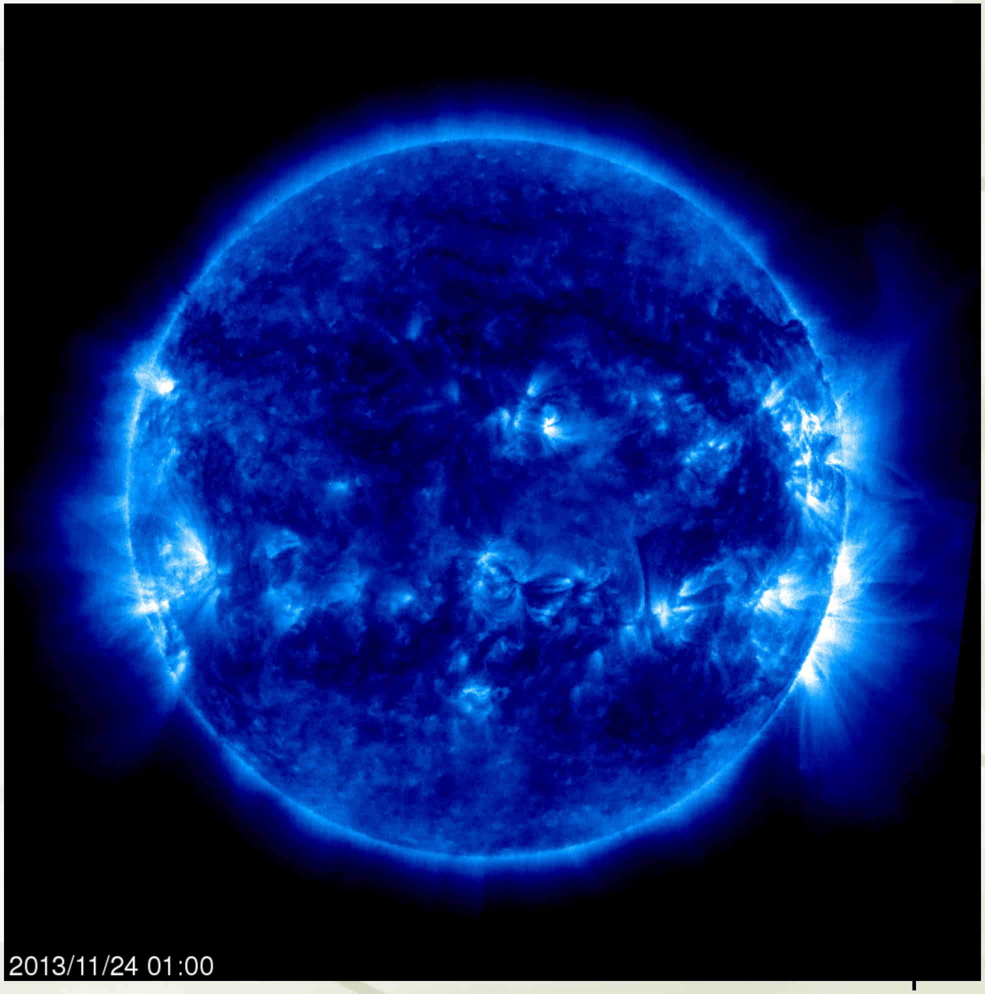
Magnetic Helicity and the Galactic Dynamo

With Amir Jafari (U Sask.)

