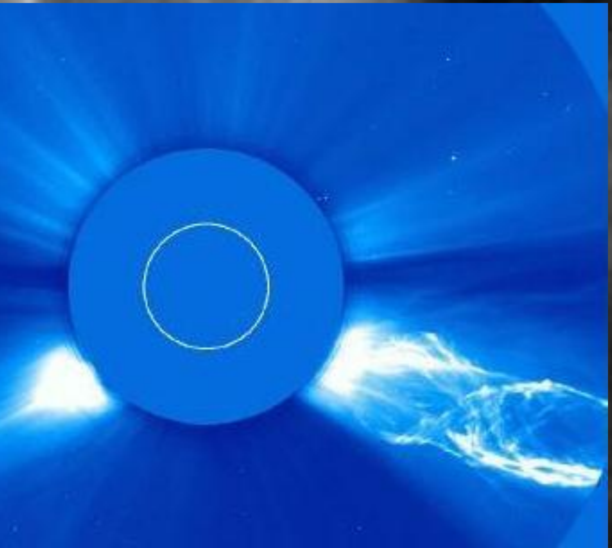
Detection of magnetic helicity in stars and galaxies

What to expect?

Lessons from dynamo theory

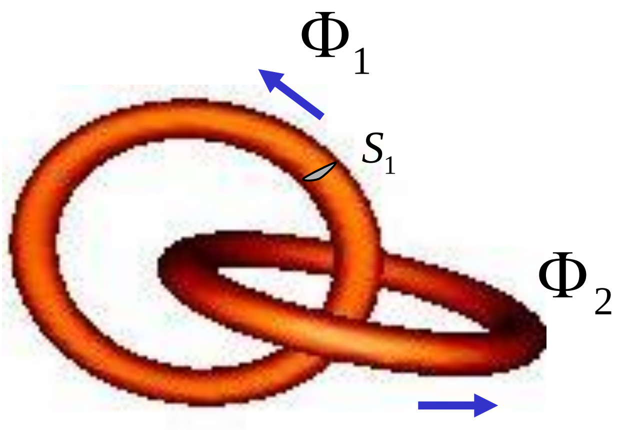
What we see in solar wind?

What we can see in galaxies…



Axel Brandenburg
(Nordita, Stockholm)

Magnetic helicity measures linkage of flux



$$H = \int_V \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}V$$

$$\mathbf{B} = \nabla\times\mathbf{A}$$

$$H = \pm 2\Phi_1\Phi_2$$

$$H_1 = \int_{L_1} \mathbf{A}\cdot\mathrm{d}\ell \;\int_{S_1} \mathbf{B}\cdot\mathrm{d}\mathbf{S}$$

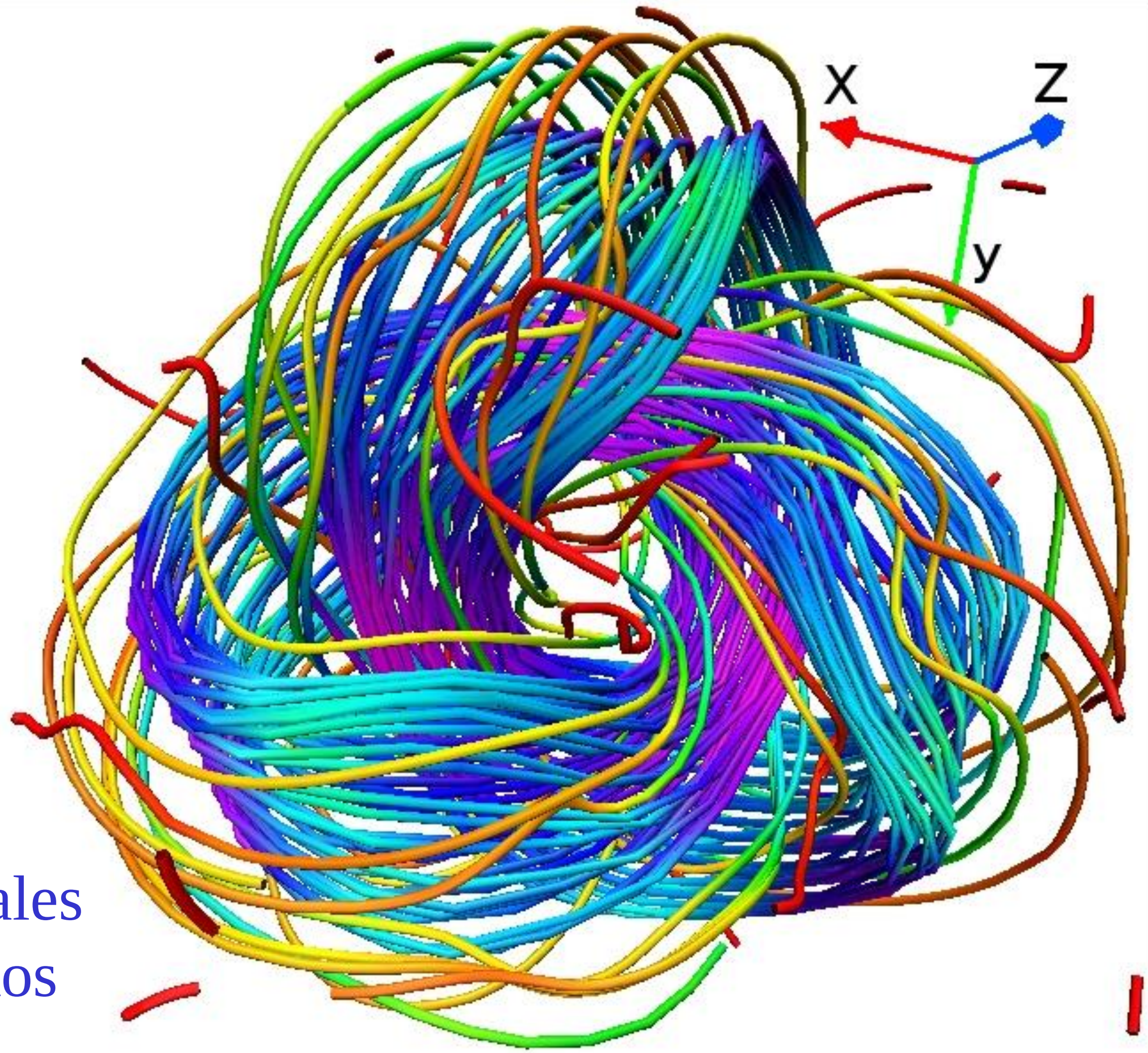
$$= \int_{S_2} \nabla\times\mathbf{A}\cdot\mathrm{d}\mathbf{S} = \Phi_2 \qquad\qquad = \Phi_1$$

Therefore the unit is
Maxwell squared

Other fun examples

Trefoil knot

$H=3\phi^2$



(Candelaresi and Brandenburg (2011))

Relevance:

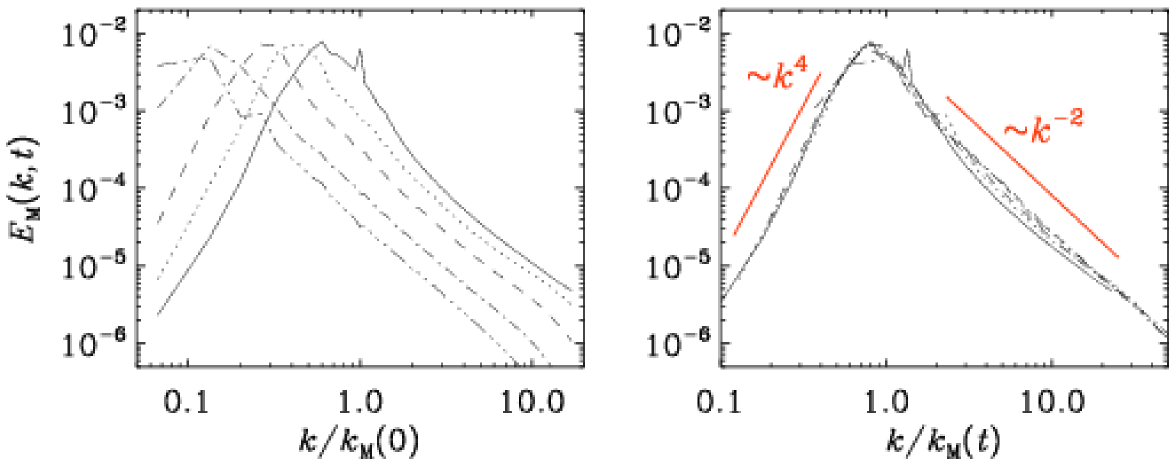
Slows down decay
Growth at large scales
Large-scale dynamos

Decaying helical fields

$$k\,|H(k)| \le 2E(k)$$

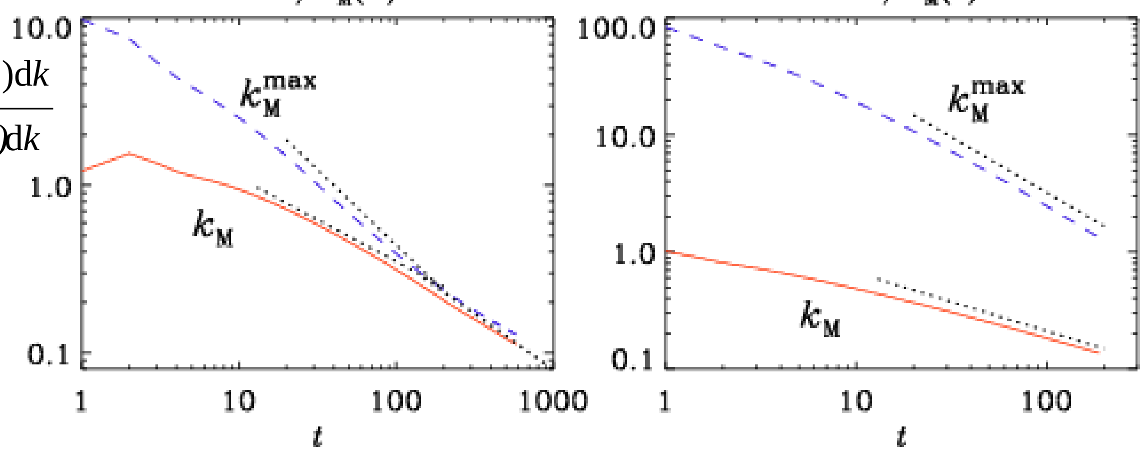
$$\int H(k)\mathrm{d}k \le 2\int k^{-1}E(k)\mathrm{d}k$$

$$k_M^{-1} \equiv \frac{\int k^{-1}E(k)\mathrm{d}k}{\int E(k)\mathrm{d}k} \ge \frac{\int H(k)\mathrm{d}k}{2\int E(k)\mathrm{d}k} \equiv \left(k_M^{\max}\right)^{-1}$$



(i) Transfer to Large scales

$$k_M^{\max} = \frac{2\int E(k)\mathrm{d}k}{\int H(k)\mathrm{d}k}$$



(ii) slow-down of decay

Tevzadze, Kisslinger, Brandenburg, Kahniashvili (2012, ApJ,)

PHYSICAL REVIEW E **84**, 016406 (2011)

Decay of helical and nonhelical magnetic knots

Simon Candelaresi and Axel Brandenburg
NORDITA, AlbaNova University Center, Roslagstullsbacken 23, SE-10691 Stockholm, Sweden and
Department of Astronomy, Stockholm University, SE-10691 Stockholm, Sweden
(Received 17 March 2011; revised manuscript received 8 May 2011; published 25 July 2011)

