A cosmic-ray current driven instability in parallel shocks

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Cosmic-ray current driven instability

- We consider the region upstream of a quasi-pll shock
- 3-component plasma
- relativistic beam of protons (Γ_b) along zeroth order field
- thermal electron/proton distribution $k_b T/mc^2 = \Theta \ll \Gamma_b$
- linear dispersion relation for circularly polarised waves
- plasma susceptibility:

$$\omega^2\chi \approx \frac{{\omega'_{\rm pb}}^2\omega'}{\epsilon\omega_{\rm c}} - \frac{{\omega'_{\rm pb}}^2\omega'}{\epsilon\omega_{\rm c}+\omega'} + \frac{c^2\omega^2}{v_{\rm A}^2} + \frac{\omega_{\rm p}^2\omega}{\epsilon\omega_{\rm c}^3}\left(c^2k^2-\omega^2\right)\left\langle u_\perp^2\right\rangle_{\rm p}$$

$$\omega\Theta_j, ck\Theta_j \ll |\omega_{cj}|$$

Cosmic-ray current driven instability

- Neglecting thermal effects, and provided $\Gamma_b \beta^2 (n_\mathrm{b}/n_\mathrm{p})(\omega_\mathrm{pp}/\omega_\mathrm{cp})^2 \gg 1$
  maximum growth rate of

$$\mathrm{Im}(\omega) = \frac{1}{2}\frac{n_\mathrm{b}}{n_\mathrm{p}}\beta_\mathrm{b}\omega_\mathrm{pp}$$

  Same result as Bell 2004

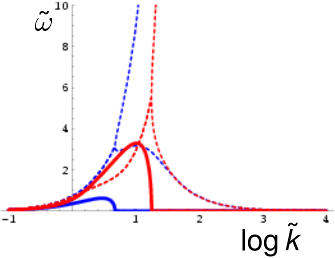
- Including thermal effects, if $\Theta_p \gg v_A/c$

$$\mathrm{Im}(\omega) = \frac{\sqrt{3}}{2}\left(\frac{n_\mathrm{b}}{n_\mathrm{p}}\right)^{\frac{2}{3}}\left(\frac{v_\mathrm{A}}{c}\right)^{\frac{2}{3}}\left(\frac{\omega_\mathrm{pp}}{\omega_\mathrm{c}}\right)^{\frac{2}{3}}\left(\frac{\beta_\mathrm{b}^2}{\langle u_\perp^2\rangle}\right)^{\frac{1}{3}}\omega_\mathrm{c}$$

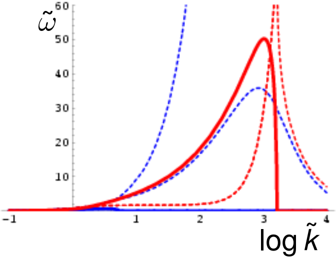
- May provide saturation mechanism in relativistic plasmas

Cosmic-ray current driven instability

e.g., $v_{\mathrm{A}} = 2 \times 10^{-5}$, $\Gamma_b = 10$, $n_{\mathrm{b}}/n_{\mathrm{p}} = 1/10$, $\epsilon = -1$, $\epsilon = +1$



$\Theta = 1/10$



$\Theta = 1/1000$

cf. BR, Kirk & Duffy 2006, PPCF

Cosmic-ray current driven instability

- saturation when currents associated with waves:

$$|\mathbf{k} \times \mathbf{B}| \approx 4\pi n_{cr} e \beta$$

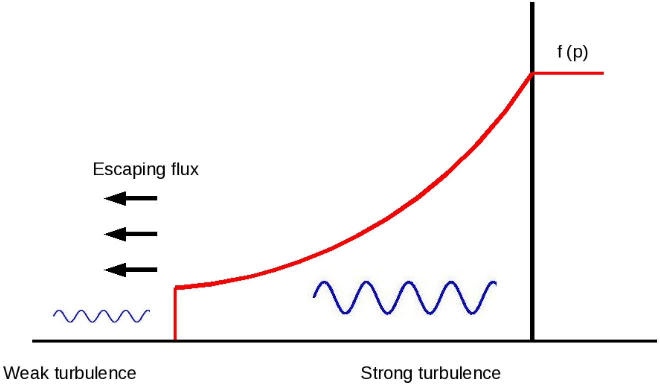
- $k \sim 1/r_g$ - saturated field energy

$$\frac{B_w^2}{8\pi} = \frac{1}{2} n_{cr} \Gamma_b m_p c^2$$

- Entire energy of the beam goes into magnetic field production
- How do we include this in acceleration models?