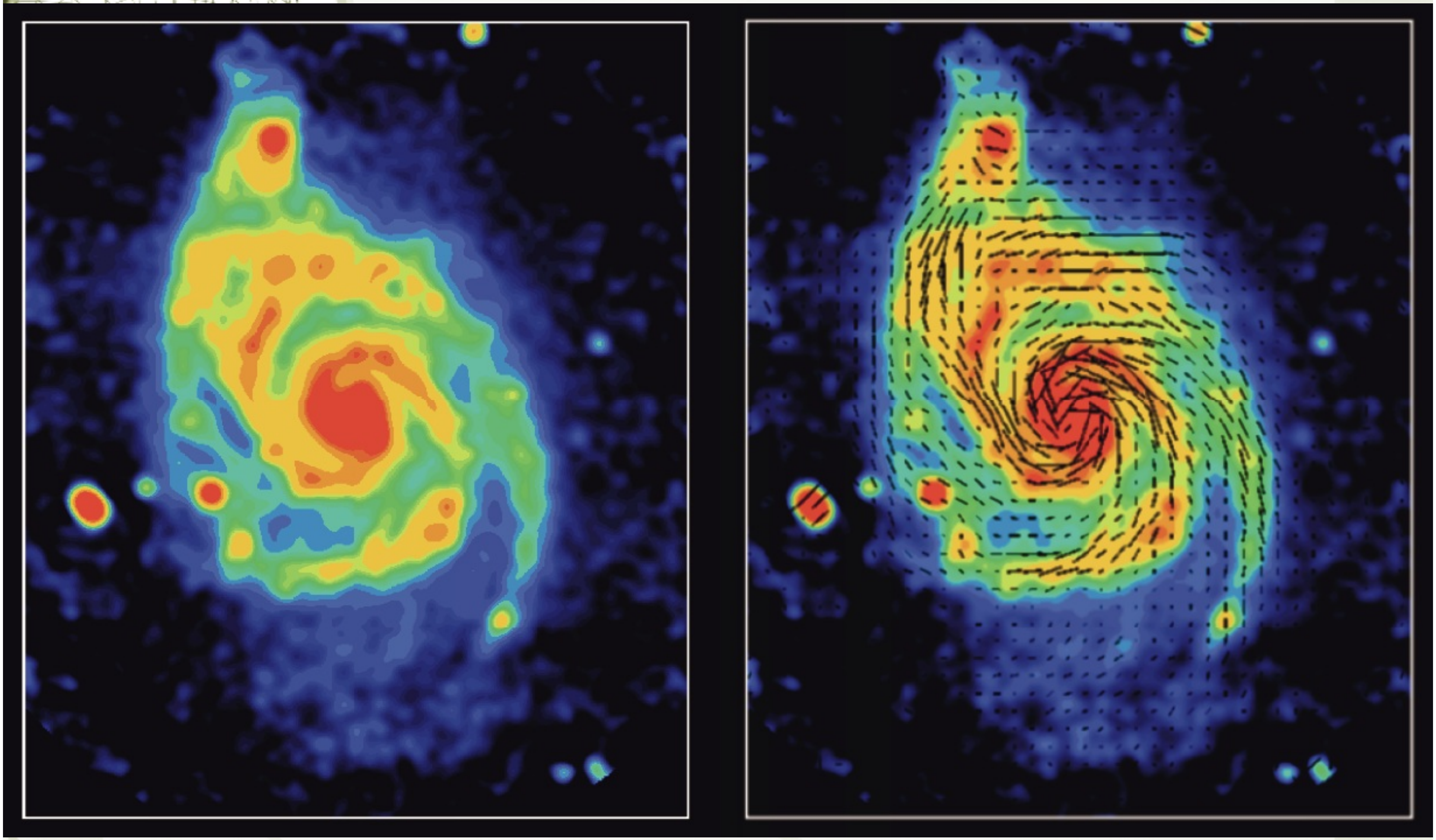
This is the closest astrophysical dynamo. ->
It is not a galaxy.



Cracow 2014

What is the Problem?

- Magnetic fields are found everywhere in the universe, in stars, in the disks of galaxies, and in the intracluster medium in giant clusters of galaxies.
- They are typically organized on the scale of the system (galaxy, star, planet..) with an energy density which is comparable to the local turbulent energy density (convection or whatever stirs the gas/fluid).
- These magnetic fields are responsible for particle acceleration. Without them, high energy astrophysics would (almost) not exist.