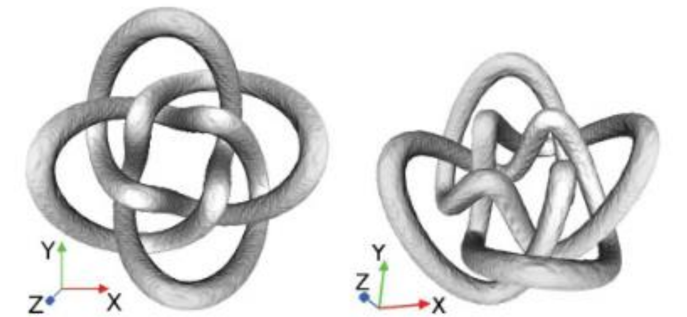
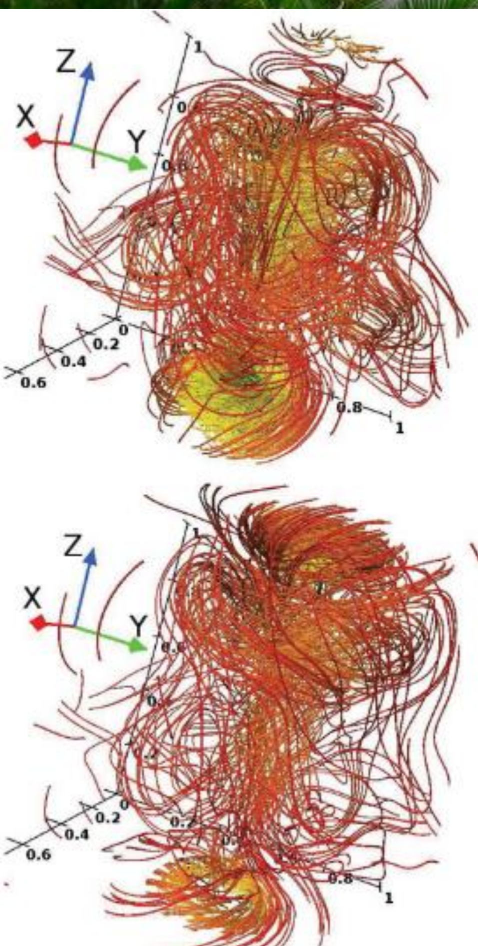
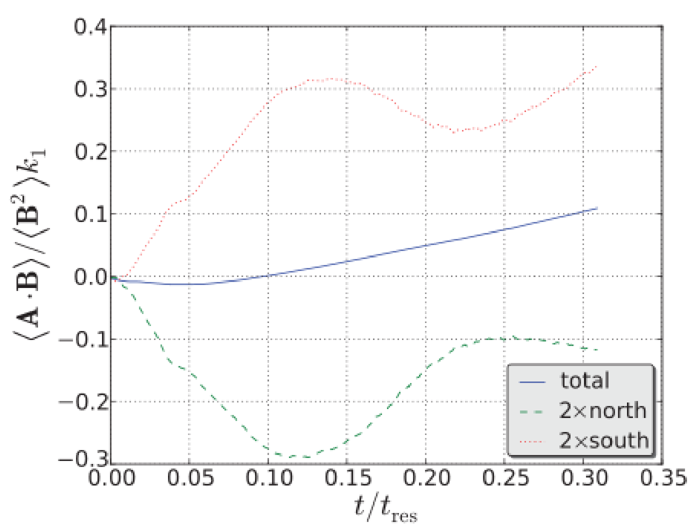
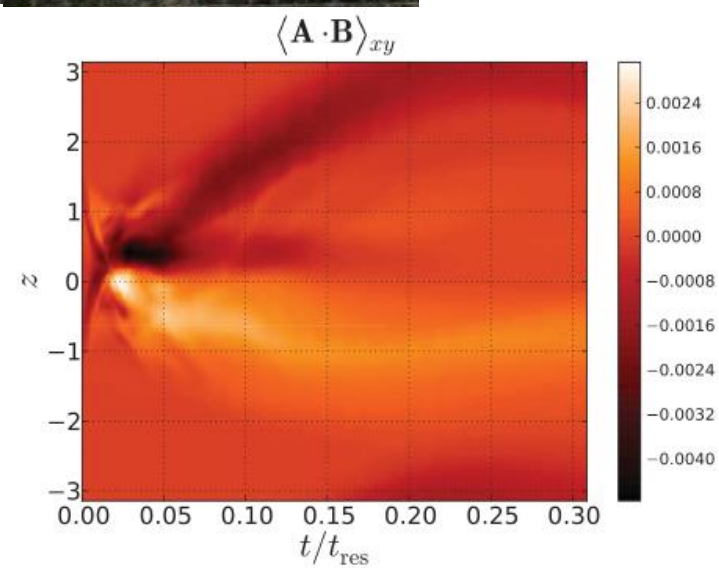


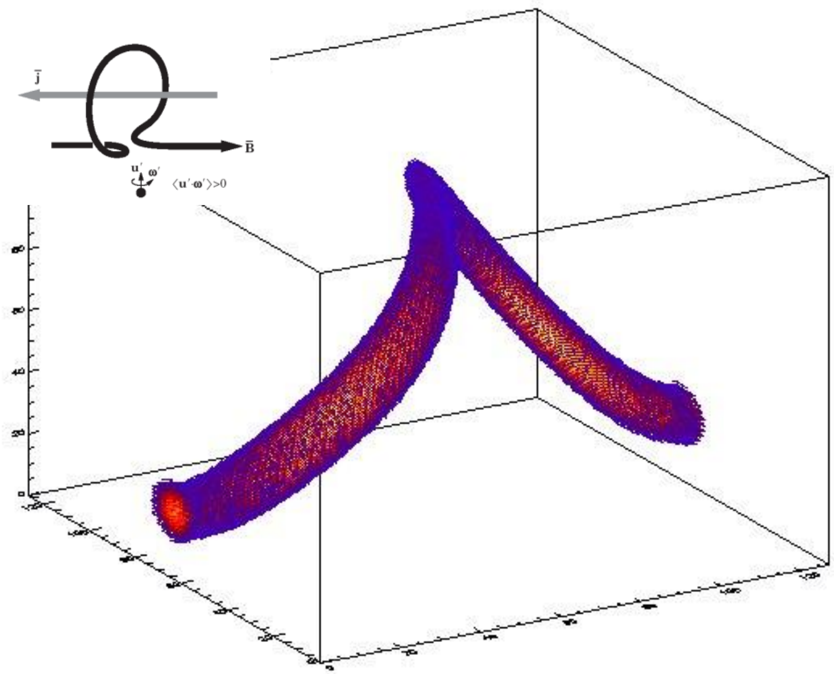
FIG. 5. (Color online) Isosurface of the initial magnetic field energy for the IUCAA knot seen from the top (left panel) and slightly from the side (right panel).



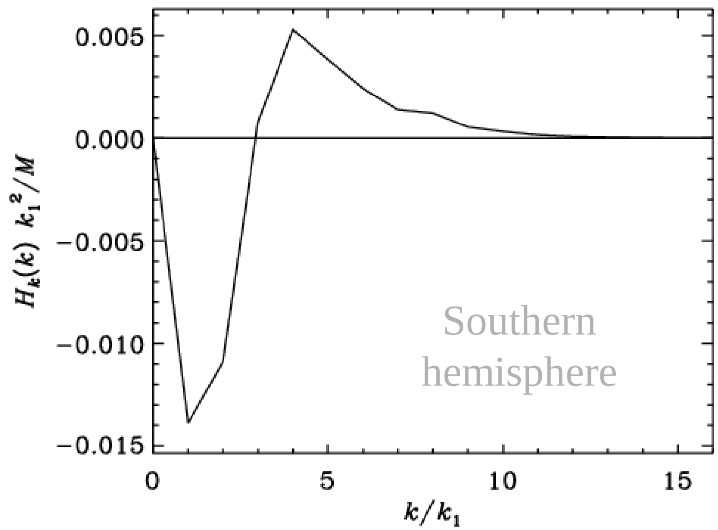




Dynamos produce bi-helical fields



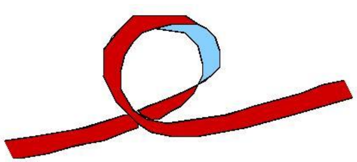
Magnetic helicity spectrum



$$\int H(k)dk = \langle \mathbf{A}\cdot\mathbf{B} \rangle$$

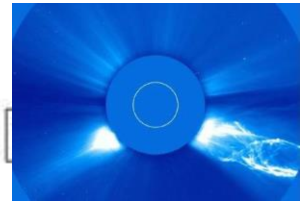
Pouquet, Frisch, & Leorat (1976)

$$\alpha = -\tfrac{1}{3}\tau\left(\langle \boldsymbol{\omega}\cdot\mathbf{u} \rangle - \langle \mathbf{j}\cdot\mathbf{b} \rangle / \rho_0\right)$$

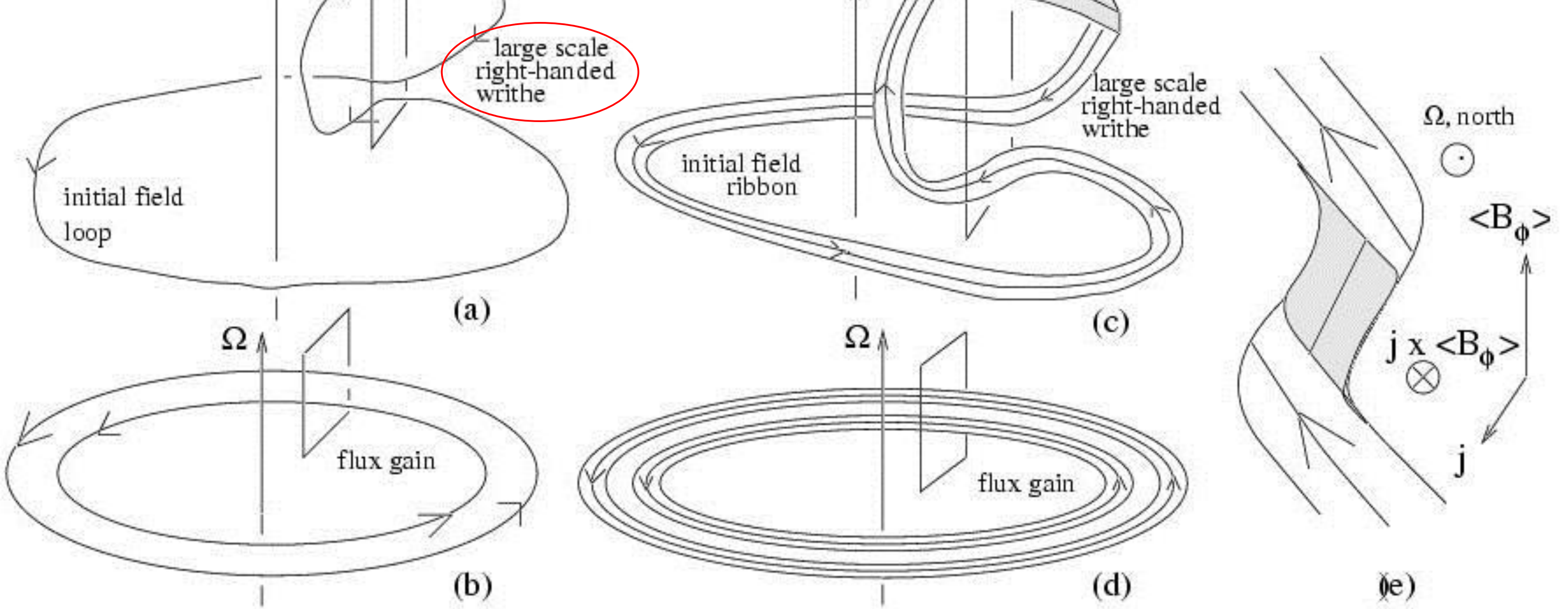


Self-inflicted twist: feedback & CMEs

=coronal mass ejection



Blackman & Brandenburg (2003)



(the whole loop corresponds to CME)

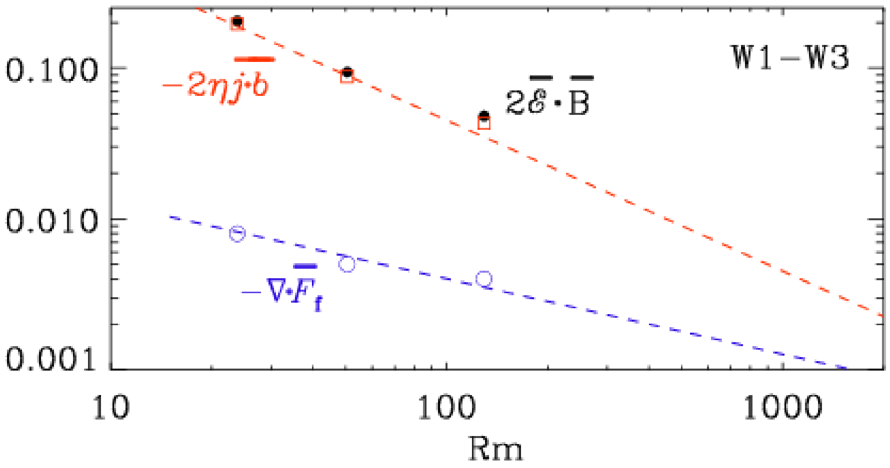
N-shaped (north)
S-shaped (south)

Magnetic helicity flux

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\right\rangle = +2\left\langle \overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\right\rangle - 2\eta\left\langle \overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\right\rangle - \nabla\cdot\boldsymbol{\mathcal{F}}_{\mathrm{m}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \mathbf{a}\cdot\mathbf{b}\right\rangle = -2\left\langle \overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\right\rangle - 2\eta\left\langle \mathbf{j}\cdot\mathbf{b}\right\rangle - \nabla\cdot\boldsymbol{\mathcal{F}}_{\mathrm{f}}$$

- EMF and resistive terms still dominant

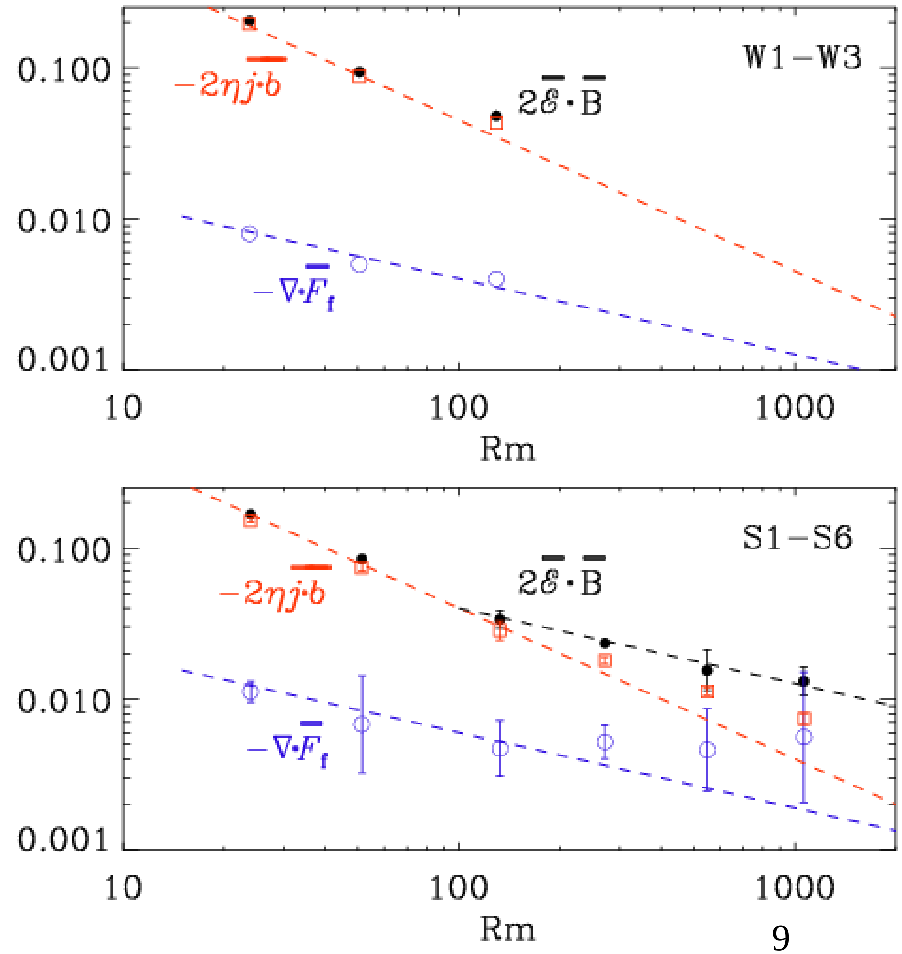


Magnetic helicity flux

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\overline{\mathbf{A}}\cdot\overline{\mathbf{B}}\rangle = +2\langle\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle - 2\eta\langle\overline{\mathbf{J}}\cdot\overline{\mathbf{B}}\rangle - \nabla\cdot\boldsymbol{\mathcal{F}}_{\mathrm{m}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\mathbf{a}\cdot\mathbf{b}\rangle = -2\langle\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}}\rangle - 2\eta\langle\mathbf{j}\cdot\mathbf{b}\rangle - \nabla\cdot\boldsymbol{\mathcal{F}}_{\mathrm{f}}$$

