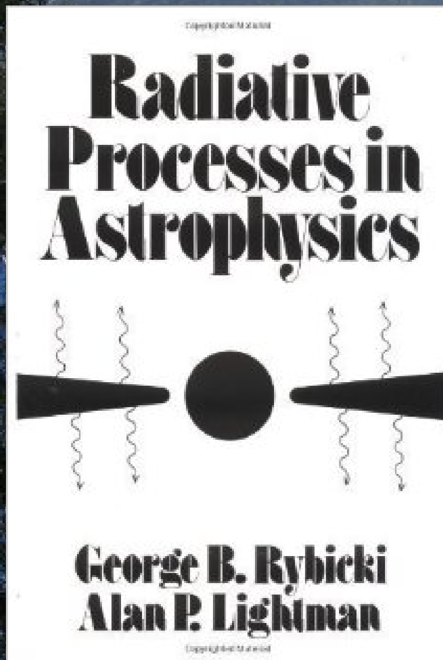
Polarization spectra of unresolved radio sources



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Based on:

"RM synthesis revisited" - Schnitzeler & Lee (submitted to MNRAS)

"Polarization spectra of unresolved radio sources" - Schnitzeler, Banfield & Lee (~ few weeks from submission)

Introduction

Question: what can we learn about distant unresolved radio sources from their polarization spectra?

Three take-home messages:

- RM synthesis as a discrete Fourier transform is an approximation (but a pretty good one!),
- At high frequencies all frequency spectra appear to be produced by 2D Gaussian sources,
- A 2D random walk model predicts a non-zero polarized flux density at low frequencies, and provides an alternative explanation to partial coverage models.

Part I: RM synthesis revisited

From **Brentjens & De Bruyn (2005)**:

$$\tilde{P}(\lambda^2) = W(\lambda^2)P(\lambda^2).$$

$$\tilde{F}(\phi) = K \int_{-\infty}^{+\infty} \tilde{P}(\lambda^2) \mathrm{e}^{-2\mathrm{i}\phi(\lambda^2-\lambda_0^2)}\, \mathrm{d}\lambda^2 \quad (25)$$

$$R(\phi) = K \int_{-\infty}^{+\infty} W(\lambda^2) \mathrm{e}^{-2\mathrm{i}\phi(\lambda^2-\lambda_0^2)}\, \mathrm{d}\lambda^2. \quad (26)$$

If $\phi\delta\lambda^2 \ll 1$ for all channels, we may approximate the integrals in Eqs. (25) and (26) by sums:

$$\tilde{F}(\phi) \approx K \sum_{i=1}^{N} \tilde{P}_i \mathrm{e}^{-2\mathrm{i}\phi(\lambda_i^2-\lambda_0^2)} \quad (36)$$

$$R(\phi) \approx K \sum_{i=1}^{N} w_i \mathrm{e}^{-2\mathrm{i}\phi(\lambda_i^2-\lambda_0^2)} \quad (37)$$

$$K = \left(\sum_{i=1}^{N} w_i\right)^{-1}. \quad (38)$$

In these equations, λ_i^2 is λ_c^2 of channel i, $\tilde{P}_i = \tilde{P}(\lambda_i^2) = w_i P(\lambda_i^2)$, $w_i = W(\lambda_i^2)$, and K has become the sum of all weights. We have implemented Eqs. (36), (37), and (38) in our RM-synthesis software.

The largest detectable RM: $|\Phi_{\max}|\,\delta\lambda^2 \approx \sqrt{3}$

Our method for discrete frequency channels: Calculate the mean derotation vector for each channel, including channel weighting functions, then align the polarization vectors of the frequency channels.

Calculate the RM spectrum using

$$\boldsymbol{p}\,(\mathrm{RM}') = \frac{1}{N_\mathrm{c}} \sum_{\mathrm{j}=1}^{N_\mathrm{c}} \boldsymbol{p}\,(\nu_{\mathrm{c},j})\,\hat{\boldsymbol{v}}_j\,(\mathrm{RM}')_{\mathrm{derot}}$$

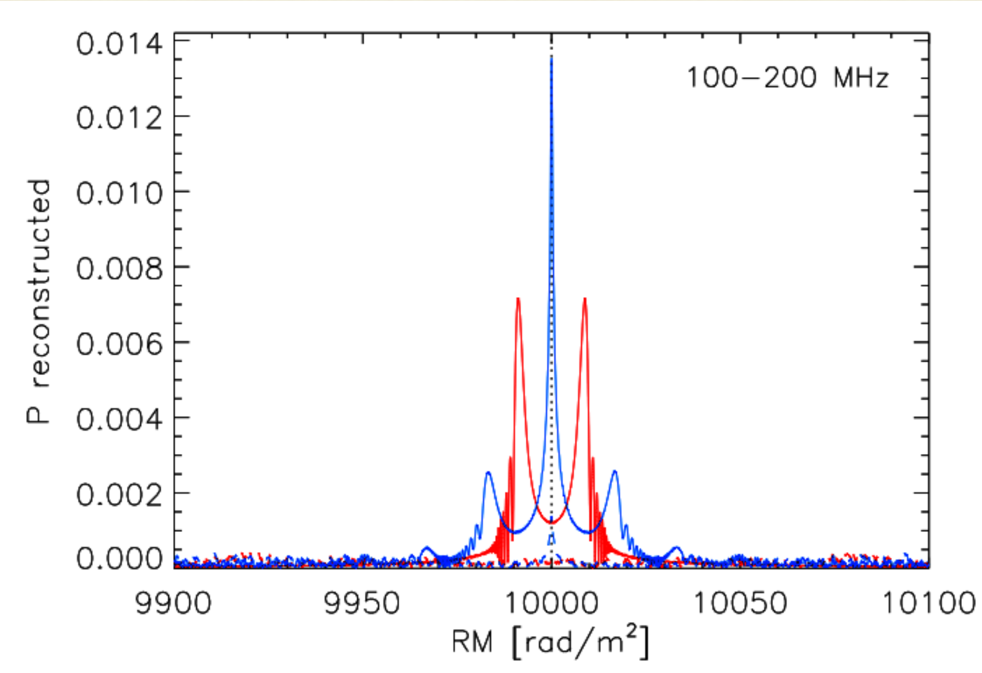
The measured polarization vectors are

$$\boldsymbol{p}\,(\nu_{\mathrm{c},j}) = \int_{-\infty}^{\infty} w_j\,(\nu)\,\boldsymbol{p}\,(\mathrm{RM})_{\mathrm{true}}\, \mathrm{e}^{2\mathrm{iRM}(c/\nu)^2}\mathrm{d}\nu$$