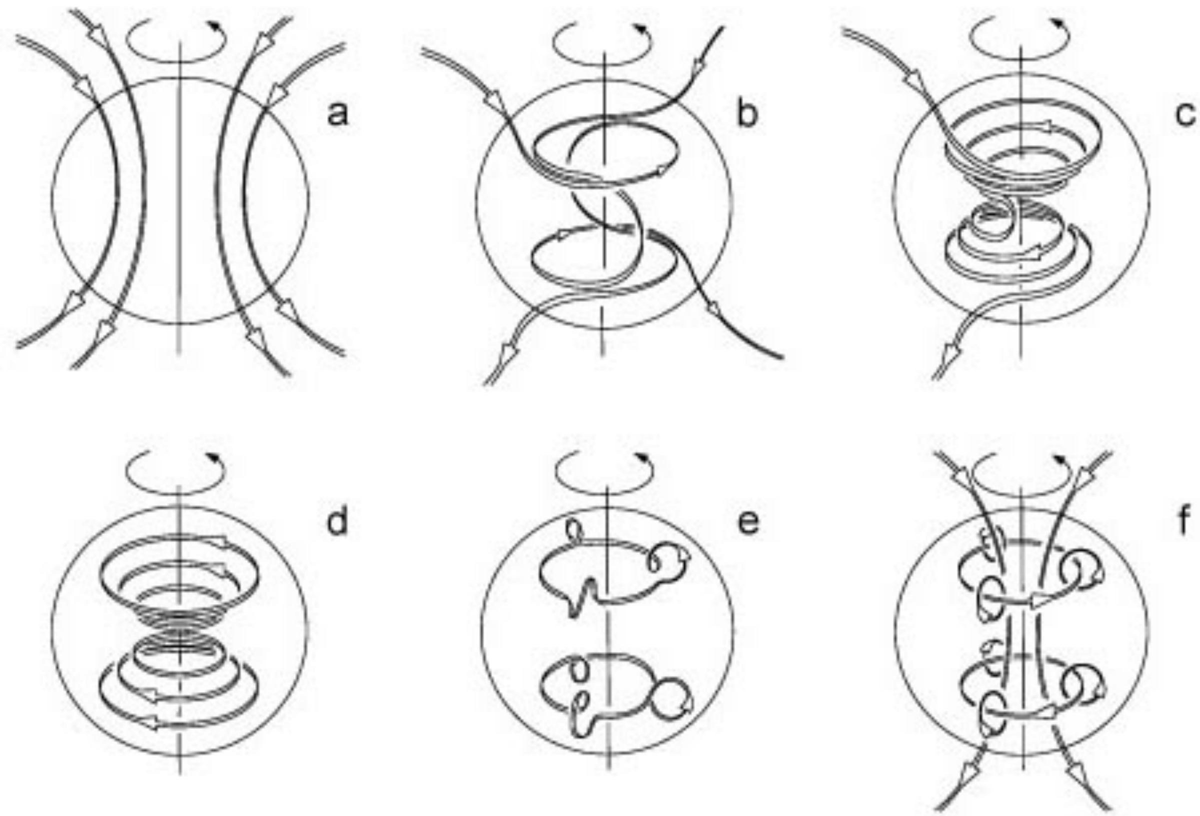
What is the Problem?

- This leads to at least two questions.
- First, how do magnetic fields come to contain a large fraction of the available energy? This is the **Small Scale Dynamo** problem.
- Second, how do they become organized on large scales, and what sets their saturation limit. This is the **Large Scale Dynamo** problem in astrophysics. (It is an example of an inverse cascade, like hurricanes.)

Small Scale Dynamo?

- In a turbulent medium, the magnetic field absorbs a fraction “of order unity” (really about 5%) of the energy cascade (Cho and Vishniac, Beresnyak)
- This leads to a linear growth of the magnetic field energy, so even infinitesimal seed fields lead to strong magnetic fields with a scale like the largest eddies in about 20 large eddy turn overs. (Linear ≠ slow!)

