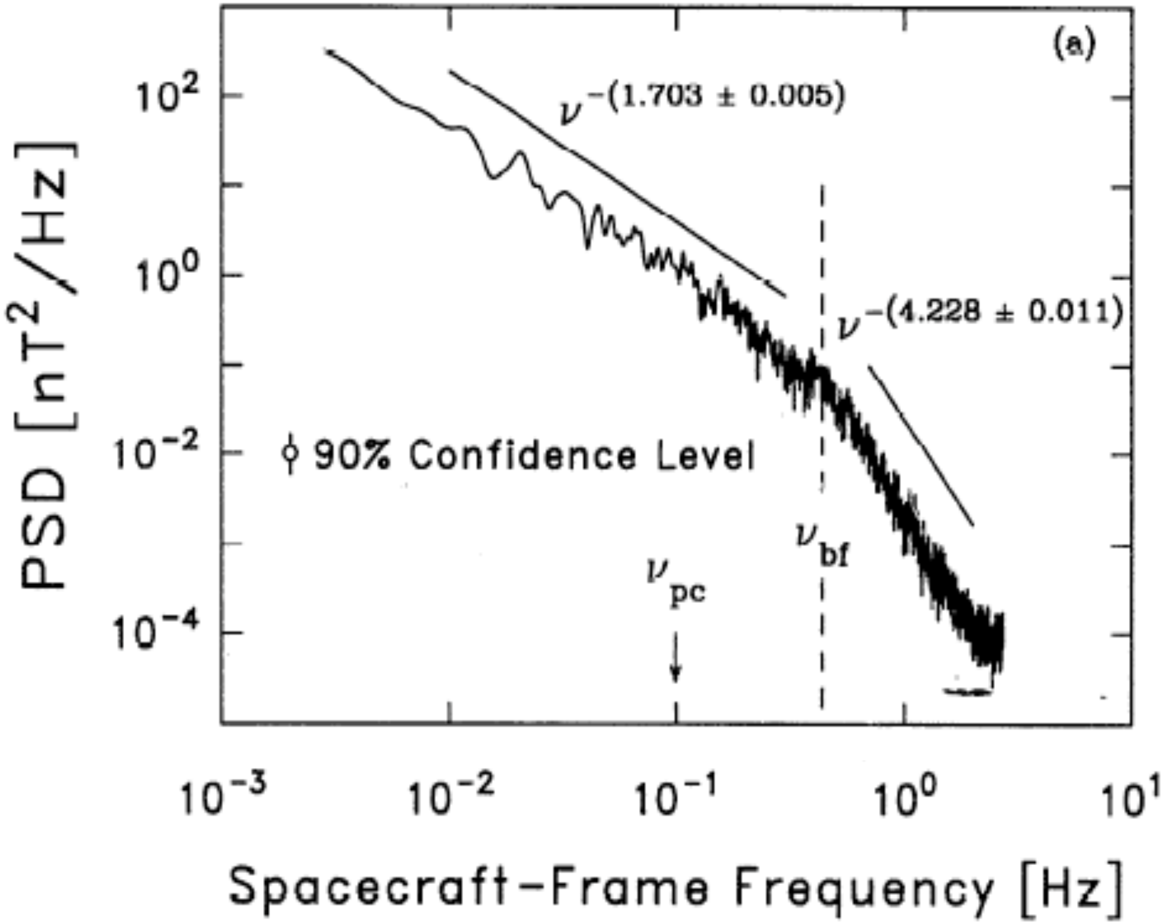
Dissipation Range of the SW: KAW?



Magnetic- and electric-field fluctuations in the solar wind at ~1 AU (19 Feb. 2002)

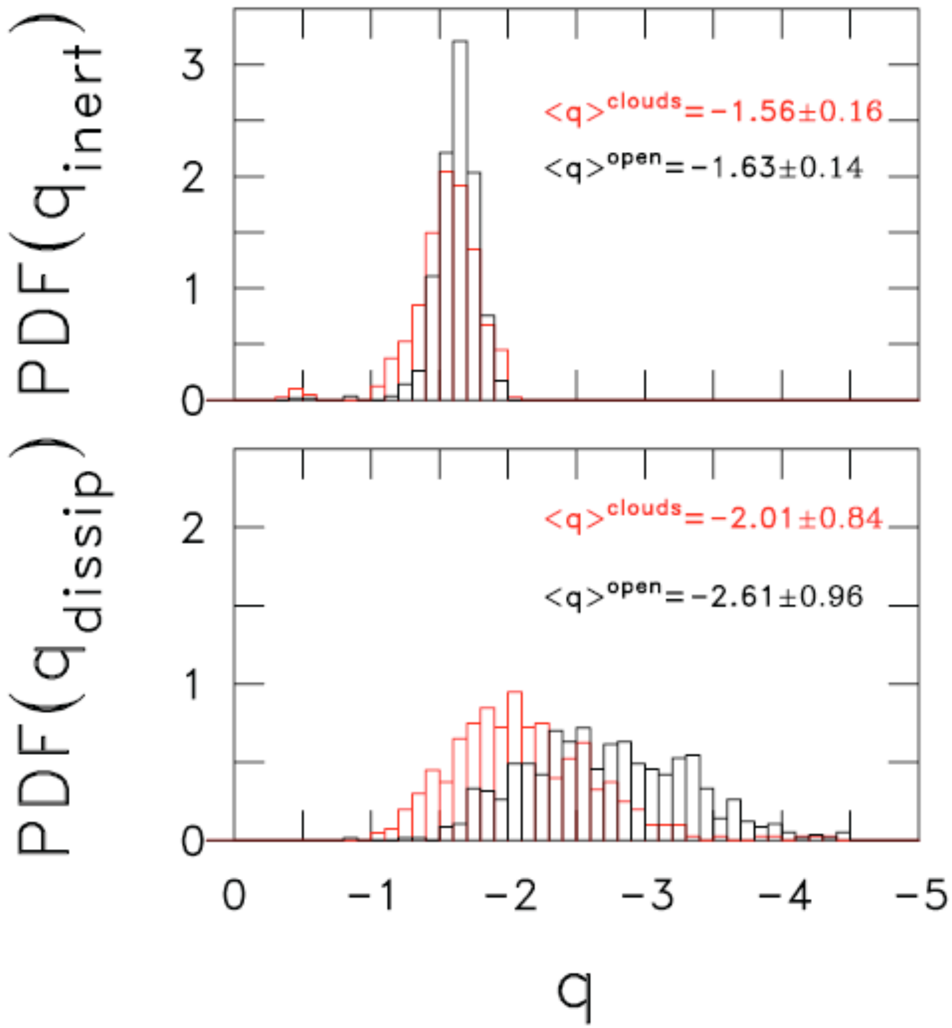
[Bale *et al*. 2005, *PRL* **94**, 215002]

Dissipation Range of the SW: No KAW?