and the (net) derotation vector of channel *j*

$$\boldsymbol{v}_j\,(\mathrm{RM}')_{\mathrm{derot}} \equiv \int_{-\infty}^{\infty} w_j\,(\nu)\, \mathrm{e}^{-2\mathrm{iRM}'(c/\nu)^2}\mathrm{d}\nu$$

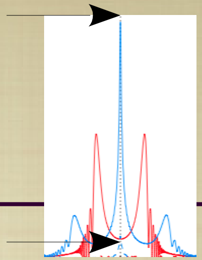
Part I: RM synthesis revisited



Input: a source emits one unit of flux density at RM = 10^4 rad/m^2.
The simulated observations span frequencies between 100 and 200 MHz with 0.1 MHz channels (solid lines), and have a top-hat response in frequency

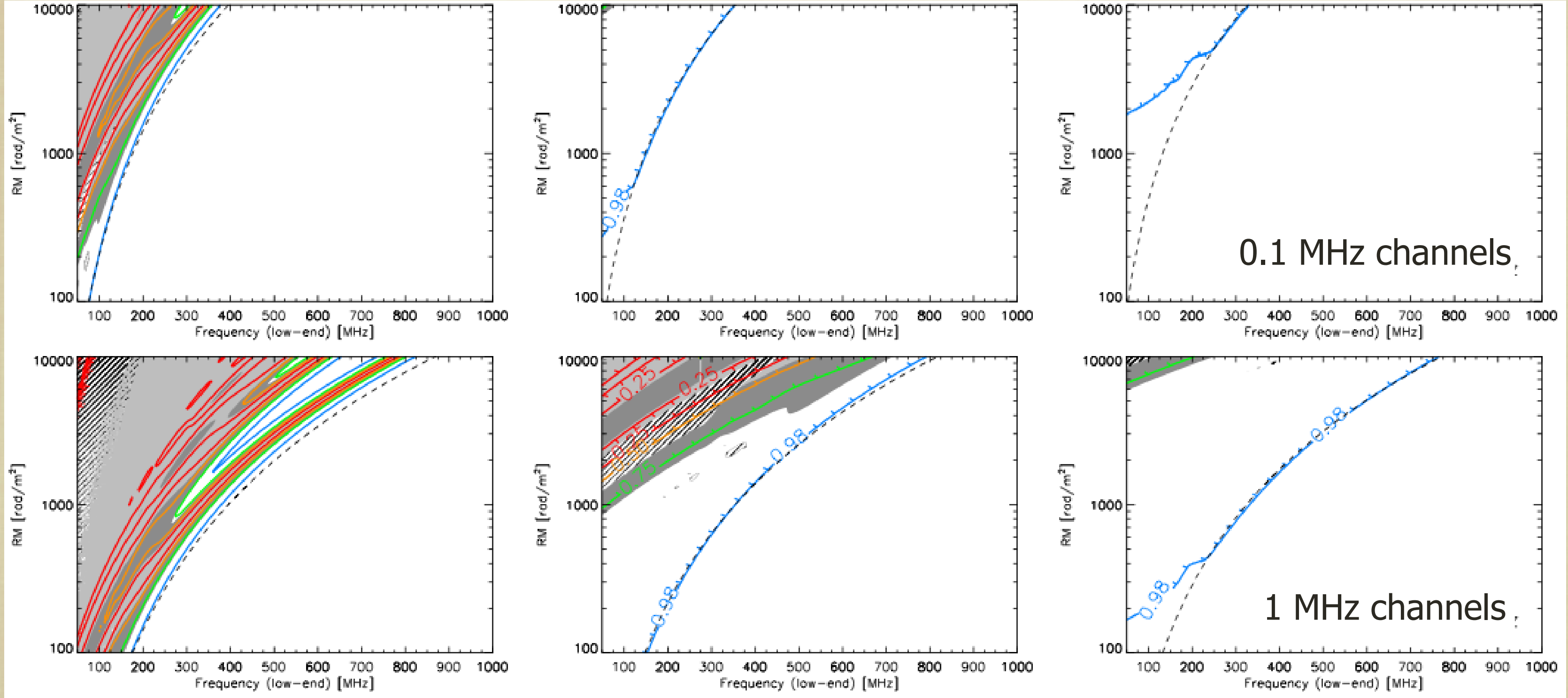
Two RM spectra calculated from these input data:
Red: formalism by Brentjens & De Bruyn (exact for top-hat response in wavelength squared)
Blue: using the response functions of the data: top-hat in frequency

How well does this work?



