Helical magnetic fields and Faraday depolarization

Cathy Horellou
Onsala Space Observatory, Chalmers University of Technology
Onsala, Sweden

Andrew Fletcher
Newcastle University, U.K.

- Based on paper by Horellou & Fletcher 2014 MNRAS, about single helical fields

- See also Brandenburg & Stepanov, 2014 ApJ, about bi-helical fields

- Use of numerical MHD simulations (work with A. Brandenburg and R. Stepanov, in prep)

Krakow, Oct 22, 2014

Magnetic helicity: a subtle quantity with profound implications

- In the **Mean-Field Dynamo theory**, kinetic helicity is crucial to amplify weak *B's*.
  - Magnetic helicity plays an important role in quenching the dynamo and setting the level at which it saturates (final *B* strength) (Brandenburg & Subramanian 2005, Phys. Report).
- Issue of **primordial B's**. A primordial helicity affects the evolution and growth of *B*.
  - Implication for early Universe physics, particle physics, Charge conjugation-Parity violation.
    Helical *B's* generated at the EW phase transition? (Durrer & Neronov 2013 A&A Rev)
- **Astrophysics**: Helical *B*'s in the Sun (Zhang+ 2014 ApJL) and young stellar objects (Chrysostomou+ 2007, Nature); in loops and arcs near the Galactic Center (Yusef-Zadeh & Morris, 1987 ApJ); in extragalactic radio jets (Gabuzda+ 2014, MNRAS; Molina+ 2014, A&A), nuclear outflows in starburst galaxies (NGC 253, Heesen+ 2011, A&A); nearby spiral galaxies (NGC 6946, Beck+ 2007 A&A); M31's synchrotron torus (Urbanik+ 1994, A&A); intracluster medium or intergalactic medium?
- Tests of Magnetic helicity based on RM gradients parallel to synchrotron polarization (Junklewitz & Ensslin 2011, A&A, Oppermann+ 2011, A&A)
- Magnetic helicity **affects the emerging polarization** – it mostly depolarizes, but can also re-polarize ("inverse depolarization", increase of %pol with lambda; Sokoloff et al. 1998 MNRAS; Horellou & Fletcher 2014, MNRAS; Brandenburg & Stepanov 2014, ApJ)

The complex polarized intensity

$$P(\lambda^2) = \int_0^{\infty} \underbrace{\varepsilon(x)}_{\text{Synchrotron emissivity}} \, \underbrace{e^{2i\chi_0(x)}}_{\textit{Intrinsic}\ \text{PA}} \, \underbrace{e^{2i\phi(x)\lambda^2}}_{\text{Faraday rotation}} \, dx$$

$$\varepsilon(x) = n_c(x)|B_\perp(x)|^{1+\alpha}$$

The Faraday dispersion function

Decomposition of the complex polarization in Faraday depth space

Observed cpolarization *Intrinsic* cpolarization Faraday rotation

$$P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} \, d\phi.$$

***P(λ²)* is simply the inverse Fourier Transform of *F*(ϕ)**

$$\begin{cases} P(\lambda^2) = \int_{-\infty}^{+\infty} F(\phi) \mathrm{e}^{2\mathrm{i}\phi\lambda^2}\, \mathrm{d}\phi. \\ F(\phi) = \frac{1}{\pi} \int_{-\infty}^{+\infty} P(\lambda^2) e^{-2\mathrm{i}\phi\lambda^2} d\lambda^2 . \end{cases}$$

RM synthesis (Brentjens & de Bruyn 2005): reconstruct $F(\phi)$ from observed $P(\lambda_i^2)$.
Inversion problem. Issue of negative λ^2

There is a Fourier relation between $P(\lambda^2)$ and $F(\phi)$.
The intrinsic polarization angle *ψ_0 may depend on ϕ.*

$$F(\phi) = |F(\phi)| e^{2\mathrm{i}\psi_0(\phi)}$$

Parametrization of the PA as a 1st-order polynomial

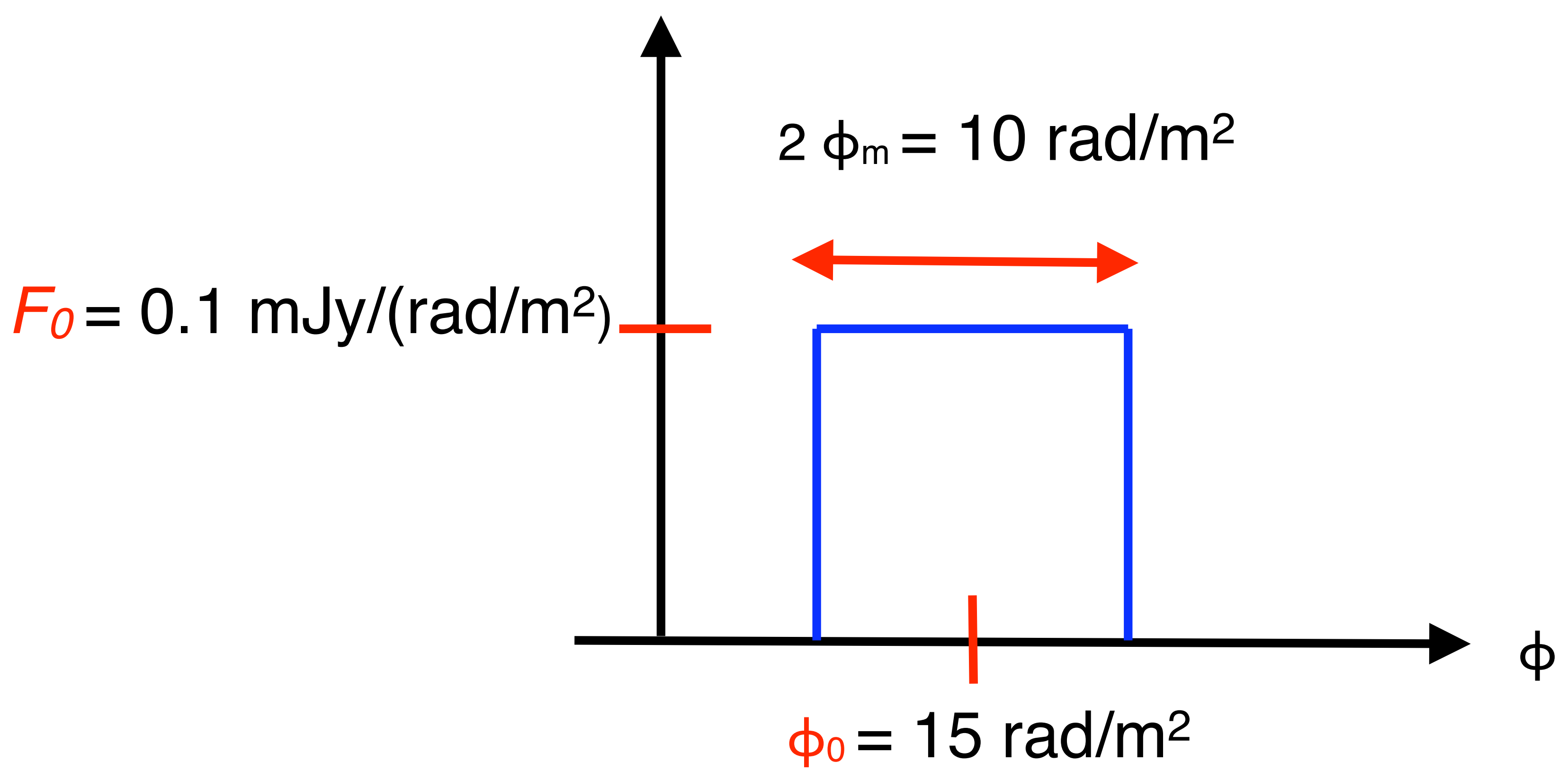
$$\psi_0 = \psi_0(\phi) = \alpha + \beta\ \phi$$