Magnetic-field fluctuations in the solar wind at ~1 AU (19 Feb. 2002)
[Leamon *et al*. 1998, *JGR* **103**, 4775]

Dissipation Range of the SW: ???



Spectral indices in the inertial and dissipation ranges

[Smith *et al*. 2006, *ApJ* **645**, L85]

Nonlinear Perpendicular Phase Mixing

IDEA #8:

DUAL (ION) ENTROPY CASCADE IN VELOCITY AND POSITION SPACE

arXiv:0806.1069

Nonlinear Perpendicular Phase Mixing

$$\frac{\partial h_i}{\partial t} + v_\| \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\mathrm{c}} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1 + \frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{n_{0i}} \int \mathrm{d}^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

Low-frequency electrostatic fluctuations

This comes from gyroaveraging

NB: In fluid models (like EMHD) these fluctuations are invisible

arXiv:0806.1069

Nonlinear Perpendicular Phase Mixing

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\mathrm{c}} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1 + \frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{n_{0i}} \int d^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

Low-frequency electrostatic fluctuations

- Potential mixes h_i via this term, so h_i developes small (perpendicular) scales in the gyrocenter space: $k_\perp \rho_i \gg 1$

arXiv:0806.1069

Nonlinear Perpendicular Phase Mixing

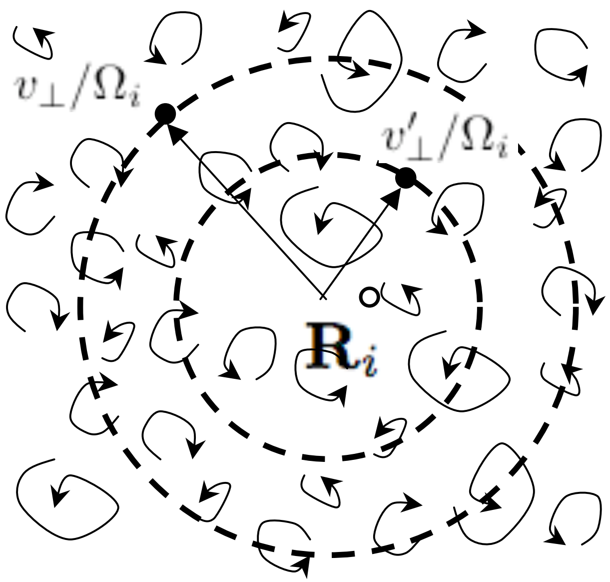
$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\mathrm{c}} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1+\frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{n_{0i}} \int \mathrm{d}^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

Low-frequency electrostatic fluctuations

- Potential mixes h_i via this term, so h_i developes small (perpendicular) scales in the gyrocenter space: $k_\perp \rho_i \gg 1$
- Two values of the gyroaveraged potential $\langle\varphi\rangle_{\mathbf{R}_i}(\mathbf{v})$ and $\langle\varphi\rangle_{\mathbf{R}_i}(\mathbf{v}')$ come from spatially decorrelated fluctuations if

$$\frac{v_\perp}{\Omega_i} - \frac{v'_\perp}{\Omega_i} \sim \frac{1}{k_\perp} \Rightarrow \frac{\delta v_\perp}{v_{\mathrm{th}i}} \sim \frac{1}{k_\perp \rho_i}$$



[The perpendicular nonlinear phase-mixing mechanism was anticipated in the work of Dorland & Hammett 1993]

arXiv:0806.1069

Entropy Cascade

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\rm c} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1 + \frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{n_{0i}} \int d^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

Low-frequency electrostatic fluctuations

- Electrostatic fluctuations come from ion-entropy fluctuations:

$$\frac{Ze\varphi(\boldsymbol{k})}{T_{0i}} \sim \frac{v_{\rm th\mathit{i}}^3}{n_{0i}} \frac{1}{\sqrt{k_\perp \rho_i}} \left(\frac{\delta v_\perp}{v_{\rm th\mathit{i}}}\right)^{1/2} h_i(\boldsymbol{k}) \sim \frac{v_{\rm th\mathit{i}}^3}{n_{0i}} \frac{h_i(\boldsymbol{k})}{k_\perp \rho_i}$$

- Entropy is conserved, so use const-flux argument:

$$\frac{m_i v_{\rm th\mathit{i}}^8}{n_{0i}} \frac{h_{i\lambda}^2}{\tau_\lambda} \sim \varepsilon$$

- Nonlinear decorrelation time:

$$\tau_\lambda \sim \left(\frac{\rho_i}{\lambda}\right)^{1/2} \frac{\lambda^2}{c\varphi_\lambda / B_0}$$

arXiv:0806.1069

Entropy Cascade

$$\frac{\partial h_i}{\partial t} + v_\parallel \frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\rm c} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1+\frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{1}{n_{0i}}\int d^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

Low-frequency electrostatic fluctuations

- Electrostatic fluctuations come from ion-entropy fluctuations:

$$\frac{Ze\varphi(\boldsymbol{k})}{T_{0i}} \sim \frac{v_{\rm th\it i}^3}{n_{0i}}\frac{1}{\sqrt{k_\perp\rho_i}}\left(\frac{\delta v_\perp}{v_{\rm th\it i}}\right)^{1/2} h_i(\boldsymbol{k}) \sim \frac{v_{\rm th\it i}^3}{n_{0i}}\frac{h_i(\boldsymbol{k})}{k_\perp\rho_i}$$