The α-Ω Dynamo – Twisting & Shear



Love, J. J., 1999. Astronomy & Geophysics, 40, 6.14-6.19.

We need some equations!

- A highly conducting fluid, i.e. most astrophysical plasmas, has

$$\vec{E} \approx -\vec{v} \times \vec{B}$$

- Consequently the induction equation becomes

$$\partial_t \vec{B} \approx \nabla \times \left(\vec{v} \times \vec{B} \right)$$

This has some unlikely consequences

- The magnetic flux through a fluid element is fixed for all time – grossly inconsistent with solar observations ("flux freezing").
- On the other hand, in a turbulent fluid, the combination of stochastic wandering of field lines plus an infinitesimal resistivity leads to a magnetized version of Richardson diffusion. (Lazarian & Vishniac 1999; Eyink et al. 2013)

How do we move from the dynamo picture to a dynamical model?

- Mean field dynamo theory – the large scale field is considered to be a dynamical object affected by some average of turbulent, eddy scale effects.

$$\partial_t \vec{B} = -\left(\vec{V}\cdot\nabla\right)\vec{B} + \left(\vec{B}\cdot\nabla\right)\vec{V} + \nabla\times\left\langle \vec{v}\times\vec{b}\right\rangle$$

advection stretching Coherent electric field due to a net contribution from eddies

Why is the electromotive force not zero?

- Basic method of evaluation – assume it is approximately zero and calculate the correction.
- Expand the quantity as a Taylor series in time and truncate after the first term. Use the eddy turnover time as the time interval. In other words

$$\left\langle \vec{v} \times \vec{b} \right\rangle \approx \left\langle \partial_t \vec{v} \times \vec{b} \right\rangle \tau_c + \left\langle \vec{v} \times \partial_t \vec{b} \right\rangle \tau_c + (\text{higher order corrections})$$

- Leaving out a **few** steps, we can write this result in terms of scalars (schematically) as

  Electromotive force = [(current helicity – kinetic helicity) x magnetic field x coherence time] + [cross helicity x coherence time x shear]+ [drift terms (e.g. buoyancy)]+[dissipative terms]

  $\text{current helicity} \equiv \vec{j} \cdot \vec{b}$ ←Related to a conserved quantity

  $\text{cross helicity} \equiv \vec{v} \cdot \vec{b}$ ← Not conserved, not useful

  $\text{kinetic helicity} \equiv \vec{v} \cdot \left(\nabla \times \vec{v} \right)$ ←Not conserved (imposed by environment).

Kinematic (conventional) Dynamo Theory

- Cross-helicity is a conserved quantity in incompressible MHD with zero dissipation. That means it is almost as conserved as energy, i.e. not at all. Also, it drives the toroidal field, which we don't need. ➔ We ignore this term.
- Kinetic helicity is not conserved, but may be imposed by the environment. We keep it. (?)
- Current helicity is not conserved, but is related to magnetic helicity, which is. This is taken to be a magnetic back reaction. Drop it (?!?) or model it as suppression when the field is strong.

What is the kinetic helicity?

- This is a pseudo-scalar. Its value is nonzero iff there is symmetry breaking in all three directions. Differential rotation breaks symmetry in the rφ plane. So we estimate the kinetic helicity as

$$h_k \sim \frac{v^2}{L}(\Omega\tau_c)$$

How fast does the large scale magnetic field grow?

- If we put all this together, we get a linear equation for the large scale magnetic field. When the dissipation term is beaten out by the combined effects of helicity and shear, we get an exponential growth rate of

$$\Gamma_{dynamo} \sim \left(h_k \tau_c \frac{S}{L} \right)^{1/2} \sim \frac{\lambda_{eddy}}{L} \Omega$$

This is the "kinematic dynamo".

Can this explain the solar dynamo?

- We need to add in a nonlinear saturation mechanism, i.e. buoyant losses and backreaction from the large scale fields.
- We need to make a model that includes the meridional flows and the detailed rotational profile of the Sun.
- We need to model “alpha” or the kinetic helicity times the coherence time everywhere.