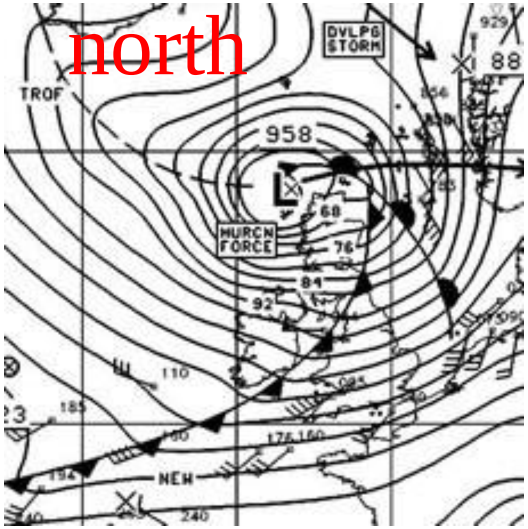
Gauge-invariant in steady state!

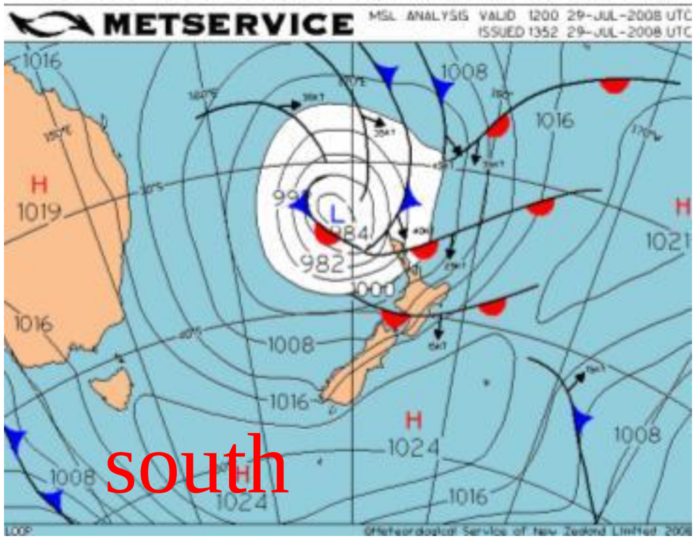


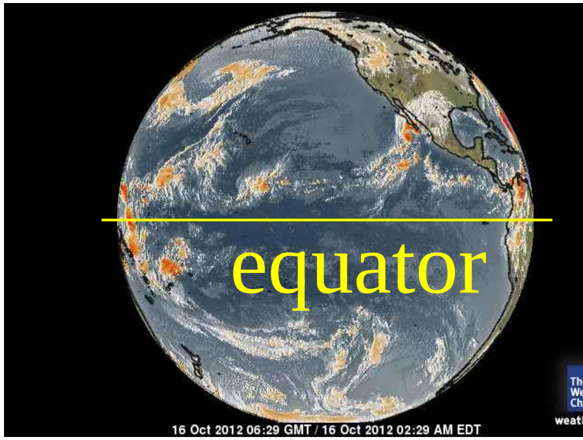
- EMF and resistive terms still dominant
- Fluxes import at large Rm ~ 1000
- Rm based on k_f
- Smaller by 2π

Del Sordo, Guerrero, Brandenburg (2013, MNRAS 429, 1686)

Northern/southern hemispheres







g $\downarrow$ $\uparrow$ Ω

$$\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle < 0$$

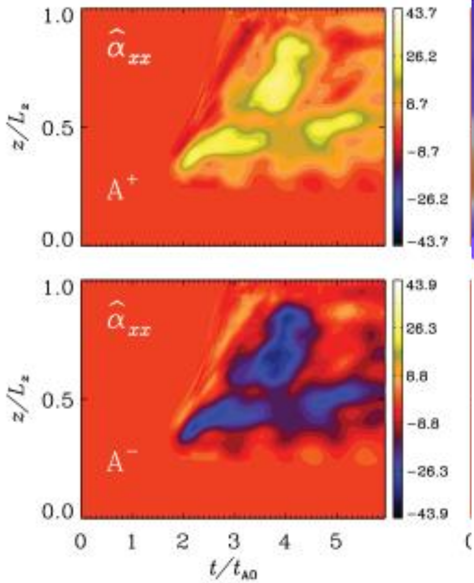
g $\uparrow$ $\uparrow$ Ω

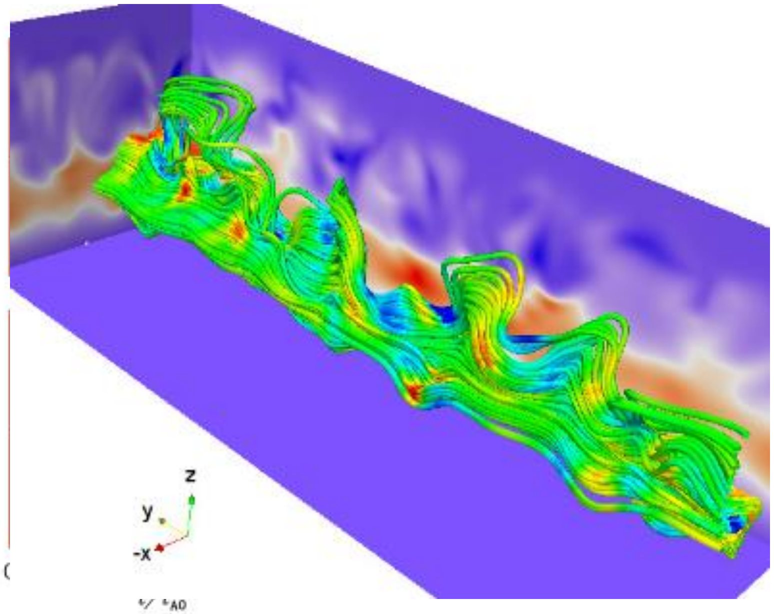
$$\langle \boldsymbol{\omega} \cdot \mathbf{u} \rangle > 0$$

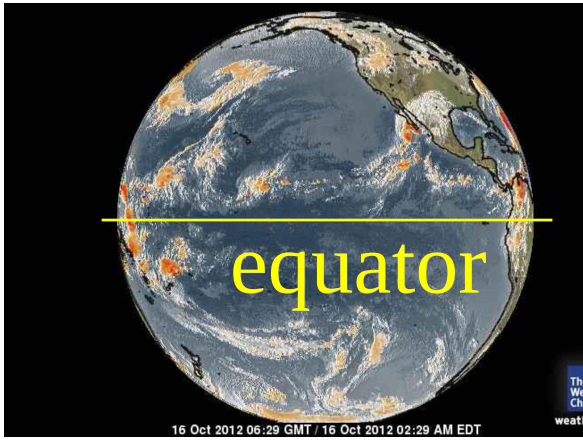
Cyclones:
Down: faster
Up: slower

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

Northern/southern hemispheres







PHYSICAL REVIEW E **84**, 025403(R) (2011)

Spontaneous chiral symmetry breaking by hydromagnetic buoyancy

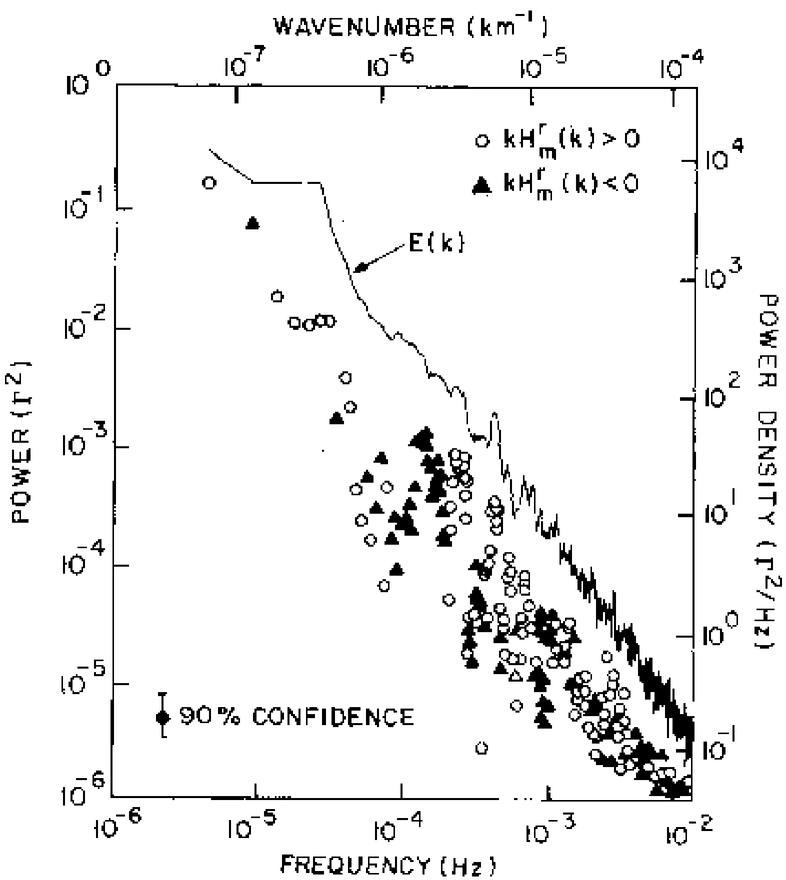
Piyali Chatterjee,[1] Dhrubaditya Mitra,[1] Axel Brandenburg,[1,2] and Matthias Rheinhardt[1]
[1] *NORDITA, AlbaNova University Center, Roslagstullsbacken 23, SE-10691 Stockholm, Sweden*
[2] *Department of Astronomy, Stockholm University, SE-10691 Stockholm, Sweden*
(Received 10 November 2010; revised manuscript received 24 June 2011; published 12 August 2011)

Lessons from dynamo theory

- Helicity
  - *Not just a measure of complexity*
  - *Critically important in dynamos*
- To confirm observationally
  - *Opposite signs at different scales*
  - *Opposite signs in different hemispheres*

(i) Helicity from solar wind: in situ

Matthaeus et al. (1982)



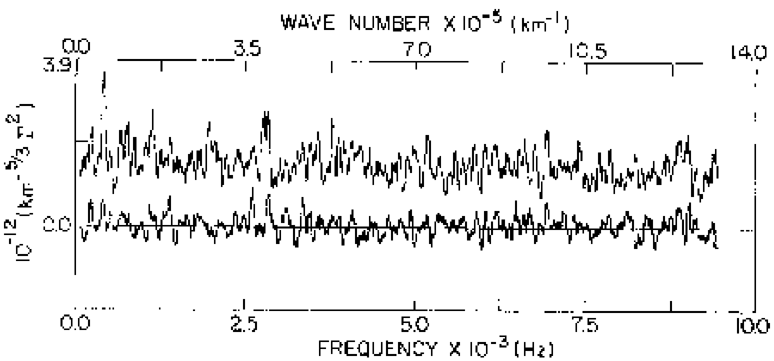
Measure correlation function

$$M_{ij}(\mathbf{r}) = \langle B_i(\mathbf{x}) B_j(\mathbf{x}+\mathbf{r}) \rangle$$

In Fourier space, calculate

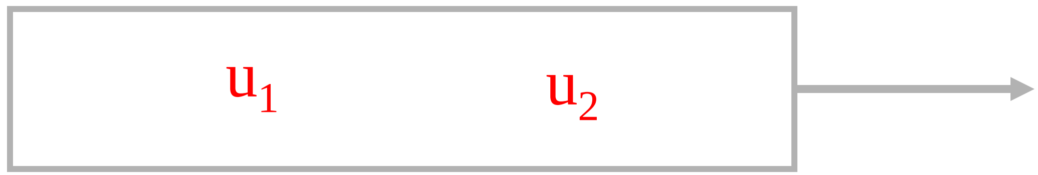
magnetic energy and helicity spectra

$$M_{ij}(k) = \left(\delta_{ij} - \hat{k}_i \hat{k}_j\right) E(k) - i\varepsilon_{ijk} k_k H(k)$$