- Entropy is conserved, so use const-flux argument:

$$\frac{m_i v_{\rm th\it i}^8}{n_{0i}}\frac{h_{i\lambda}^2}{\tau_\lambda} \sim \varepsilon$$

$$c\varphi_\lambda/B_0 \sim v_{\rm th\it i}^4 h_{i\lambda}\lambda/n_{0i}$$

- Nonlinear decorrelation time:

$$\tau_\lambda \sim \left(\frac{\rho_i}{\lambda}\right)^{1/2}\frac{\lambda^2}{c\varphi_\lambda/B_0} \sim \frac{\rho_i^{1/2}\lambda^{1/2}n_{0i}}{v_{\rm th\it i}^4 h_{i\lambda}}$$

arXiv:0806.1069

Entropy Cascade

$$\frac{\partial h_i}{\partial t} + v_{\parallel}\frac{\partial h_i}{\partial z} + \frac{c}{B_0}\{\langle\varphi\rangle_{\boldsymbol{R}_i}, h_i\} - \left(\frac{\partial h_i}{\partial t}\right)_{\rm c} = \frac{\partial}{\partial t}\frac{Ze\langle\varphi\rangle_{\boldsymbol{R}_i}}{T_{0i}} F_{0i}$$

$$\left(1+\frac{\tau}{Z}\right)\frac{Ze\varphi}{T_{0i}} = \sum_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\frac{1}{n_{0i}}\int d^3\boldsymbol{v}\, J_0\left(\frac{k_\perp v_\perp}{\Omega_i}\right) h_i(\boldsymbol{k})$$

Low-frequency electrostatic fluctuations

We get the following set of scaling relations:

$l_0 = m_i n_{0i} v_{\rm thi}^3/\varepsilon$

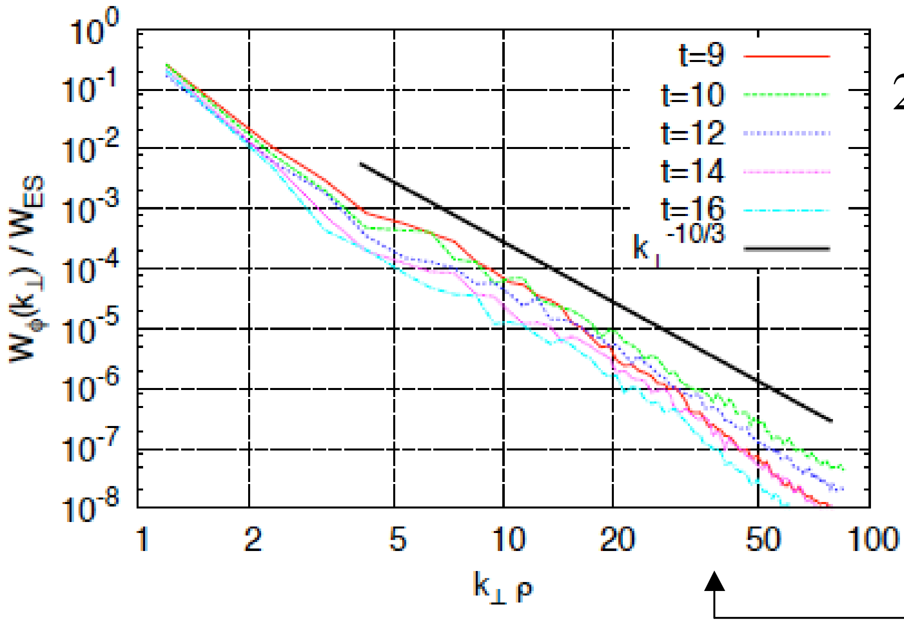
$$\frac{Ze\varphi_\lambda}{T_{0i}} \sim \frac{\lambda^{7/6}}{\rho_i^{5/6} l_0^{1/3}} \quad \Rightarrow \text{spectrum} \sim k_\perp^{-10/3}$$

$$h_{i\lambda} \sim \frac{n_{0i}}{v_{\rm thi}^3}\frac{\rho_i^{1/6}\lambda^{1/6}}{l_0^{1/3}} \quad \Rightarrow \text{spectrum} \sim k_\perp^{-4/3}$$

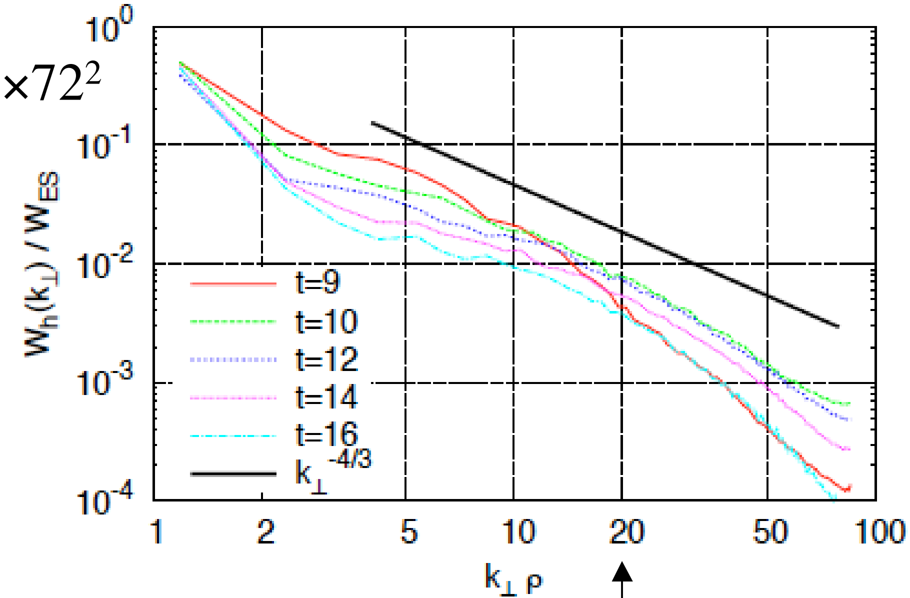
$$\tau_\lambda \sim \frac{l_0^{1/3}\rho_i^{1/3}\lambda^{1/3}}{v_{\rm thi}}$$

arXiv:0806.1069

Entropy Cascade: GK 4D DNS by T. Tatsuno



$256^2 \times 72^2$



$l_0 = m_i n_{0i} v_{\mathrm{th}i}^3/\varepsilon$

$$\frac{Ze\varphi_\lambda}{T_{0i}} \sim \frac{\lambda^{7/6}}{\rho_i^{5/6} l_0^{1/3}} \quad \Rightarrow \text{ spectrum} \sim k_\perp^{-10/3}$$