- If we do all this, then we can produce mean field models that imitate the solar cycle and move the zone of erupting, buoyant magnetic field from high latitudes towards the equator.
- Success!! But does it mean anything? Probably not.

- In fact, one can achieve a pretty good match even if the underlying physics is simply **wrong.**

Reasons to worry:

1. Not unique

2. Numerical simulations have their own problems, but they show little or no correlation between the kinetic helicity and the electromotive force.
3. Alpha suppression, i.e. the backreaction from the current helicity shuts down the dynamo early. (Gruzinov & Diamond 1994)

Alpha Suppression ? (Symmetry in action!)

- The magnetic helicity, $\vec{A}\cdot\vec{B}$, is a conserved topological quantity. It is the "twistiness" of the magnetic field. In the coulomb gauge the current helicity is roughly proportional to it. It can be moved around, but not destroyed. If we separate out the contribution from the eddy scales, h, (as opposed to the large scales) we can show that

$$\partial_t h + 2\vec{B}\cdot\left\langle \vec{v}\times\vec{b}\right\rangle = -\nabla\cdot\vec{j}_h$$

Alpha suppression (continued)

- *If* we can neglect the RHS of this equation, then running a dynamo produces an accumulation of current helicity which counteracts the kinetic helicity and turns the dynamo off when the large scale field is still weak. (Gruzinov and Diamond 1994)
- **The kinematic dynamo dies tragically young.**

How can we save the dynamo?

- First, we should remember that we are in a trap of our own making. Nature has no problem with the large scale dynamo. It is not completely clear, but apparently numerical simulations do not either.
- Second, we ignored the magnetic helicity flux. It gets dragged around. It diffuses. Most importantly, it can be driven by differential rotation even if it is zero everywhere initially!

What is the magnetic helicity flux?

- The eddy scale magnetic helicity has a flux given by

$$\vec{j}_h = \left\langle \vec{a} \times \left(\vec{v} \times \vec{b} + \nabla \varphi \right) \right\rangle - 2 S_{ij} \left\langle \vec{b} \nabla^{-2} \left(a_{i,j} \right) \right\rangle$$

- We can use this to calculate the leading order terms (and we have done so) but we also note that a simple dimensional estimate is.....

$$j_h \sim \langle v^2 \rangle \langle b^2 \rangle (S \text{ or } \Omega \ ) \tau_c^2$$

- Here I have assumed that the shear and rotation are small. (They can be large.)
- This is a pseudo-vector (does not reverse under parity). It can only point in the direction of the rotation, or the local vorticity (due to differential rotation). Both components will be present, in general.

- We can solve a simple version of the dynamo problem by ignoring the kinetic helicity term altogether and assuming that the parallel component of the electromotive force is

$$\left\langle \vec{v} \times \vec{b} \right\rangle \approx \frac{-2\vec{B}}{B^2} \nabla \cdot \vec{j}_h$$

- The implication is that dynamo growth is fast when the field is weak, and slows until magnetic field loss or dissipation balances growth.

Can we really do this?

- These simple estimates suggest that the kinetic helicity is dominated by the current helicity after one eddy turn over time. Ignoring it should be OK.
- There is no kinematic dynamo in realistic systems.
- When the large scale field is very weak, life is more complicated, because the magnetic helicity can accumulate ($\partial_t h \neq 0$) but this expression becomes valid as the field strength grows.

What does this mean for the solar dynamo?

- We don't know. This has never been implemented. The closest thing to it are the recent calculations by Kosovichev and collaborators, but they only included advective and diffusive magnetic helicity flux. (BTW, their models for the solar dynamo look great, despite leaving out the dominant transport term.)
- Numerical dynamos in a box show that this mechanism works (Shapovalov and Vishniac 2011/2013)

How about a toy model?

- If we use this expression for the magnetic helicity flux for turbulence in a box, the dynamo growth rate drops as the magnetic field grows.
- Eventually turbulent dissipation can compete with the dynamo and we reach saturation.
- At this point we get a crude estimate of the saturation Alfven speed.