→ Should be done with Ulysses data away from equatorial plane

Measure 2-point correlation tensor



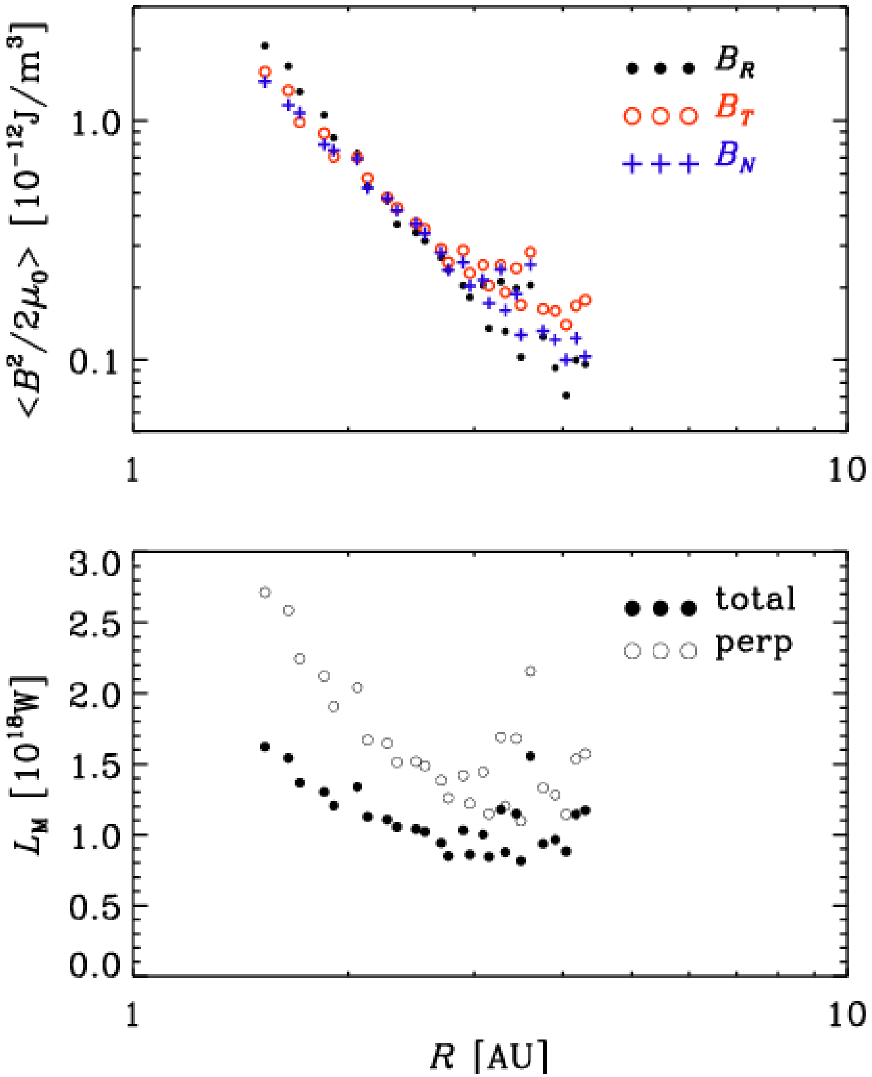
Taylor hypothesis: $R = R_0 - u_R t$

$$\tilde{B}_i(k_R) = \int e^{\mathrm{i}k_R R} B_i(R)\,\mathrm{d}R, \quad i = R, T, N,$$

$$M_{ij}^{\mathrm{1D}}(k_R) = \tilde{B}_i(k_R)\tilde{B}_j^*(k_R),$$

$$H(k_R) = 4\,\mathrm{Im}\left\langle \tilde{B}_T(k_R)\tilde{B}_N^*(k_R)\right\rangle / k_R$$

Ulysses: scaling with distance

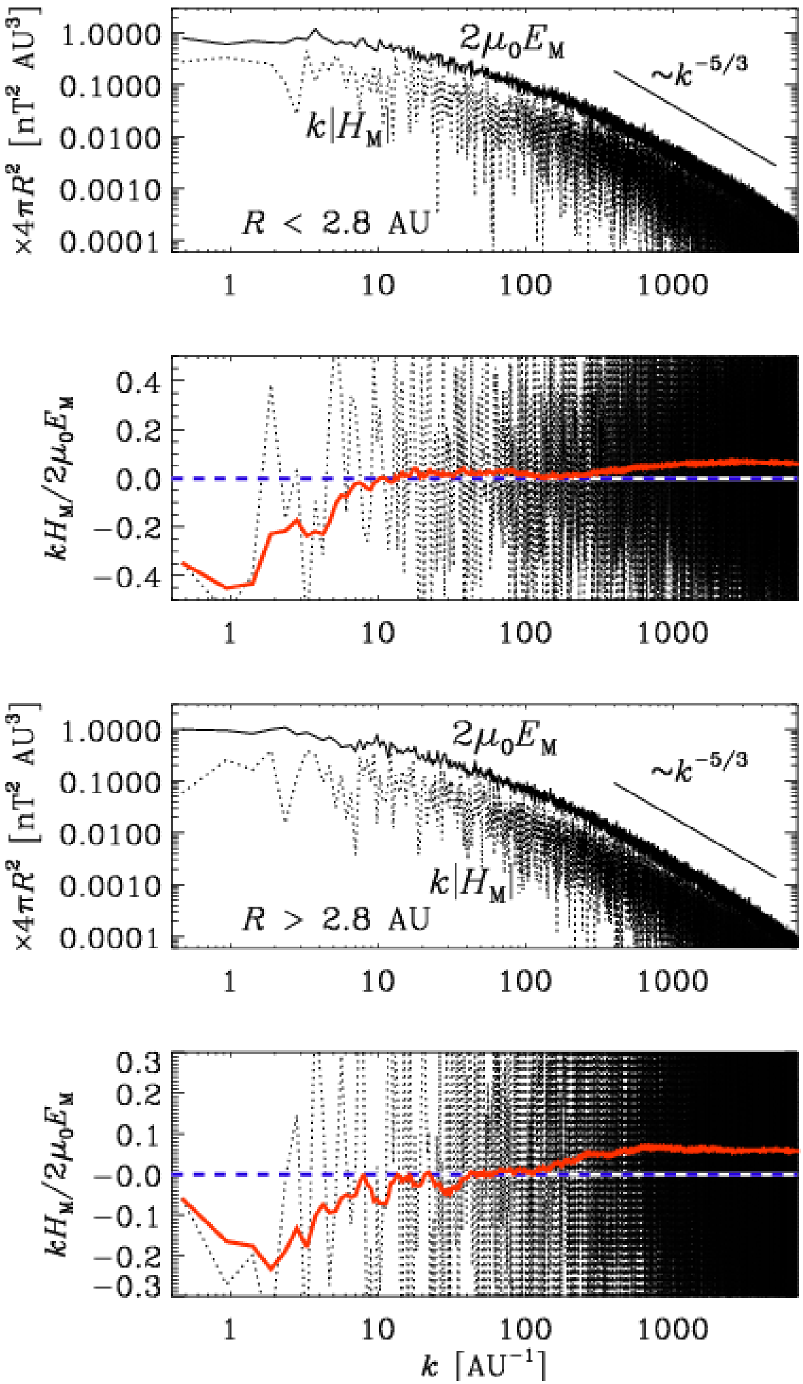


Vector helium magnetometer
2 sec resolution
10 pT sensitivity (0.1 μG)

* Fairly isotropic
* Falls off faster than R^{-2}
* Need to compensate before R averaging

$$L_M = 4\pi R^2 u_R \left\langle B^2 / 2\mu_0 \right\rangle$$

Power similar to US consumption
Energy density similar to ISM

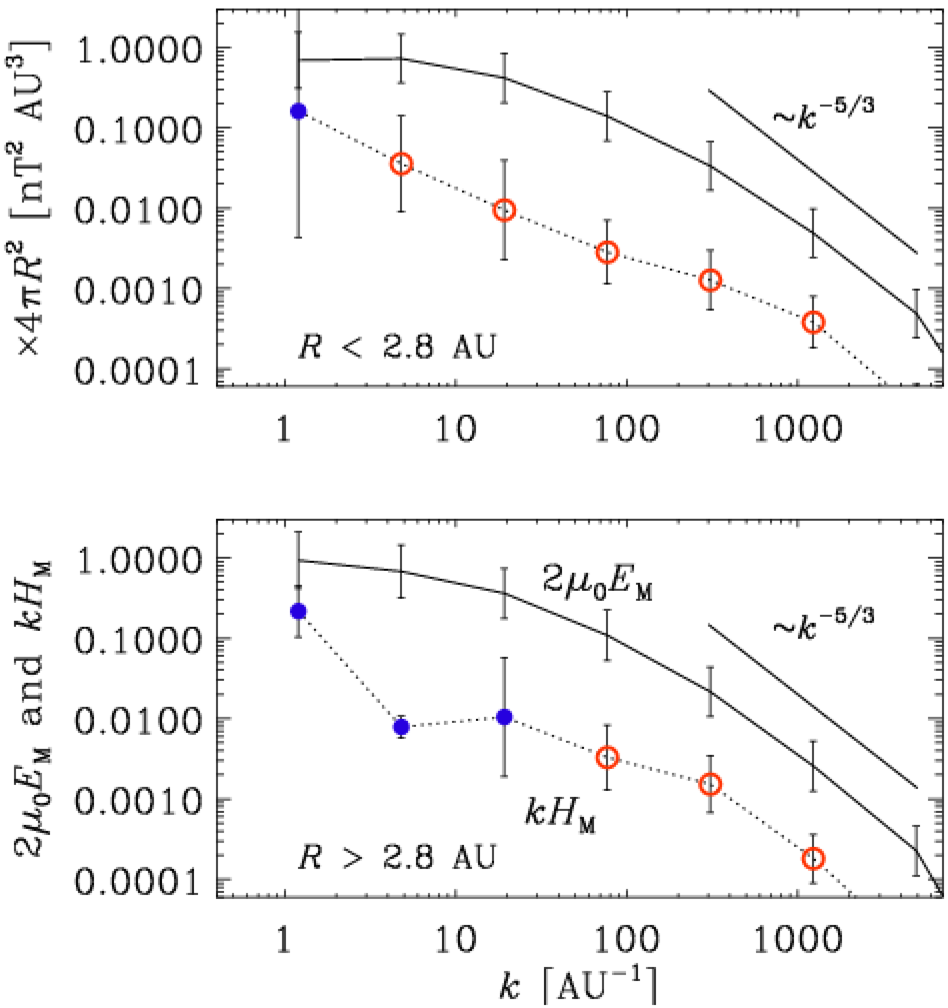


Noisy helicity from Ulysses

- Taylor hypothesis
- Roundish spectra
- Southern latitude with opposite sign
- Positive *H* at large *k*

Brandenburg, Subramanian, Balogh, & Goldstein (2011, ApJ 734, 9)