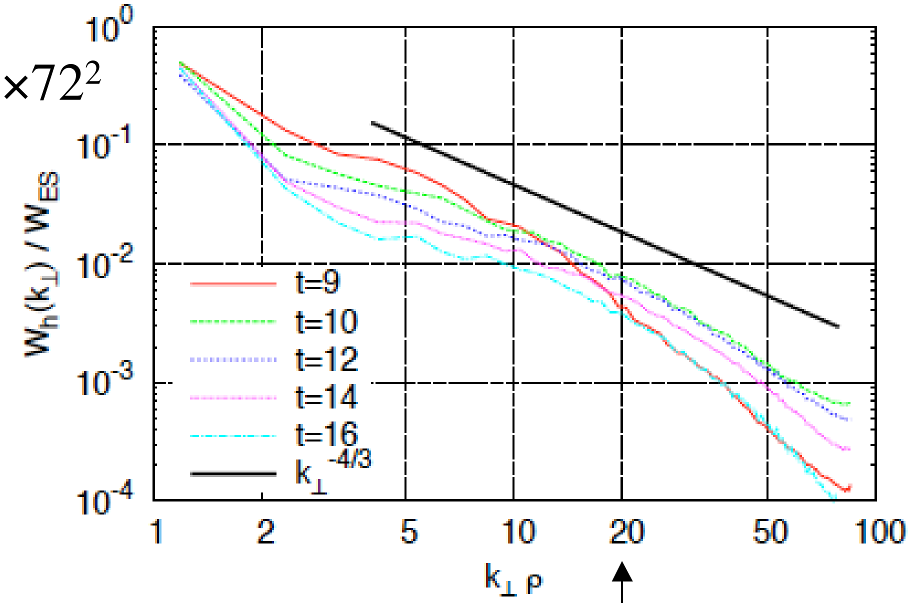
$$h_{i\lambda} \sim \frac{n_{0i}}{v_{\mathrm{th}i}^3} \frac{\rho_i^{1/6} \lambda^{1/6}}{l_0^{1/3}} \quad \Rightarrow \text{ spectrum} \sim k_\perp^{-4/3}$$

$$\tau_\lambda \sim \frac{l_0^{1/3} \rho_i^{1/3} \lambda^{1/3}}{v_{\mathrm{th}i}}$$

arXiv:0806.1069

Entropy Cascade: GK 4D DNS by T. Tatsuno

$256^2 \times 72^2$



Similar (density) spectra also reported in 3D ITG/ETG tokamak flux-tube GK simulations by Görler & Jenko (2008)

$$l_0 = m_i n_{0i} v_{\text{th}i}^3/\varepsilon$$

$$\frac{Ze\varphi_\lambda}{T_{0i}} \sim \frac{\lambda^{7/6}}{\rho_i^{5/6} l_0^{1/3}} \quad \Rightarrow \text{spectrum} \sim k_\perp^{-10/3}$$

$$h_{i\lambda} \sim \frac{n_{0i}}{v_{\text{th}i}^3} \frac{\rho_i^{1/6}\lambda^{1/6}}{l_0^{1/3}} \quad \Rightarrow \text{spectrum} \sim k_\perp^{-4/3}$$

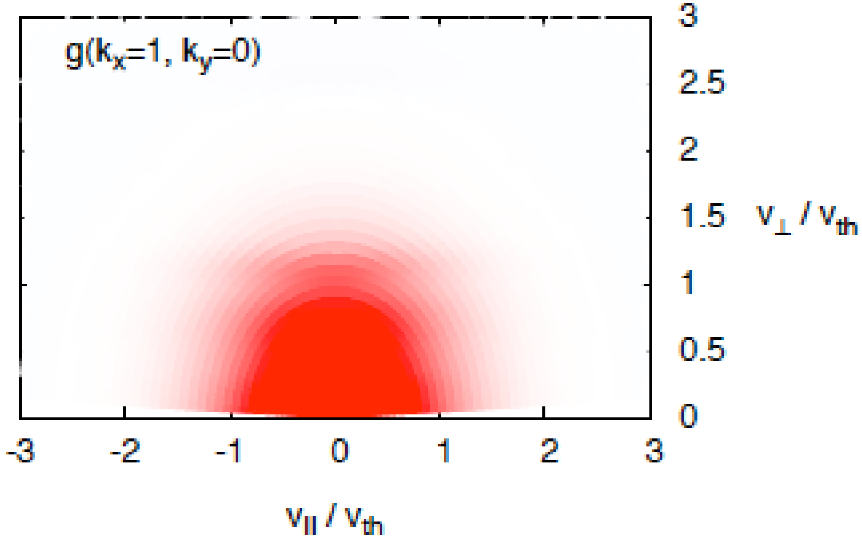
$$\tau_\lambda \sim \frac{l_0^{1/3}\rho_i^{1/3}\lambda^{1/3}}{v_{\text{th}i}}$$

arXiv:0806.1069

Entropy Cascade: GK 4D DNS by T. Tatsuno

Distribution function develops small-scale structure in velocity space

$$\frac{\delta v_\perp}{v_{\mathrm{th}i}} \sim \left(\frac{\nu_{ii}}{\omega}\right)^{1/2} \sim \frac{1}{k_\perp \rho_i}$$