Characteristic Toroidal Magnetic Fields

- For slow rotators (using the expression given earlier) like a galaxy this leads to

$$B_T \sim \rho^{1/2} L S$$

- This ignores the role of magnetic buoyancy, which in real objects pushes out the magnetic field. Allowing for buoyancy moderated by turbulent drag, this implies

$$B_T \sim \rho^{1/2} L_P S$$

that is, we replace the box height with the pressure scale height.

For a spiral galaxy like ours:

- This is an Alfven speed of a few kilometers per second, or a magnetic field of a few microgauss.
- This is indistinguishable from ~equipartition with the local turbulent energy density, but arises from a different cause.

What if.....

- There is a background magnetic field?

-- We get a similar contribution but with

$$\langle b^2 \rangle \rightarrow \frac{\left(\vec{k} \cdot \vec{B}\right)^2}{k^2}$$

In this case the numerator is always the square of the dynamical rate and the magnetic helicity flux does not increase with the strength of the background field.

What if…..

- Rotation is fast ($\Omega\tau_c \gg 1$)?

-- The turbulence is stretched in the vertical direction by that same factor. The terms that contribute to rotationally driven magnetic helicity flux all have a factor of

$$\frac{k_z^2}{k^2}$$

The rotationally driven magnetic helicity flux decreases as rotation increases. The dynamo becomes dominated by the shear.

What if…..

- Shear is strong($S\tau_c \gg 1$)?

--Turbulent eddies are sheared so that the radial wavelength and radial field components decrease as the shear increases. Since every term depends on the square of the radial wavelength, or $\langle b_r^2 \rangle$, or $\langle v_r^2 \rangle$ this means that the magnetic helicity flux is inversely proportional to the shear.

Characteristic Magnetic Fields

- Consequently, in the limit of strong rotation and shear the dynamo is independent of the shear and the rotation.
- In this limit the saturation magnetic field is

$$B_T \sim \rho^{1/2} L_z \tau_c^{-1}$$

- Again, this looks like equipartition, but the energy of the toroidal field is taken from the local shear.

- For a star, limited by buoyant losses, the magnetic “luminosity” is a set fraction of the total luminosity – everything scales with the convective flux and the large scale magnetic field energy scales with the turbulent energy density.