Compare the pol. flux densities at the RM of the source:

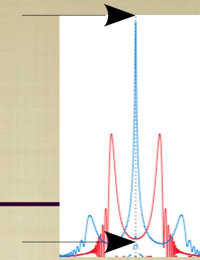
100 MHz bandwidth | 500 MHz bandwidth | 1000 MHz bandwidth



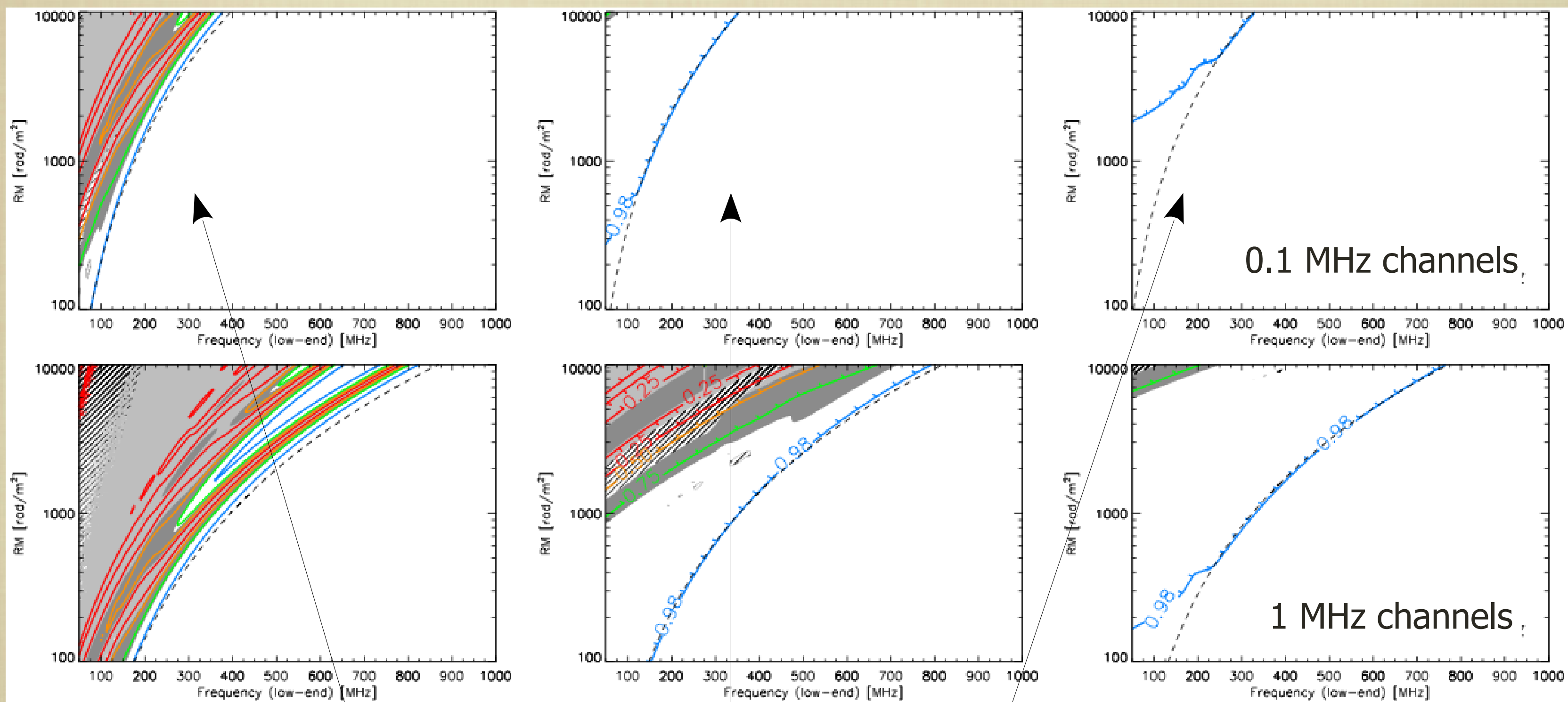
Flux ratio contours: $P_{B\&dB} / P_{new}$ = 0.98 (blue), 0.75 (green), 0.5 (orange), 0.25 (red)

White/light/dark grey backgrounds: in the RM spectrum calculated using B&dB $P(RM_{source})$/ (max. P in the RM spectrum) is > 0.95, >0.5 and < 0.95, or < 0.95, respectively

How well does this work?



Compare the pol. flux densities at the RM of the source:



Flux ratio contours: P / P = 0.98 (blue), 0.75 (green), 0.5 (orange), 0.25 (red)

$$\mathrm{RM} \approx 1.44 \times 10^4 \ \mathrm{rad\ m^{-2}} \left(\frac{\nu_{\mathrm{low}}}{\mathrm{GHz}}\right)^{\frac{5}{2}} \left(\frac{\nu_{\mathrm{high}}}{\mathrm{GHz}}\right)^{\frac{1}{2}} \left(\frac{\delta\nu}{\mathrm{MHz}}\right)^{-1}$$

White/lig … dB

P(RM_{source} … , respectively

Part II: Polarization signatures of point sources

We developed new models for sources with large-scale or turbulent magnetic fields, and calculated polarization spectra across a wide range of frequencies (200 MHz – 10 GHz).

Four source types with large-scale fields:

- Burn jet
- Gaussian jet
- Cylinder (any inclination)
- Ellipsoid (any inclination)

Sources with turbulent fields

Part II: Polarization signatures of point sources

The rules of the game:

1. The volume emissivity and the shape of the spectrum are constant throughout the source,
2. The emission is synchrotron-thin (no absorption),
3. The intrinsic position angle is constant throughout the source,
4. Wavelength-independent depolarization is constant throughout the source,
5. The Faraday-rotating gas is outside the source, and is non-relativistic.

This set of rules simplifies interpreting spectra in terms of the geometry of the source.