Bi-helical fields from Ulysses



Taylor hypothesis

Broad k bins

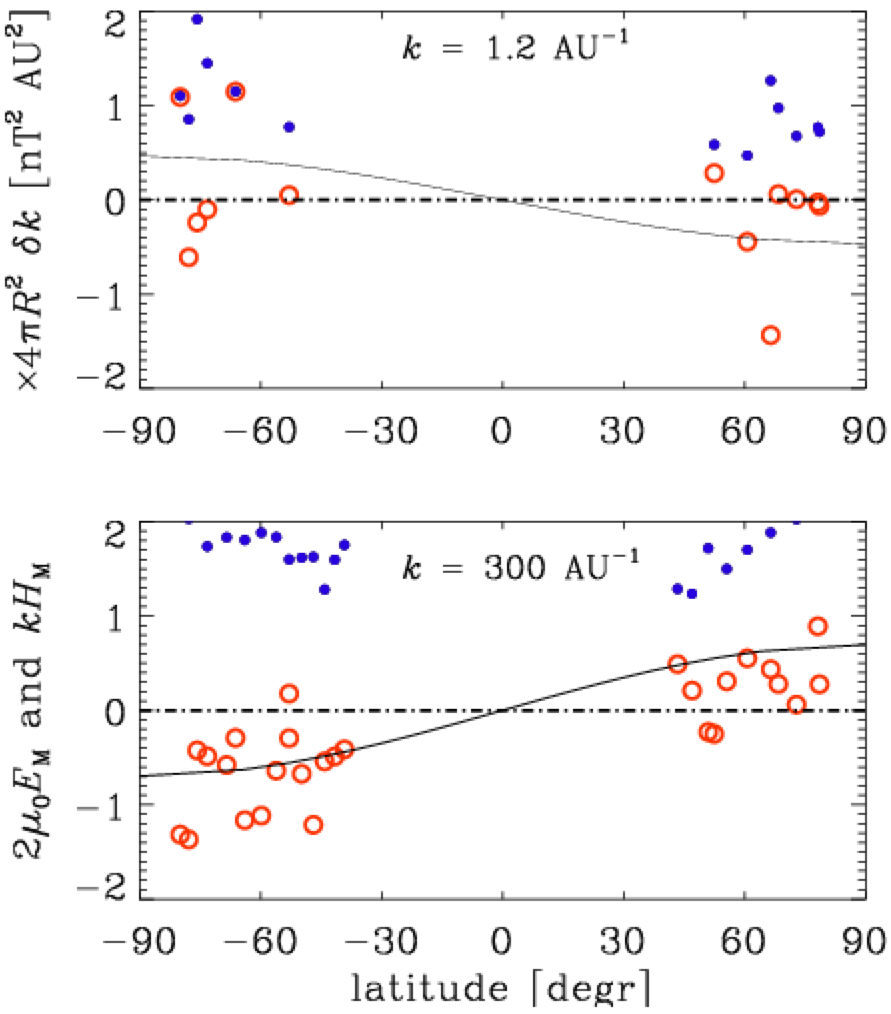
Southern latitude with opposite sign

Small/large distances

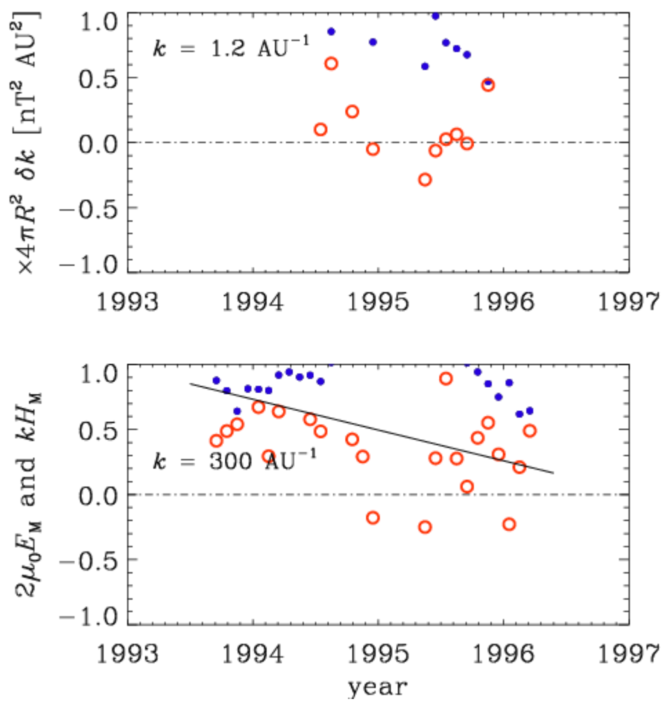
Positive H at large k

Break point with distance to larger k

Latitudinal scaling and trend



1. Antisymmetric about equator
2. Decline toward minum

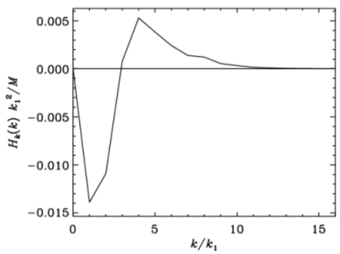


Comparison

Southern hemisphere

Results for $\frac{1}{2}\mathcal{L}_{\mathrm{H}}^{\pm}T_{\mathrm{cyc}}$ in Units of Mx^2 cycle^{-1}

Distance	Large Scales	Small Scales
$R < 2.8$ AU	-0.9×10^{45}	$+0.3 \times 10^{45}$
$R > 2.8$ AU	-1.3×10^{45}	$+0.03 \times 10^{45}$

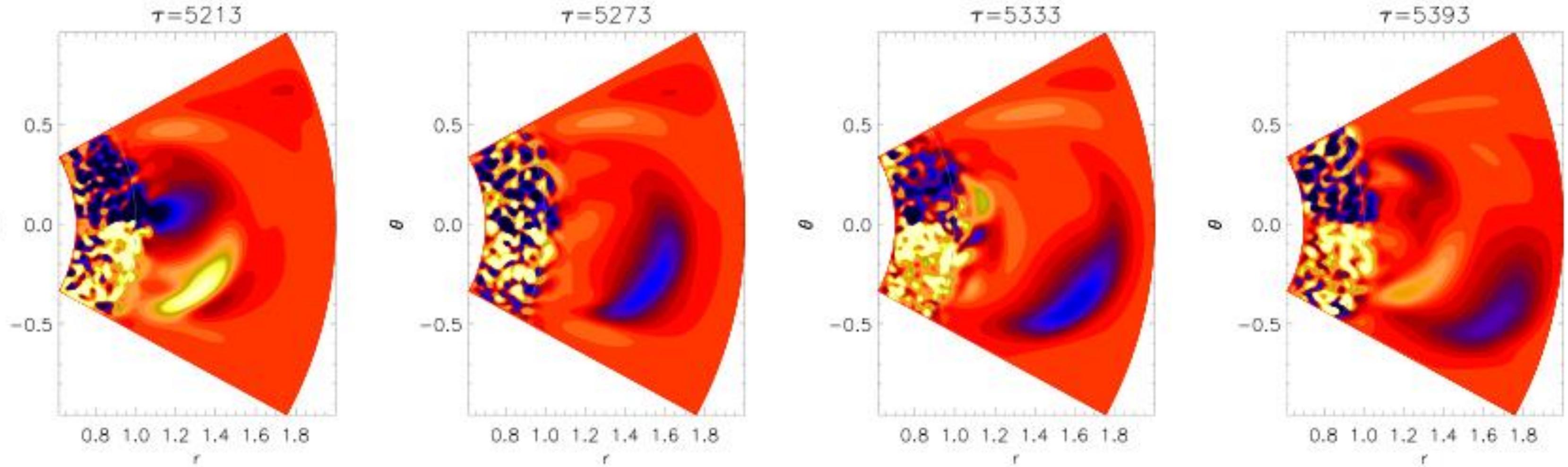


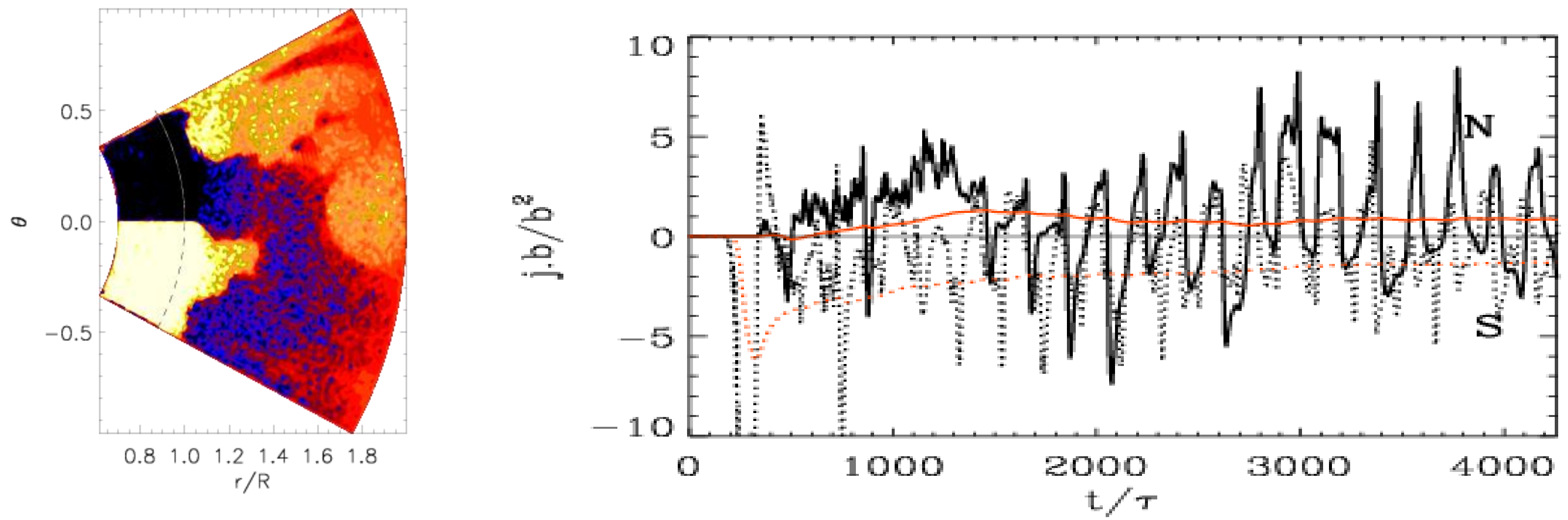
- Field in solar wind is clearly bi-helical
- ...but not as naively expected
- Need to compare with direct and mean-field simulations
- Recap of dynamo bi-helical fields

Helicity	*LS*	*SS*
Dynamo	+	-
Solar wind	-	+

Shell dynamos with ~CMEs

Warnecke, Brandenburg, Mitra (2011, A&A, 534, A11)





Strong fluctuations, but positive in north

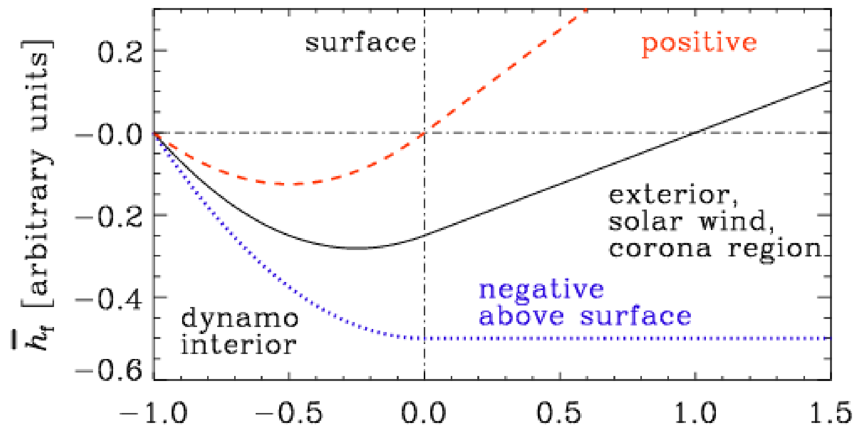
To carry negative flux: need positive gradient

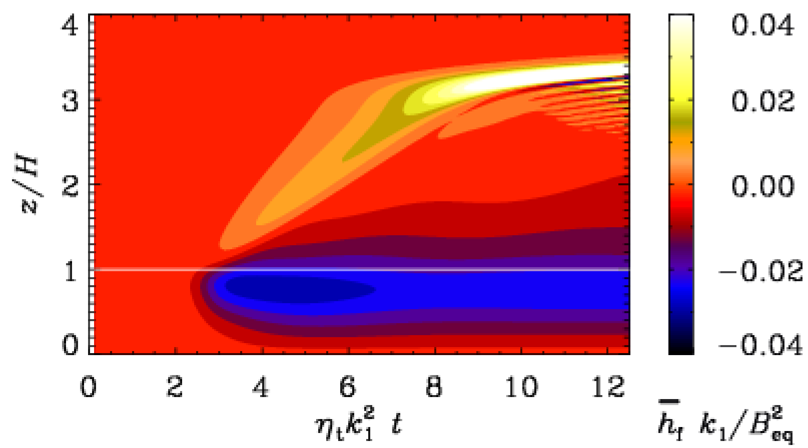
$$\overline{\mathbf{F}}_\mathrm{f} = -\kappa_h \nabla \overline{h}_\mathrm{f}$$

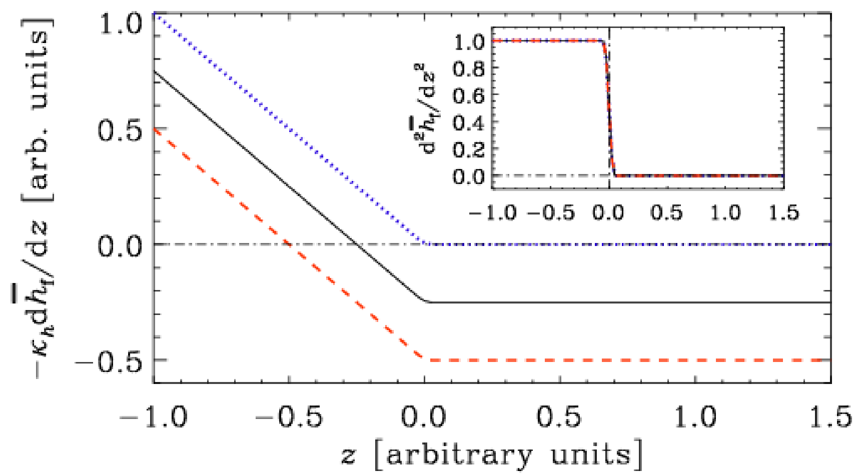
Brandenburg, Candelaresi, Chatterjee
(2009, MNRAS 398, 1414)

$$\frac{\mathrm{d}\overline{h}_\mathrm{m}}{\mathrm{d}t} = +2\alpha\overline{\mathbf{B}}^2 - 2\eta_t \overline{\mathbf{J}}\cdot\overline{\mathbf{B}} - \nabla\cdot\overline{\mathbf{F}}_\mathrm{m}$$

$$\frac{\mathrm{d}\overline{h}_\mathrm{f}}{\mathrm{d}t} = -2\alpha\overline{\mathbf{B}}^2 + 2\eta_t \overline{\mathbf{J}}\cdot\overline{\mathbf{B}} - \nabla\cdot\overline{\mathbf{F}}_\mathrm{f}$$







Sign reversal makes sense!