[rad] [rad] [m²] [rad/m²]

α = 0 rad

β = 0, ± 0.2 m^2

A simple model of the Faraday dispersion function: a uniform slab



Parametrization of the PA as a 1st-order polynomial

$$B = \begin{pmatrix} B_\perp \cos(\alpha + k_H z) \\ B_\perp \sin(\alpha + k_H z) \\ B_{||} \end{pmatrix}$$

Helicity parameter

$$\psi_0 = \alpha + k_H z = \alpha + \beta\phi, \tag{16}$$

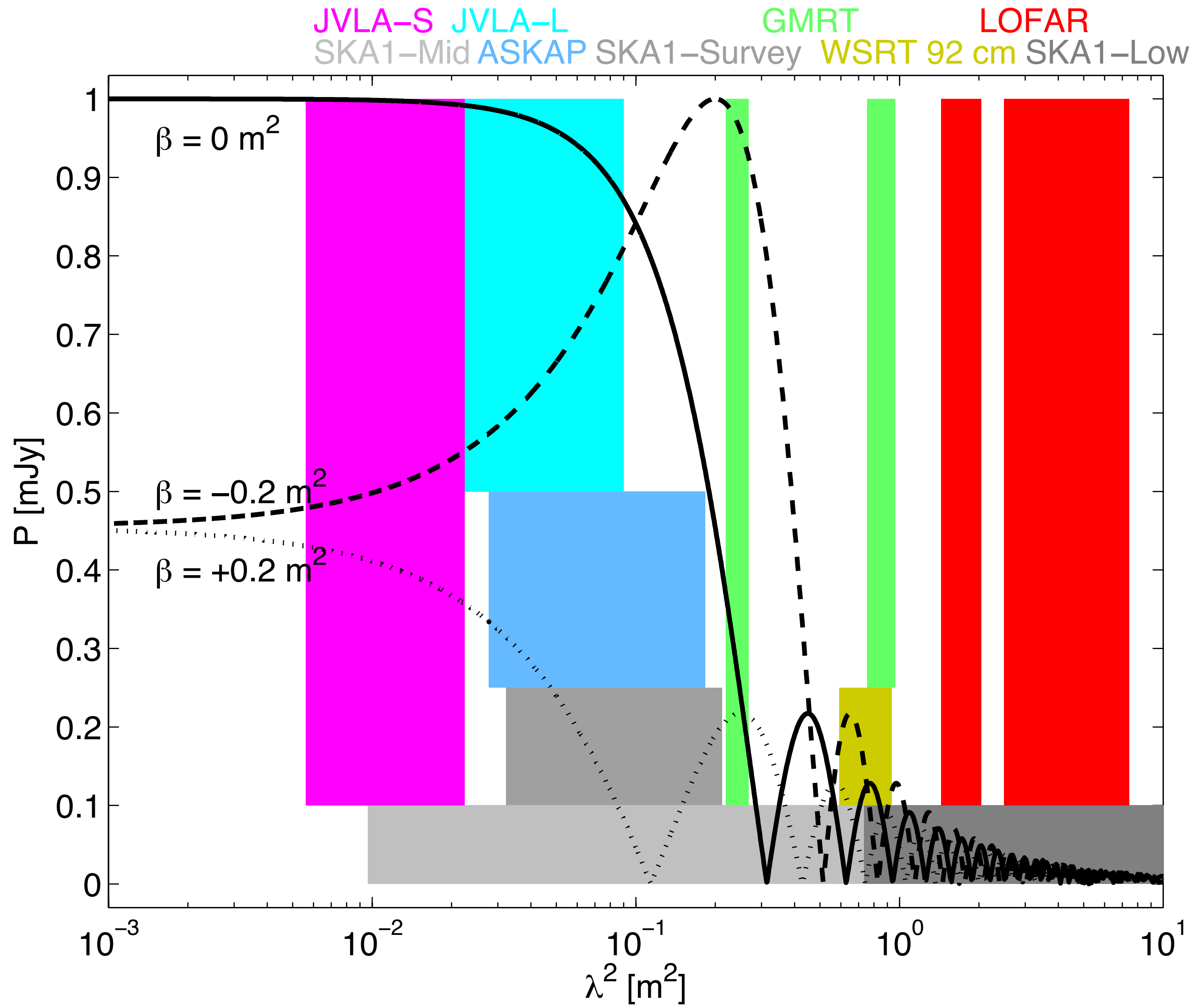
where

β >0 : Helicity and $B_{||}$ have same sign → more depolarization

β <0 : Helicity and $B_{||}$ have opposite sign → less depolarization

$$\begin{aligned}\beta &= \frac{k_H}{0.81 n_e B_{\|}} \\ &= 0.086\,\mathrm{m}^2 \left(\frac{k_H}{2\pi\ \mathrm{rad\ \ kpc}^{-1}}\right)\left(\frac{0.03\,\mathrm{cm}^{-3}}{n_e}\right)\left(\frac{3\,\mu\mathrm{G}}{B_{||}}\right)\ .\end{aligned} \tag{17}$$