The total flux density emitted over all RM is constant:

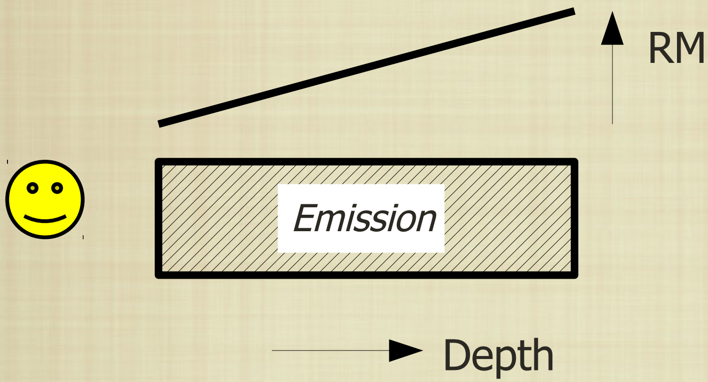
$$\int_{-\infty}^{\infty} |\boldsymbol{P}_{\mathrm{em}}\,(\mathrm{RM})|\,\mathrm{dRM}$$ = 1000 polarized flux density units

→ a source which emits over a larger range in RM will have a smaller peak polarized flux density in the RM spectrum.

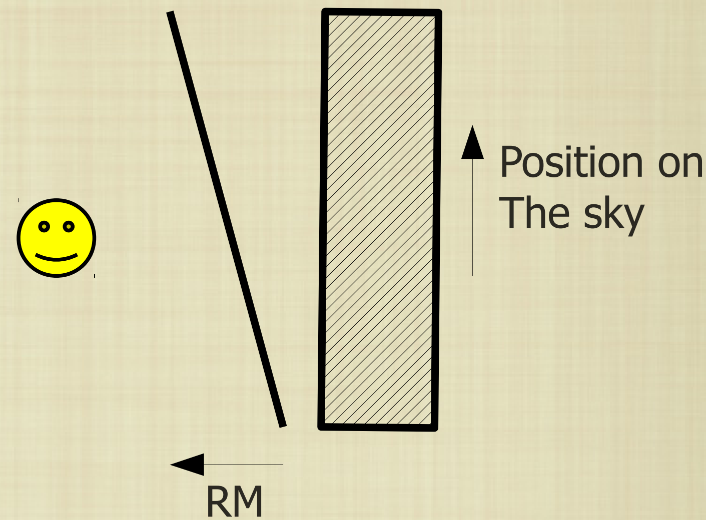
We include depolarization across the 1 MHz channels in our sim.

The Burn jet

Burn slab: emission and rotation are mixed



Burn jet: Faraday screen in front of the jet.



Both types of sources are described by

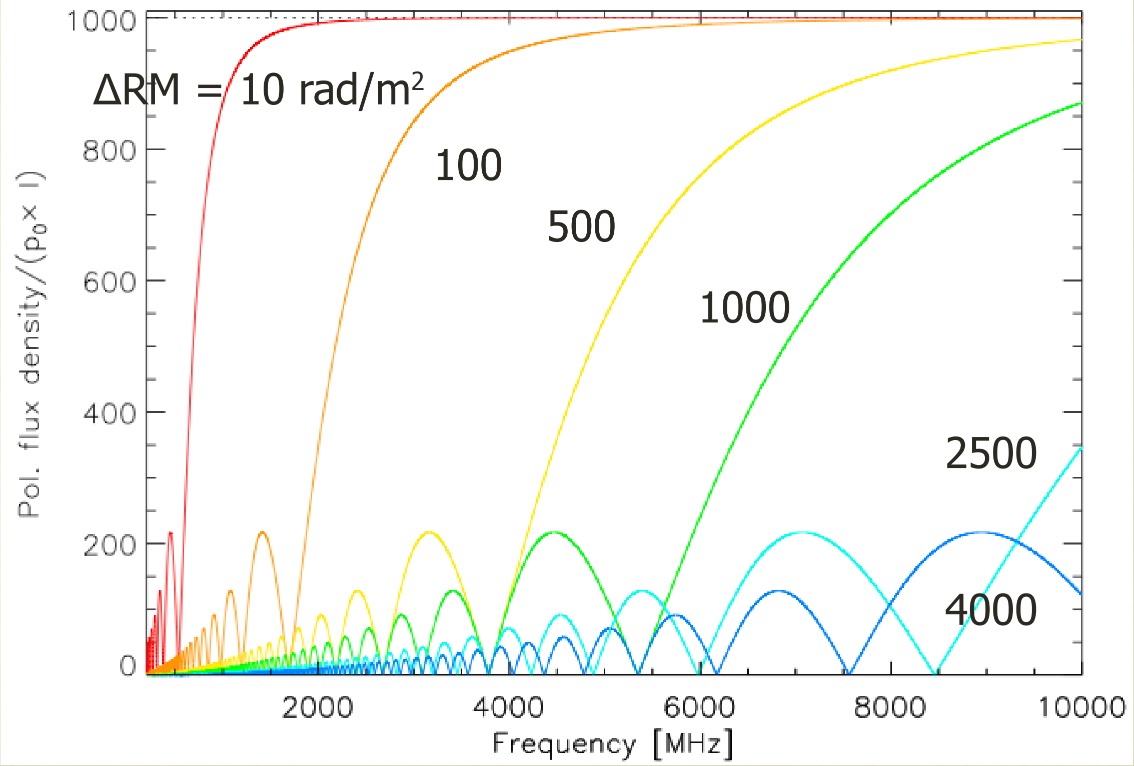
$$\boldsymbol{P}_{\rm obs}(\nu) = (p_0 \times I)\,{\rm sinc}\left(\Delta{\rm RM}\,({\rm c}/\nu)^2\right){\rm e}^{2{\rm i}{\rm RM}_{\rm c}({\rm c}/\nu)^2}$$

where ΔRM = the total range in RM with emission; RM_c is the mean RM of the emission (Burn 1966).

Note: any source with |P(RM)| = constant is a Burn jet

Spectra of Burn jets

The polarized flux density spectrum of a Burn jet with different values of ΔRM:



A 2D Gaussian jet

Idea: a 2D Gaussian source on the sky, with a linear RM gradient in front of it.