Cosmic-ray modified shocks

- Particles *escape* beyond some boundary



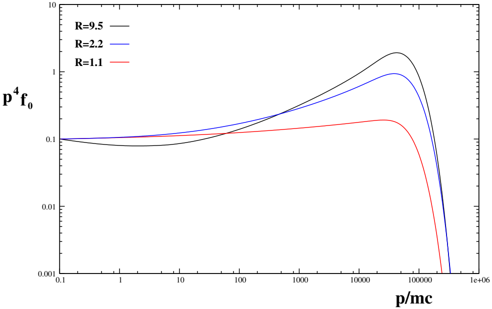
- escaping particles drive magnetic field growth

Steady-state solutions

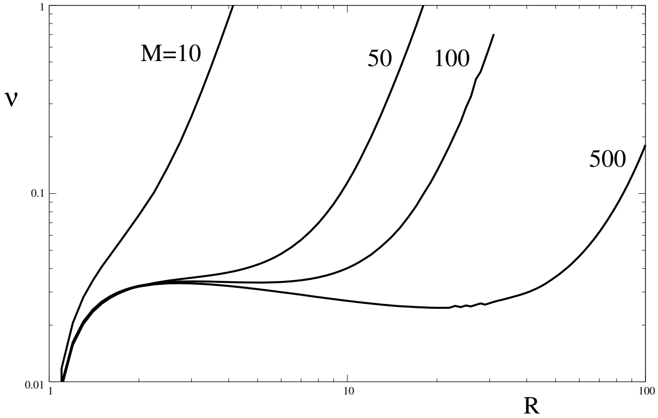
- coupled hydrodynamic - kinetic equations

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial x}\left(uf - \kappa(x,p)\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial p^3}\left(p^3 f \frac{\partial u}{\partial x}\right)$$

$$L_{\rm esc} = \frac{\kappa(p^*)}{u_0}, \quad \text{e.g.} \quad p^* = 10^5$$



Injection efficiency $\nu - R$ diagram



$u_0 = 5000\text{km s}^{-1}$, $n_0 = 1\ \text{cm}^{-3}$, $p^* = 10^3$

$$\nu = \frac{4\pi}{3}\frac{mc^2}{\rho_0 u_0^2} p_0^4 f_0(p_0),$$

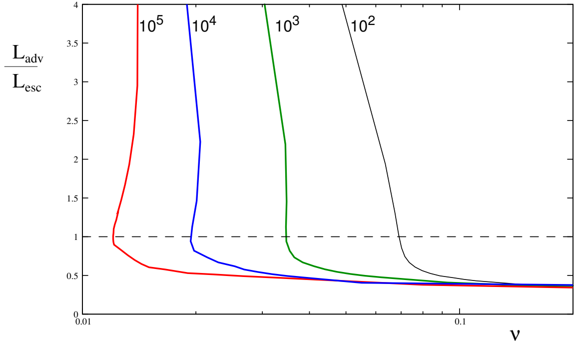
Maximum momentum

- what is the location of escape boundary - transition zone from weak to strong turbulence
- $L_{\mathrm{esc}} = \kappa(p^*)/u_0$ determines maximum confined energy
- diffusive current at escape boundary drives nonresonant instability

$$j_{\mathrm{cr}}(-L_{\mathrm{esc}}) = -4\pi e \int_{p_0}^{\infty} \kappa \frac{\partial f}{\partial x} p^2 \mathrm{d}p$$

- How do we determine L_{esc} in a self-consistant manner?
- Calculate CR flux (and p^*) from numerical SS solution
- Transition zone determined from condition $L_{\mathrm{adv}} \equiv u_{\mathrm{sh}}/\Gamma_{\mathrm{max}} \sim L_{\mathrm{esc}}$

Maximum momentum



BR, Kirk & Duffy, ApJ submitted

- Maximum energy can be calculated as a function of injection parameter

Summary

- Efficient acceleration of particles at collisionless shocks connected with magnetic field amplification
- nonresonant mode (Bell 2004) appears to be of greatest importance in efficient cosmic-ray accelerating shocks
- reduced diffusion coefficients leads to more rapid acceleration
- field amplification incorporated using free escape boundary $L_{\rm esc}$
- maximum energy determined by boundary position - calculated self-consistently
- system is self-organising - maximum energy function of injection