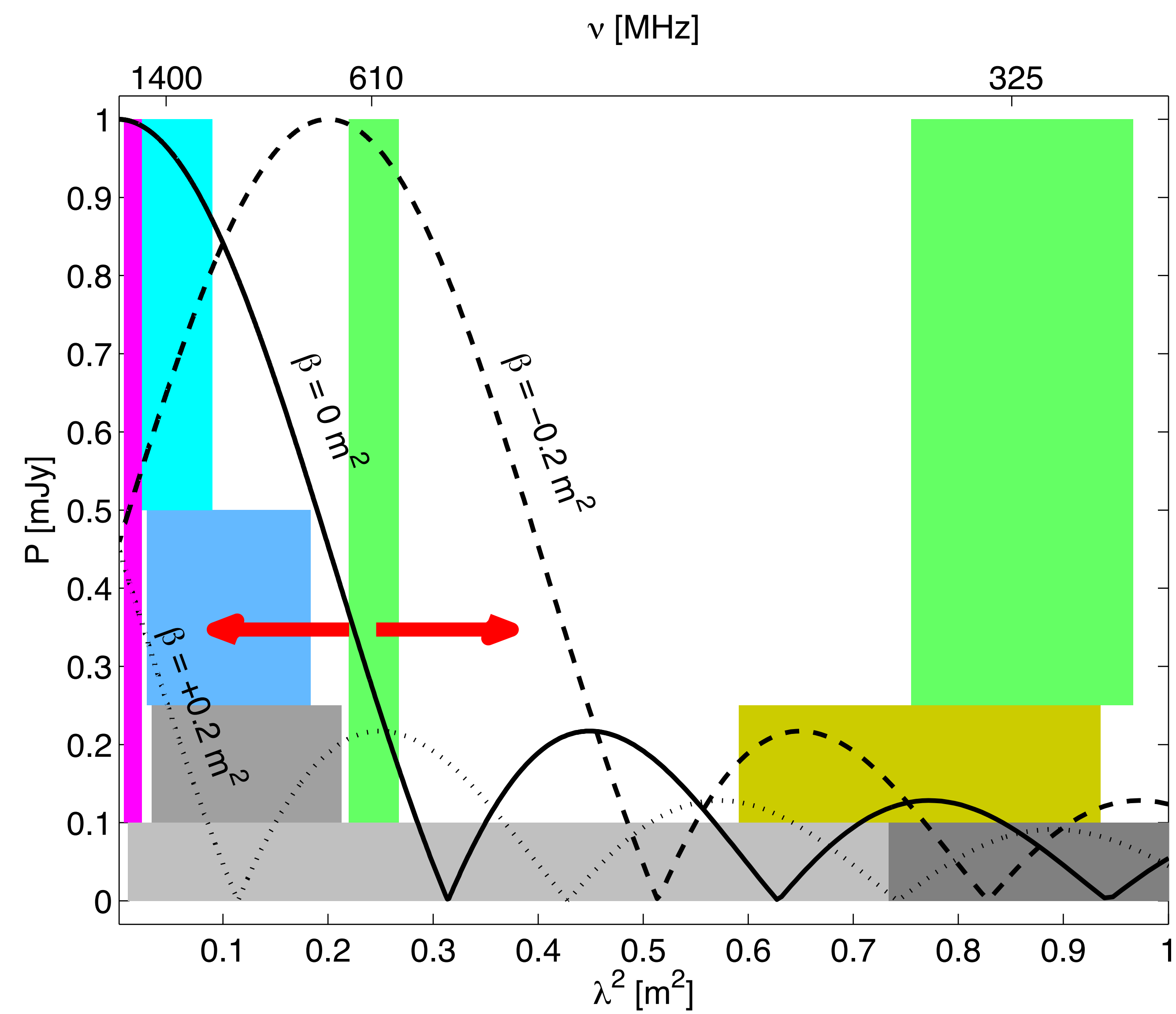
For an helical field with $k_H \simeq 2\pi$ rad kpc^{-1}, we have $\beta \simeq 0.1$ m^2.



$$P(\lambda^2; \mathbf{p}) = 2\phi_{\mathrm{m}}\, F_0 \,\mathrm{sinc}[2\phi_{\mathrm{m}}(\lambda^2 + \beta)]e^{2\mathrm{i}\psi(\lambda^2;\mathbf{p})}$$

where

$$\psi(\lambda^2; \mathbf{p}) = \alpha + (\lambda^2 + \beta)\phi_0$$



Fisher analysis: To estimate the *precision* that can be achieved on model parameters & *the covariance matrix* for a certain data set.

$$\mathcal{F}_{jk} = -\frac{\partial^2 \ln \mathcal{L}}{\partial p_j \partial p_k} = \frac{1}{2}\frac{\partial^2 \chi^2}{\partial p_j \partial p_k}$$

Model of the Faraday dispersion function: Top-hat or Gaussian.
Data: *Q* and *U* at different wavelengths (as in Table).