arXiv:0806.1069

Entropy Cascade: GK 4D DNS by T. Tatsuno

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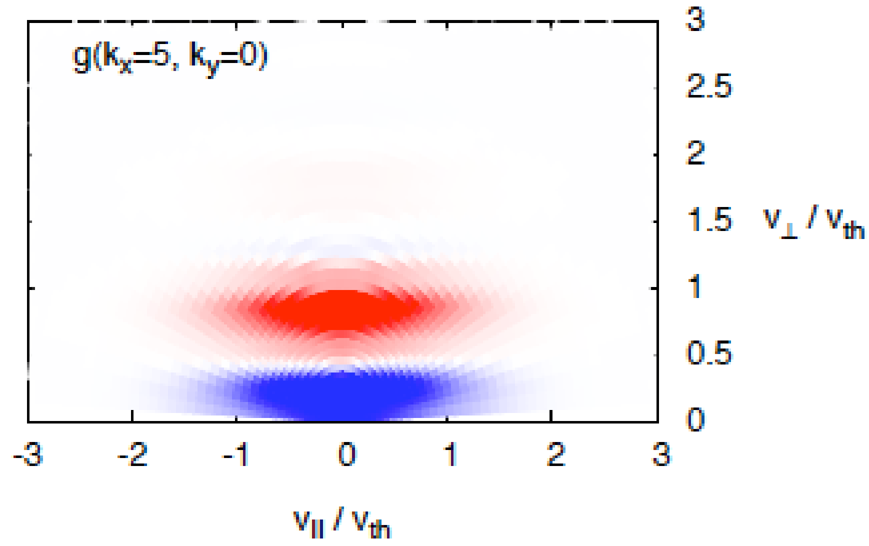

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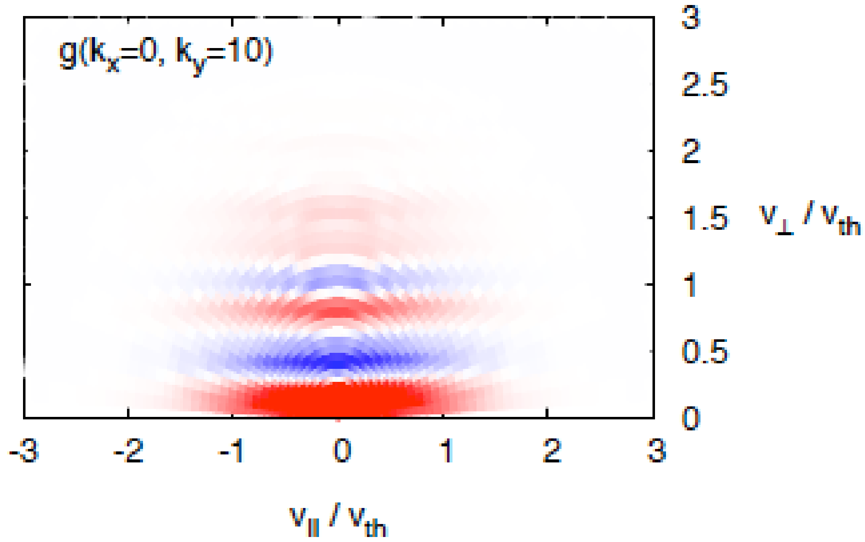

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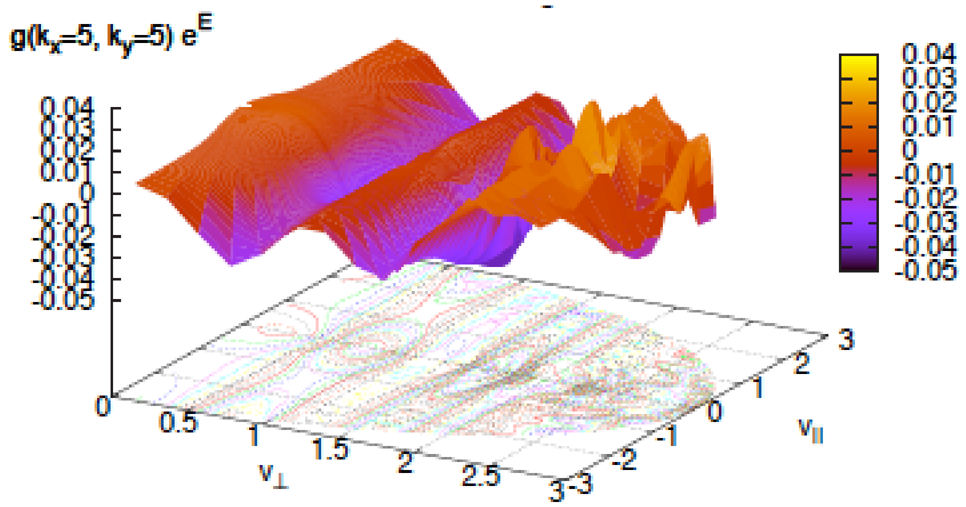

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G. Plunk has developed a *"kinematics of phase-space turbulence"* to quantify perpendicular velocity-space structure via Hankel transforms and derived scaling relations à la K41

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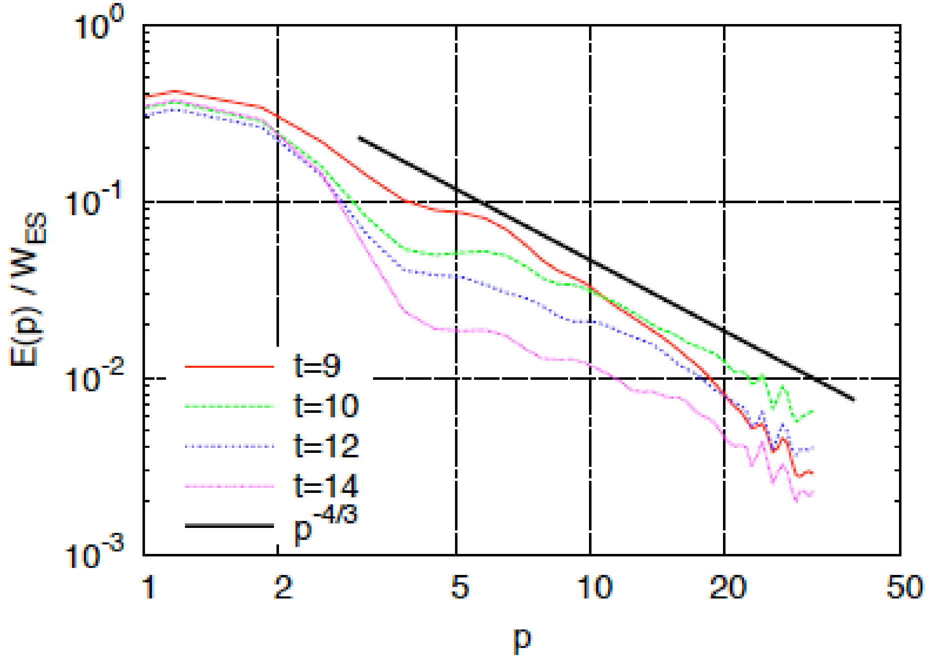