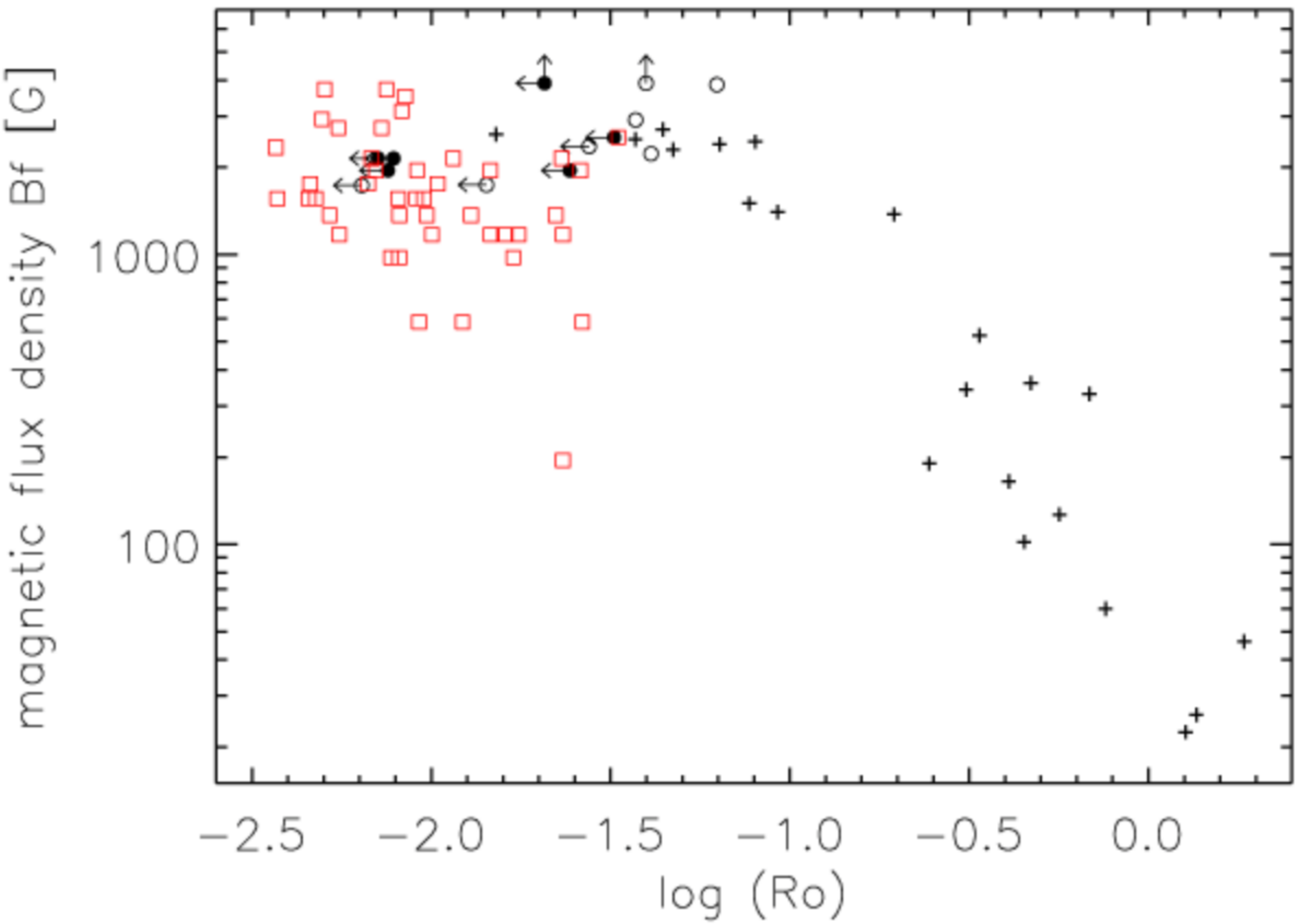


Figure 19: Magnetic fields as a function of Rossby number. Crosses are sun-like stars Saar (1996a, 2001), circles are M-type of spectral class M6 and earlier (see Reiners *et al.*, 2009a). For the latter, no period measurements are available and Rossby numbers are upper limits (they may shift to the left hand side in the figure). The black crosses and circles follow the rotation-activity relation known from activity indicators. Red squares are objects of spectral type M7 – M9 (Reiners and Basri, 2010) that do not seem to follow this trend ($\tau_{\rm conv}$ = 70 d was assumed for this sample).