Similar method for solar surface

$$\left\langle \hat{B}_i(\boldsymbol{k},t)\hat{B}_j^*(\boldsymbol{k}',t)\right\rangle = \Gamma_{ij}(\boldsymbol{k},t)\delta^2(\boldsymbol{k}-\boldsymbol{k}'),$$

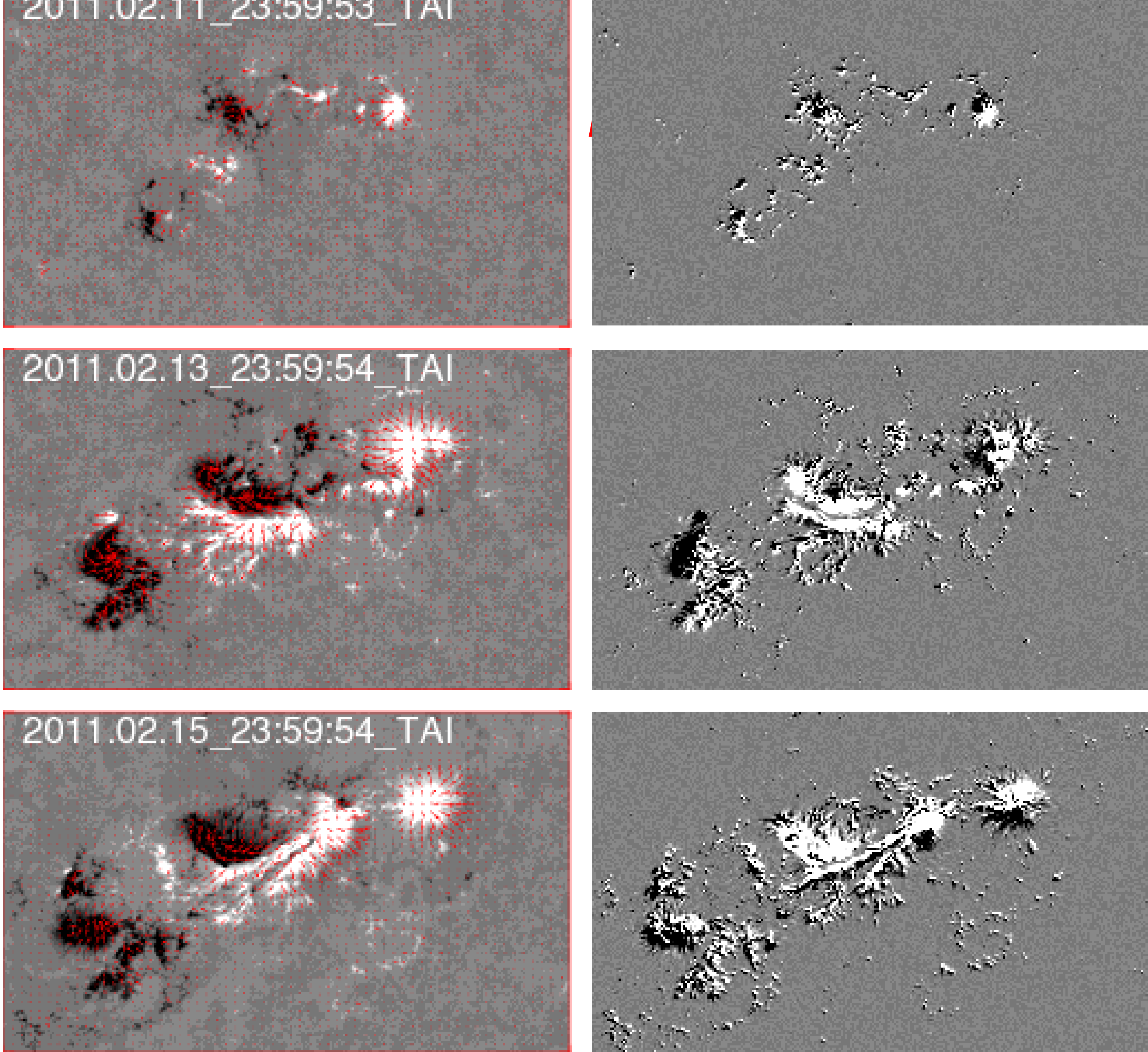
$$\Gamma_{ij}(\boldsymbol{k},t) = \frac{2E_M(k,t)}{4\pi k}(\delta_{ij}-\hat{k}_i\hat{k}_j)+\frac{iH_M(k,t)}{4\pi k}\varepsilon_{ijk}k_k,$$

$$\begin{pmatrix} (1-\cos^2\phi_k)2E_M & -\sin 2\phi_k E_M & -ik\sin\phi_k H_M \\ -\sin 2\phi_k E_M & (1-\sin^2\phi_k)2E_M & ik\cos\phi_k H_M \\ ik\sin\phi_k H_M & -ik\cos\phi_k H_M & 2E_M \end{pmatrix}$$

$$2E_M(k) = 2\pi k\,\mathrm{Re}\left\langle \Gamma_{xx}+\Gamma_{yy}+\Gamma_{zz}\right\rangle_{\phi_k},$$

$$kH_M(k) = 4\pi k\,\mathrm{Im}\left\langle \cos\phi_k\Gamma_{yz}-\sin\phi_k\Gamma_{xz}\right\rangle_{\phi_k},$$

Zhang, Brandenburg, Sokoloff (2014, ApJL, 784, L45)


2011.02.11_23:59:53_TAI
2011.02.13_23:59:54_TAI
2011.02.15_23:59:54_TAI

Results & realizability

$$L_M = \int k^{-1} E_M(k)\, dk \Big/ \int E_M(k)\, dk. \quad (11)$$

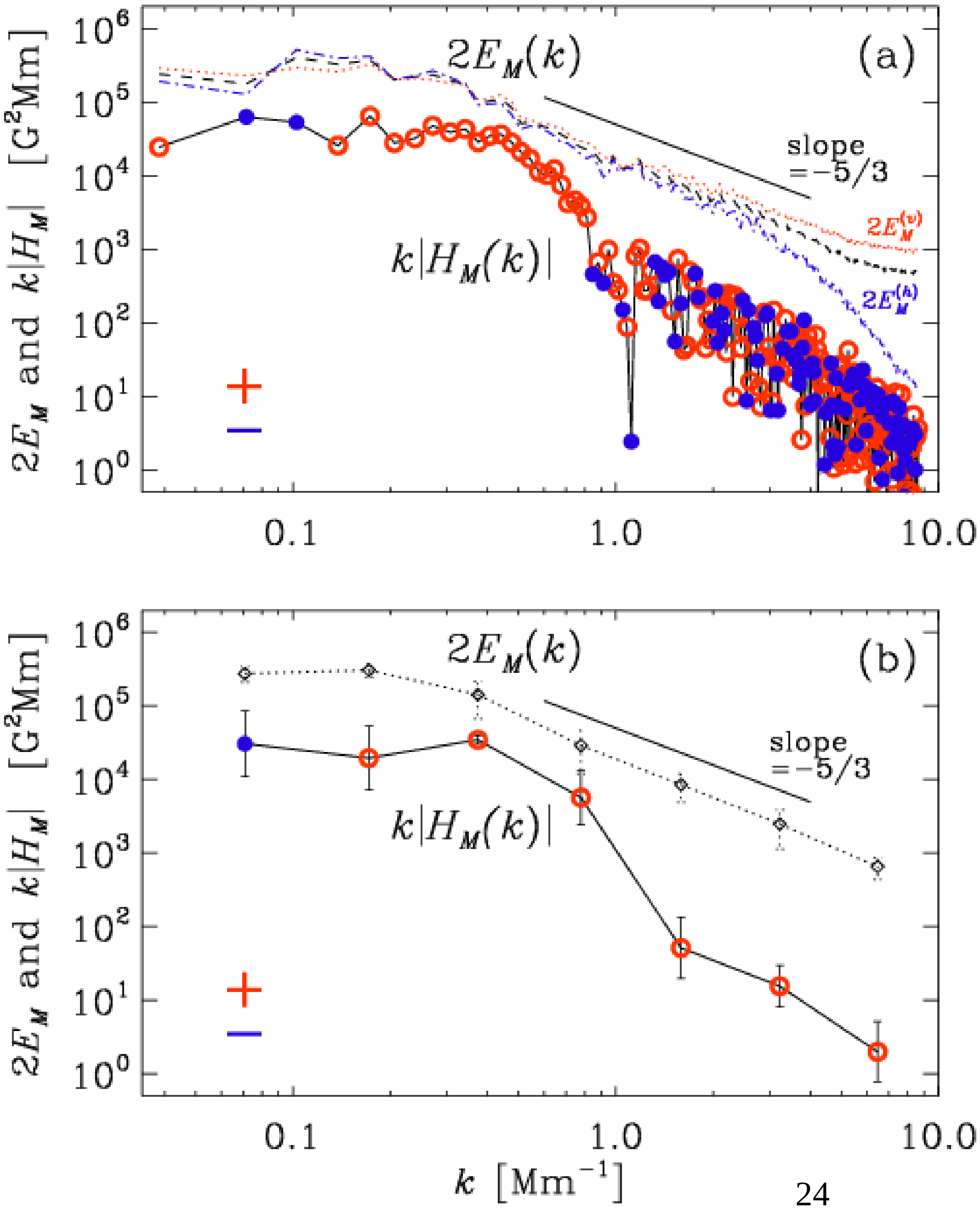
The realizability condition of Equation (8) can be rewritten in the integrated form (e.g. Kahniashvili et al. 2013) as

$$\mathcal{H}_M = \int H_M\, dk \le 2 \int k^{-1} E_M(k)\, dk \equiv 2 L_M \mathcal{E}_M. \quad (12)$$

In particular, we have $|\mathcal{H}_M(t)| \le 2 L_M \mathcal{E}_M(t)$. This allows us then to define the relative magnetic helicity,

$$r_M = \mathcal{H}_M / 2 L_M \mathcal{E}_M, \quad (13)$$

30,000 G^2Mm/(2 6Mm 70,000 G^2)=0.04

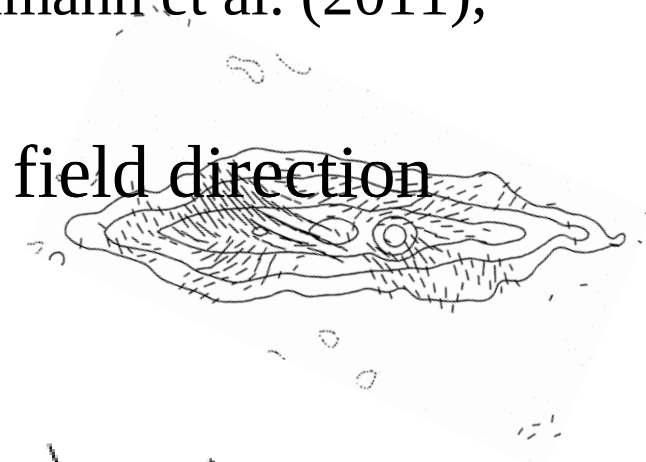


- Isotropy
- Positive hel.
- Expected for south

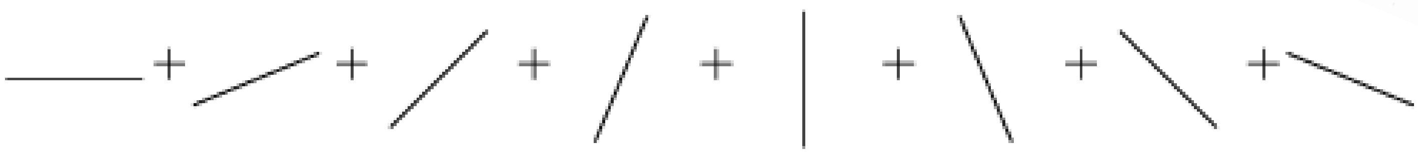
(ii) Galactic context: synchrotron radiation & Faraday rotation

Brandenburg & Stepanov (2014, ApJ 786, 91)

- Volegova & Stepanov (2010), Oppenmann et al. (2011), Horellou & Fletcher (2014)
- Polarization vector → magnetic field direction



- Faraday depolarization



  - Headache for observers → short λ
  - Now: use λ dependence
- Application to edge-on galaxies

Polarized synchrotron emission

$$I(\lambda^2) = \int_0^\infty \varepsilon(z,\lambda)\,\mathrm{dz}$$

$$P(\lambda^2) = p_0 \int_0^\infty \varepsilon e^{2i(\psi+\phi\lambda^2)}\,\mathrm{dz}$$

$$p = p_0 e^{2i\psi}$$

complex polarized emissivity

$$\psi = \arctan B_y / B_x - \pi/2$$

intrinsic polarization

$$\mathcal{B} = B_x + \mathrm{i}B_y = |B_\perp| e^{i\psi_B}$$

$$\phi(z) = -K\int_0^z n_\mathrm{e} B_z\, dl$$

$$K = 0.81\,\mathrm{rad\,m^{-2}cm^3\mu G^{-1}pc^{-1}}$$

$$\phi(z) = -K n_\mathrm{e} B_z z$$

$$\psi + \phi\lambda^2 = \psi + kz$$

$$k = -K n_\mathrm{e} B_z \lambda^2$$

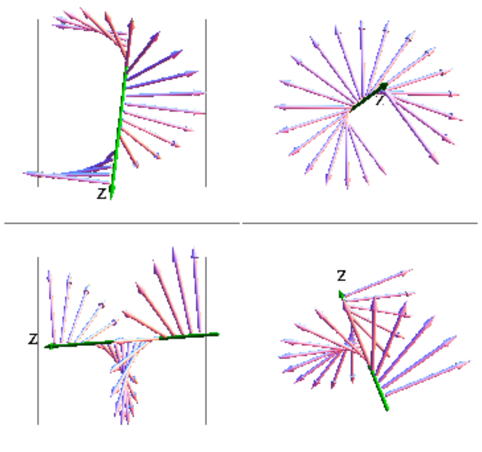
Helical (swirling) magnetic fields

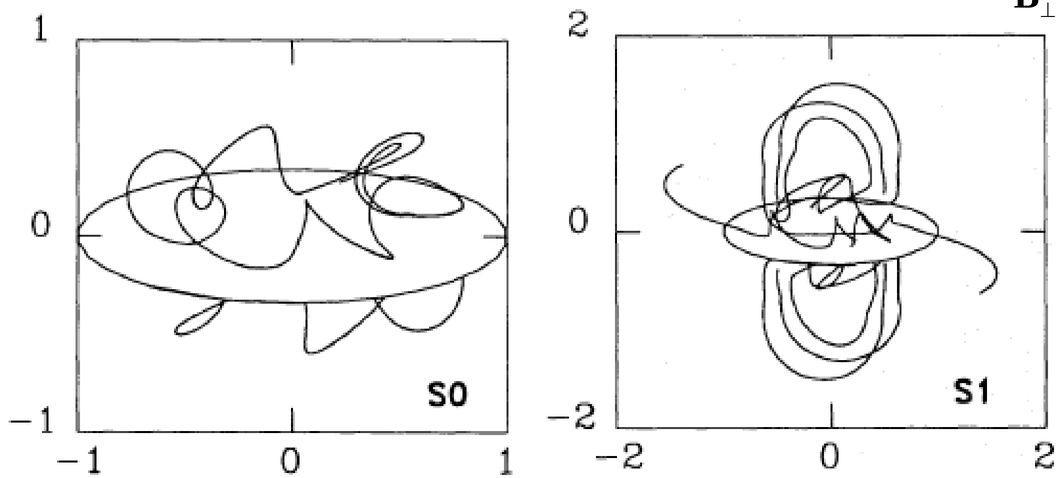
$$B_x + \mathrm{i}B_y = |B_\perp| e^{ikz+\psi_0}$$