The monochromatic polarization vector is

$$\boldsymbol{P}_{\rm obs}(\nu) = \frac{(p_0 \times I)}{2\pi\sigma_y\sigma_z}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} {\rm e}^{-\frac{1}{2}\left[\left((y-y_0)/\sigma_y\right)^2+((z-z_0)/\sigma_z)^2\right]}\,{\rm e}^{2{\rm iRM}(y,z)({\rm c}/\nu)^2}\,{\rm d}y{\rm d}z$$

$$= (p_0 \times I)\,{\rm e}^{-\ 2\Delta{\rm RM}^2({\rm c}/\nu)^4\ +\ 2{\rm iRM_c}({\rm c}/\nu)^2}$$

where

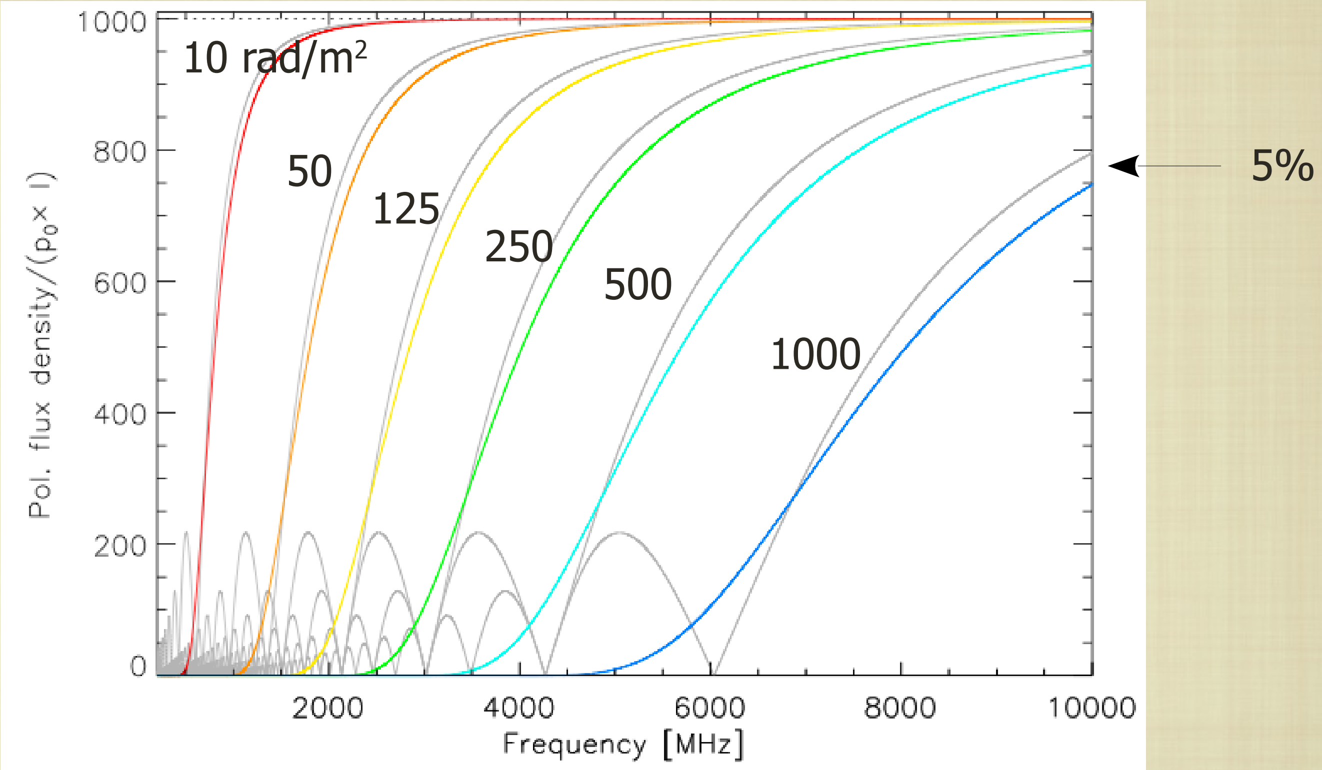
$$\Delta{\rm RM}^2 = \left(\frac{\partial{\rm RM}}{\partial y}\sigma_y\right)^2 + \left(\frac{\partial{\rm RM}}{\partial z}\sigma_z\right)^2$$

is the total RM difference across the major and minor axis of the Gaussian

Leahy et al. (1986), Johnson et al. (1995), and Sokoloff et al. (1998): uniform source with a linear RM gradient, which is observed with a Gaussian beam.

Spectra of 2D Gaussian jets

Overplot spectra for Gaussian jets on top of Burn jets of the same length (2x3 sigma; grey)

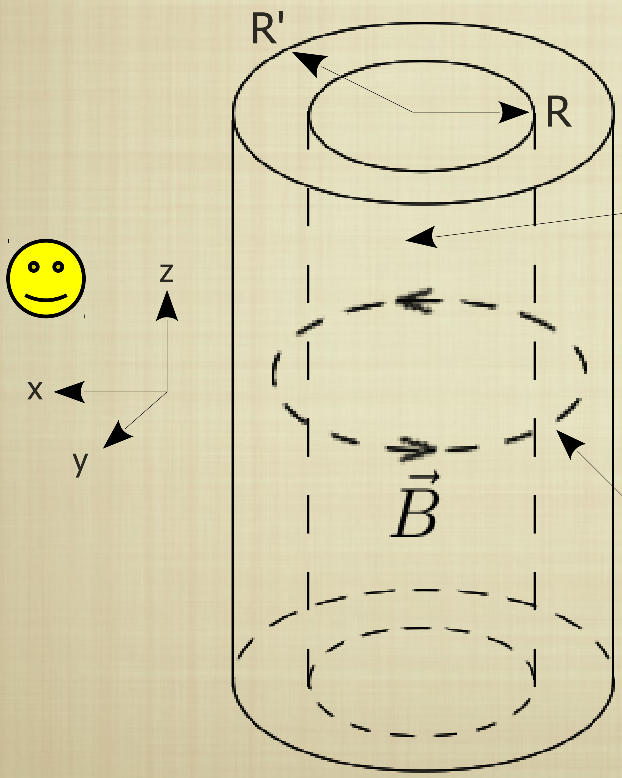


At high frequencies these two different types of sources produce very similar spectra!