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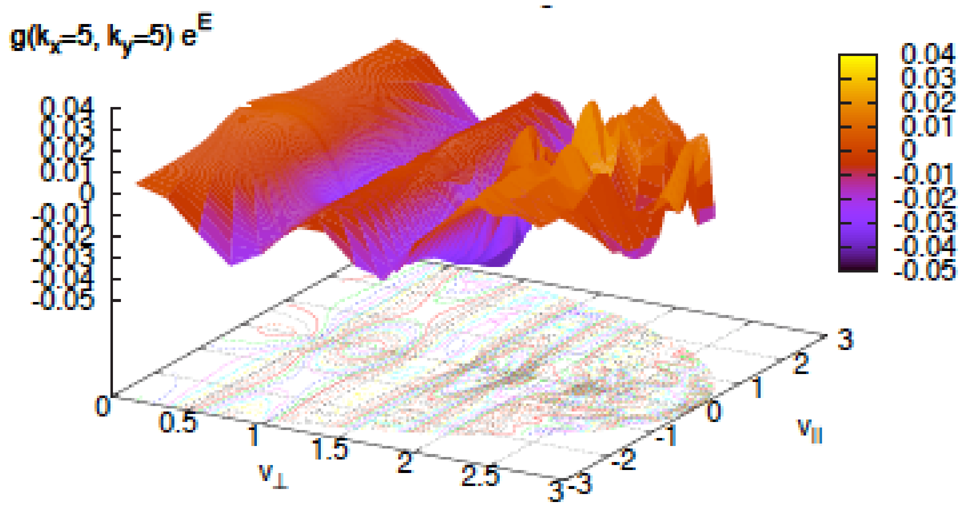
Distribution function develops small-scale structure in velocity space



$$\frac{\delta v_\perp}{v_{\mathrm{th}i}} \sim \left(\frac{\nu_{ii}}{\omega}\right)^{1/2} \sim \frac{1}{k_\perp \rho_i}$$

$$\tau_\lambda \sim \frac{l_0^{1/3} \rho_i^{1/3} \lambda^{1/3}}{v_{\mathrm{th}i}}$$

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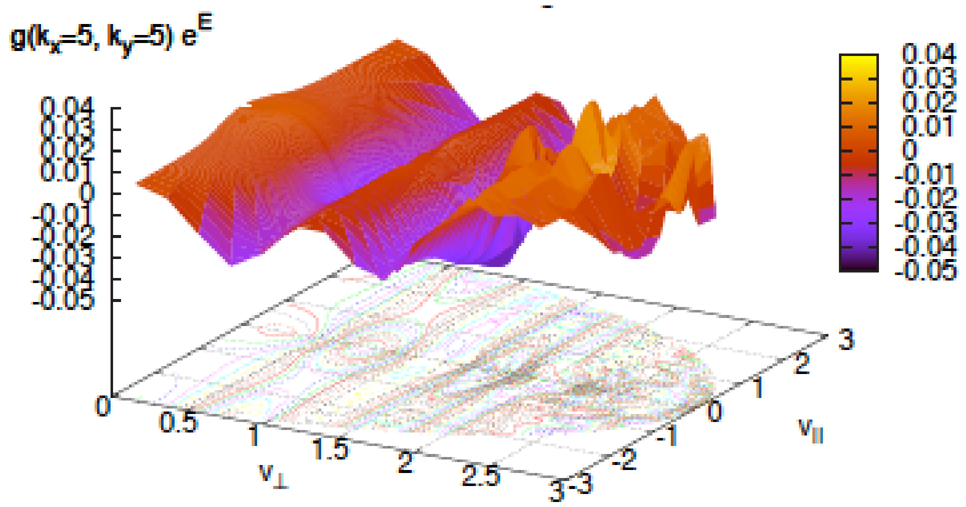
Distribution function develops small-scale structure in velocity space



$$\frac{\delta v_{\perp c}}{v_{\mathrm{th}i}} \sim \frac{1}{k_{\perp c}\rho_i} \sim (\nu_{ii}\tau_{\rho_i})^{3/5} \sim \frac{l_0^{1/5}\rho_i^{2/5}}{\lambda_{\mathrm{mfp}}^{3/5}}$$

$$\tau_{\rho_i} \sim (m_i n_{0i}\rho_i^2/\varepsilon)^{1/3}$$ characteristic time at the ion gyroscale

$$l_0 = m_i n_{0i} v_{\mathrm{th}i}^3/\varepsilon$$

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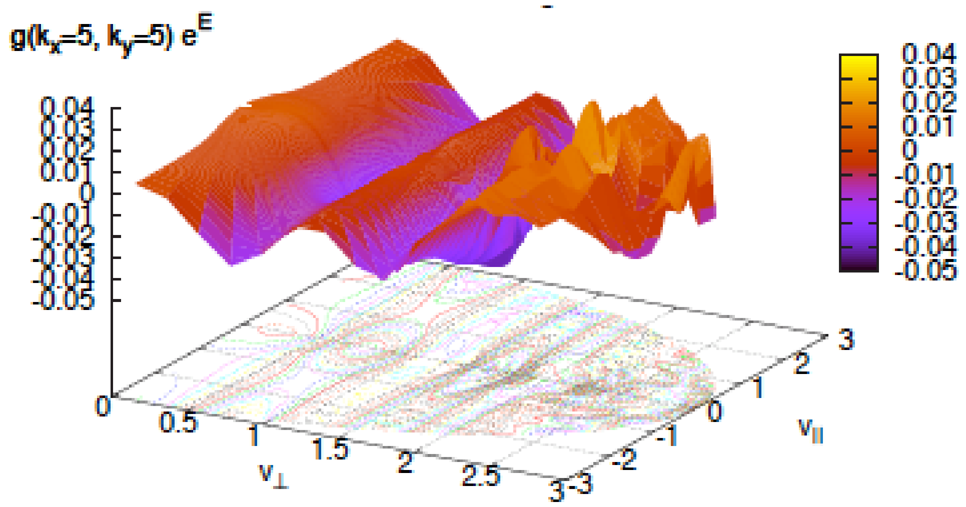
Phase-Space Cutoff