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{Q_i - Q_{\mathrm{mod}}(\lambda_i; p_1, \ldots, p_P)}{\sigma_i}\right)^2 + \left(\frac{U_i - U_{\mathrm{mod}}(\lambda_i; p_1, \ldots, p_P)}{\sigma_i}\right)^2 . \tag{7}$$

The Fisher matrix elements can be written as

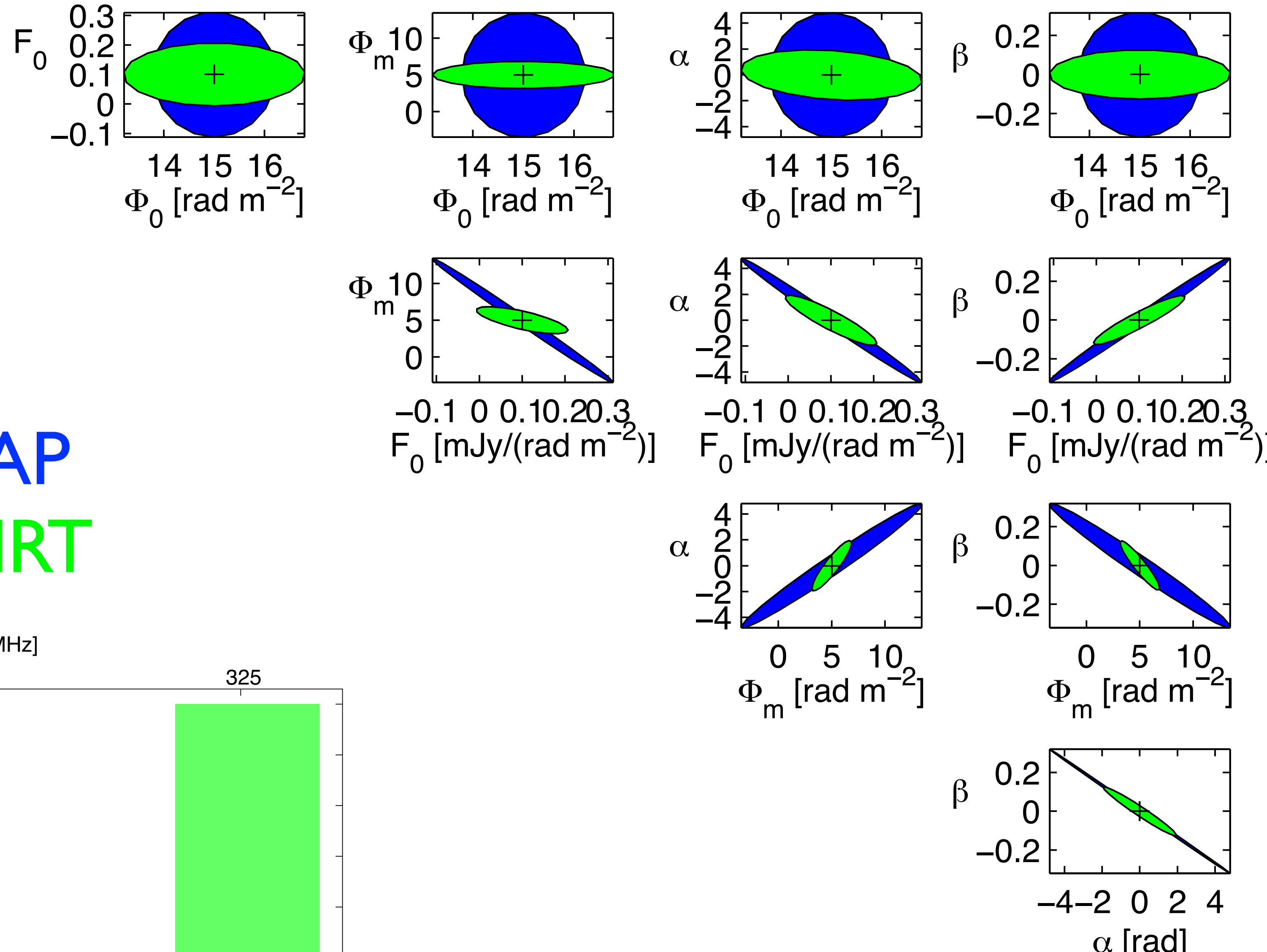
$$\mathcal{F}_{jk} = \sum_{i=1}^{N} \frac{1}{\sigma_i^2} \left(\frac{\partial Q_{\mathrm{mod}}(\lambda_i^2; p_1, \ldots, p_P)}{\partial p_j} \frac{\partial Q_{\mathrm{mod}}(\lambda_i^2; p_1, \ldots, p_P)}{\partial p_k} + \frac{\partial U_{\mathrm{mod}}(\lambda_i^2; p_1, \ldots, p_P)}{\partial p_j} \frac{\partial U_{\mathrm{mod}}(\lambda_i^2; p_1, \ldots, p_P)}{\partial p_k} \right) . \tag{8}$$

The covariance matrix is the inverse of the Fisher matrix:

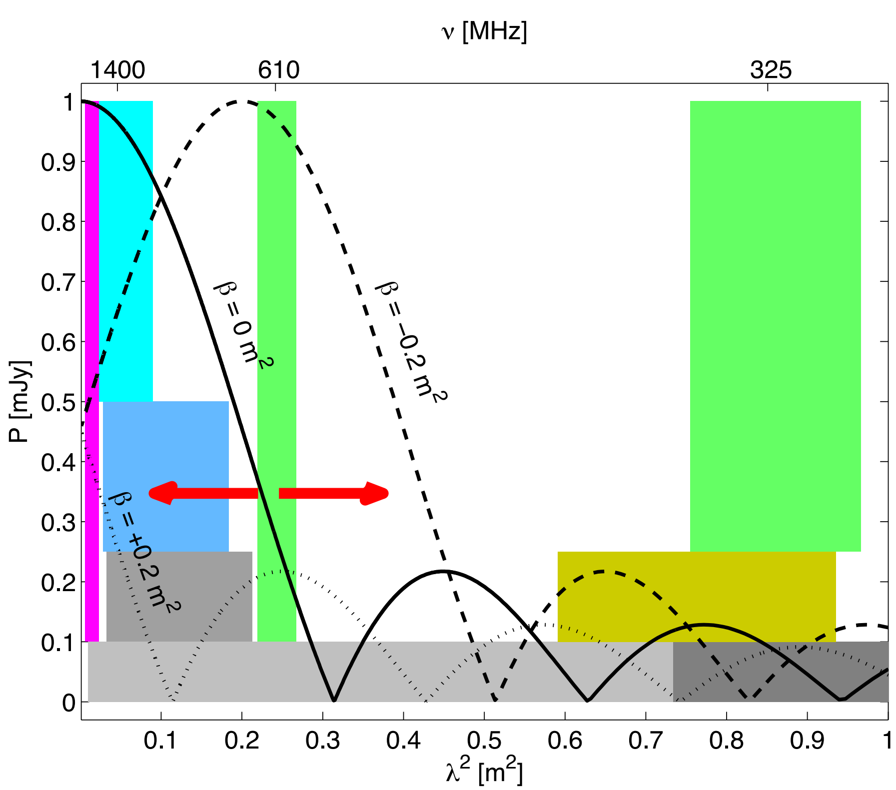
$$\sigma_{jk}^2 = (\mathcal{F}^{-1})_{jk} . \tag{9}$$

Instrument	Note	Frequency band [MHz]	Sensitivity [mJy]	Channel width [MHz]	Integration time
JVLA	S-band	2000-4000	0.3	2	10 min
JVLA	L-band	1000-2000	0.6	1	10 min
ASKAP		700–1800	2.5	1	10 min
SKA1	Survey	650–1670	0.3	1	10 min
SKA1	**Mid**	350–3050	0.1	1	10 min
SKA1	**Low**	50–350	0.08	1	10 min
GMRT		580–640	0.5	1	1 hr
GMRT		305–345	**3.8**	1	1 hr
WSRT	**92 cm**	310–390	3.9	1	1 hr
LOFAR	HBA2	210–250	**2.6–6.0**	**1**	**1 hr**
LOFAR	HBA1	110–190	**1.6**	**1**	**1 hr**

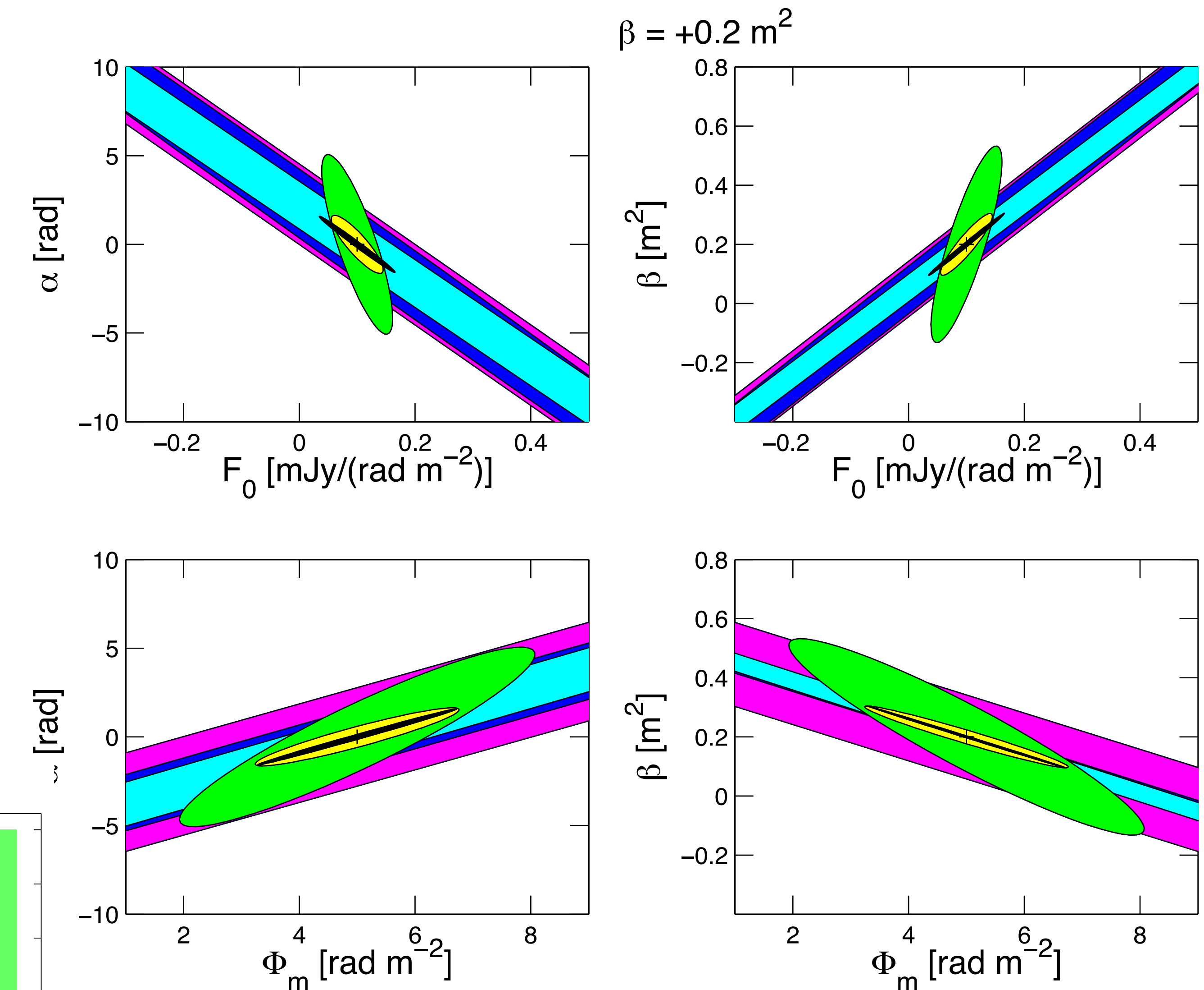
68.3% confidence regions
$\beta = 0\ m^2$