The difference between each pair of curves increases with increasing ΔRM.

Cylinder with a wrapped-around field

Idea: Take two coaxial cylinders of radius R and R'. The inner cylinder is the source, Faraday rotation occurs in the boundary layer between the inner and outer cylinders. These cylinders do not have to lie in the plane of the sky.



Emission in the inner cylinder:

$$\boldsymbol{P}_{\mathrm{em}}(y,z) = \int \epsilon_\nu(x,y,z)\sqrt{1-(y/R)^2}/|\cos\theta|\,\mathrm{d}x$$

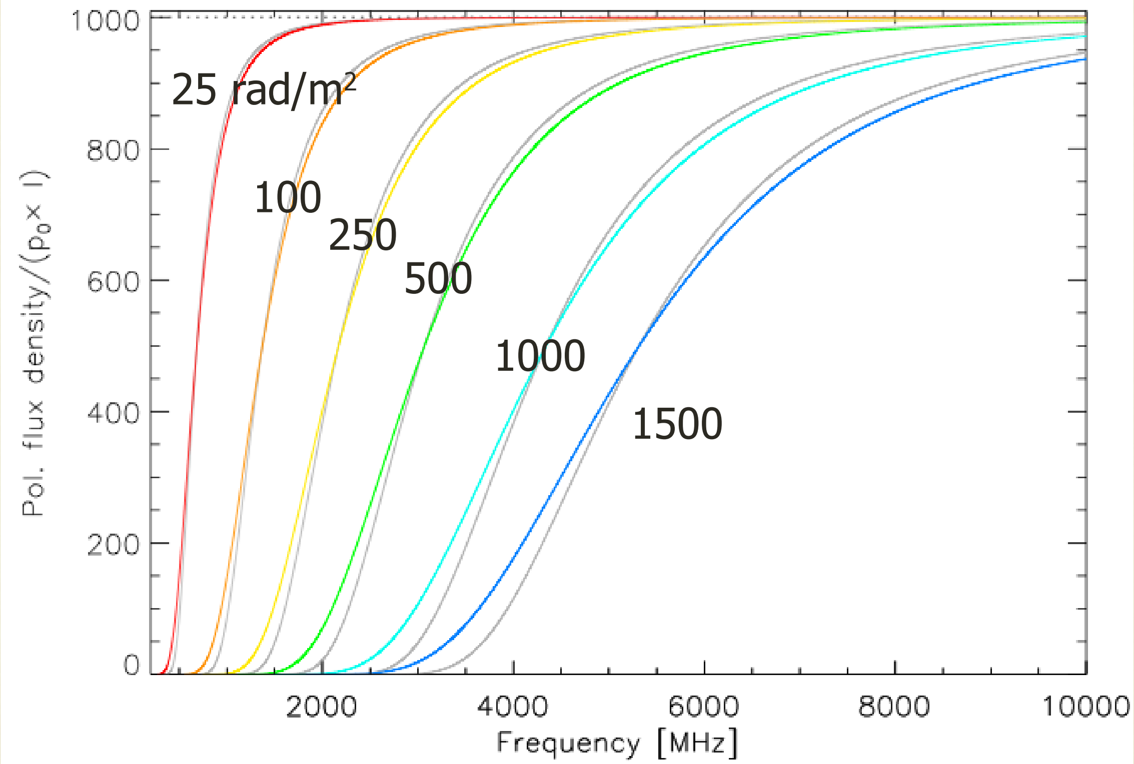
ε is the volume emissivity, θ the inclination angle of the cylinder w.r.t. the plane of the sky

Faraday rotation in the boundary layer:

$$\frac{\mathrm{RM}(y,z)}{\mathrm{RM}_{\max}} = \frac{y}{R}\frac{\ln\left(\frac{R'+\sqrt{(R')^2-y^2}}{R+\sqrt{R^2-y^2}}\right)}{\ln\left(\frac{R'}{R}+\sqrt{\left(\frac{R'}{R}\right)^2-1}\right)}$$

Spectra of cylinders with wrapped fields

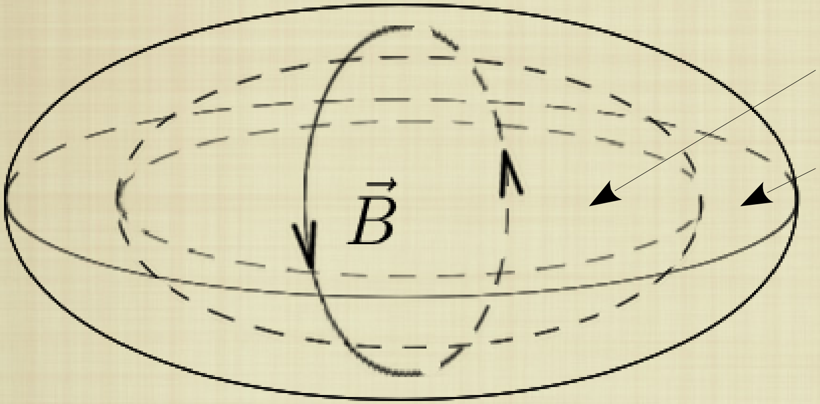
Overplot spectra for different RM_max on top of spectra for Gaussian jets:



The radius of the outer cylinder is 10% larger than the radius of the inner cylinder; thin boundary layers (R' ≈ R) produce similar results

Spheroid with a wrapped-around field

Idea: Take two coaxial ellipsoids. The inner ellipsoid is the source, Faraday rotation occurs in the boundary layer between the inner and outer ellipsoids.