Distribution function develops small-scale structure in velocity space



$$\frac{\delta v_{\perp c}}{v_{\mathrm{th}i}} \sim \frac{1}{k_{\perp c}\rho_i} \sim \mathbf{Do}^{-3/5}$$

x- and *v*-space resolution are related

$$\mathrm{Do} = \frac{1}{\nu_{ii}\tau_{\rho_i}}$$

$$\tau_{\rho_i} \sim \left(m_i n_{0i} \rho_i^2/\varepsilon\right)^{1/3}$$

characteristic time at the ion gyroscale

Dorland Number

cf. $k_c L \sim \mathrm{Re}^{3/4}$ in Kolmogorov fluid turbulence

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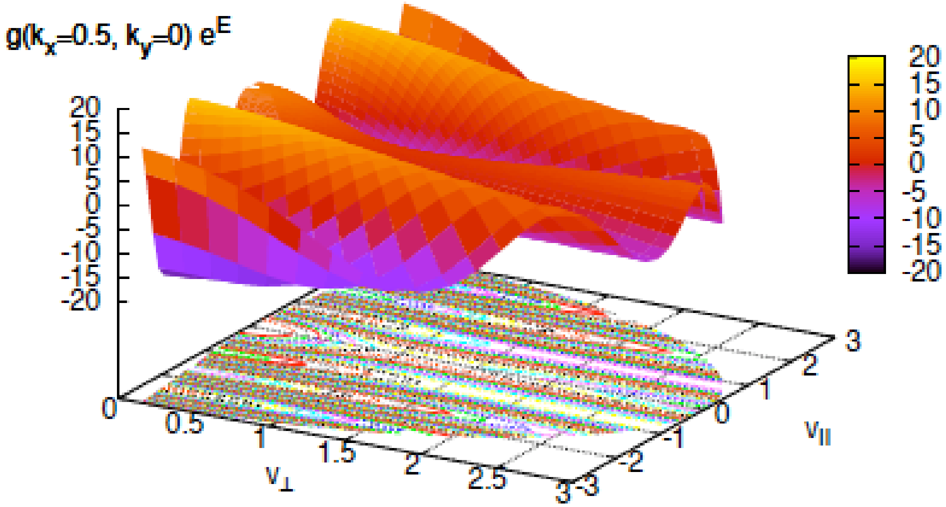
Linear Parallel Phase Mixing

Parallel phase mixing is due to the "ballistic response":

$$\frac{\partial h_i}{\partial t} + v_\| \frac{\partial h_i}{\partial z} + \cdots = 0$$

$$h_i \propto e^{ik_\| v_\| t}$$



$$\frac{\delta v_\|}{v_{\mathrm{th}i}} \sim \frac{1}{k_\| v_{\mathrm{th}i} t} \sim 1$$

after $t \sim \tau_\lambda$

if linear propagation time ~ nonlinear decorrelation time

("critical balance")

So the nonlinear perpendicular phase mixing dominates

arXiv:0806.1069

Dissipation Range With and Without KAW

With KAW

$$E_E(k_\perp) \propto k_\perp^{-1/3}$$
$$E_B(k_\perp) \propto k_\perp^{-7/3}$$
$$E_n(k_\perp) \propto k_\perp^{-7/3}$$

High-frequency, electromagnetic, fluid-like (EMHD)

Without KAW

Low-frequency, electrostatic, purely kinetic (GK ions)

$$E_E(k_\perp) \propto k_\perp^{-4/3}$$
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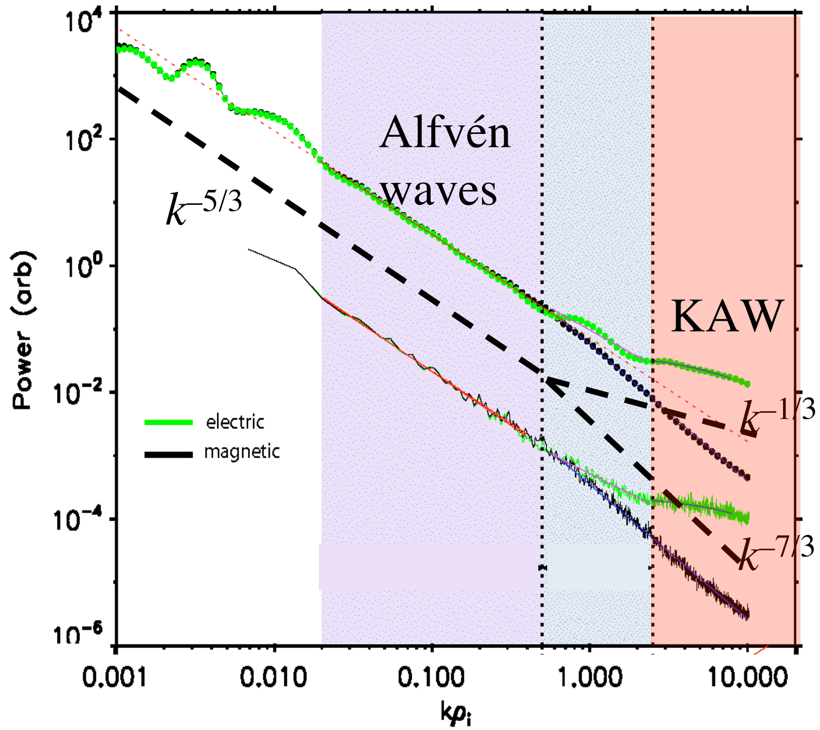
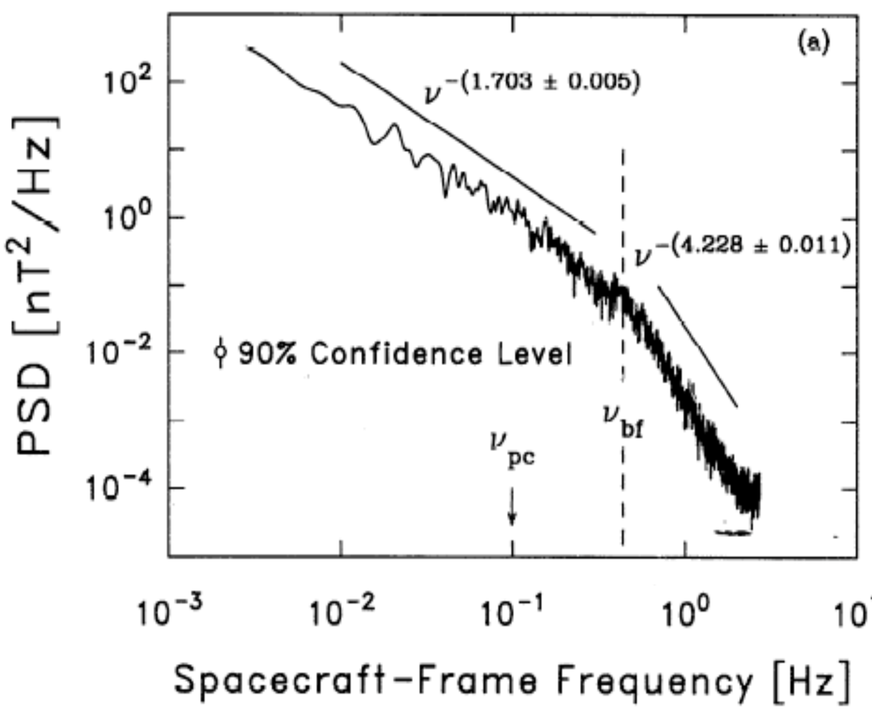