$$\mathbf{B}_\perp = B_\perp\left(\cos(kz+\psi_0), -\sin(kz+\psi_0), 0\right)$$

$$\nabla\times\mathbf{B}_\perp = k\mathbf{B}_\perp$$

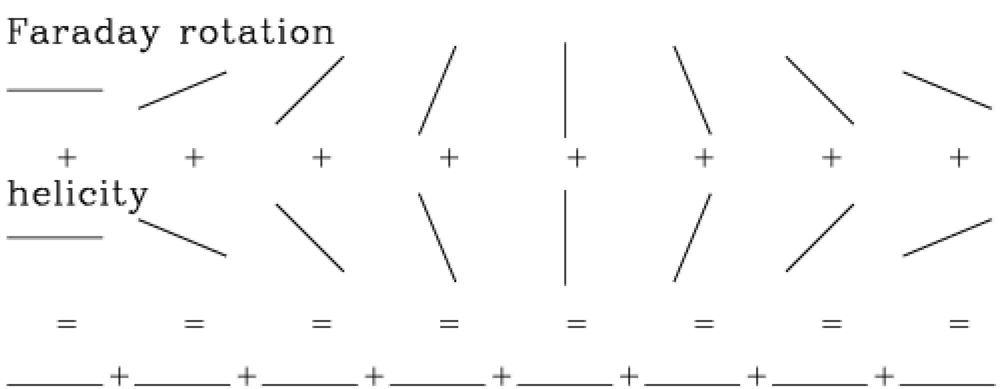
$$\mathbf{B}_\perp\cdot\nabla\times\mathbf{B}_\perp = kB_\perp^2$$





Brandenburg & Donner (1990)

Rotation from swirl compensates Faraday rotation



See also Sokoloff et al. (1998)

Scales and applications

$$z \leftrightarrow \phi$$

$$k \leftrightarrow \lambda^2$$

- L = 1 kpc → k = 6 kpc^{-1} → λ=30 cm
- L<0.1 kpc → k > 60 kpc^{-1} → λ=1 m
- Assuming B = 3 μG, n_e=0.03 cm^{-3}

λ coverage only possible with SKA: 2 cm – 6m

Stokes Q&U for singly helical field

Stokes Q and U parameters $$P = Q + iU$$

$$Q = p_0 \int_{-\infty}^{\infty} \varepsilon \cos 2\left(\psi + \phi\lambda^2\right) dz$$

$$U = p_0 \int_{-\infty}^{\infty} \varepsilon \sin 2\left(\psi + \phi\lambda^2\right) dz$$

Intrinsic polarized emission from B

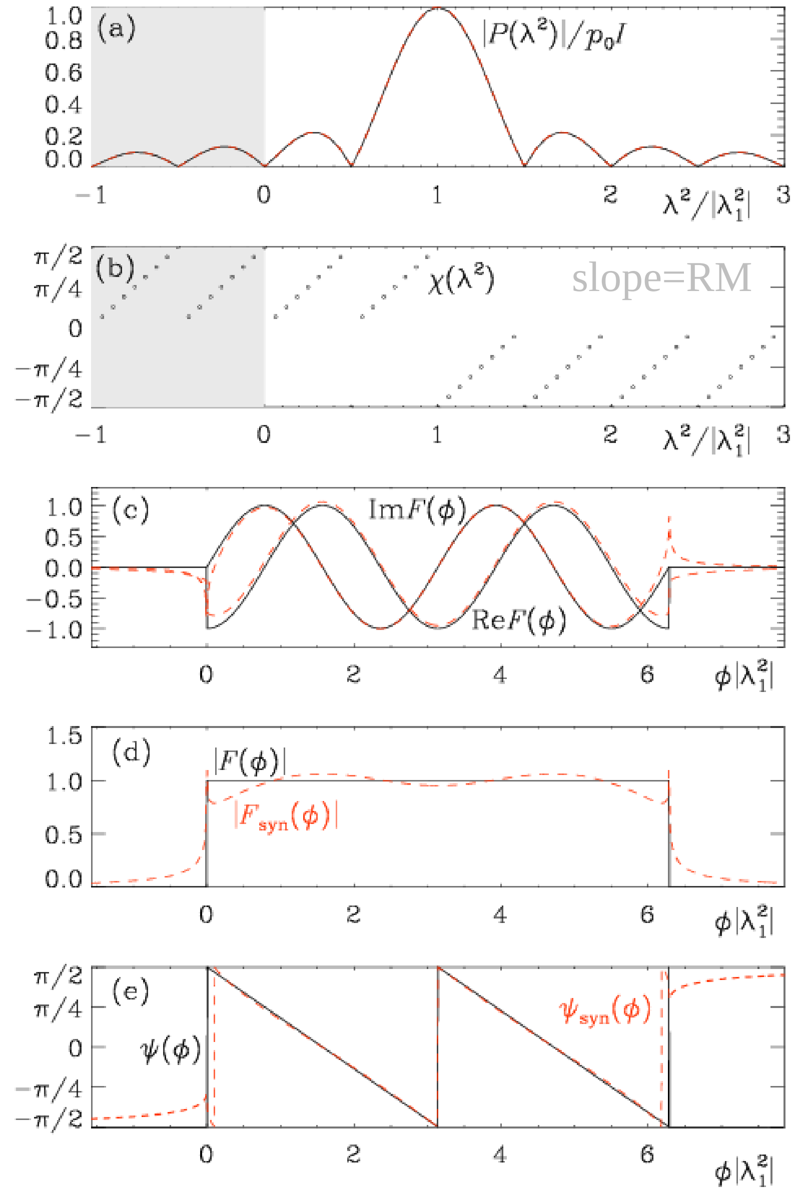
$$B_x + \mathrm{i}B_y = |B_\perp| e^{i\psi_B}, \quad \psi = \psi_B + \tfrac{1}{2}\pi$$

Cancellation condition

$$\psi = -kz, \quad \phi = -Kn_{\mathrm{th}} B_z z$$

Helical field w/ positive helicity

$$\mathbf{B} = \begin{pmatrix} B_1 \cos kz \\ -B_1 \sin kz \\ B_0 \end{pmatrix}$$



ON THE DEPOLARIZATION OF DISCRETE RADIO SOURCES BY FARADAY DISPERSION

B. J. Burn

(Received 1965 July 7)

$P(\phi)$ for its intrinsic polarization. Defining the 'Faraday dispersion function' as $F(\phi)=E(\phi)P(\phi)$, we obtain the Fourier transform relation

$$P(\lambda^2)=\int_{-\infty}^{\infty} F(\phi)e^{2i\phi\lambda^2}\,d\phi. \qquad (11)$$

It would be very convenient to be able to invert this transform and so obtain the Faraday dispersion function from the relation

$$F(\phi)=\pi^{-1}\int_{-\infty}^{\infty} P(\lambda^2)e^{-2i\phi\lambda^2}\,d(\lambda^2). \qquad (12)$$

However, to evaluate this integral we must know $P(\lambda^2)$ for $\lambda^2<0$, and this is not an observable quantity. It is readily seen from equation (11) that this is the

Only works if RM > 0 and k > 0

Peak determined by single parameter

$$\lambda_1^2 = -k / K n_{\rm th} B_0 \propto k / \mathrm{RM}$$

But difficult/impossible to recover F(ϕ)

(Burn 1966)

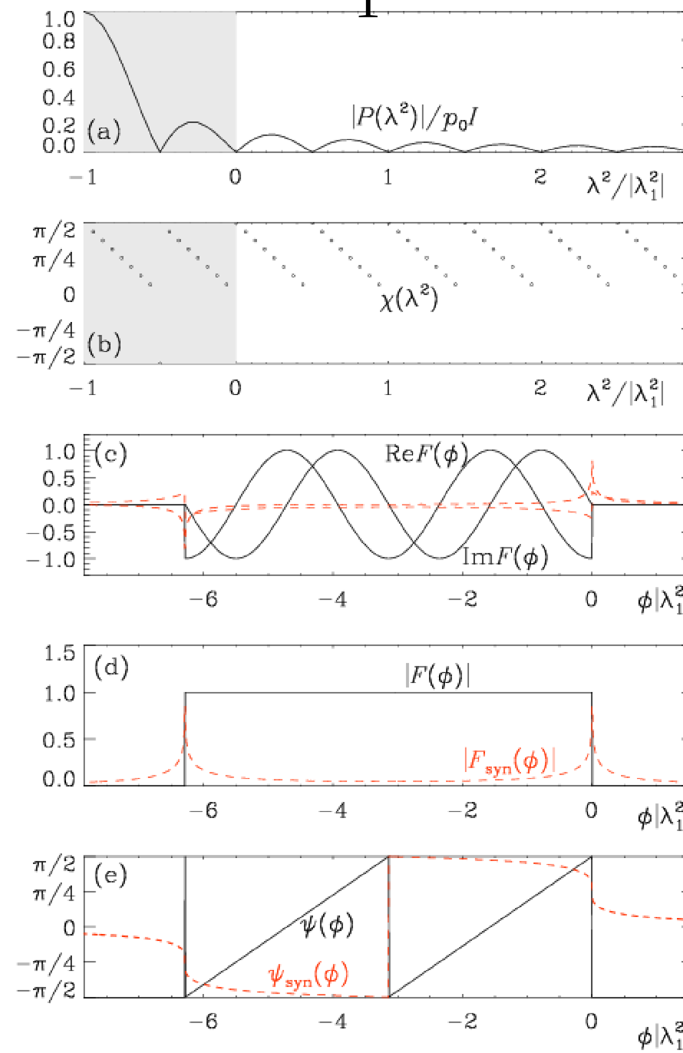
$$P(\lambda^2) = \int_{-\infty}^{\infty} F(\phi) e^{2i\phi\lambda^2}\, d\phi$$

$$F(\phi) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(\lambda^2) e^{-2i\phi\lambda^2}\, d(2\lambda^2)$$

Positivity:

$$F_{\rm syn}(\phi) = \frac{1}{2\pi} \int_{0}^{\infty} P(\lambda^2) e^{-2i\phi\lambda^2}\, d(2\lambda^2)$$

$$\lambda_1^2 < 0$$

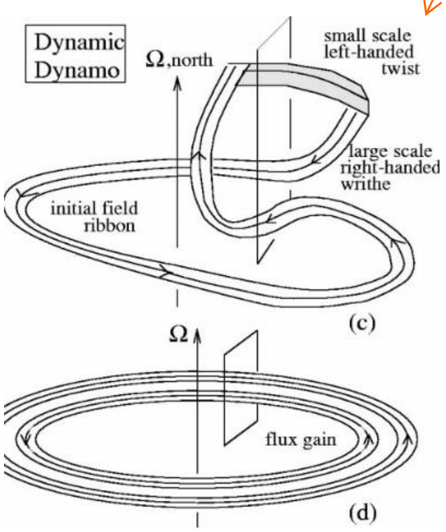


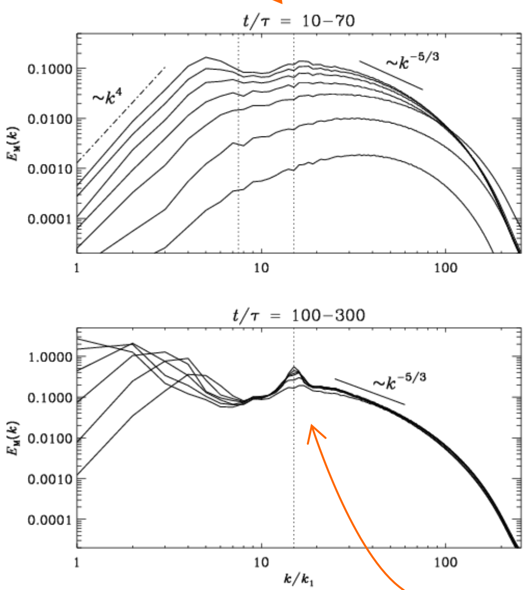
Brandenburg & Stepanov (2014, ApJ 786, 91)

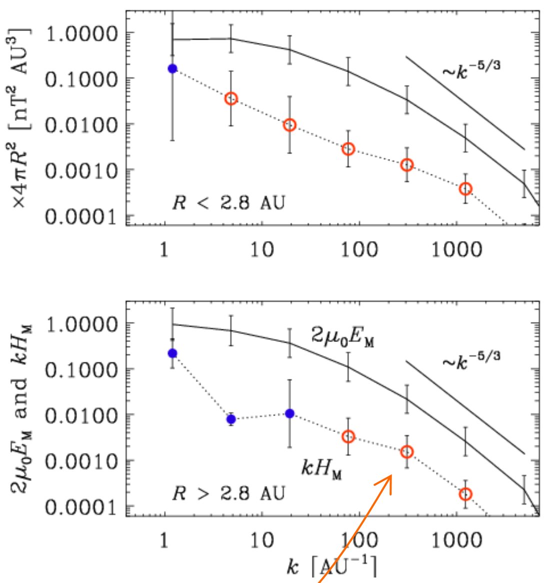
Expect bi-helical fields

- Magnetic helicity conserved
- Inverse cascade produces small-scale waste!
- Opposite sign of helicity (or k)