Individual lines of sight:
Emitted polarized flux density in inner ellipsoid: analytical integration
Faraday rotation in boundary: numerical

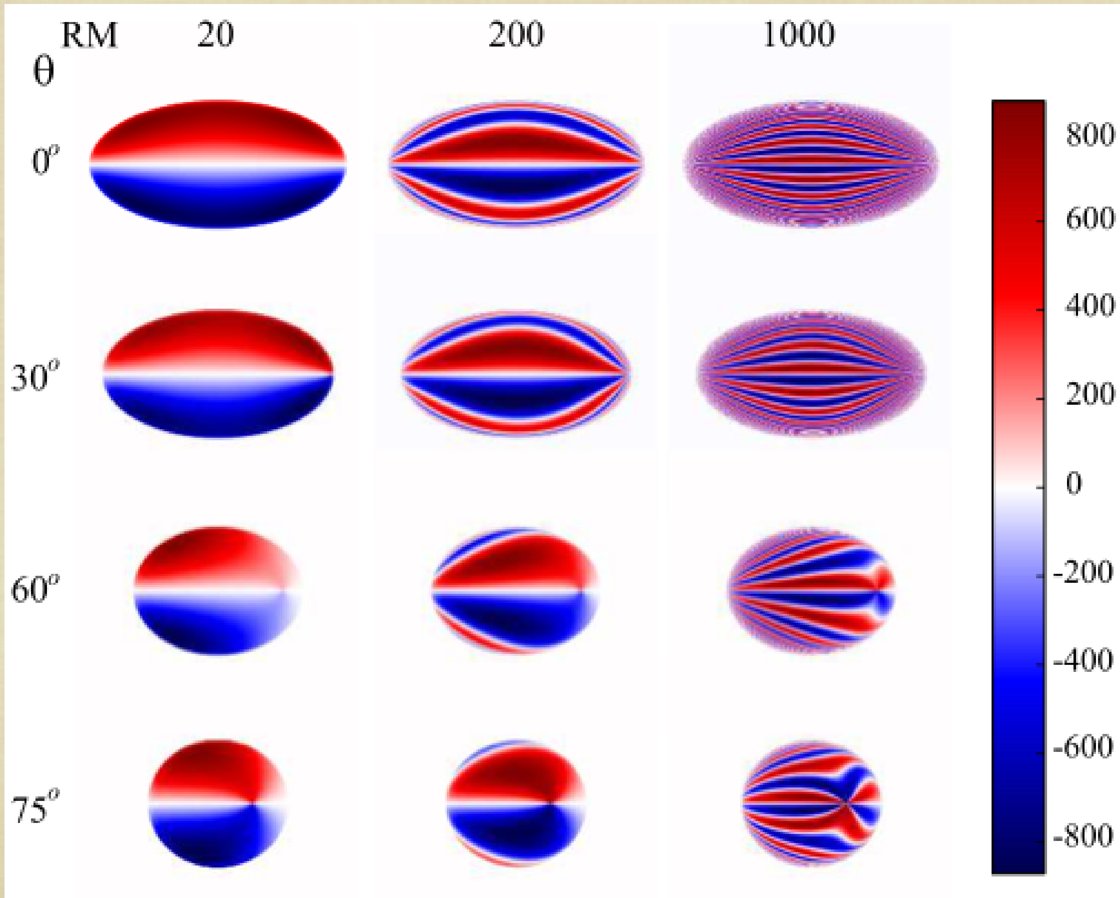
Source as a whole: use numerical integration to combine contributions from different sightlines

The framework we developed can handle ellipsoids with any axis ratio. To simplify our analysis we analyse spheroids, which are ellipsoids with a circular cross-section.

Ellipsoid with a wrapped-around field

Stokes Q across the surface of a spheroid with axis ratio 1:1:2 (arbitrary flux density units)