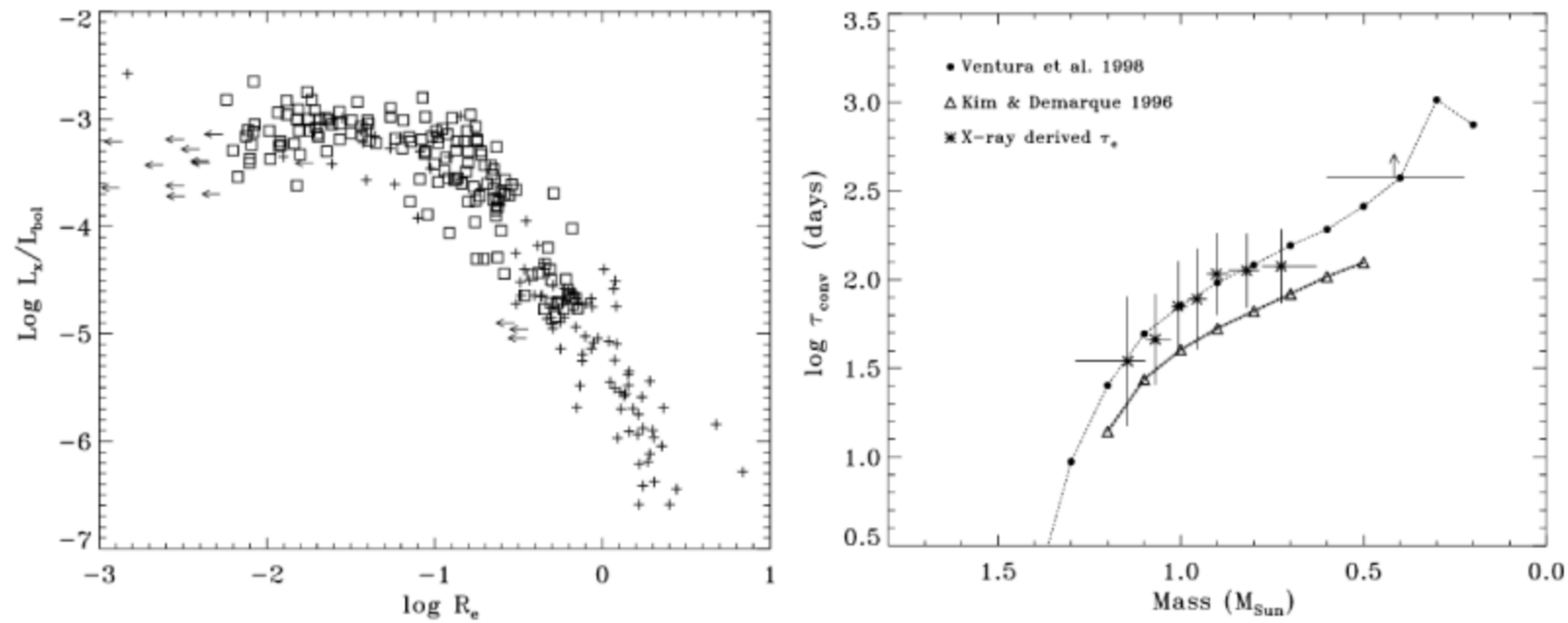


Figure 18: *Left panel:* Rotation-activity relation showing the normalized X-ray luminosity as a function of Rossby number. *Right panel*: Empirical turnover time chosen to minimize the scatter in the rotation-activity relation (from Pizzolato *et al.*, 2003, reprinted with permission © ESO).

Back to galaxies…

- This same line of reasoning can be applied to disks (accretion & galactic)
- Galaxies form from unmagnetized gas (unless rather speculative physics models are invoked).
- During, or before, disk formation the small scale dynamo creates eddy sized magnetic fields and random noise on larger scales generates a Poisson spectrum of radial magnetic fields.
- This stage takes a couple dozen eddy turn over times, roughly a galactic rotation.

Galaxies

- Since this takes place in a shearing disk the resulting azimuthal fields are an order of magnitude stronger (about 10^{-7}G).
- At the same time the spontaneous magnetic helicity flux is producing a large current helicity, separated in the vertical direction.

Galaxies

- The magnetic field couples efficiently to the accumulated magnetic helicity in the turbulent disk and grows to saturation (observed strengths) in another full rotation. (This stage actually starts before the saturation of the previous stage.)

Galaxies

- The complete time to saturation is at most few rotations, which is fast enough that finding a galaxy which is clearly not fully magnetized will be quite challenging. (Such fields will not be organized on a global scale though. The typical domain will be annular, with a radial extent like the disk thickness.)

Summary

- We are proposing a new version of mean field dynamo theory, in which dynamo action is driven by local accumulations of magnetic helicity. These accumulations are generated by turbulence in a differentially rotating system, even in the absence of a large scale magnetic field.
- This model naturally explains the dependence of magnetic field on stellar spin rates and the typical magnetic field strength in disk galaxies.
- As an additional benefit, it also explains why galaxies acquire strong magnetic fields so quickly.