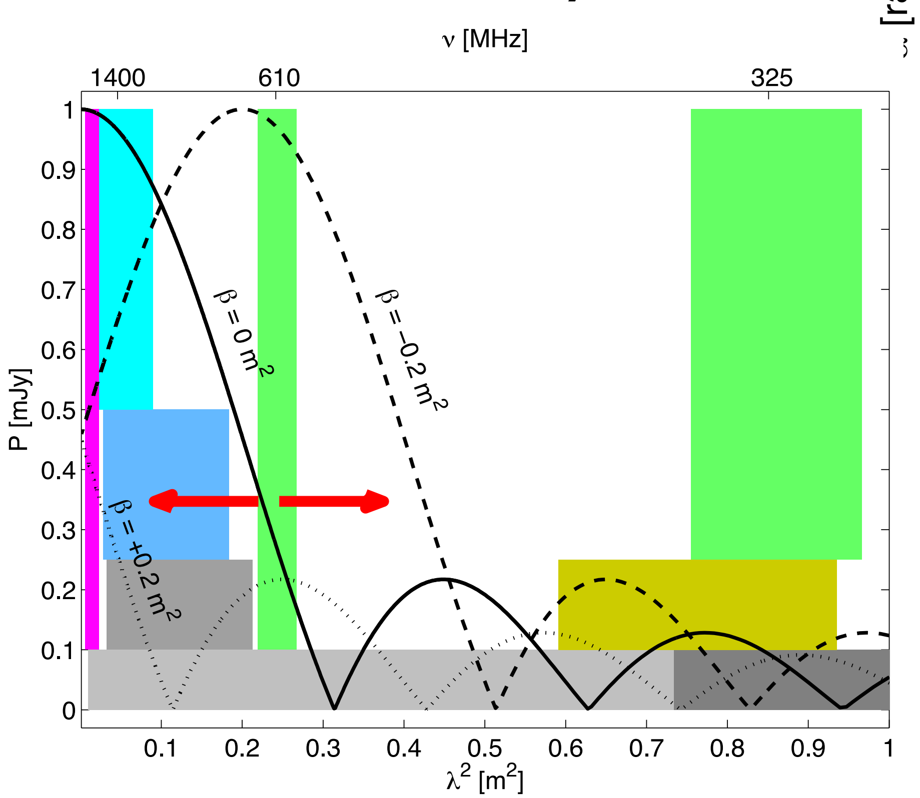
Blue: ASKAP
Green: GMRT



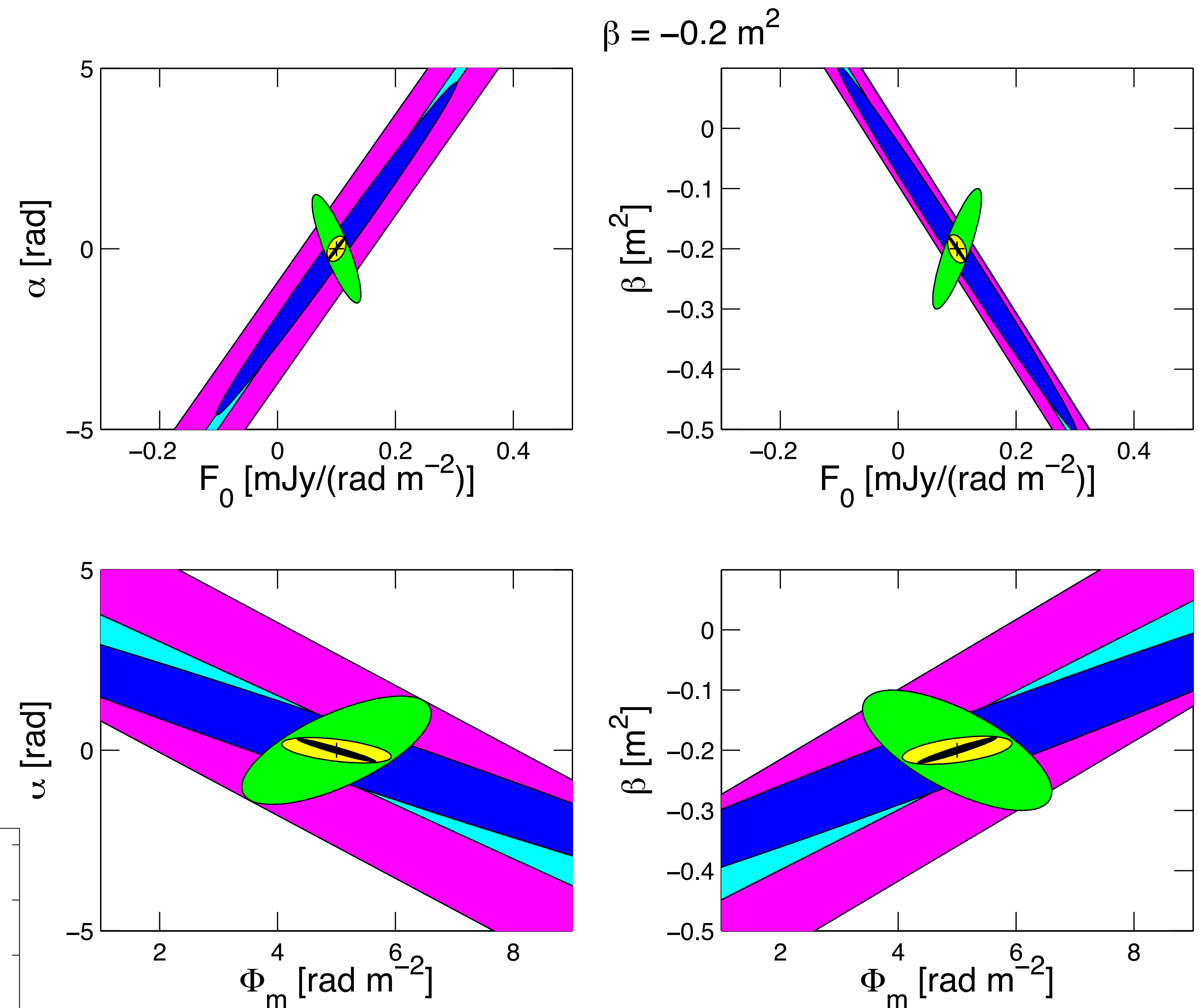
68.3% confidence regions
$\beta = +0.2\ m^2$