[Bale *et al.* 2005, *PRL* **94**, 215002]



[Leamon *et al.* 1998, *JGR* **103**, 4775]

Dissipation Range of the Solar Wind

With KAW

$$E_E(k_\perp) \propto k_\perp^{-1/3}$$
$$E_B(k_\perp) \propto k_\perp^{-7/3}$$
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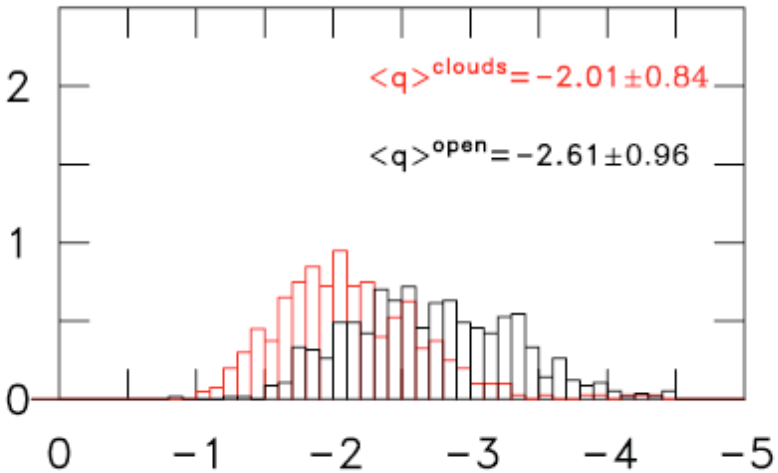
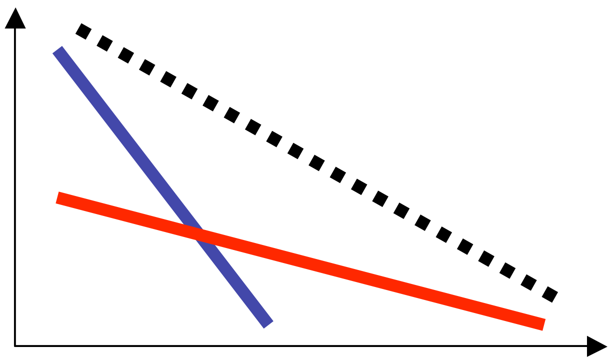


[Smith *et al*. 2006, *ApJ* **645**, L85]



Variable spectral index in the dissipation range may be due to superposition of KAW and no KAW cascades

arXiv:0704.0044

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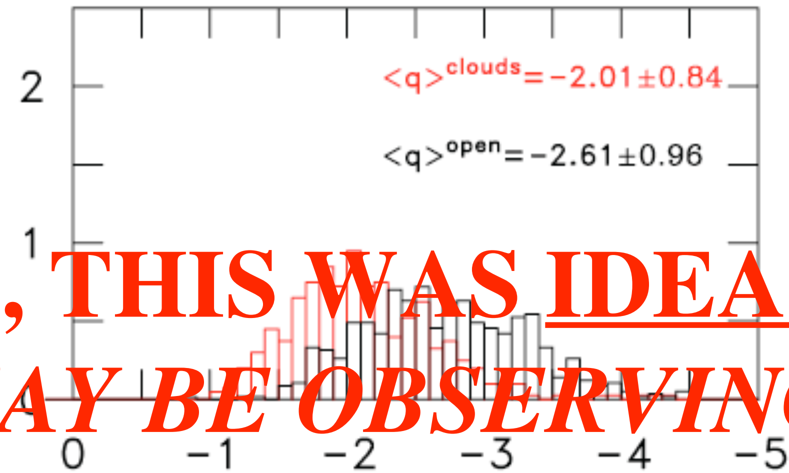
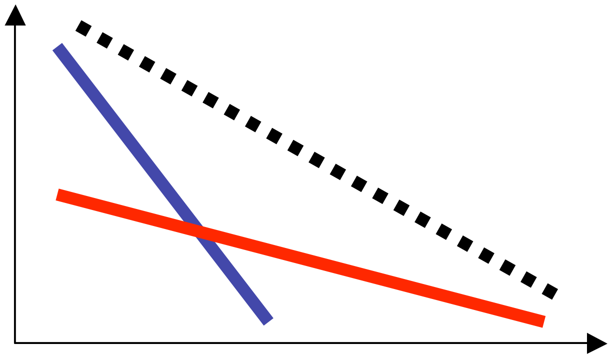


[Smith *et al.* 2006, ApJ **645**, L85]

SO, THIS WAS <u>IDEA #9</u>:

WE MAY BE OBSERVING THE ENTROPY CASCADE



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