Blackman & Brandenburg (2003)

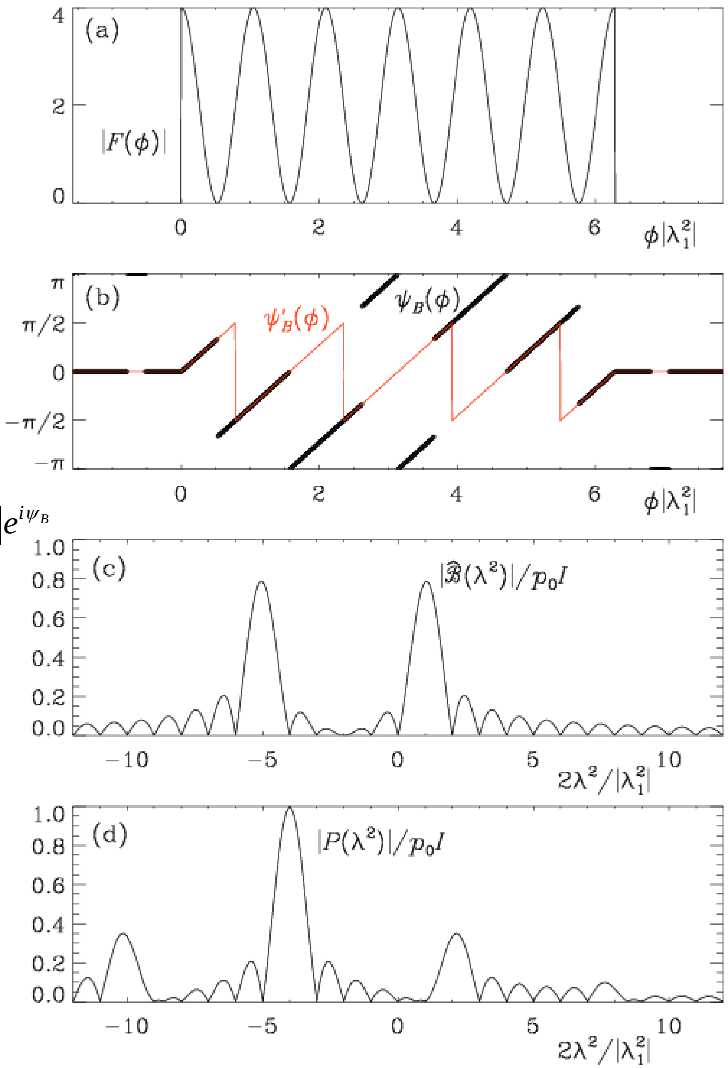






π ambiguity lead to "line splitting"

$\boldsymbol{\mathcal{B}} = B_x + \mathrm{i}B_y = |B_\perp| e^{i\psi_B}$



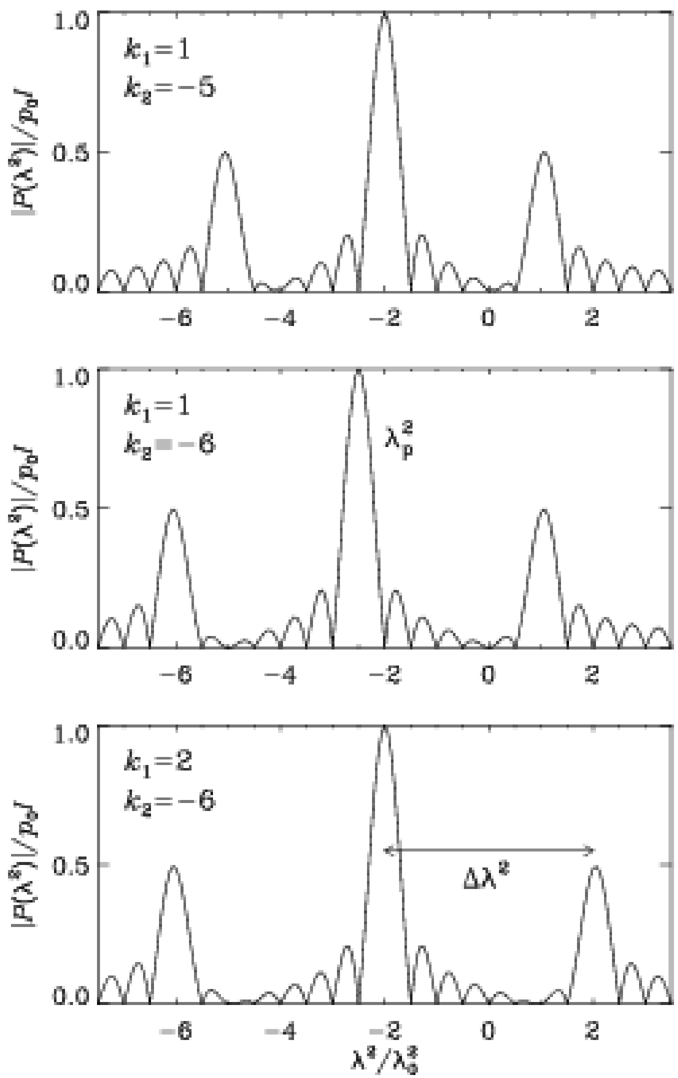
Peaks at k_1=1 and k_2=-5

translate to k_1+k_2 = -4
and to k_1-k_2 = 6

(i) peak in P at -4
peak separation 6

(ii) in Faraday dispersion:
frequency 6
-2x phase gradient -4

π ambiguity: other examples



Peaks at k_1=1 and k_2=-5
1 and -6, or, 2 and -6

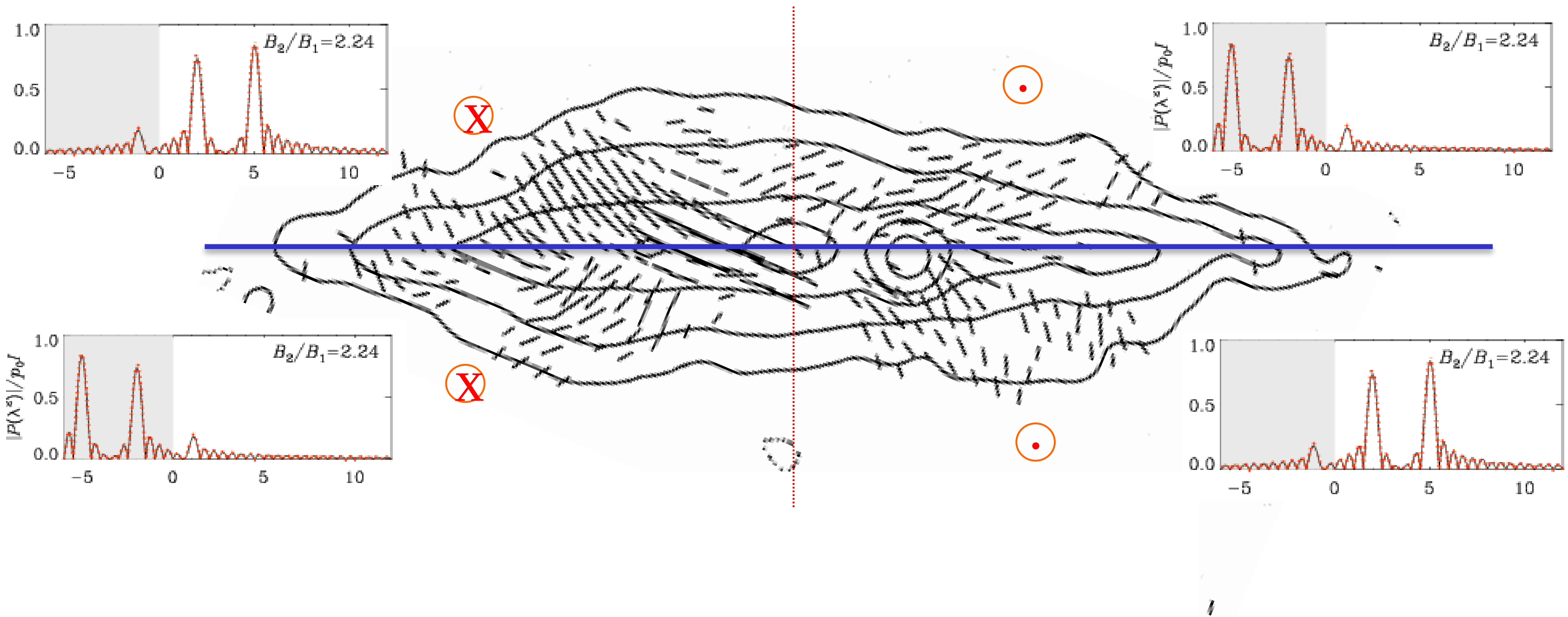
translate to k_1+k_2 =-4
and k_1-k_2 =6

Table 1. Summary of the three examples shown in Fig. 4 for RM $>$ 0 with bi-helical magnetic fields of wavenumbers k_1 and k_2, the corresponding values of $k_\pm = k_1 \pm k_2$, the peak wavenumber k_p, the peak separation Δk, the phase gradient (ϕ derivative, indicated by ∇ for brevity), and corresponding values for λ_p^2 and $\Delta\lambda^2$. All values of k are normalized by k_0 and all values of λ^2 are normalized by λ_0^2.

k_1	k_2	k_+	k_-	k_p	Δk	$\nabla\psi'_B$	λ_p^2	$\Delta\lambda^2$	$\nabla\psi$
1	−5	−4	6	−4	6	2	−2	3	2
1	−6	−5	7	−5	7	2.5	−2.5	3.5	2.5
2	−6	−4	8	−4	8	2	−2	4	2

Hopes for SKA

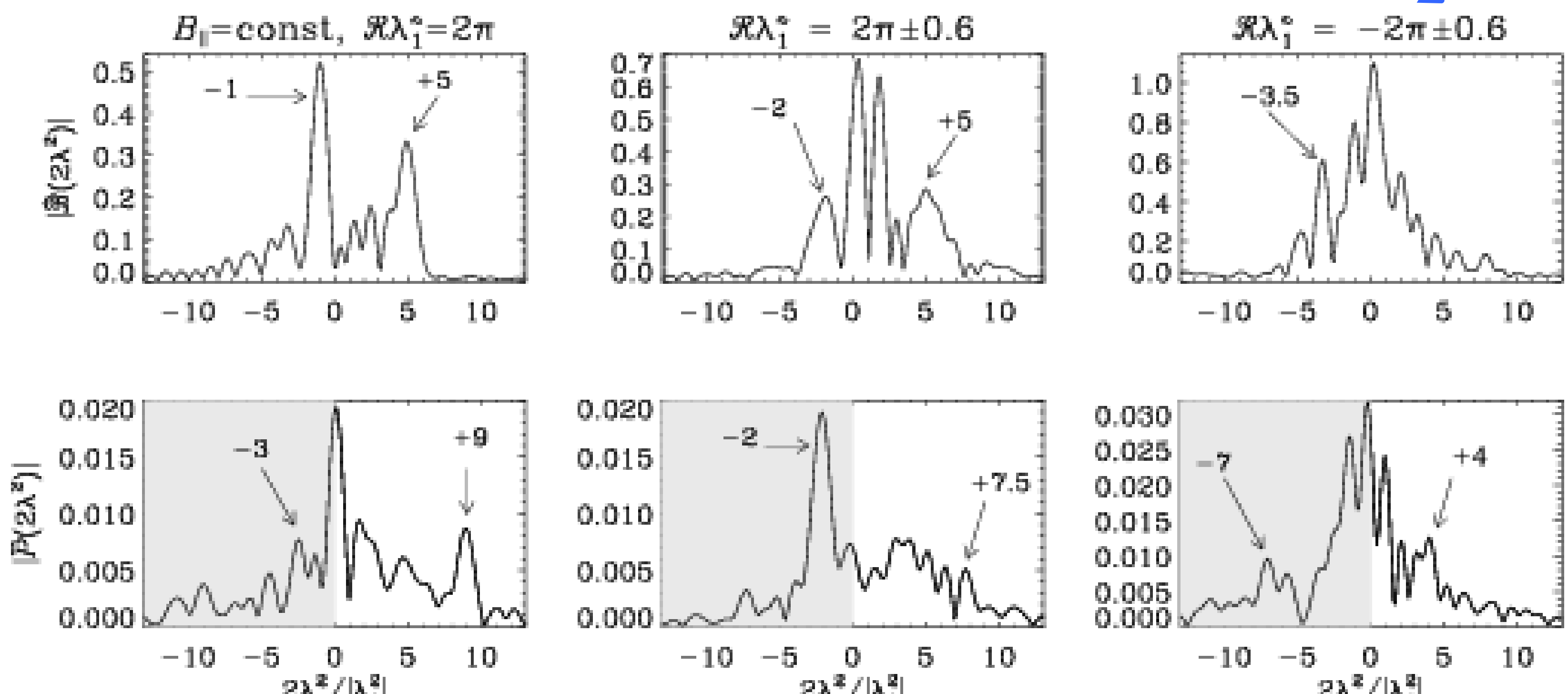
- RM synthesis: measure magnetic helicity
- Need line of sight component: edge-on galaxy
- Expect polarized intensity only in 2 quadrants
- 2 characteristic peaks



Reality less straightforward

- Turbulent dynamo: k_f=5, k_1=-1
- More than just 2 scales
- ϕ not linear in z

select



Expect -2, +10

Expect -10, 2

Conclusions

Vetenskapsrådet

- Magnetic helicity
  - Essential for dynamo
  - Expect bi-helical
- Solar wind: yes, but reversed!
- Galaxies: yes, in theory

Brandenburg & Stepanov (2014, ApJ 786, 91)