Blue: ASKAP
Green: GMRT
Yellow: ASKAP + GMRT
Black: SKA1-Survey



68.3% confidence regions
β= -0.2 m^2



Blue: ASKAP
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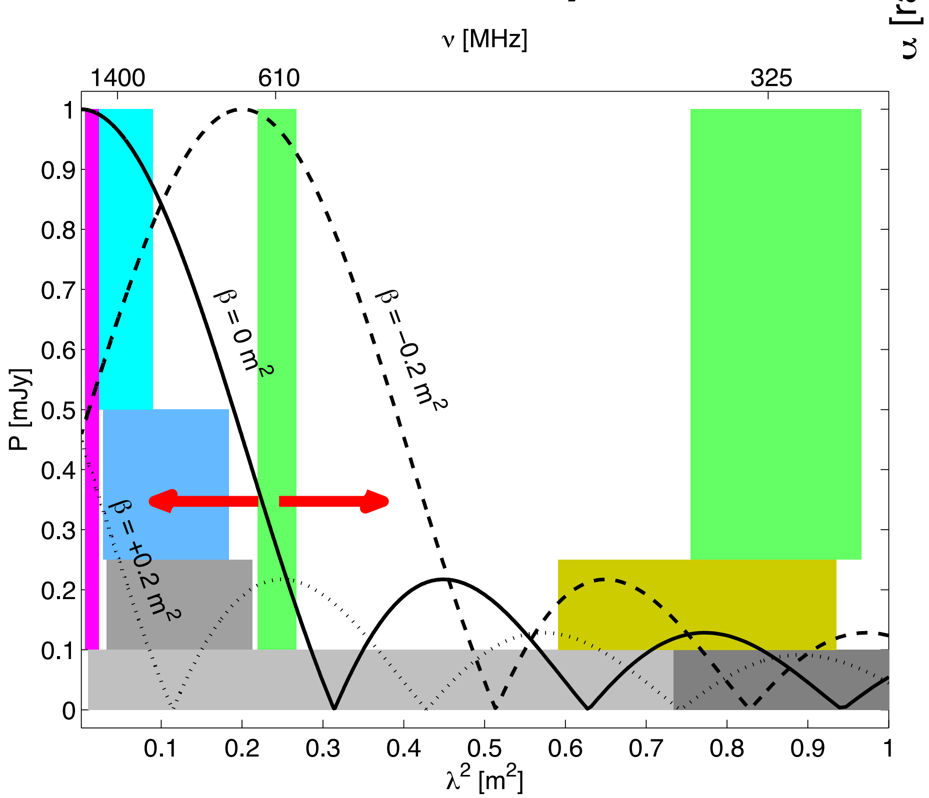
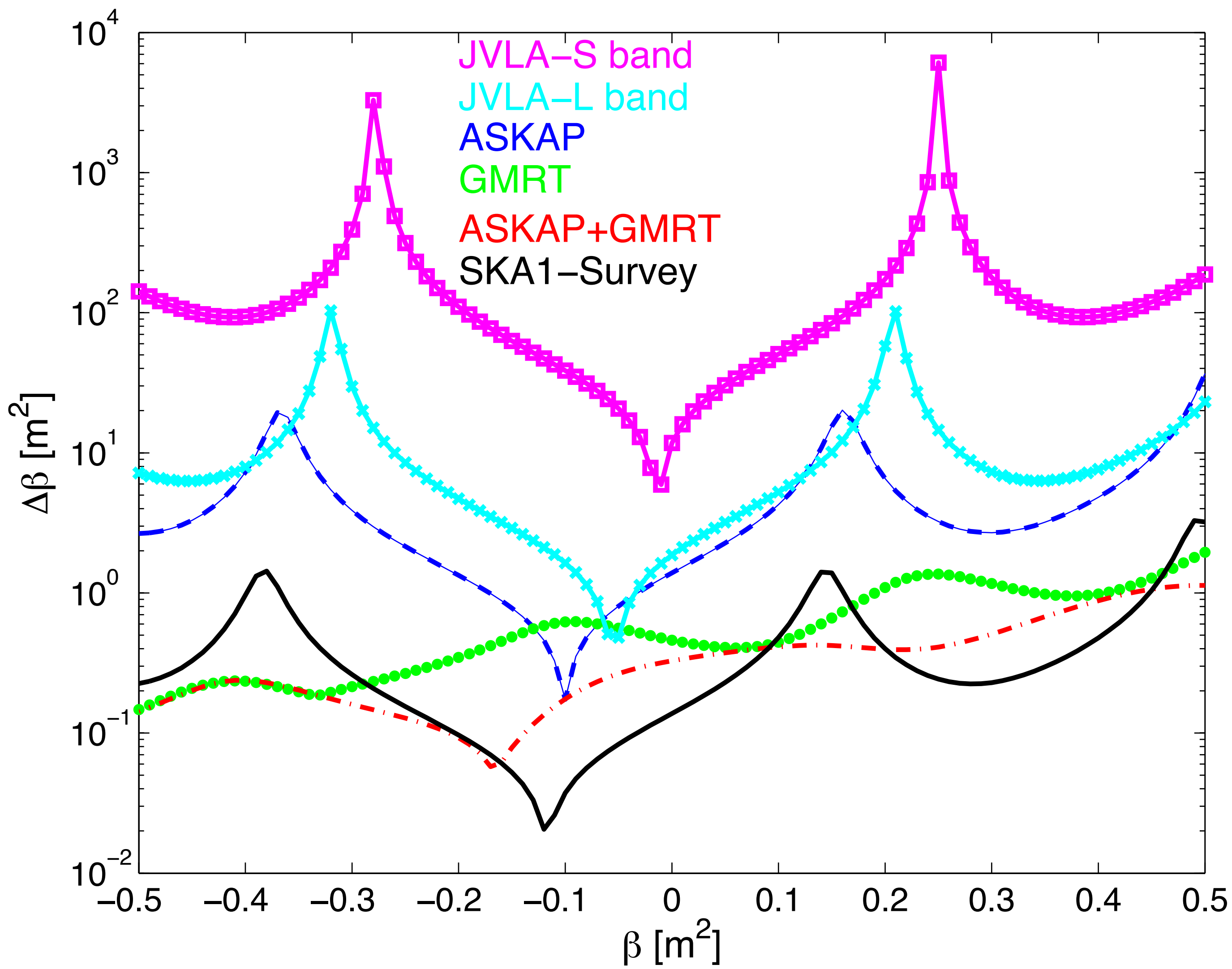
Yellow: ASKAP + GMRT
Black: SKA1-Survey