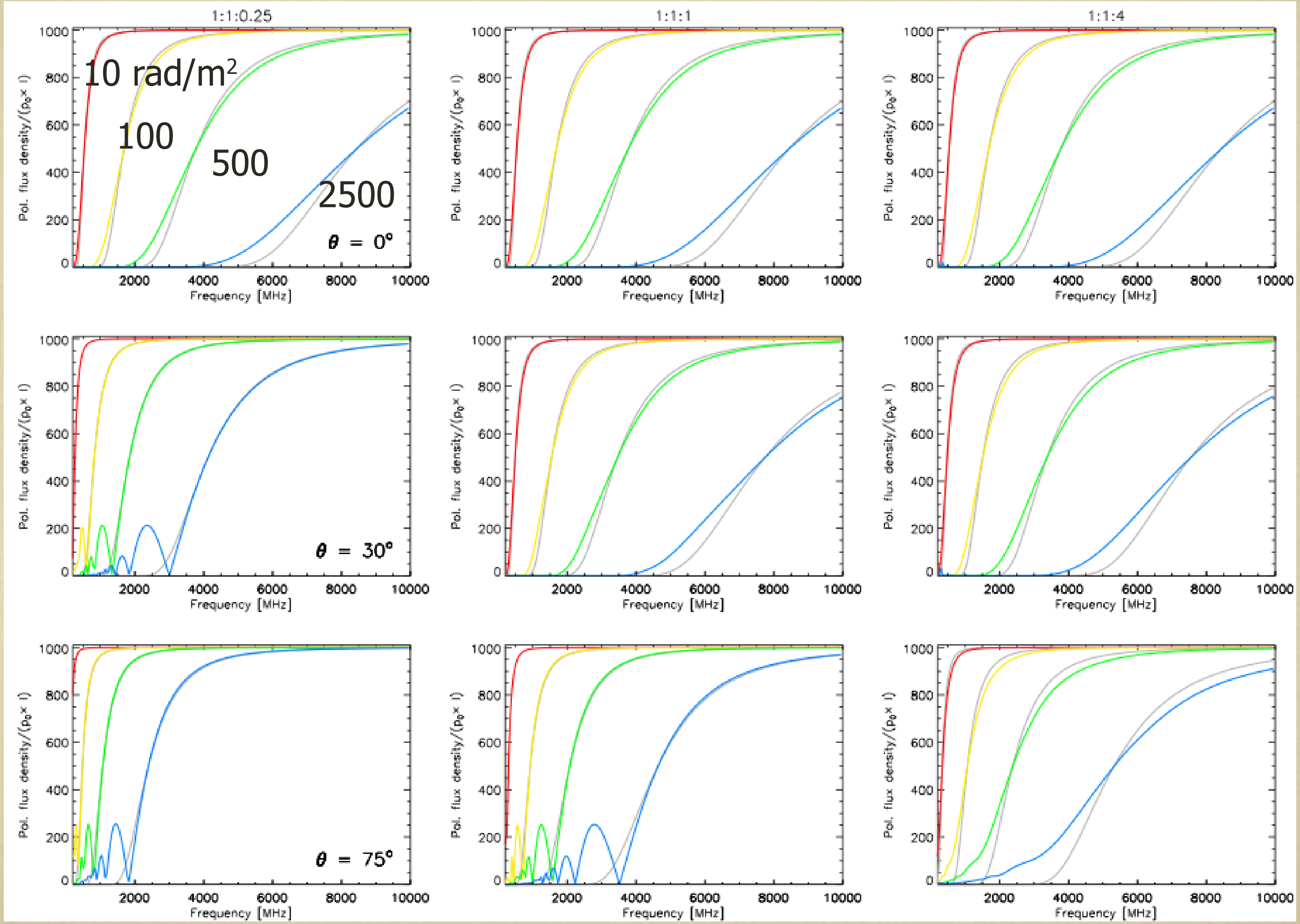
Increasing inclination angle



Increasing maximum RM

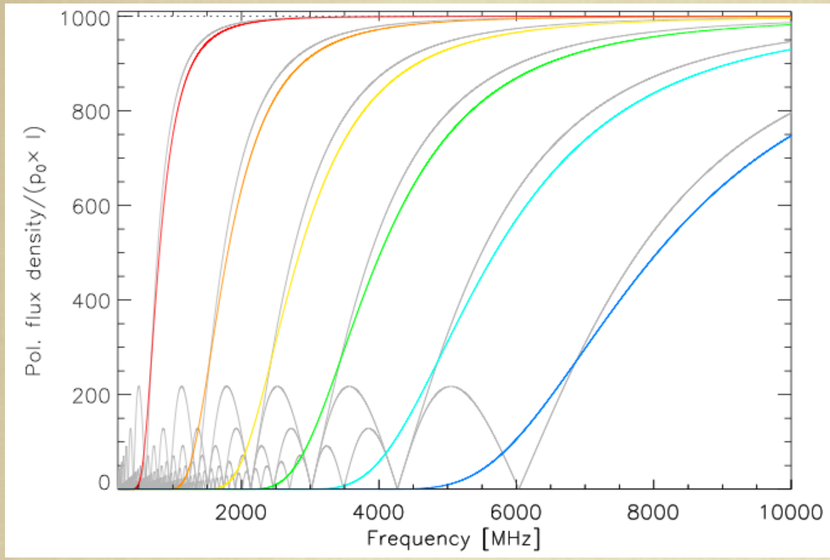
Spectra of spheroids with wrapped fields

Increasing inclination angle

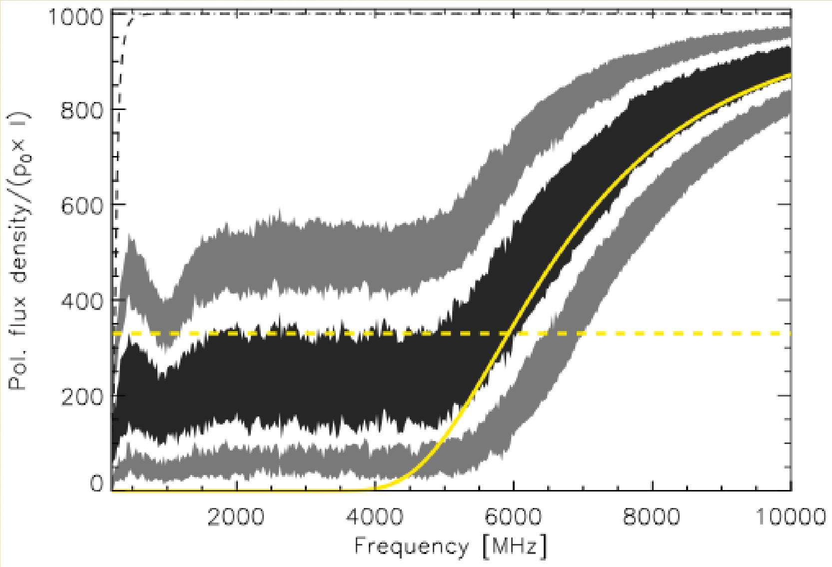


Axis ratio (oblate 1:1:0.25 – sphere – prolate 1:1:4)

The big picture



Sources with large-scale fields
(shown previously in my talk)
+ Burn slabs
(emission and rotation are mixed)



Uniformly emitting source with a
turbulent foreground screen.
Ten sightlines; large field coherence length
Solid yellow line: Burn depolarization curve

Sources with large-scale fields and sources with turbulent foregrounds show a similar drop-off at high frequencies; spectra for sources with turbulent fields level off at lower frequencies.

Not covered in my talk (but in the paper)

- How we simulated turbulent foregrounds
- The Burn depolarization law for depolarization in a turbulent foreground requires field coherence lengths << source size
- Partial coverage models vs. random walk models
- RM spectra of sources with large-scale or turbulent magnetic fields

→ Please come and talk to me! (schnitzeler@mpifr-bonn.mpg.de)

THANK YOU FOR YOUR ATTENTION!