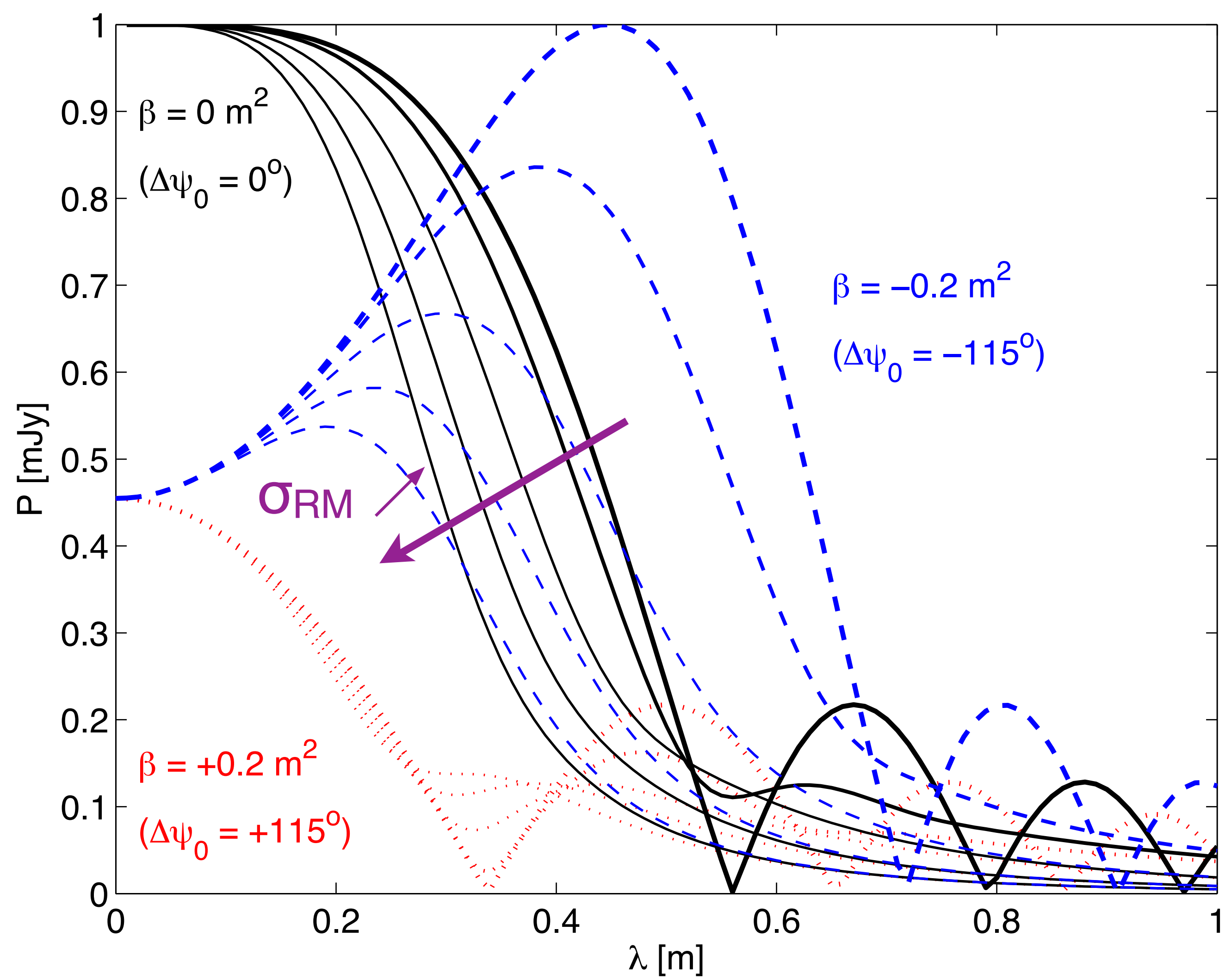
Uncertainty on the β parameter versus β



$$\beta = \frac{k_H}{0.81 n_e B_{\|}}$$

Rotation along the los of B_{perp}

Including Faraday dispersion **attenuates the effect**



From top to bottom: σ_{RM} increases from 0 to 10 rad/m^2 by steps of 2.5

Summary

• Helicity is a fundamental property.

• We have investigated the detectability of a 1st-order polynomial variation of the intrinsic polarization angle with Faraday depth ϕ.

• Combining multifrequency datasets (as ASKAP & GMRT) is a powerful way to increase precision on the derived parameters, before SKA becomes operational.

• Faraday dispersion attenuates the variations and shifts the peak of polarized intensity.

•Surveys will be biased toward sources with negative β-parameters because of the increased depolarization for positive β's.

• Need to define an estimator to measure magnetic helicity in data (both from simulations and observations).